A New Resistive Response to 3-D Fields in Low Rotation H-modes

by

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Low Torque H Modes are Susceptible to Error Fields

- Applying a **static** error field destabilizes a **rotating** 2/1 tearing mode:

  1.8kHz Fourier decomposed fast visible imaging

**DIII-D**

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Low Torque H Modes are Susceptible to Error Fields

- Applying a static error field destabilizes a rotating 2/1 tearing mode:
  - Tearing $\beta_N$ limit falls with rotation:

![Graph showing the effect of rotation on $\beta_N$ limit]
Low Torque H Modes are Susceptible to Error Fields

- Applying a static error field destabilizes a rotating 2/1 tearing mode:
  - Tearing $\beta_N$ limit falls with rotation
  - Error field brakes plasma, accessing instability $\rightarrow$ mode grows & locks

![Graph showing the relationship between $\beta_N$, torque, and rotation](image)

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Error Field Thresholds Exhibit $\beta$ and Torque Dependence

- Field thresholds reach optimal I coil correction level of 1.3G as $\beta_N$ rises
- Torque dependence

Measure as 2/1 resonant boundary field including ideal response via overlap integral with IPEC dominant mode
Error Field Thresholds Exhibit $\beta$ and Torque Dependence

- Field thresholds reach optimal coil correction level of 1.3G as $\beta_N$ rises.
- Torque dependence explained by proximity to natural tearing $\beta$ limit:
  - $\beta_{N-TM-limit} = 2.2 + 0.32T_{NBI}$

What is nature of plasma response?

$\beta_{N-TM-limit} = 2.2$ at zero torque
Magnetic Probing Data & Modeling Suggest Both Ideal and Resistive Responses Occurring

- $\beta_N$ dependence $\rightarrow$ ideal response
Magnetic Probing Data & Modeling Suggest Both Ideal and Resistive Responses Occurring

- $\beta_N$ dependence $\rightarrow$ ideal response
- Rotation dependence $\rightarrow$ resistive?
  - Break down of screening response?

Response to 10 Hz probing field

Plasma Response (a.u.)

$\beta_N$

DIII-D
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- $\beta_N$ dependence $\rightarrow$ ideal response
- Rotation dependence $\rightarrow$ resistive?
  - Break down of screening response?
  - *Explore with MARS-F modeling…*

Response to 10 Hz probing field

- High rotation
- Low rotation

Plasma Response (a.u.)

Considered case

Linear calculation with resistivity

Many thanks to Y Liu
Magnetic Probing Data & Modeling Suggest Both Ideal and Resistive Responses Occurring

- $\beta_n$ dependence $\rightarrow$ ideal response
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  - Found to be an effect in MARS-F:

Response to 10 Hz probing field

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Experiment case: strong shielding

Plasma Response (a.u.)

Vacuum response: No shielding

Decreasing rotation

0.1%/$\Omega_{\exp}$
1%/$\Omega_{\exp}$
10%/$\Omega_{\exp}$

Normalized flux

Field at rational surface (logarithmic scale)
Fitting Confirms “Two Knobs” Needed to Explain Plasma Response – *not simply ideal or resistive*

- Simple 1-D fit of response to tearing \(\Rightarrow\) limit proximity does not do good job
- 2-D fit shows \(\beta\) and rotation response needed

Rotation dependence identified – indicative of resistive response

\[
\begin{align*}
\text{Field Amplification (au)} & \\
\beta_N & \\
\text{Rotation Corrected} & \\
\end{align*}
\]

\[
\begin{align*}
y & = 1.35x^{-0.55} \\
\beta_N \text{-mode} - \beta_N \text{-TM-limit} & \\
\end{align*}
\]
• So, low torque H modes exhibit an increased response to 3-D fields due ideal and resistive effects
  – Ideal: increases with $\beta_N$
  – Resistive: decreased screening at low rotation

• This brakes the plasma to access natural tearing instability

• What does this imply for error field sensitivity & tearing mode $\beta$ limits in devices like ITER?
Extrapolate to ITER by Measuring Density and $B_T$
Scaling of Threshold in Torque Free H-modes

- **ITER baseline-like SND at $\beta_N = 1.8$ but $q_{95} \sim 4.3$**
  - ITER heating systems low in torque
    - ‘torque-free’ reasonable approximation
  - Enables rotation to be treated as hidden variable

- **H mode scalings broadly consistent with previous Ohmic scalings…**
  - Linear in density (within error bars)
  - Inverse with $B_T$
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**But 7 times lower threshold!**

- As expected of course:
  - Increased ideal & resistive response
  - More braking to trigger mode
New ITER H mode Error Field Threshold Scaling

Infer size scaling from dimensional invariance to obtain:

\[
\frac{B_{\text{pen}}}{B_T} = (1.72 - [\beta_N - 1.8]) \times \left( \frac{n_e / 10^{20} \text{ m}^{-3}}{R / 6.2 \text{ m}} \right)^{0.725} \left( \frac{B_T}{5.3T} \right)^{1.02} \times 10^{-4}
\]

- DIII-D threshold of \(1.4 \times 10^{-4}\) scales to \(1.7 \times 10^{-4}\) in ITER

- Lower than projections for ITER low density Ohmic phase
  - Ohmic threshold of \(2.9 \times 10^{-4}\) for I-coil-like fields in these variables

Note: ITER was designed to minimize \(m=1,2,3\) fields
  - We now understand \(m=4-8\) are key harmonics driving ideal response

Important to re-evaluate ITER’s error field and its correction in the relevant parameters for the ITER baseline scenario
Conclusions

• Plasma resistive response becomes important in low torque H modes close to tearing stability limits
  – Error fields open the door to tearing $\beta$ limit via braking

• New threshold scalings predict error fields are a major concern for torque free H modes, even at low $\beta_N$
  – Implications for ITER & future low rotation devices
Extrapolating to ITER

- Rotation is key, but not predicted for ITER – how to scale?
  - Solution: treat rotation as hidden variable in torque-free H modes
  - As for Ohmic regimes – implicit in threshold scalings
  - Possible for ITER H mode, as ITER has low (≈zero) torque
  - Valid provided rotation fn(ρ*, ν*, β) does not change from DIII-D range to ITER.
  - Measure scaling with main plasma parameters

- Use dimensional scaling as for Ohmic plasma:
  \[ \frac{B_{\text{pen}}}{B_T} \propto n^{\alpha_n} R^{\alpha_R} B^{\alpha_B} q^{\alpha_q} \quad \{ \times \text{some fn (β) if varied} \} \]
  - \[ \alpha_R = 2\alpha_n + 1.25\alpha_B \]
  from dimensional considerations, as for confinement  
  [Connor and Taylor NF 17 1047]
To Extrapolate to ITER

• ITER’s error correction system is based on vacuum 2/1 field
  – Had quoted as this (1.1G/kA in I coils) – but this is not correct physics!

• Actual q=2 field includes plasma response → higher harmonics matter
  – IPEC calculates at 3.26G/kA for similar DIII-D plasmas

• But ITER needs an estimate for tolerable external field – solution:
  – IPEC identifies a dominant field component at the boundary:
    • All other components give an order of magnitude lower response
  – Calculate overlap integral of I coils with this: 1.57G/kA (Used this talk)
    • Provides component of external field that generates q=2 response
  – Other error sources of ITER can be mapped to this with IPEC
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**Warning:** This is probably incomplete!
- Experimentally we know structure of field matters:
  - DIII-D I coils still leave 60% of field uncorrected
  - Likely: response of other surfaces & modes matter (eg q=3, NTV…?)
Allowing for Intrinsic Error

- DIII-D I coil cannot correct intrinsic error perfectly
  - Different harmonic content adds to field

- Consider fields as distributions of normal magnetic field at boundary:
  - Intrinsic error composed of two components: \( B_E = B_{EN} + B_{EA} \)
    - \( B_{EN} \) ‘non-aligned’ has zero overlap with I coil
    - \( B_{EA} \) aligned part – adds linearly = −ve I coil field for optimal correction
  - Torque ~ \( B^2 \approx (B_i - B_{Ioptimal})^2 + B_{EN}^2 \)
    - Deduce \( B_{EN} \) from density limits with no I coil & optimal I coil correction
      - Density limit scales as \( |B| \sim \sqrt{T} \)
      - Ratio of density limits of 0.61 gives \( B_{EN} = 0.61 \cdot B_{Ioptimal} \)

- Consistently captures threshold between zero & optimal I coil correction, and the asymptote to high I coil current
  - Though variations in harmonic content with mix may alter this further
Key Physics – later…

• **Error field effects are about ideal and resistive responses**
  - Ideal governs how fields permeate a rotating plasma
    - Screening currents prevent tearing
    - Drives kink distortion – increases with beta
  - Local resistive response ultimately will always manifest itself as field progresses towards penetration threshold
    - Resistive response governs criteria for mode formation

• **Resistive response critically dependent on further parameters**
  - Lower rotation \(\Rightarrow\) less screening \(\Rightarrow\) increased tearing & greater torque at rational surfaces
  - \(\Delta'\), by definition, governs plasma tearing response to residual field

Low rotation and \(\Delta'\) stability is the region expected for ITER
How Rotation is Buried in Extrapolation to Next Steps

- Plasma rotation & torque are key determinants of field threshold
  - H modes: usually driven rotation; ITER rotation uncertain
  - Ohmic regimes: no injected torque, self generated rotation
    - Rotation then becomes a hidden variable, implicitly varying
    - Adopt same approach for torque free H modes

- Use dimensional scaling as for Ohmic plasma:
  \[
  \frac{B_{\text{pen}}}{B_T} \propto n^{\alpha_n} R^{\alpha_R} B^{\alpha_B} q^{\alpha_q}
  \]
  \[
  \alpha_R = 2\alpha_n + 1.25\alpha_B \quad \text{from dimensional considerations, as for confinement}
  \]
  \[\text{[Connor and Taylor NF 17 1047]}\]

But COMPASS-D behaved differently…
Tearing Stability is a Concern at Low Rotation

Tearing $\beta$ limits fall with rotation

- Interpreted as rotation shear changing tearing stability $\Delta'$:

\[
\frac{1}{\sqrt{2} \pi} \frac{\Omega}{q} \left( \frac{B_0}{B_{BS, Sauter}} \right) \quad \text{vs. rotation}
\]

\[
\frac{1}{\sqrt{2} \pi} \frac{\Omega}{q} \left( \frac{B_0}{B_{BS, Sauter}} \right) \quad \text{vs. rotation shear}
\]

Rotation shear provides correlation in ‘No visible trigger’ cases and improves correlation in triggered cases

Error Field Threshold is All About Plasma Response and Rotation

• Apply small field:
  → Plasma rotation leads to shielding currents
  → But residual plasma response → small island forms
    • Response depends on tearing stability, 1/Δ’, and rotation
    • Island couples viscously to bulk plasma, slipping past
      → viscous torque depends further on rotation and viscosity
  → EM torque between island and 3d field
    • Self consistent high shielding nearly suppressed state with π/2 to EF

• Increase field:
  → Response grows → Increased torque → Island phase to EF closes
  → Island bigger still → Increased response…
    • Eventually reach bifurcation as braking enables more tearing & torque

[Fitzpatrick NF 1993]
Plasma Rotation Leads to Shielding of 3-D Fields

Image currents inhibit tearing response

3d Field
(think of as equivalent to currents + and −)

Shielding response if ideal MHD

Bulk Plasma Rotation

[Image: Fitzpatrick NF 1993]
Shielding is Imperfect – Residual Island Depends on Rotation & $\Delta'$

- Torque balance established between viscous drag and EM torque on island

3d Field (think of as equivalent to currents + and –)

Island

Island current perturbations (+ and –)

Bulk Plasma Rotation
Island Torque Balance and ‘Penetration’ Depend on Plasma Response and Rotation

Less 3d Field
→ Less torque,
→ Viscosity wins
→ More shielding

3d Field
(think of as equivalent to currents + and –)

Island current perturbations (+ and –)

Bulk Plasma Rotation

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[Fitzpatrick NF 1993]
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3d Field (think of as equivalent to currents + and −)

More 3d Field
→ More torque,
→ Phase dragged
→ More response

Less 3d Field

Island

Island current perturbations (+ and −)

Bulk Plasma Rotation

Bulk Plasma Slowed

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Island Torque Balance and ‘Penetration’ Depend on Plasma Response and Rotation

- Ultimately bifurcates to locked state, as phases align, island grows & couples strongly to field.

More 3d Field
- More torque,
- Phase dragged
- More response

Bulk Plasma Slowed
Island Torque Balance and ‘Penetration’ Depend on Plasma Response and Rotation

Island response depends on how easy it is to drive tearing instability $\rightarrow 1/\Delta'$ & how much plasma forces it out of phase with 3d field $\rightarrow \tau_V$ and $\omega$

Less 3d Field
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More 3d Field
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