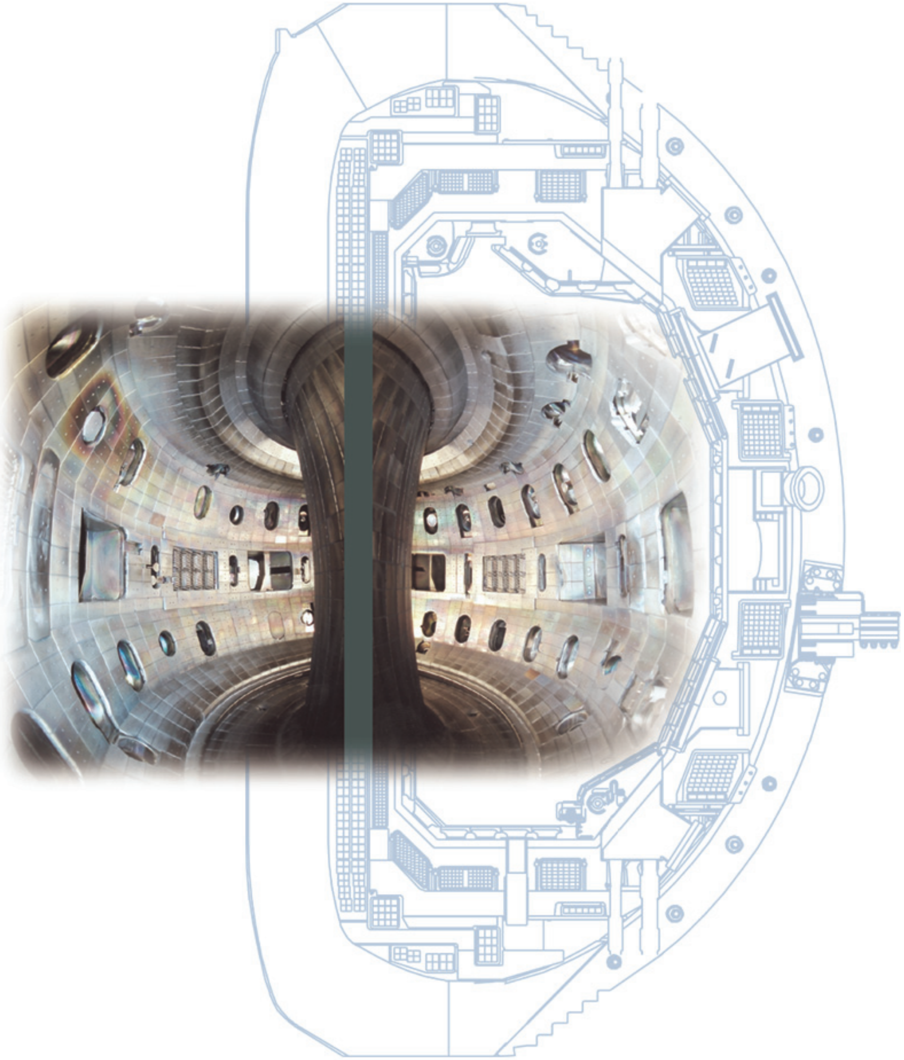
A New View of the Sawtooth Instability and its Relation to the Internal Kink

A Tale of Two Discharges



by
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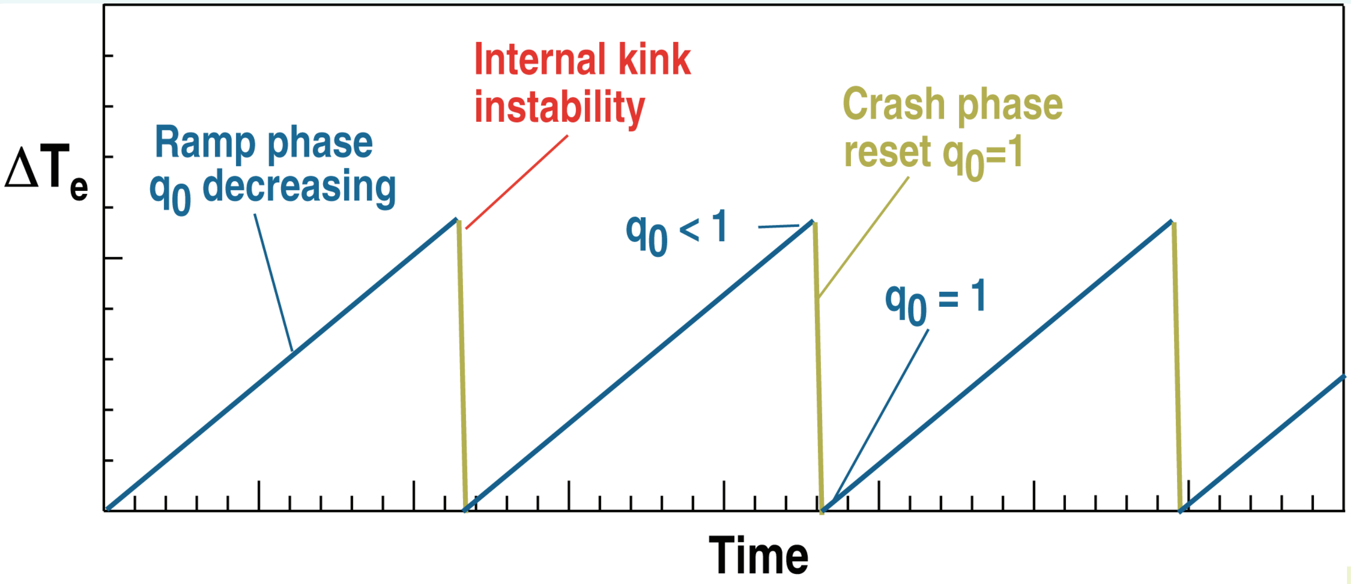
Sawtooth is a Repetitive Oscillation Consisting of a Slow Ramp Followed by a Rapid Crash: First Observed in T_e

- **Conventional picture: q_0 drops during the ramp from current diffusion**
  - ⇒ Ideal stability continually degrades
- **Degrading ideal stability overcomes non-ideal stabilizing effects**
  - – Sawtooth crash believed to be triggered by n=1 m=1 internal kink mode



(Lazarus, et al., Phys. Plasmas 14, 055701 (2007)

- **DIII-D experiments with Bean and Oval cross sections produced very different sawtooth characteristics**

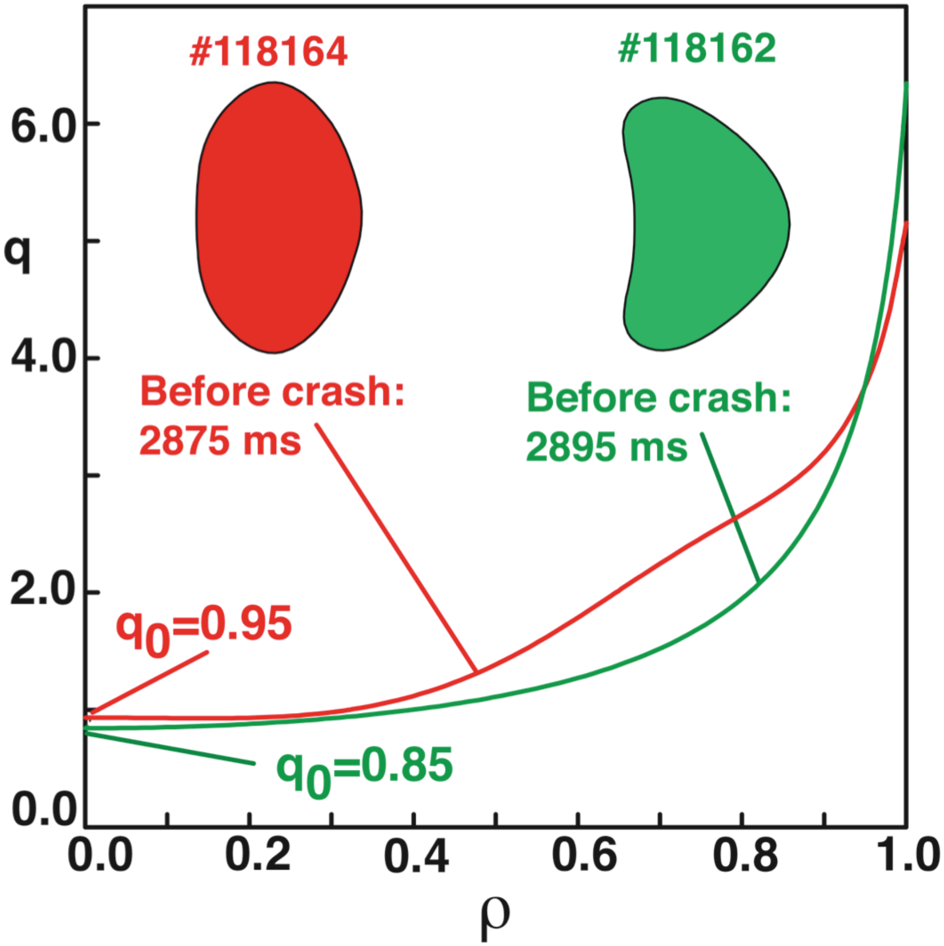
First time sawteeth were followed through multiple cycles with complete equilibrium profile data and analyzed with an MHD stability code

Despite Similar Discharge Conditions Bean and Oval had Different Transport which Produced Different Profiles

- **Profile change is largely due to large change in χ_e**
  - Oval had very poor electron confinement inside q=1

    ⇒ **Discharges had very different stability and different sawteeth**

- **Equilibrium diagnostics enabled highly accurate q profile reconstruction and fluctuations**
  - T_e, T_i, n_e, n_i, Z_{eff}, field line pitch
- **Both discharges had $q_0 < 1$ immediately before the crash**
  - $q_0 \sim 0.95$ for the Oval and $q_0 \sim 0.85$ for the Bean
- **q then returned to near one after each crash**



Experiments in Bean and Oval Cross Sections Showed Very Different Sawtooth Behavior

Bean Crash

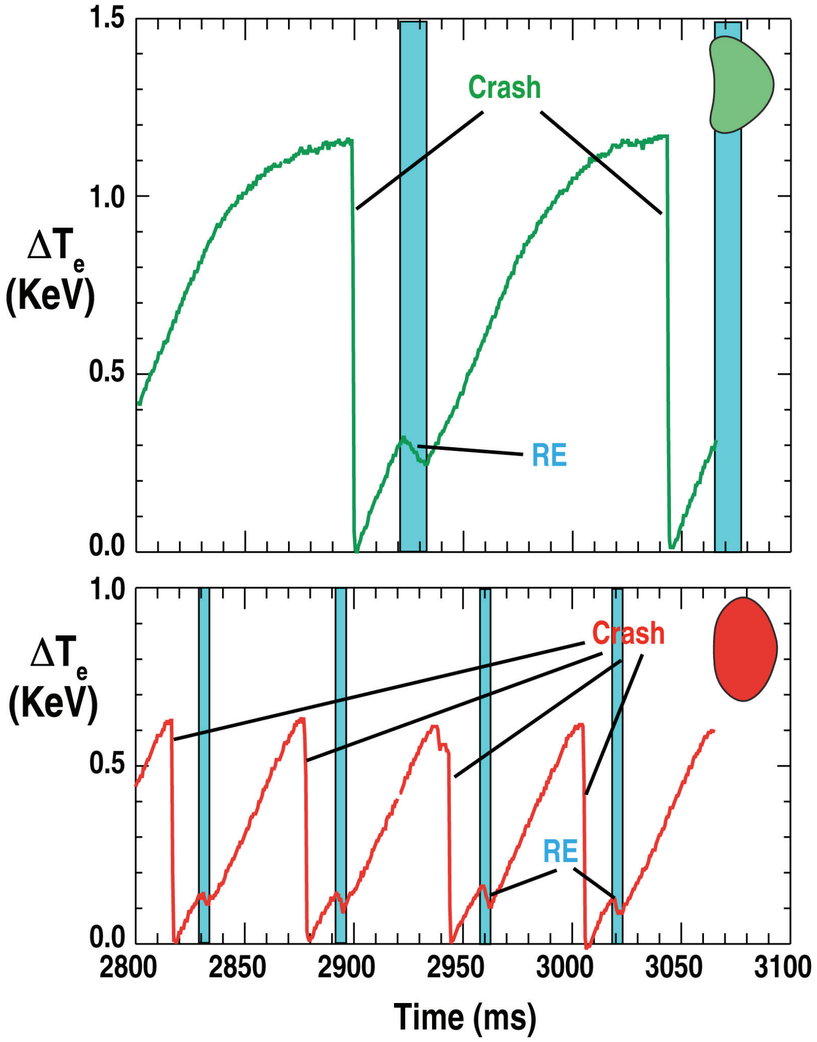
- Distinct reconnection event observed
- B_θ crash time ~ T_e crash time

Oval Crash

- No distinct reconnection event observed
- B_θ crash time >> T_e crash time

Both discharges exhibited MHD relaxation events (REs) a few tens of ms after the crash

- Similar characteristics to the Oval sawtooth crash



High quality equilibrium reconstructions enabled a detailed and systematic stability analysis through multiple cycles

Theme: Detailed Analysis of the Experiments Leads to a New View of the Sawtooth

- **Quasi-interchange may provide alternative explanation for different crash characteristics**
- **Time development of equilibrium quantities show several key features of the conventional wisdom are not valid**
  - Ideal stability for Bean improves through the ramp
- **Crash trigger is Quasi interchange in Oval and internal kink in Bean**
- **MHD relaxation events look like mini sawtooth crashes**
  - Underlying unstable quasi-interchange as in Oval sawtooth crash

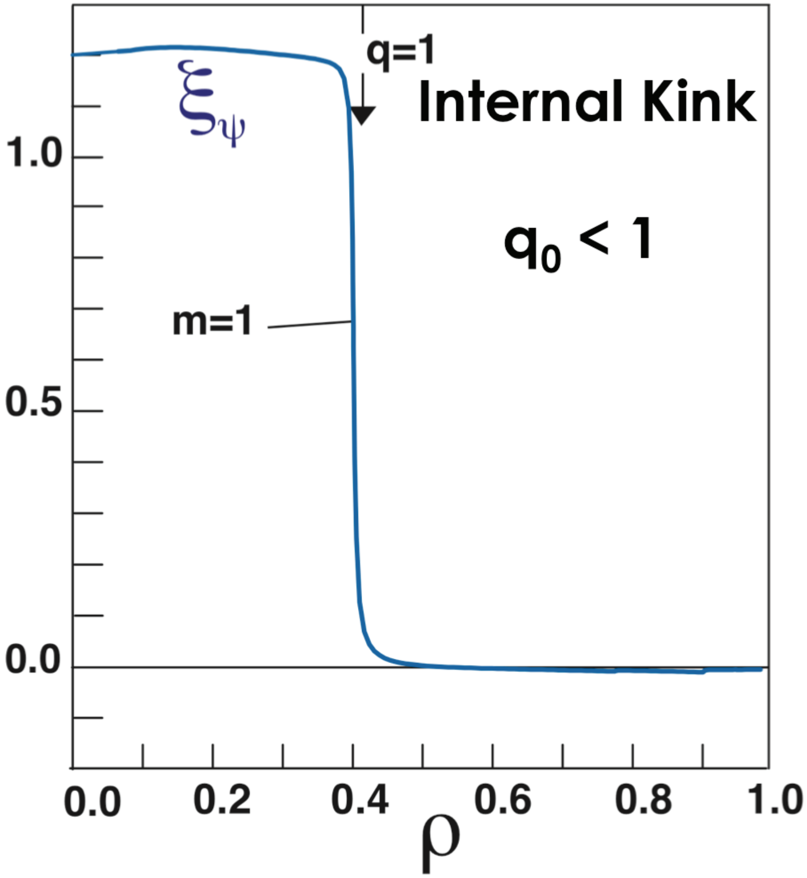
Conjecture: Crash type is determined by underlying linear ideal mode structure

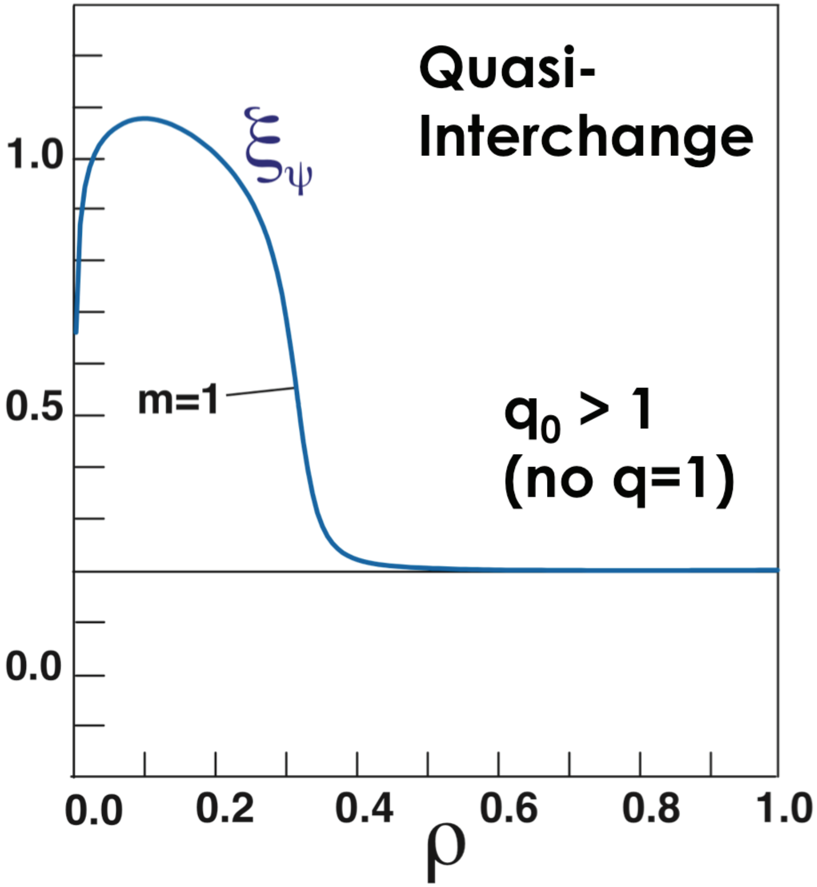
- **Mechanism**
  - Linear mode determines the nonlinear mode
  - Nonlinear flow pattern is a key element to the reconnection rate
- **Model proposed for quasi-interchange sawtooth when $q_0 < 1$**

Ideal n=1 Quasi-interchange Proposed as Alternative Sawtooth Model 1985 to Explain Giant Sawteeth in JET

- Internal kink radial displacement ξ_ψ is a Top-hat structure
- Quasi-Interchange (Wesson 1985) decays smoothly

Normal "radial" component: $\xi_\psi \equiv \xi \cdot \nabla\psi / |\nabla\psi|$

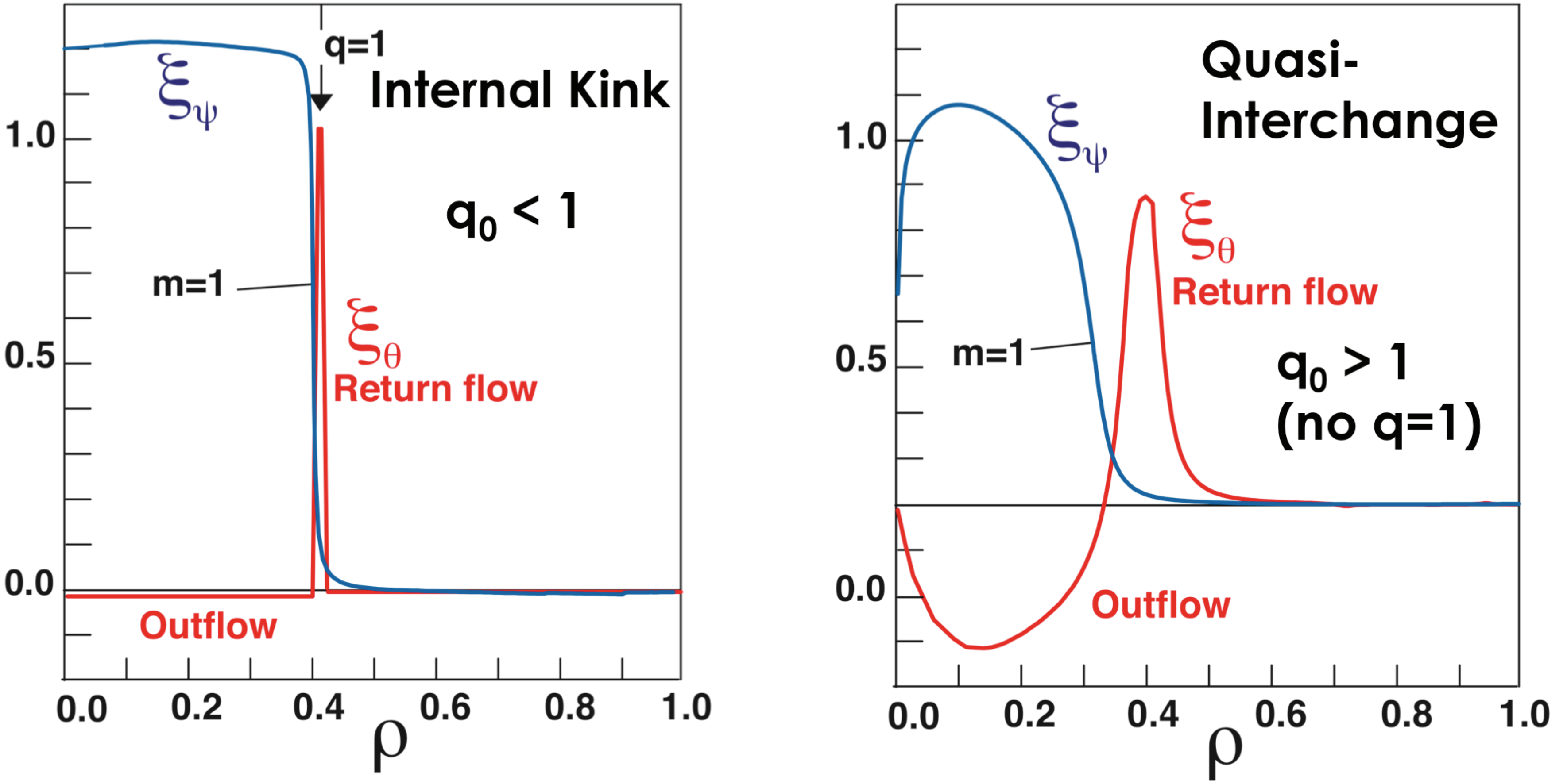




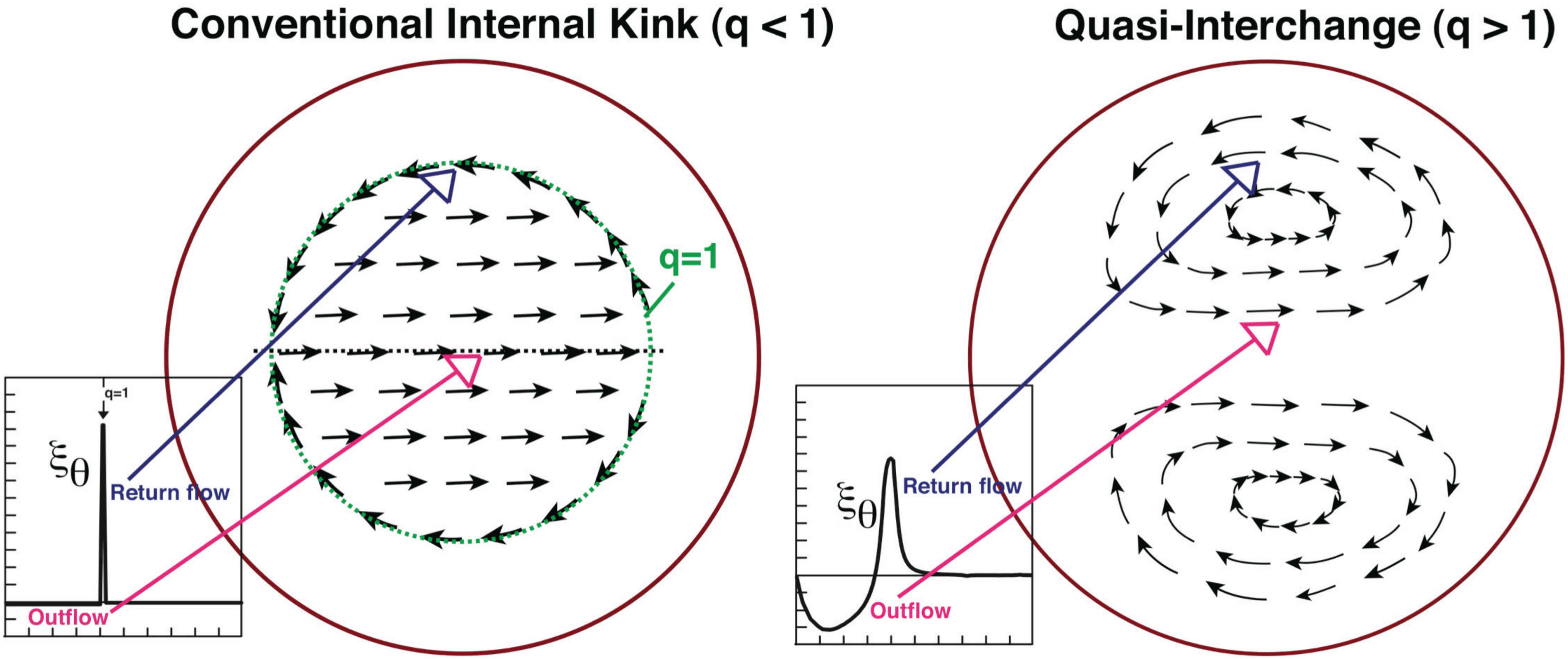
Ideal n=1 Quasi-interchange Proposed as Alternative Sawtooth Model 1985 to Explain Giant Sawteeth in JET

- **Poloidal displacement component shows distinct difference**

Poloidal component: $\xi_\theta \equiv \xi \cdot (\nabla\psi \times \nabla\phi) / |B_{pol}^2|$



Conventional Internal Kink Return Flow Concentrated at q=1 But Quasi-Interchange Shows Broad Return Flow



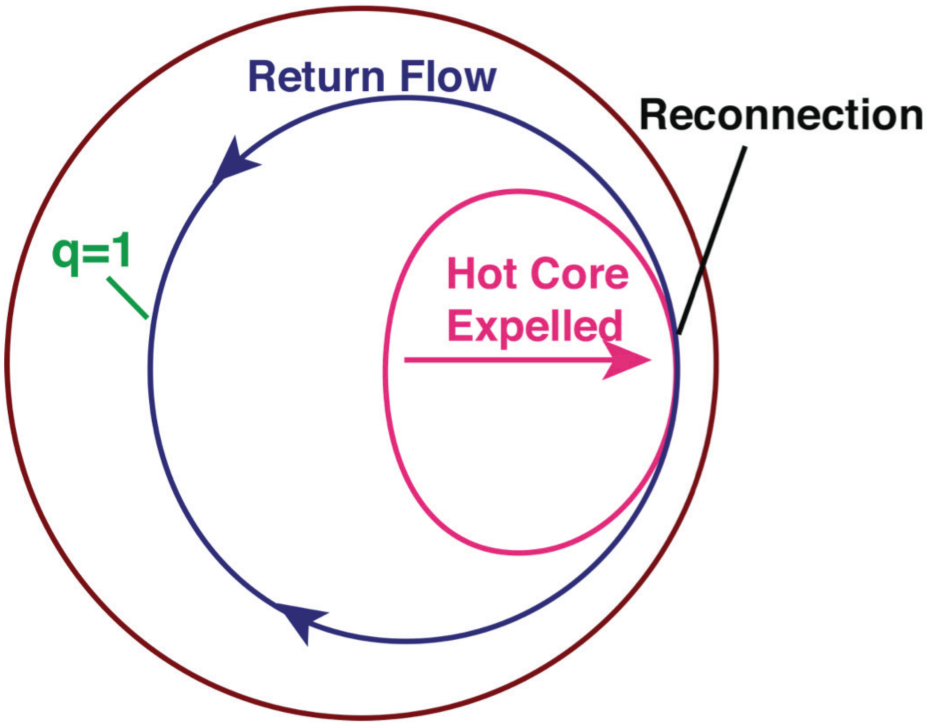
Conventional Internal Kink (q < 1)

Quasi-Interchange (q > 1)

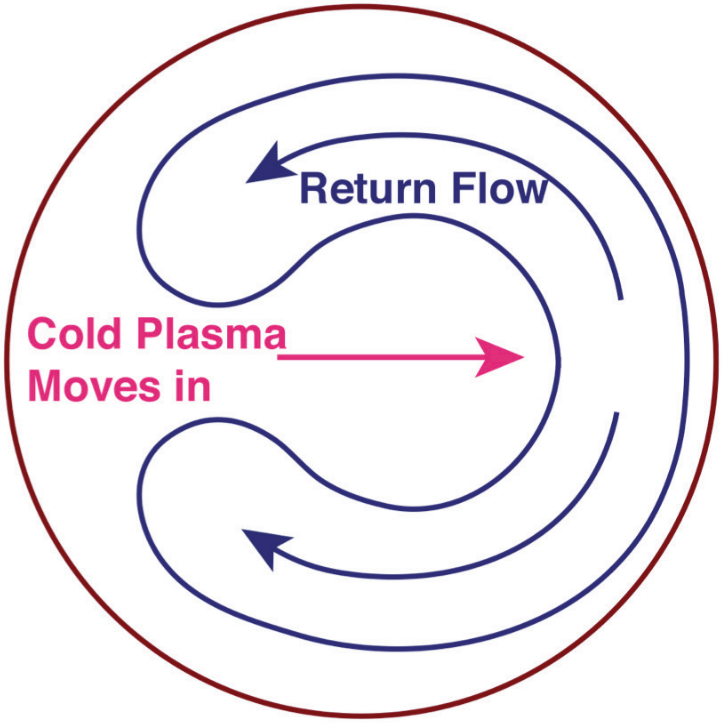
- Conventional internal kink moves as a rigid shift of the core to the q=1 surface with a return flow along the q=1 surface
- Quasi-interchange is a Rayleigh Taylor-like convection cell

Wesson Conjecture: Linear Mode is Reflected in Nonlinear Behavior and Results in Very Different Crash

Conventional Internal Kink



Quasi-Interchange

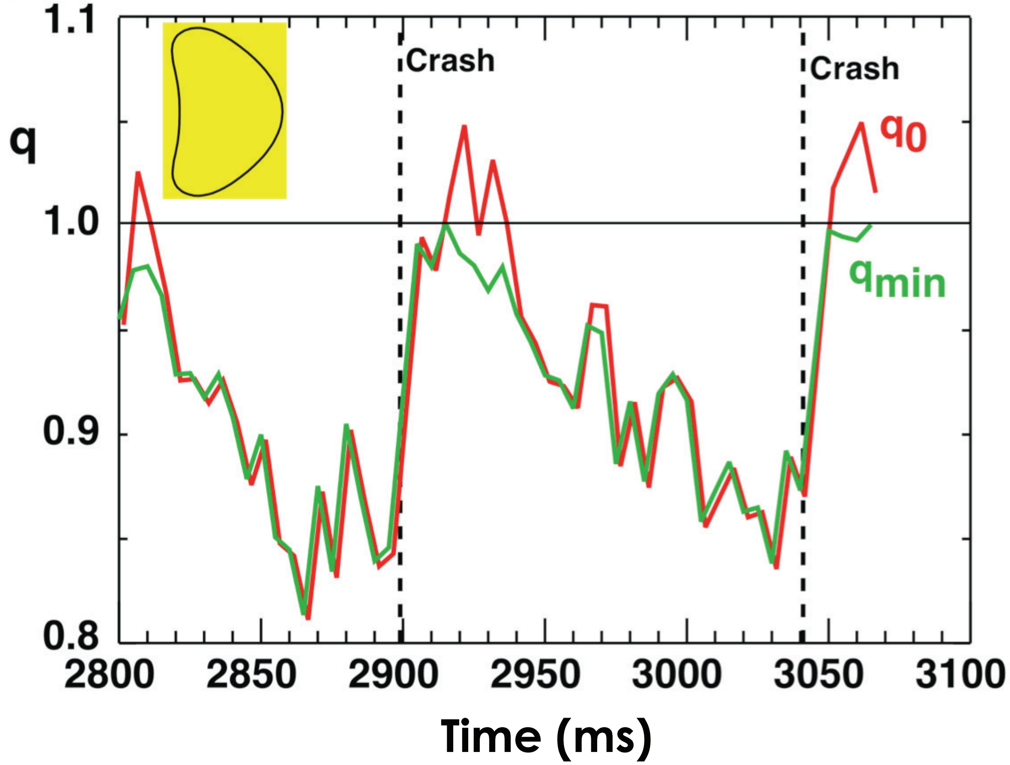


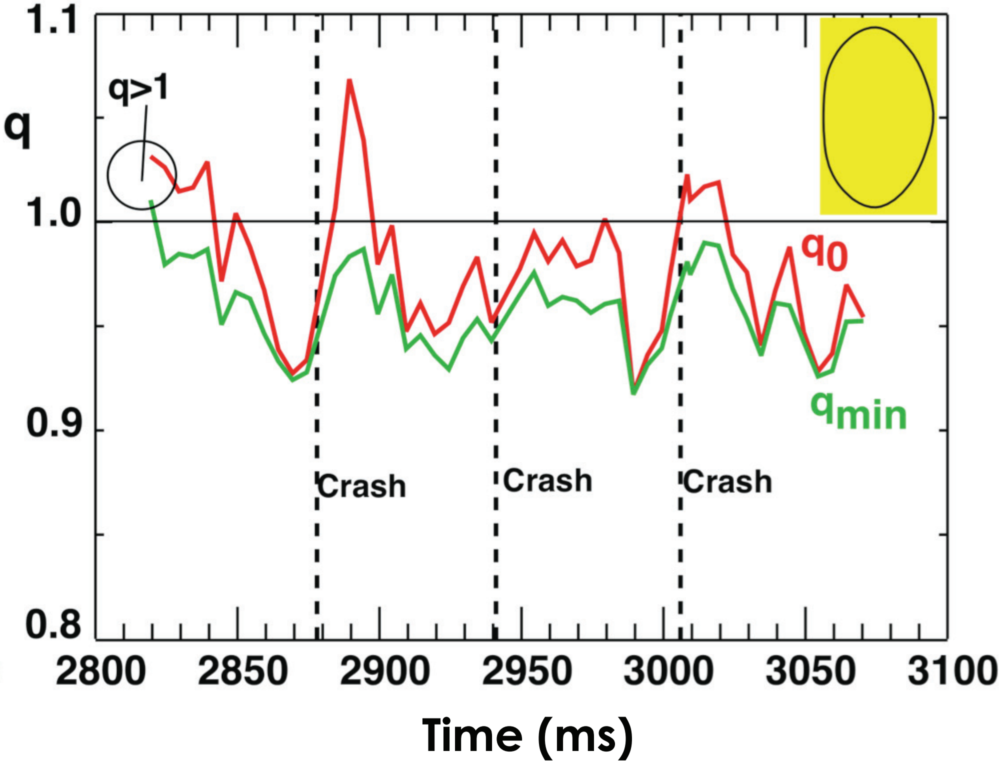
- Hot core slams in to q=1
- Reconnection at q=1 redistributes plasma
  - ⇒ Fast T_e collapse and fast B_θ change to reset q=1

- Cold plasma moves in to core on ideal time scale
  - ⇒ Fast T_e collapse
- Slow current diffusion during subsequent ramp resets q
  - ⇒ Slower B_θ change

Equilibria Reconstructed for 20–30 Times per Cycle Through Several Successive Cycles in Both Discharges

- **Expected decaying trend found in q_0 and q_{min} through each sawtooth cycle for both discharges**
  - But q_0 and q_{min} return to near one after the crash in both discharges
  - Off-axis minimum in q after the crash for the Bean as well as the Oval

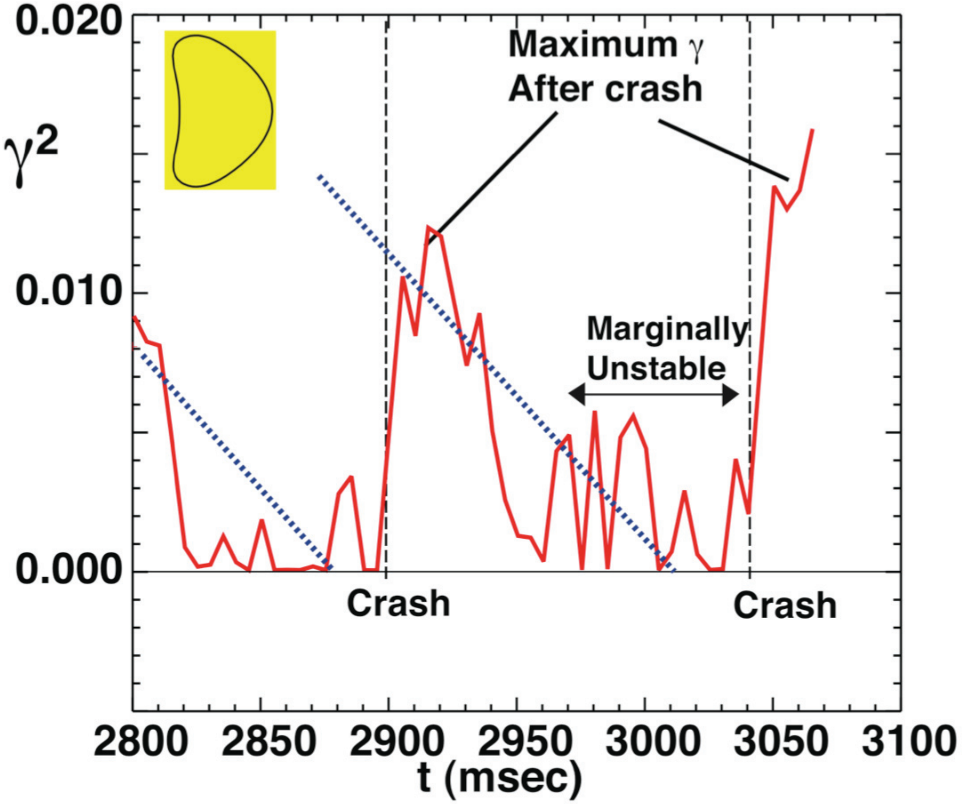




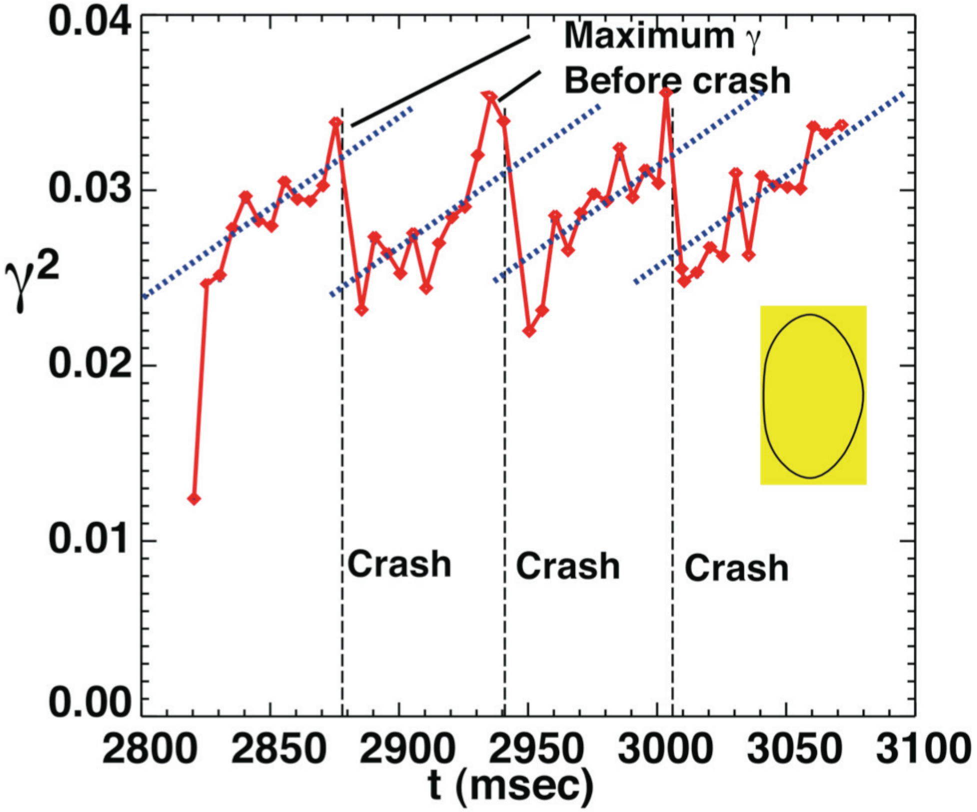
- **Ideal n=1 stability analysis was then performed for each equilibrium**

Conventional Picture where Ideal Stability Continually Degrades During Ramp Does not Always Hold

- Growth rate decreases with dropping q_0



- Growth rate increases with dropping q_0



- Bean is at most marginal for the last two thirds of the ramp

Oval is unstable to ideal quasi-interchange and Bean is unstable to ideal internal kink at the respective crash times

Difference in Ideal Stability Between Bean and Oval Provides A Clue to the Differences in B_θ Crash Rates

- **Bean crash in B_θ is rapid (<500 μs)**
  Oval B_θ crash (~5 ms) is much slower than both Oval T_e crash and the Bean case B_θ and T_e crash
  - For the oval the rise in q takes 5 to 10 ms after the crash in T_e

- **Two different crash types with different underlying linear modes**
  - Quasi-interchange in Oval
  - Conventional internal kink in the Bean

Quasi-interchange is the key to the crash in the Oval

But what does it mean when $q_0 < 1$

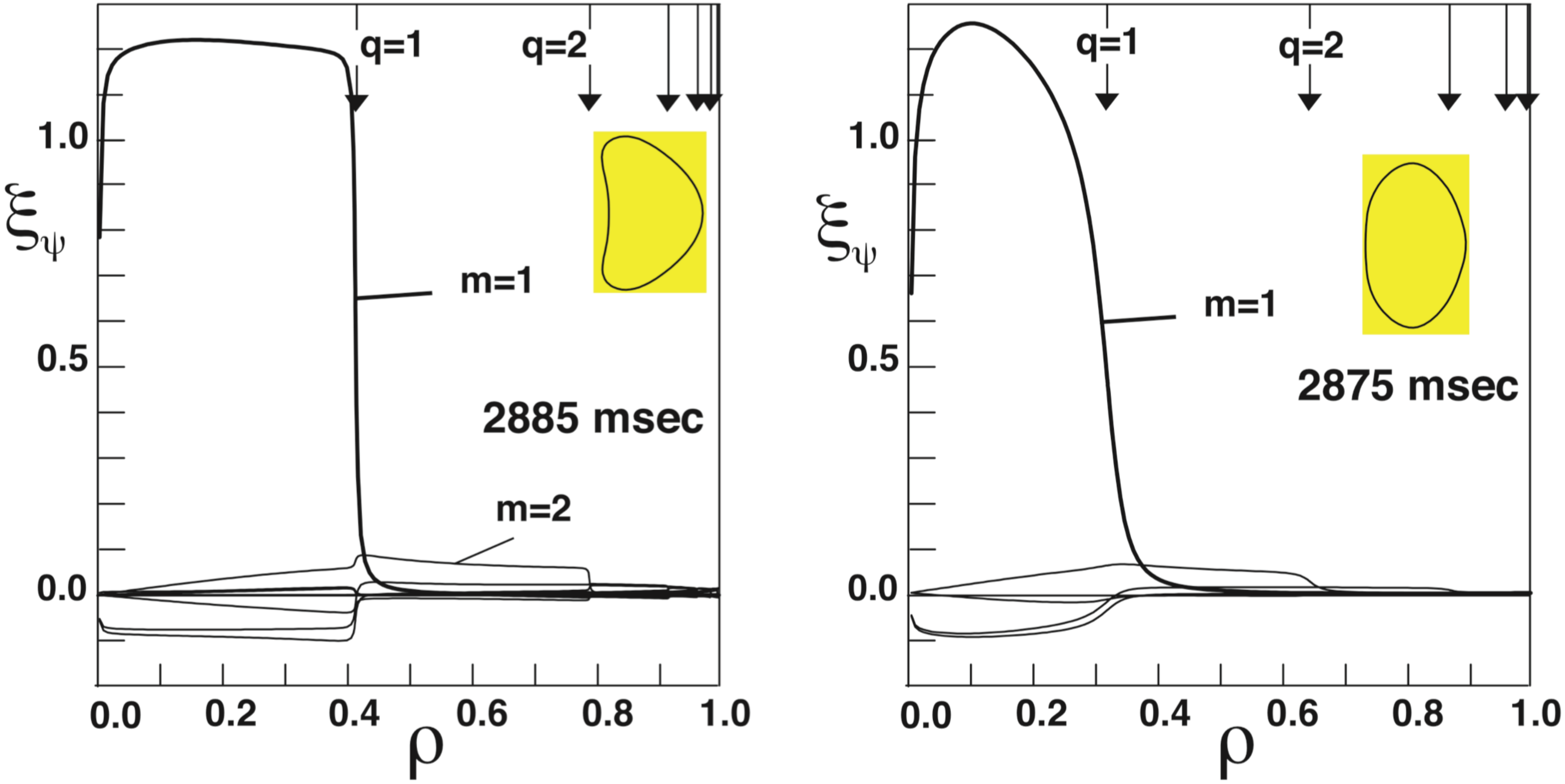
Both the Bean and Oval discharges have $q_0 < 1$ at the time of the crash

Quasi-interchange should be defined by poloidal flow pattern and not whether q_0 is above or below one

This is what is important for the ultimate consequences

Underlying Ideal Mode for Oval Discharge Just Before Crash Time Deviates from Conventional Internal Kink

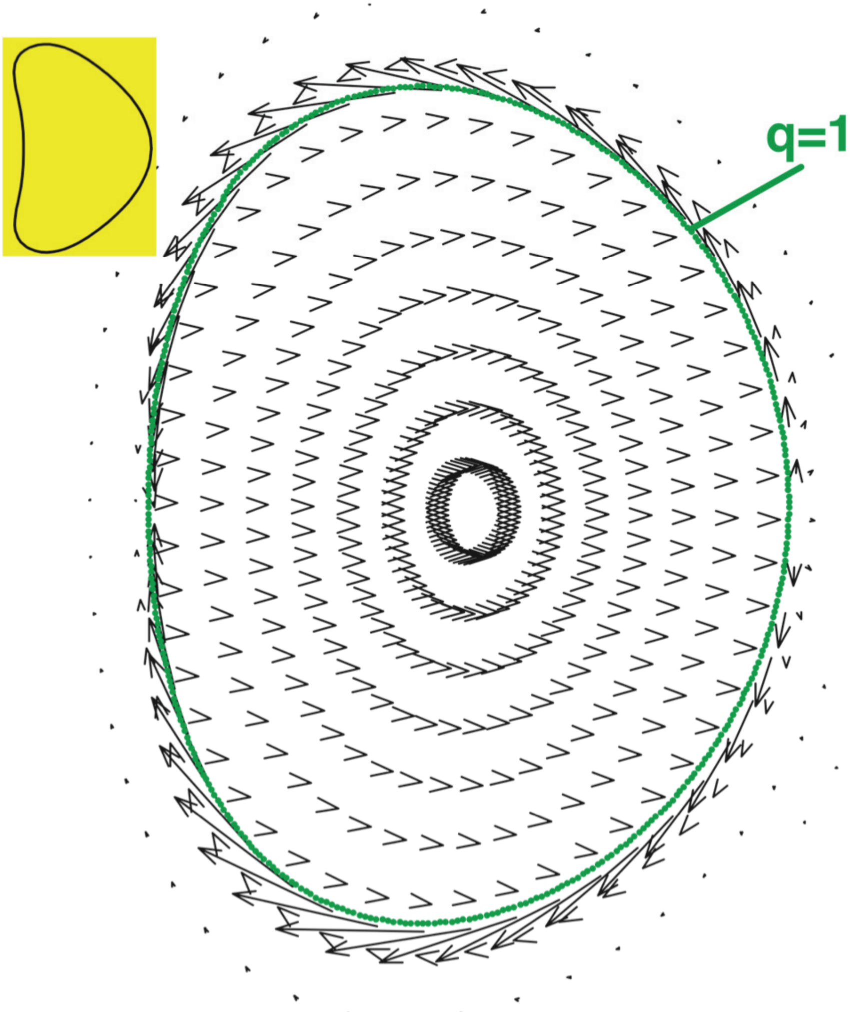
- **Mode in Oval case has parabolic normal displacement ξ_ψ expected for quasi-interchange: For Bean it is a Top-Hat rigid shift**

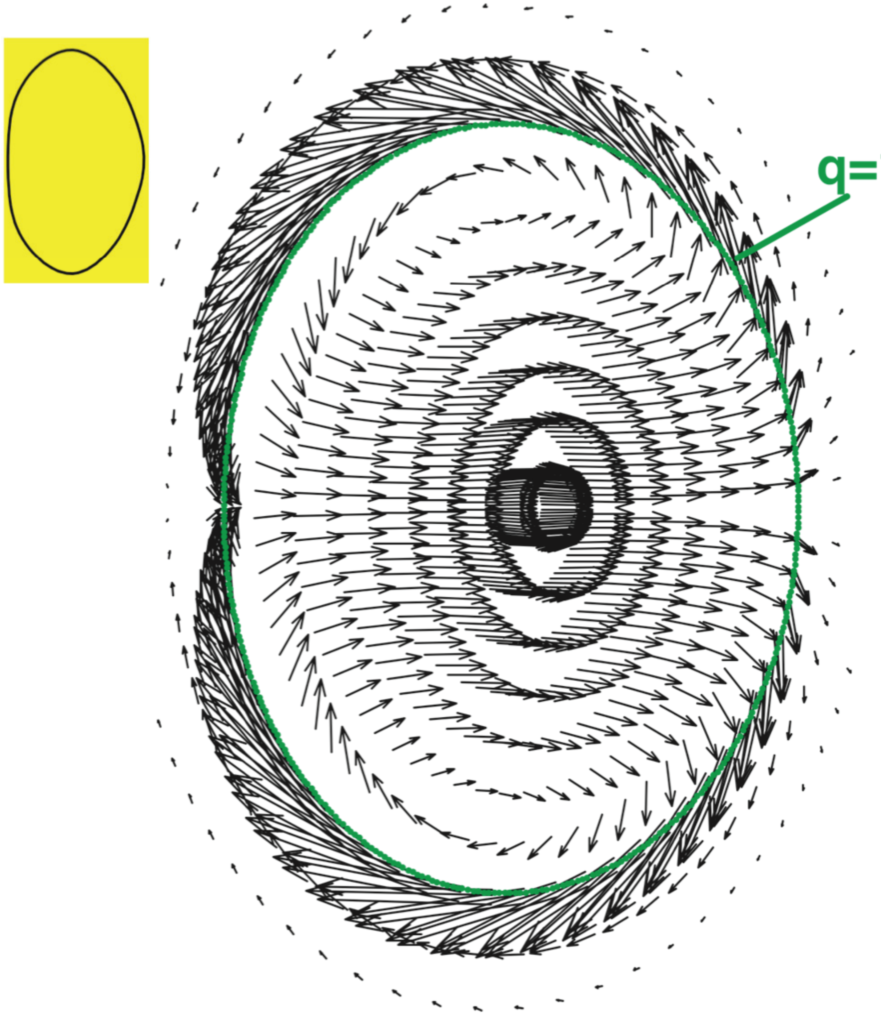


Radial mode structure ξ_ψ is not always a good indicator of a quasi-interchange
⇒ **Flow pattern is a much better discriminator**

Expanded Views of Flow Pattern in Poloidal Plane Show Rigid Shift in Bean And Convective Flow Pattern in Oval

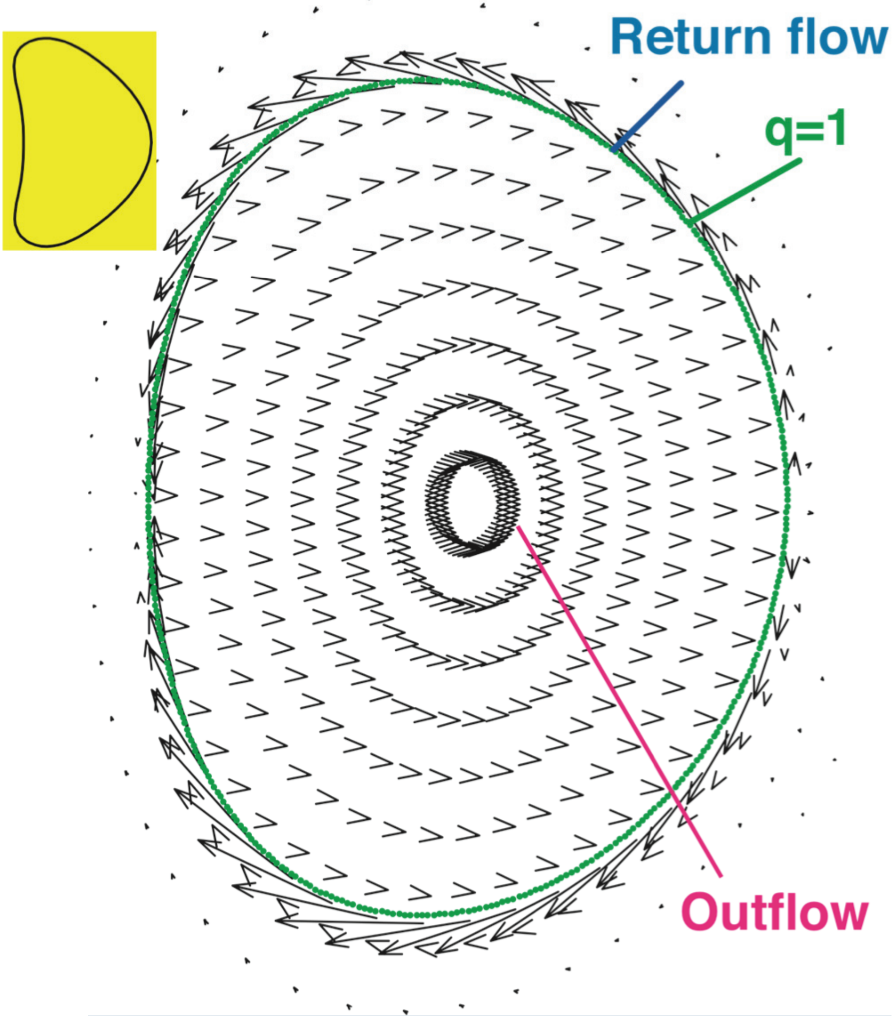
- For $q_0 < 1$ structure is confined largely in and around core

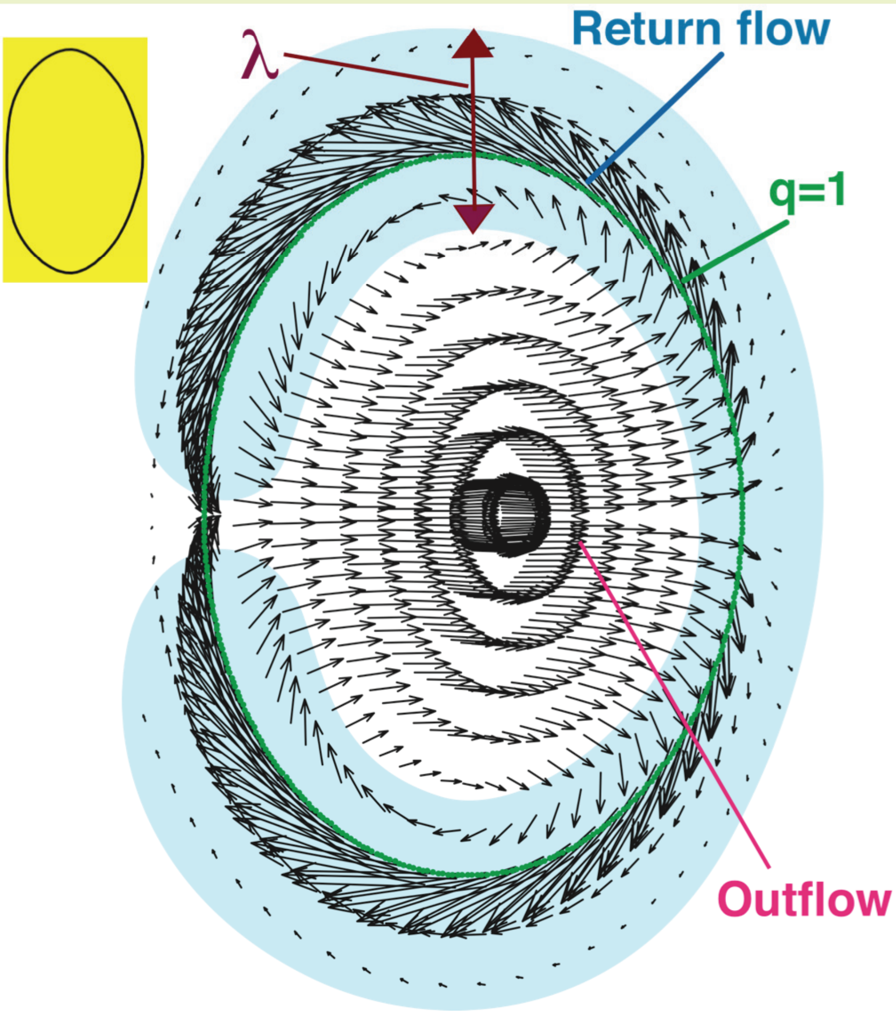




Expanded Views of Flow Pattern in Poloidal Plane Show Rigid Shift in Bean and Convective Flow Pattern in Oval

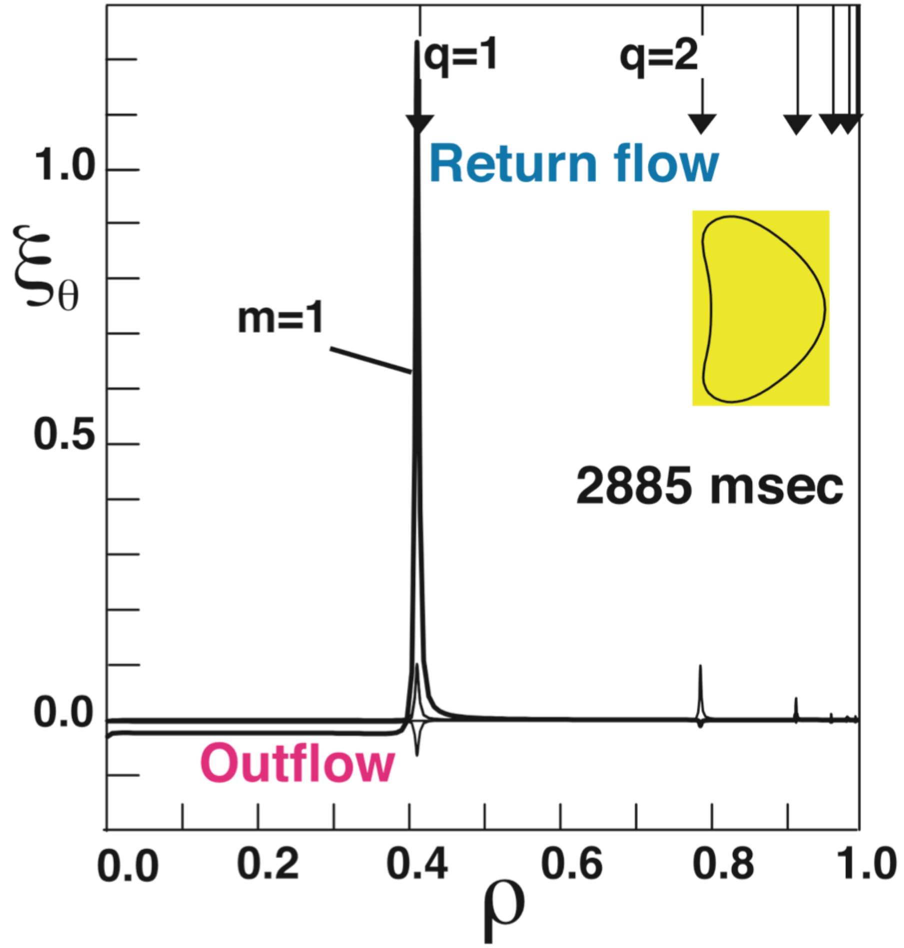
- **Oval has the broad return flow expected of a quasi-interchange**
  - Centered around but not confined to q=1

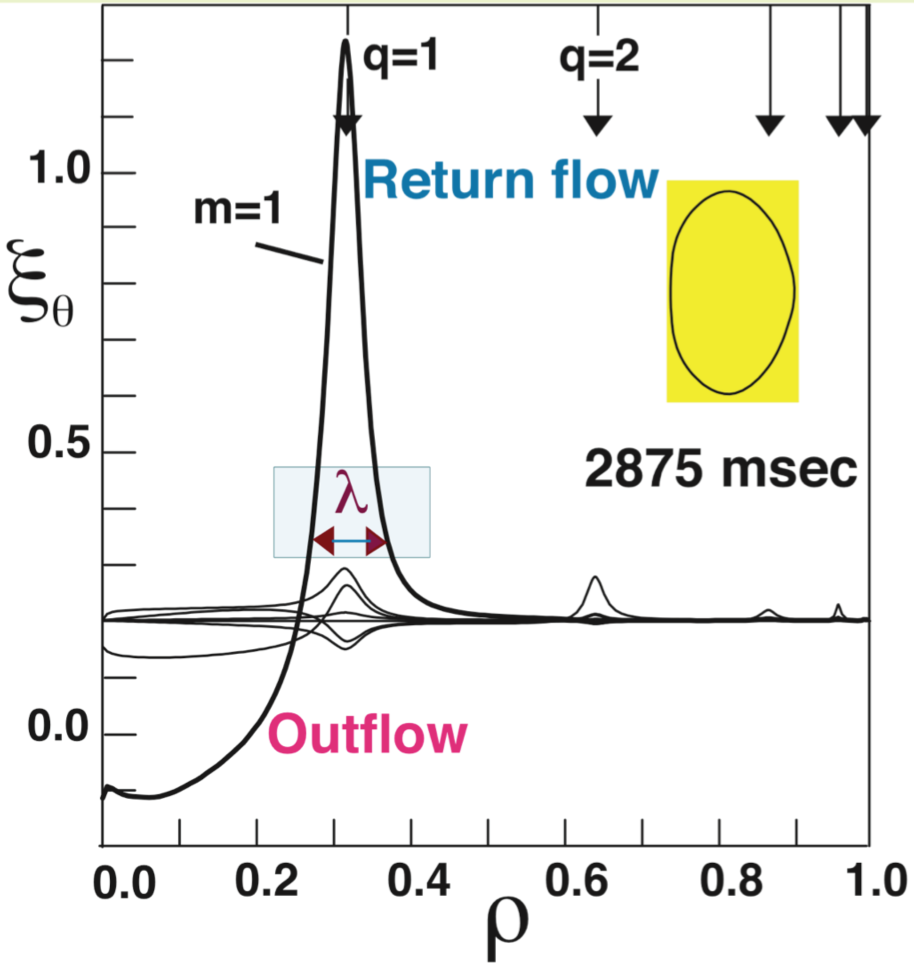




Conventional Internal Kink and Quasi-Interchange are Most Easily Identified by Poloidal Displacement

- **Finite width λ and sign reversal of ξ_θ implies convective cell in Oval**
  - All return flow in Bean is at q=1
- **Use this to distinguish quasi-interchange from internal kink**

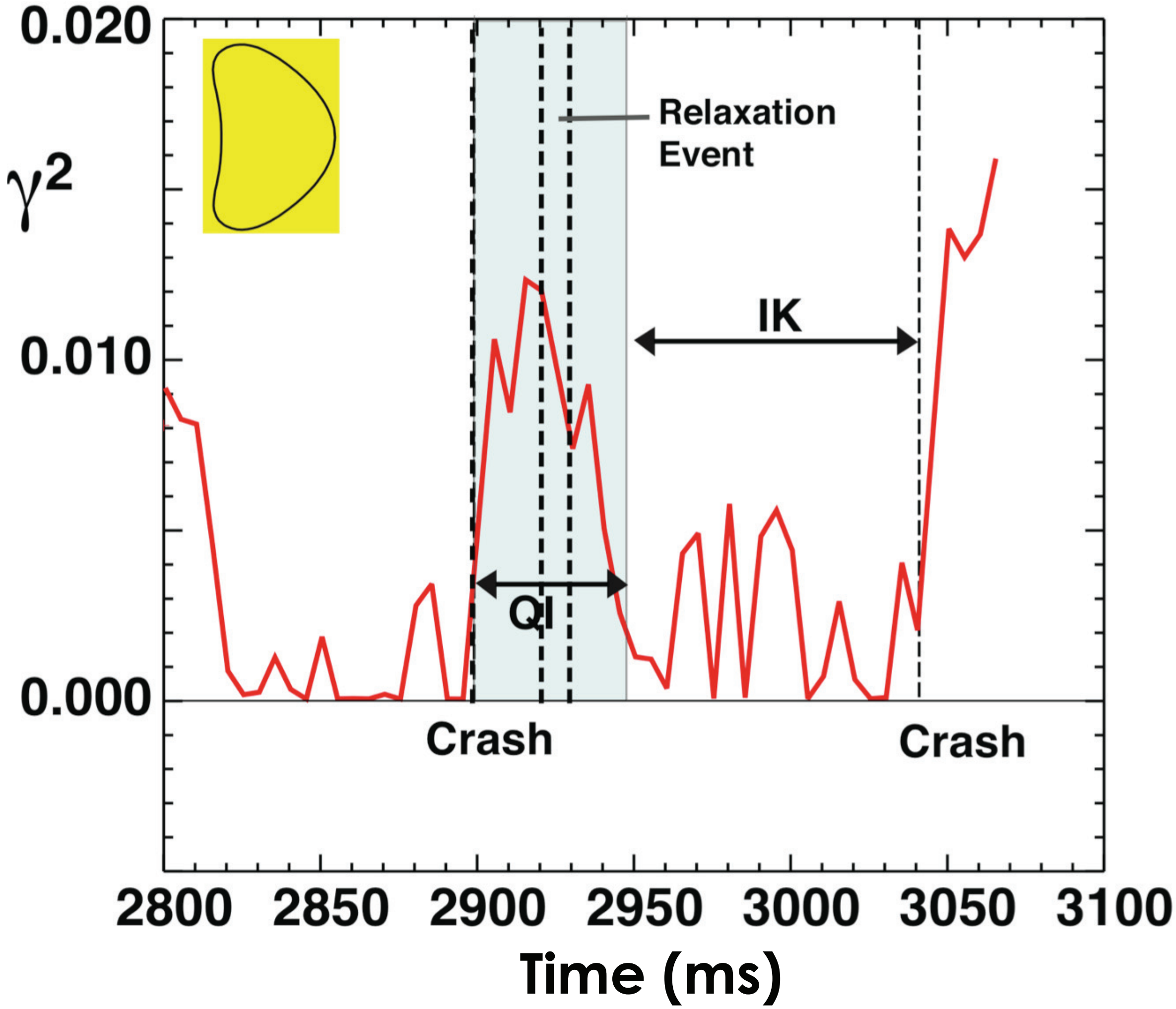




Bean Also Unstable to Quasi-interchange Just After Sawtooth Crash and Through Relaxation Events

- **Relaxation event bursts last ~1-5 ms**

No observable reconnection just as in the Oval sawtooth crash

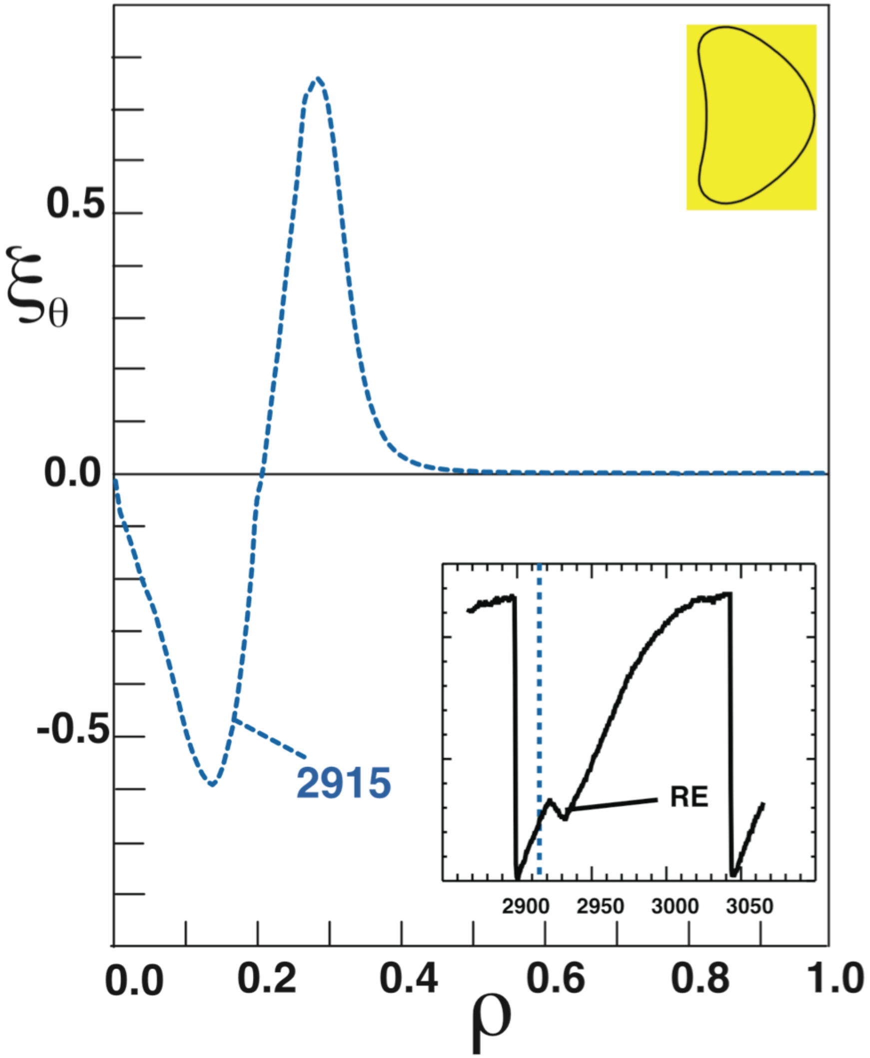


- **Bean case transitions from quasi-interchange mode to marginal internal kink after MHD relaxation event**

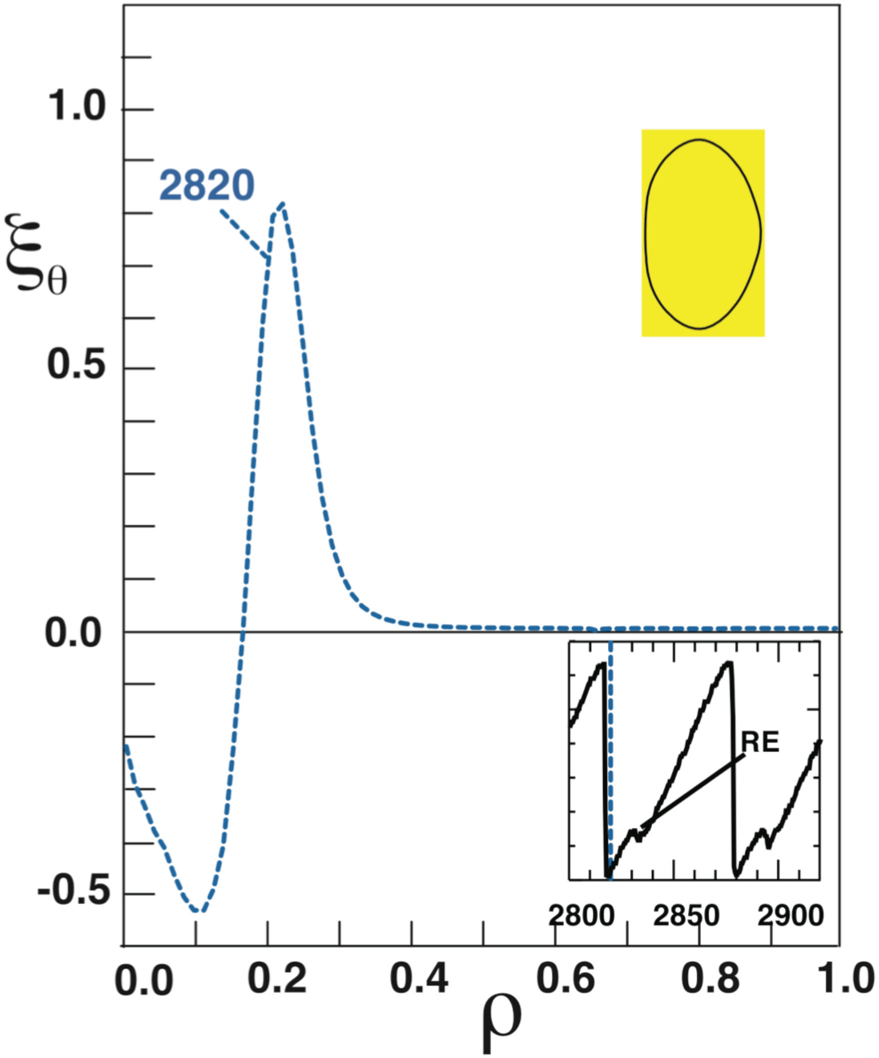
- **Oval case is a quasi-interchange mode at each time including during the MHD relaxation events**

Mode Structure is Quasi-Interchange from the Crash Through the MHD Relaxation Event in Both Discharges

Quasi-interchange at beginning of Bean ramp

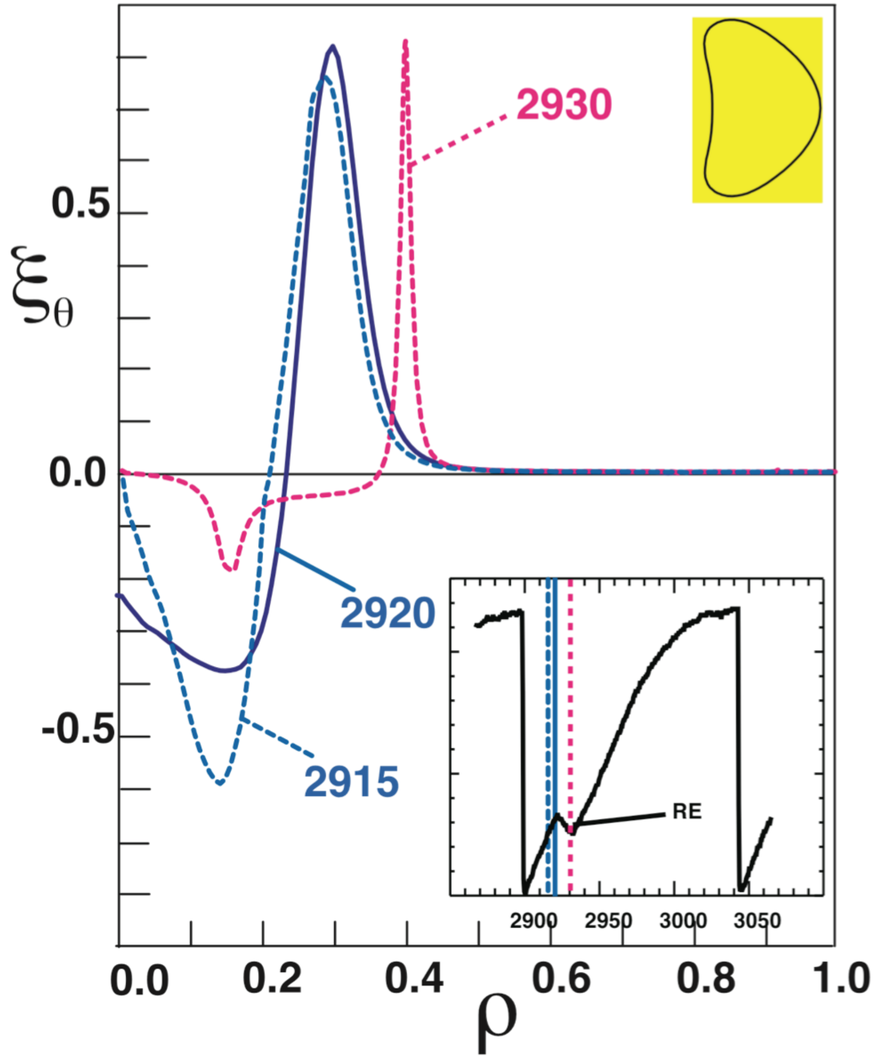


Quasi-interchange at beginning of Oval ramp

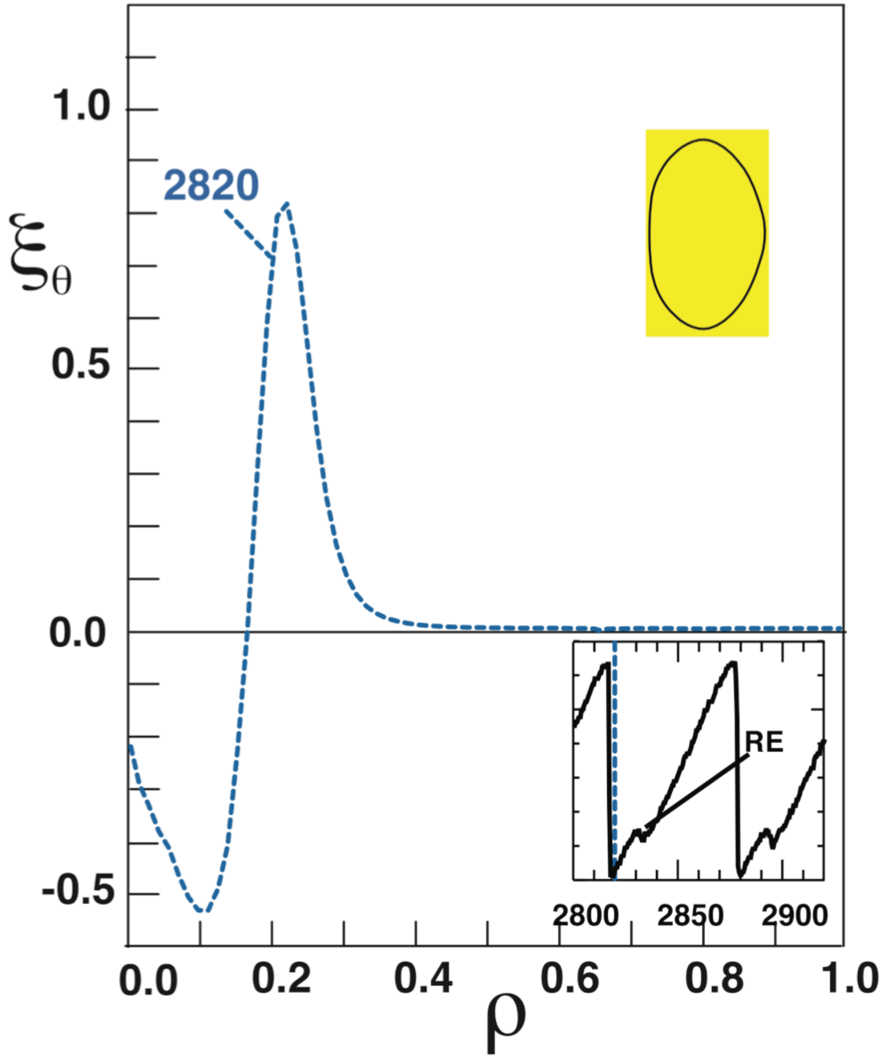


Mode Structure is Quasi-Interchange from the Crash Through the MHD Relaxation Event in Both Discharges

Quasi-interchange through Bean relaxation event

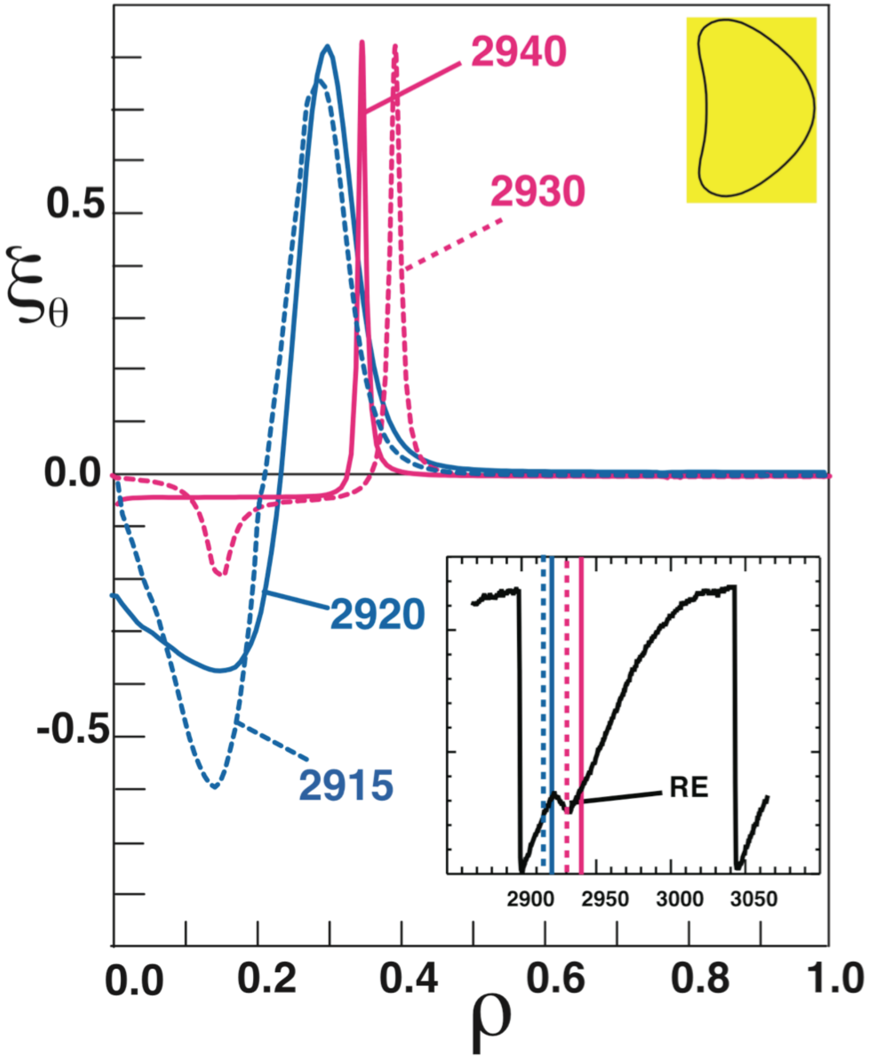


Quasi-interchange at beginning of Oval ramp

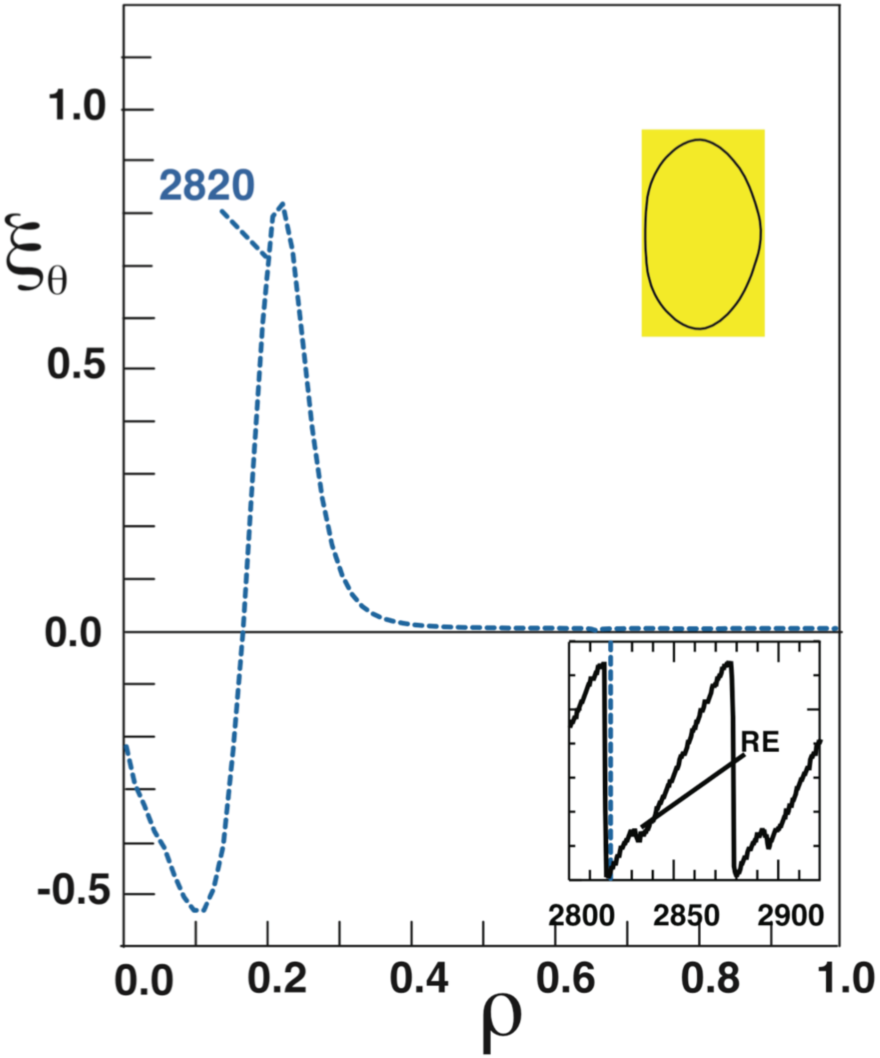


Mode Structure is Quasi-Interchange from the Crash Through the MHD Relaxation Event in Both Discharges

Transition to localized return flow at q=1 after Bean relaxation event

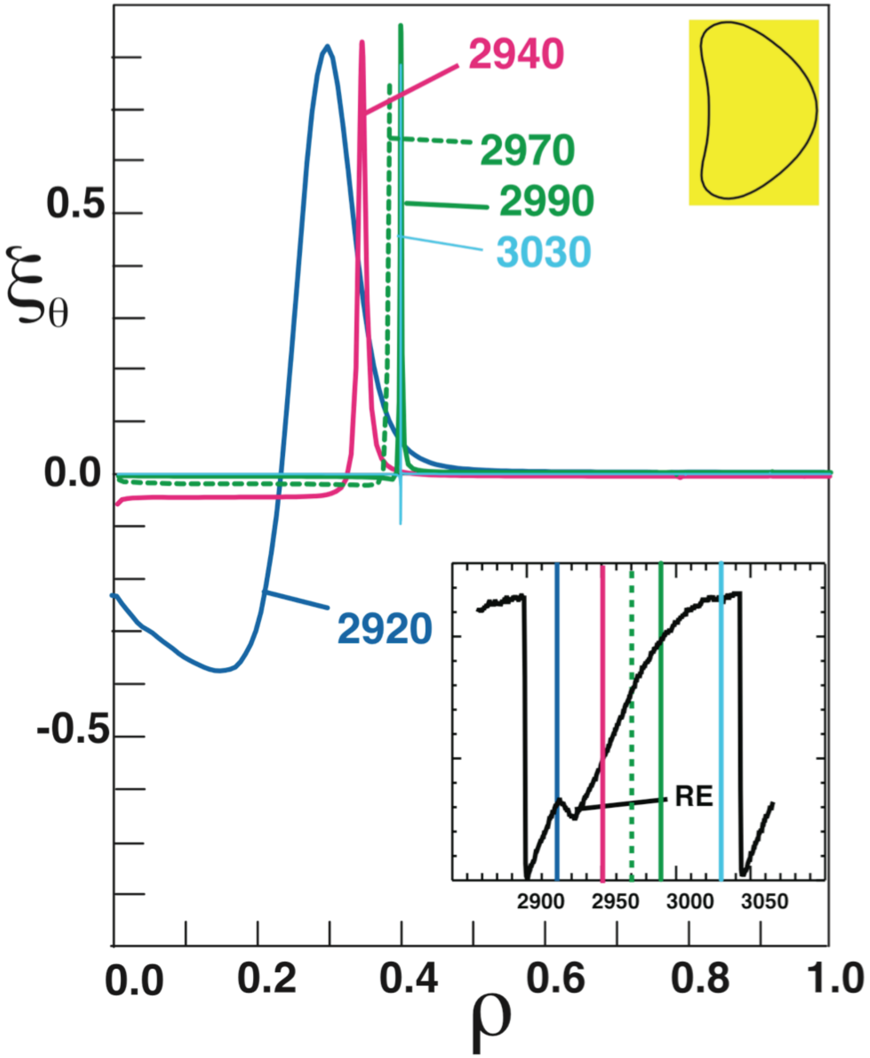


Quasi-interchange at beginning of Oval ramp

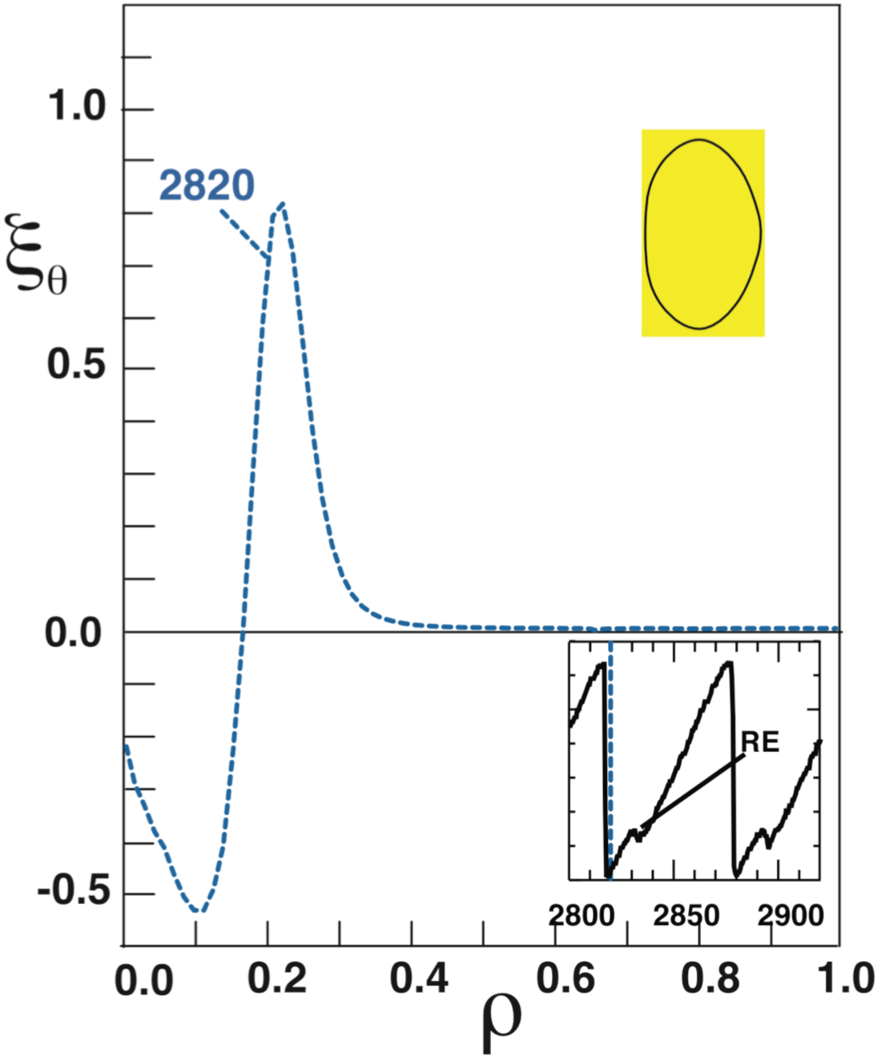


Mode Structure is Quasi-Interchange from the Crash Through the MHD Relaxation Event in Both Discharges

Localized return flow at q=1 remains through the end of the Bean ramp



Quasi-interchange at beginning of Oval ramp



Mode Structure is Quasi-Interchange from the Crash Through the MHD Relaxation Event in Both Discharges

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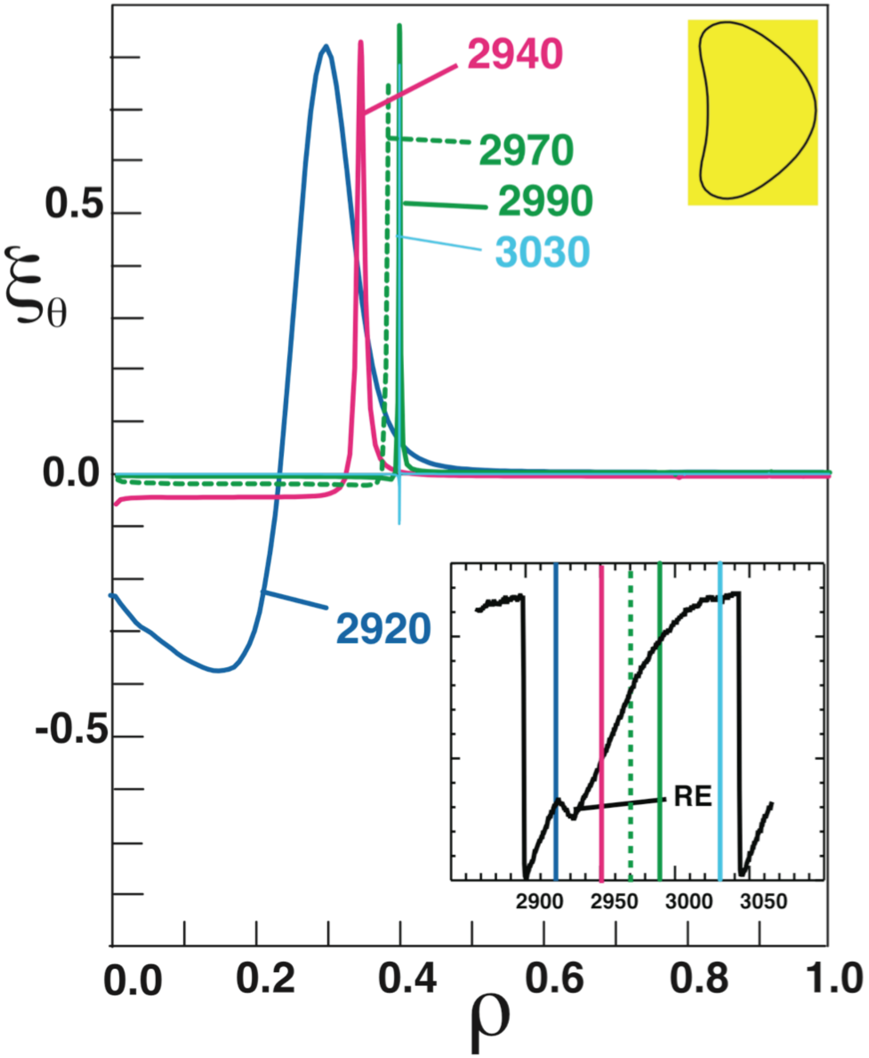
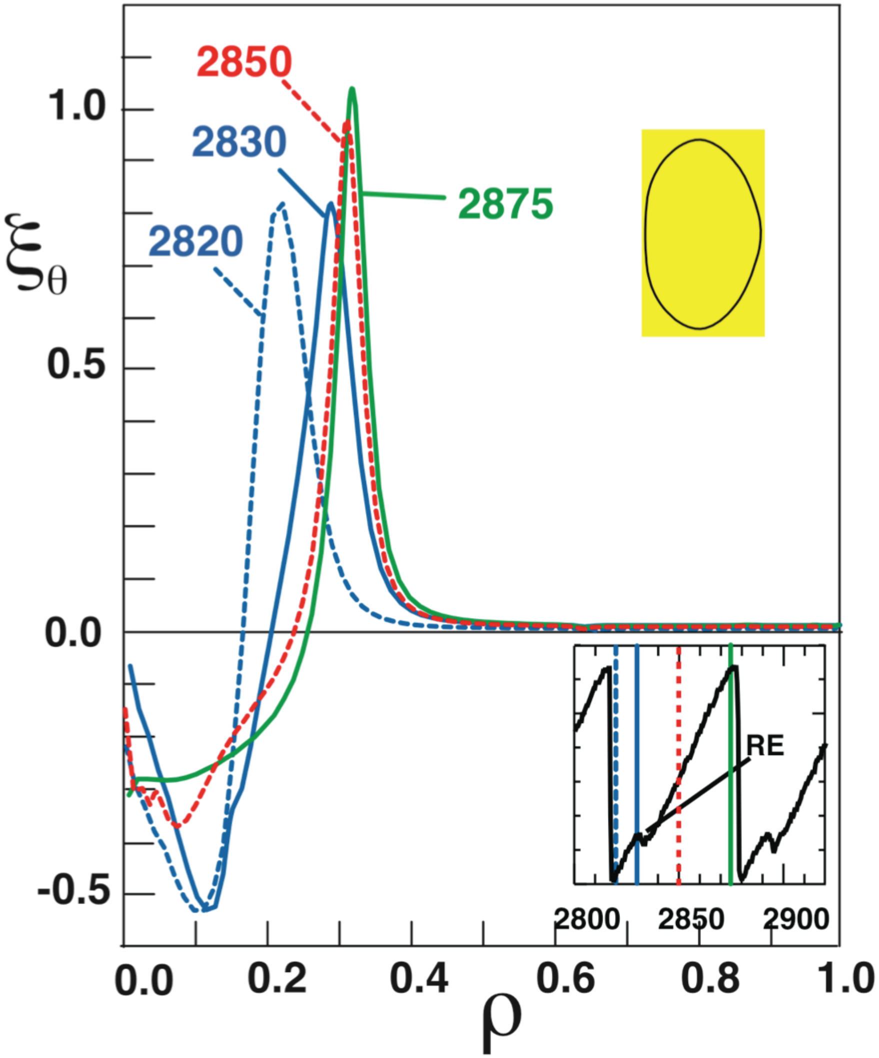


Quasi-interchange remains throughout Oval ramp



Different Crash Types in Bean and Oval Result from Different Underlying Linear Modes

- **Ideal quasi-interchange underlies the Oval sawtooth crash and both the MHD relaxation events**

Internal Kink: (Kadomtsev)

Reconnection at q=1

- Crash in T_e when hot core is expelled
  - ⇒ Fast reconnection with B_θ and T_e crashes tied

Reconnection width δ from balance of incoming flow, reconnection rate, and return flow implicitly assumes return layer width $\lambda = \delta$

Quasi-interchange: (Wesson)

Crash in T_e is not associated with reconnection

- Fast crash in T_e due to ideal motion of cold bubble moving in
- Slow B_θ crash and resetting of q through next ramp phase from resistive diffusion

$\lambda \sim r_1$ = width of return flow

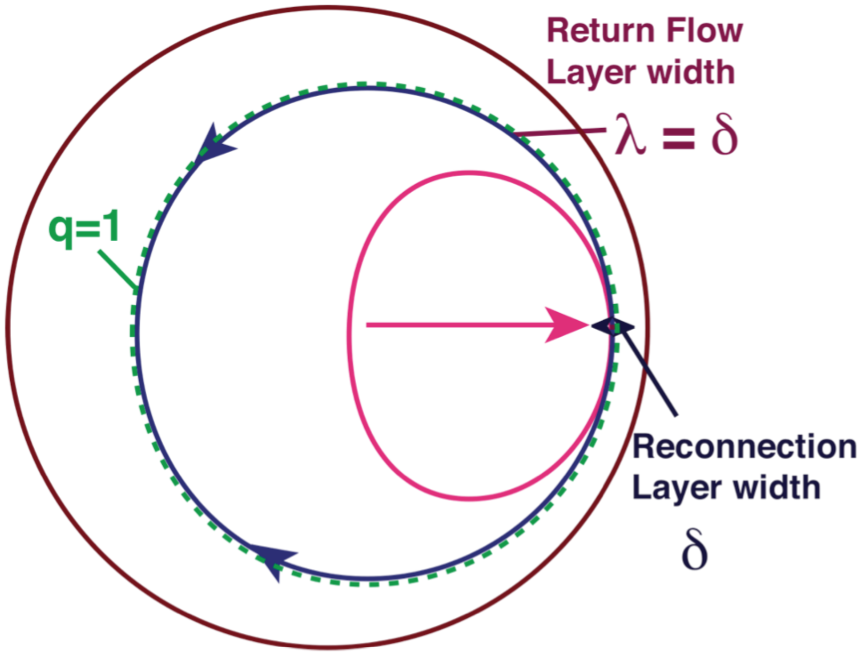
$\lambda >> \delta$

B_θ Crash for Quasi-Interchange Model due to Resistive Diffusion: Crash for Internal Kink from Reconnection

- B_θ crash rates are controlled by different physics in the two cases

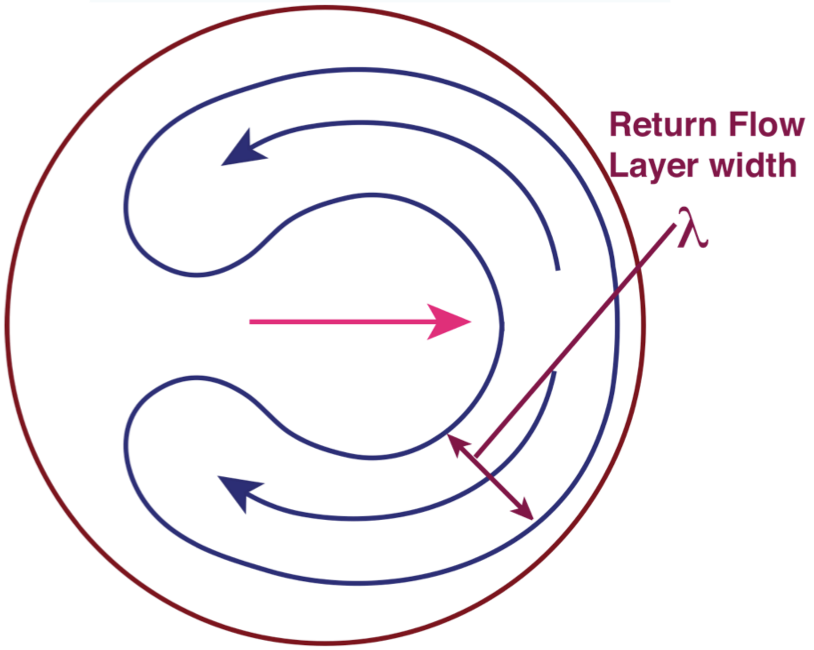
Reconnection

$$\tau_K \sim (\mu_0/\eta)\delta^2 \qquad \delta \sim \left({\tau_A}/{\tau_R}\right)^{1/2} r_1 << r_1$$



Resistive diffusion

$$\tau_W \sim (\mu_0/\eta)\lambda^2 \qquad \lambda \sim r_1$$



Wesson model with $\lambda \sim r_1$ would yield a time for the B_θ crash of $\tau_W \sim 1$ second

The Quasi-interchange mode in our case also has a q=1 surface!

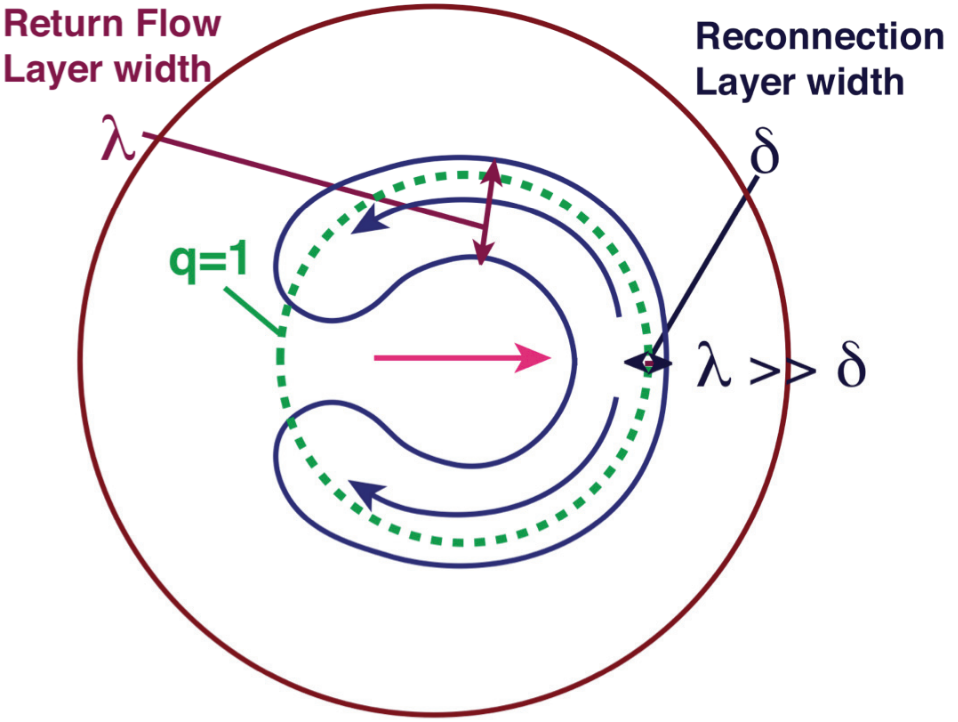
Conjecture: Quasi-interchange Crash Determined by Hybrid of Return Flow Width λ and Reconnection Layer

- **For $q_0 < 1$ a hybrid time is physically expected**
  - Core moves in with local reconnection at q=1 surface
  - Resistive diffusion and possibly additional reconnection also occur across the layer λ

Propose:

$$\tau_c \sim (\mu_0/\eta)d^2 \quad d \sim \delta^{(1-\alpha)}\,\lambda^{\alpha}$$

- Fast crash in T_e still due to cold bubble moving in rapidly
- B_θ crash faster than resistive diffusion but slower than T_e crash



Kadomtsev: reconnection

$$\alpha = 0: \quad \tau_c = \tau_K$$

Wesson: resistive diffusion

$$\alpha = 1: \quad \tau_c = \tau_W$$

Hybrid: reconnection and diffusion

$$\textbf{For} \quad \alpha = \tfrac{1}{2}: \quad \tau_c/\tau_K \sim \left(\lambda^2/\delta^2\right)^{\alpha} \sim 20-100$$

Additional Evidence that the Quasi-interchange Underlies Sawtooth Crashes in Experiments

- **Evidence of smoothly varying radial component in Oval discharge**
  - With no distinct reconnection event observed
  - Lazarus, *et al.*, Phys. Plasmas **14**, 055701 (2007)
- **In DIII-D fast ion diagnostic in the Bean and Oval discharges found the bean crash redistributes fast ions more extensively than the oval**
  - Consistent with the flow pattern in the Oval coinciding with a quasi-interchange and in the Bean with an internal kink
  - Faster reconnection in the Bean leads to higher fast energy transport

See Poster JP8.00108 by Chris Muscatello

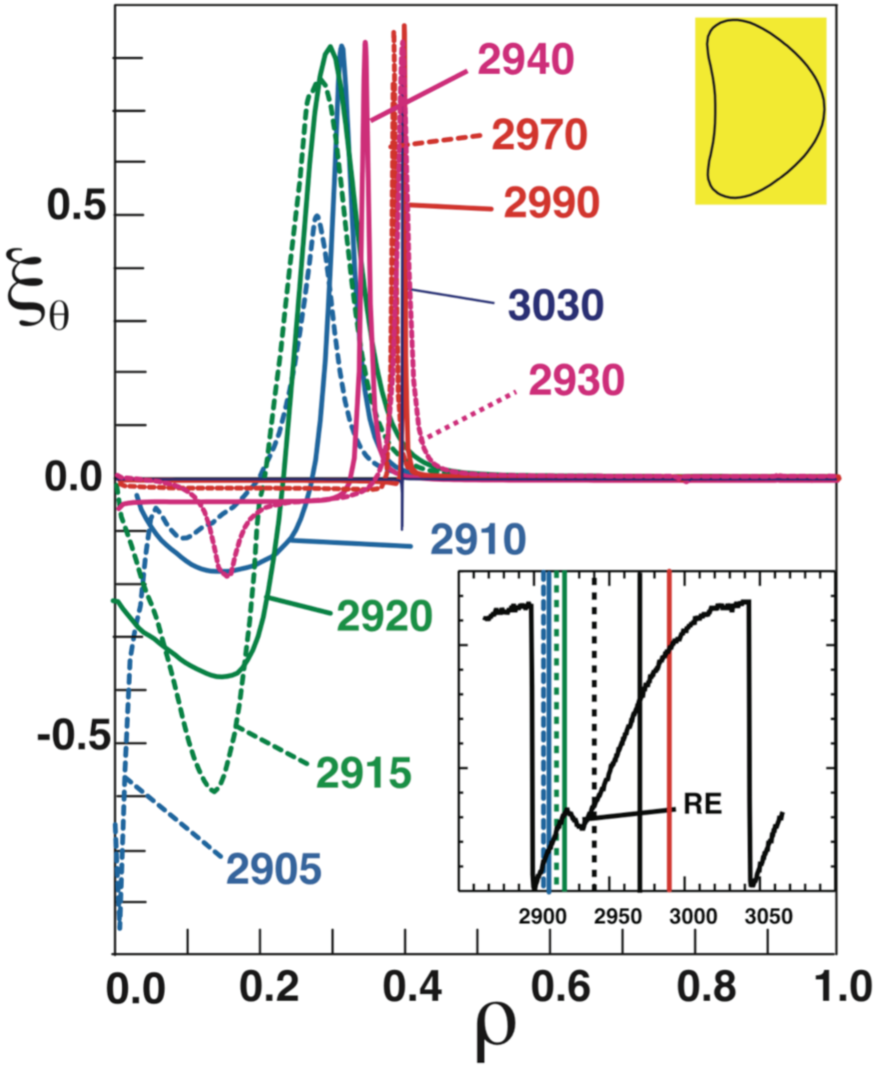
- **In JET SXR reconstructions (~1986) showed quasi-interchange flows**
  - Edwards, *et al.*, Phys. Rev. Lett. **57**, 210, (1986)
  - But reported $q_0 \sim 0.9$ so the linear mode should be the conventional internal kink (Campbell *et al.*, Phys. Rev. Lett. **60**, 2148, 1988)
  - Inconsistent with quasi-interchange unstable for $q_0 \sim 1$ and very flat or nonmonotonic q (Wesson *et al.*, Phys. Rev. Lett. **79**, 5018, 1997)

Conclusion: Sawtooth Crash does not Necessarily Follow Conventional Picture of Internal Kink Trigger

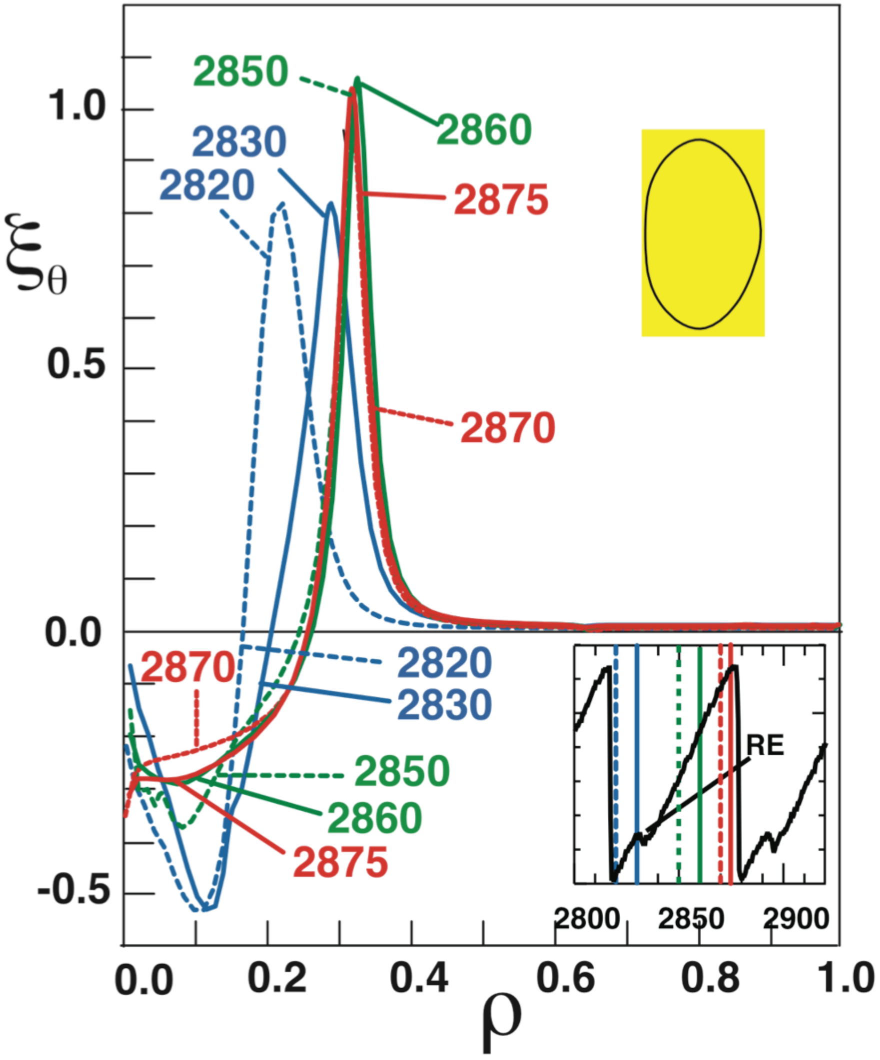
- **Ideal growth rates do not uniformly increase through ramp phase**
  - Ideal stability actually improves through the ramp phase in the bean cross section
- **Underlying ideal mode in oval at sawtooth crash is a quasi-interchange but is an internal kink in the bean**
  - Underlying modes up to and through relaxation events are quasi-interchange in both
  - Underlying mode transitions to internal kink in the bean after the relaxation event
- **Crash rate depends on the underlying mode structure:**
  - Fast reconnection in conventional internal kink case and slow current diffusion in the quasi-interchange case
- **A hybrid model for reconnection is needed to fully explain crash for the quasi-interchange with $q_0 < 1$**
  - Reconnection time is determined partly by the width of the return flow
  - Potential to explain quantitatively the difference in B_θ crash times between Bean and Oval

Mode Structure is Quasi-Interchange from the Crash Through the MHD Relaxation Event in Both Discharges

Localized return flow at q=1 remains through the end of the Bean ramp



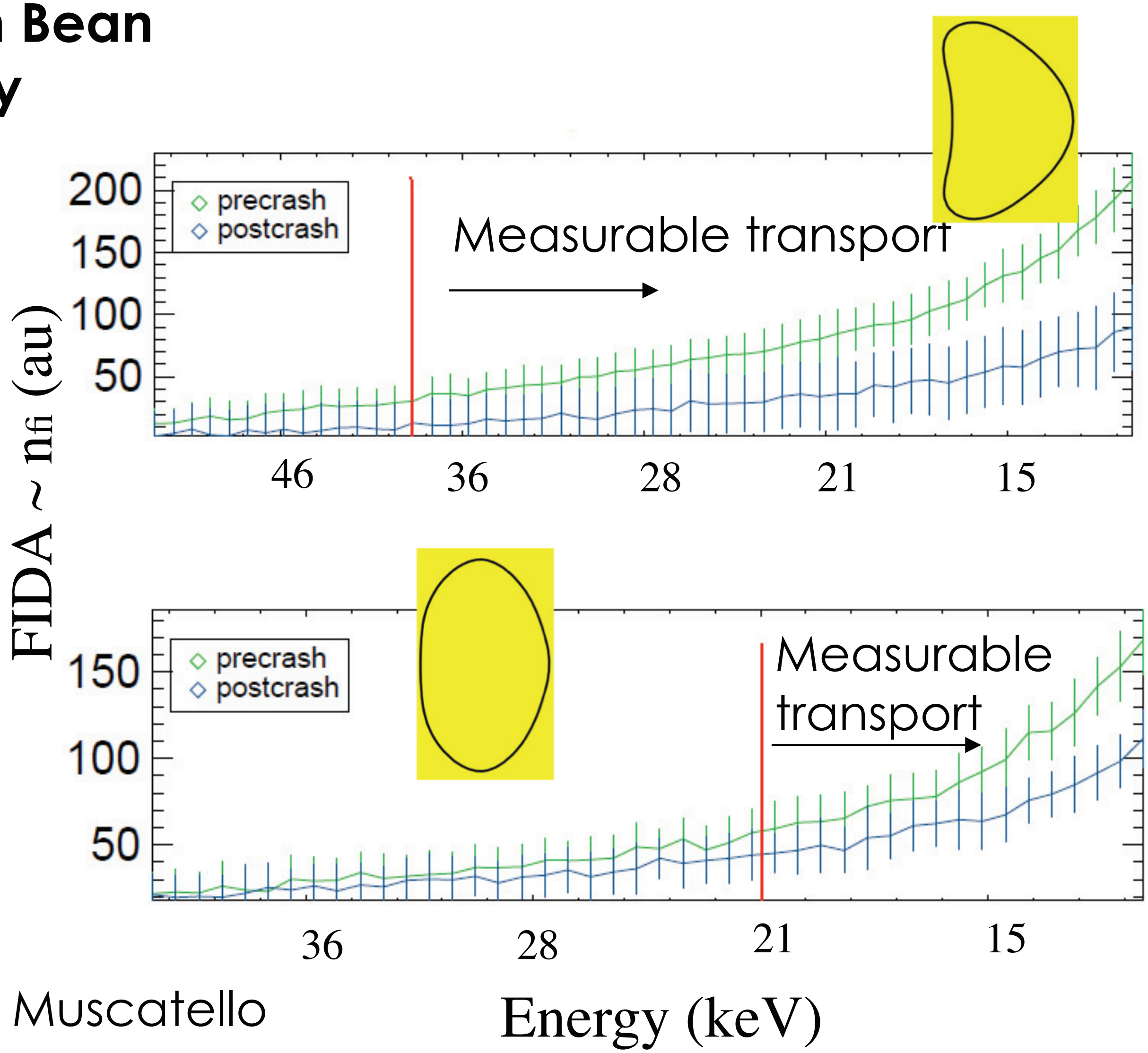
Quasi-interchange remains throughout Oval ramp



Observed Differences in Fast-Ion Transport at Crash Consistent with Different Sawtooth Reconnection

- **Faster reconnection in Bean leads to higher-energy fast-ion transport**
- **Transport cut-off determined where pre-crash and post-crash signals are statistically significant**

Δ signal = uncertainty



Poster JP8.00108 by Chris Muscatello