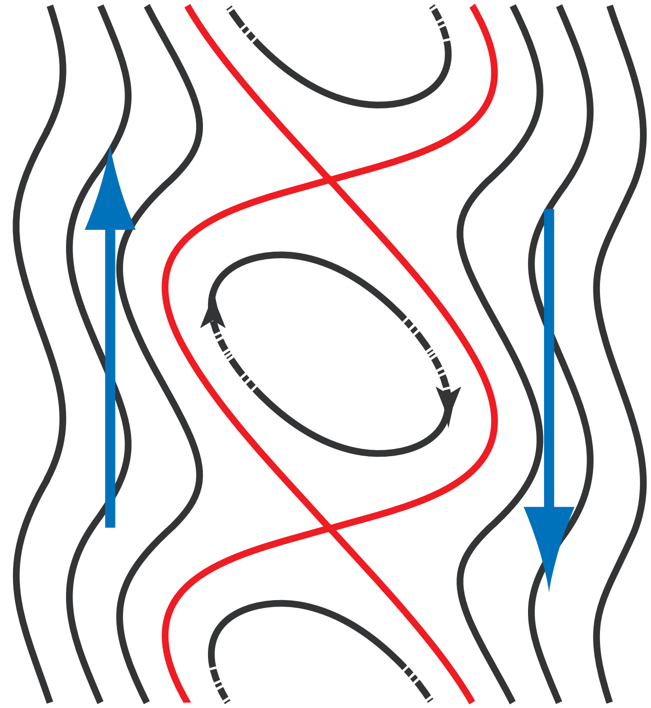
Islands in the Stream: The Effect of Plasma Flow on Tearing Stability

by
R.J. La Haye, General Atomics,
with many acknowlegments given at end of talk



Presented at the
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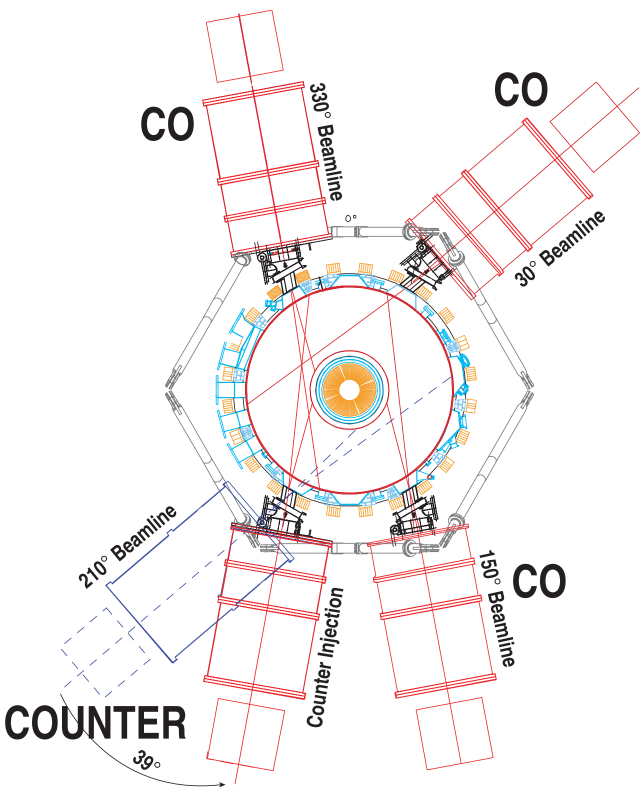
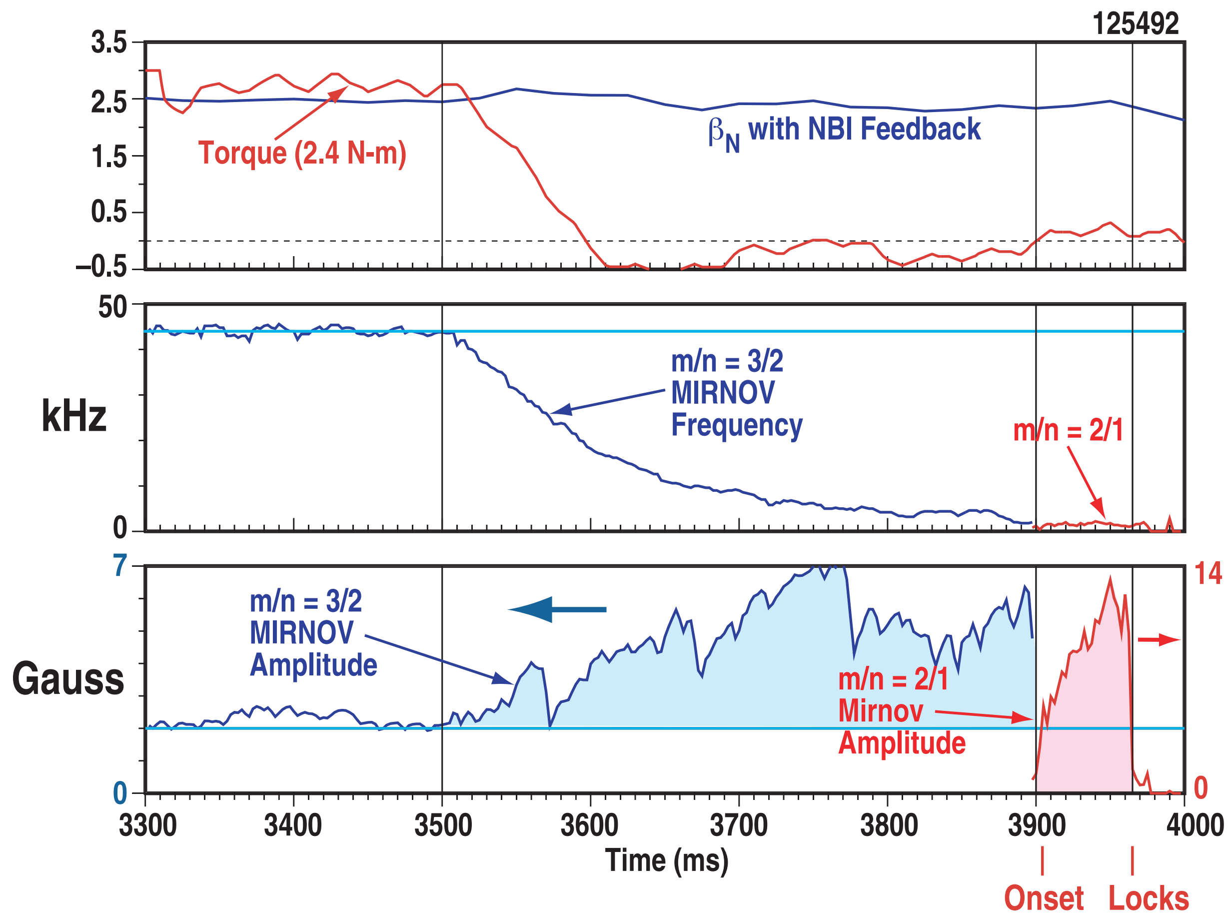
the Division of Plasma Physics
Atlanta, Georgia

November 2–6, 2009

Prologue: Decreasing Plasma Flow Impairs Tearing Stability

- **Unanticipated consequences that impact performance for future tokamaks with low rotation**



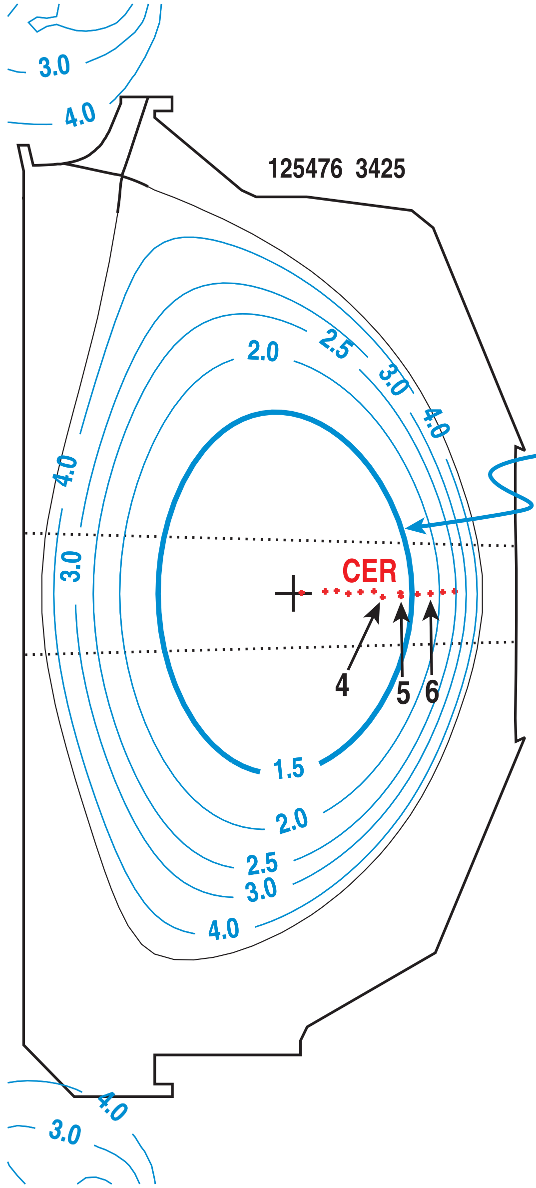


- **Existing tearing modes (m/n = 3/2 shown) get bigger ⇒ degrades confinement**
- **Otherwise stable m/n = 2/1 mode can become unstable ⇒ lock, disrupt**

DIII-D NATIONAL FUSION FACILITY

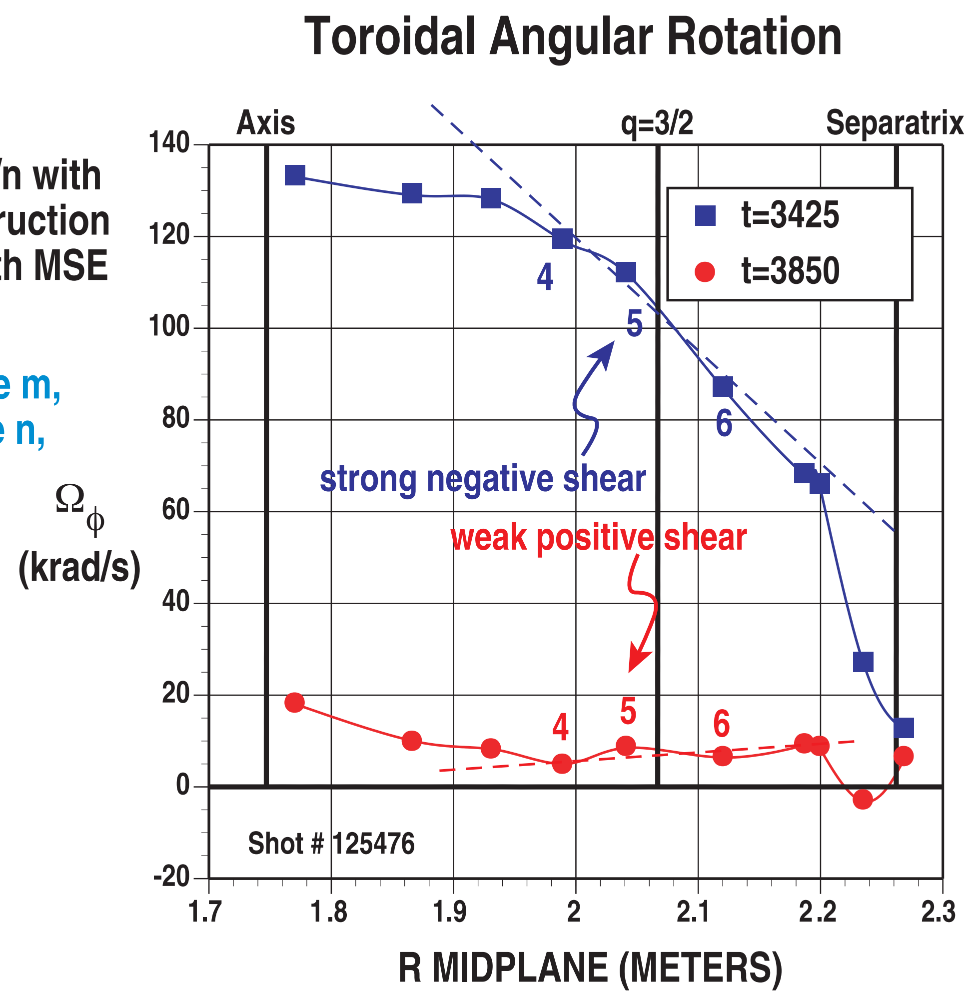
P.A. Politzer et al NF 2008

Plasma Rotation Measured by Charge Exchange Recombination (CER) of CVI Line



Locate q = m/n with MHD reconstruction code EFIT with MSE

poloidal mode m, toroidal mode n, surface (q=3/2)



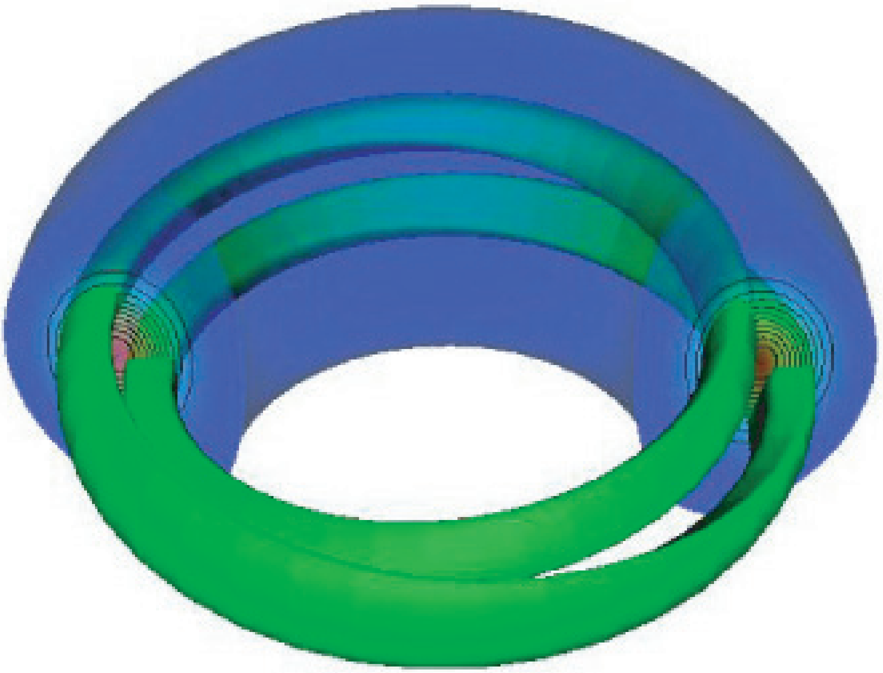
Classical Tearing Mode Island Stable ($\Delta' < 0$) with a Stable Total Plasma Current Density Profile

- **Rutherford Equation (P.H. Rutherford 1973)** $\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'$, **the effective classical tearing index**

★ for $\Delta' < 0$, the helically perturbed current has higher energy

– linearly stable as the perturbed energy $\delta W_{mag} \approx -\frac{1}{4\mu_o}\left(\frac{r\tilde{B}_r}{m}\right)^2 \Delta' r > 0$

$$\ldots \Delta' = \left.\frac{d\tilde{B}_r/dr}{\tilde{B}_r}\right|_{r_s-\varepsilon}^{r_s+\varepsilon}$$ calculable from magnetic equilibrium



- **Island in DIII-D simulated (NIMROD) for an m/n = 2/1 tearing mode**

★ decays away if classically stable

★ but can be sustained by helically perturbed bootstrap current (NEOCLASSICAL TEARING MODE)

M.S. Chu, et al, PoP 2002

Postulate: Flow Shear Effect On Classical Tearing Stability is a Key Feature of Neoclassical Tearing Mode Behavior

$$\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta' + \varepsilon^{1/2}\frac{L_q}{L_{pe}}\beta_{\theta e}\left[\frac{1}{w} - \frac{w_{marg}^2}{3w^3}\right]$$

Modified Rutherford Equation (MRE)

effective classical tearing index (Δ')

$\varepsilon = r/R$

$L_q \equiv q/(dq/dR)$

$L_{pe} \equiv -p_e/(dp_e/dR)$

$\beta_{\theta e} = \frac{2\mu_o p_e}{B_\theta^2}$

lumped small island threshold effects

- For large saturated islands ($\dot{w} = 0$)

$$0 \approx \Delta' + \varepsilon^{1/2}\frac{L_q}{L_{pe}}\frac{\beta_{\theta e}}{w}\left[1 - \cancel{\frac{w_{marg}^2}{3w^2}}\right]$$

- For onset of islands ($\dot{w} > 0$)

$$\beta_{\theta e} > \frac{3}{2}\frac{(w_{marg}/r)(-\Delta' r)}{\varepsilon^{1/2}(L_q/L_{pe})}$$

- Look for influence of flow shear on effective classical tearing Δ'

$$\Delta' r \approx -C_0 - C \times (-d\Omega_\phi/dr)\, L_s \tau_A$$

Experimental Data Examined for Trends with the Normalized Flow Shear

- existing islands ($\dot{w} \approx 0$)

$$w \propto (\tilde{B}_\theta)^{1/2} \propto \frac{1}{-\Delta' r}$$

- onset of islands ($\dot{w} > 0$)

$$\beta \propto -\Delta' r$$

with $\Delta' r$ a function of the flow shear
normalized to both the parallel magnetic shear length
and the Alfvén time

When you follow two separate chains of thought, Watson, you will find some point of intersection which should approximate the truth.

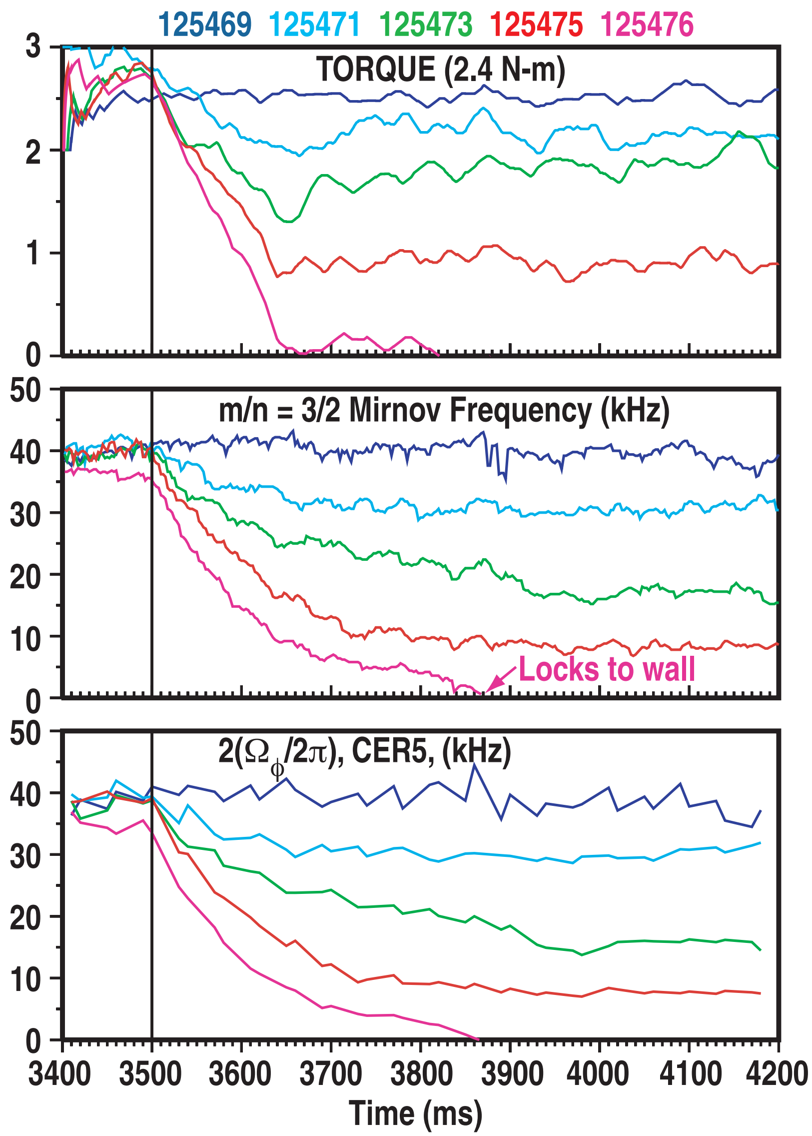
— Sherlock Holmes, *The Disappearance of Lady Francis Carfax*
Sir Arthur Conan Doyle

MODE AMPLITUDE VERSUS FLOW SHEAR

m/n = 3/2 Modes Get Larger with Lower Rotation (and Shear) as "Inferred" Δ' Less Negative

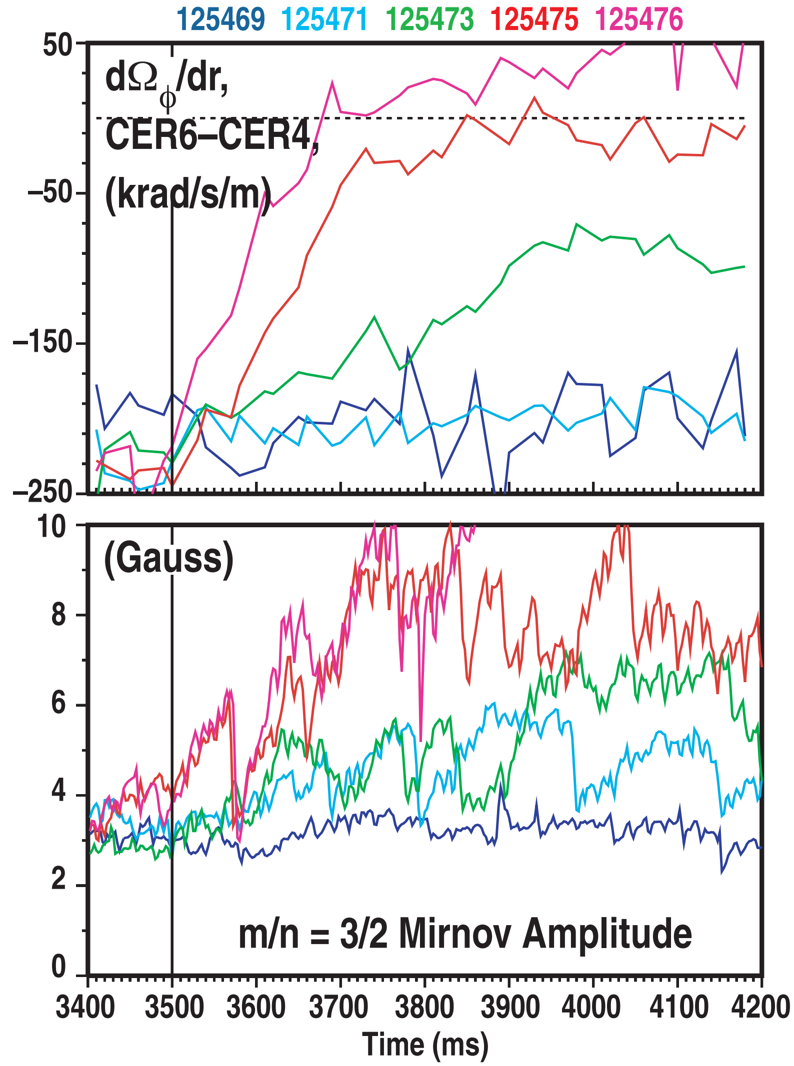
- **All co-NBI has counter beams added ($\beta_N \approx 2.6$, $\bar{n}_{13} \approx 3.5$)**
  - ★ rotation reduced

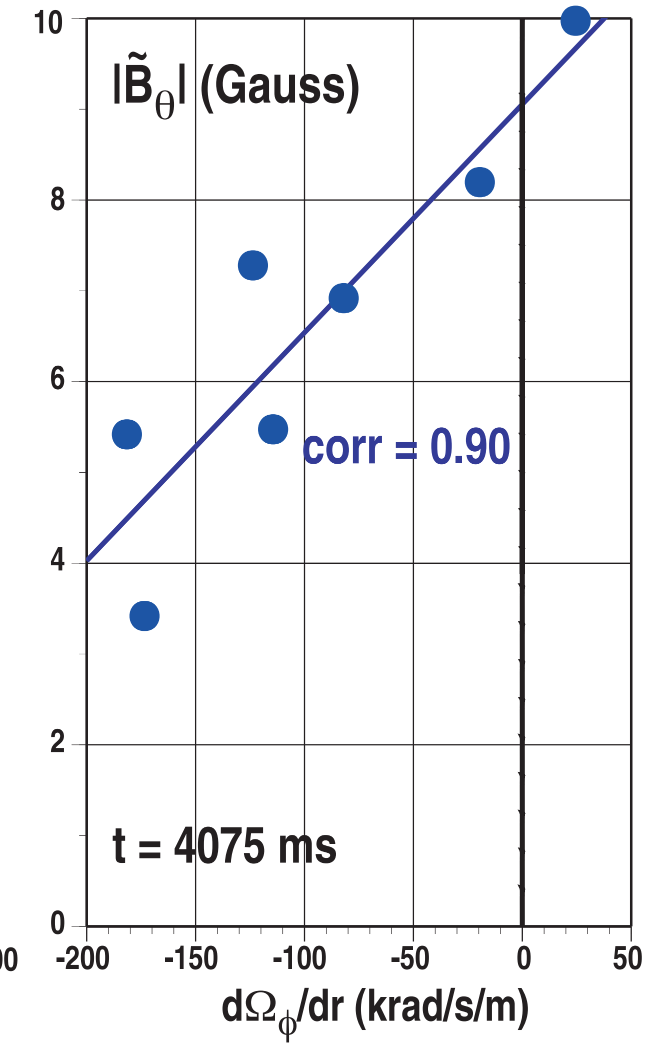
    . . .CER CVI chord 5 toroidal rotation is good measure of MIRNOV rotation



m/n = 3/2 Modes Get Larger with Lower Rotation (and Shear) as "Inferred" Δ´ Less Negative

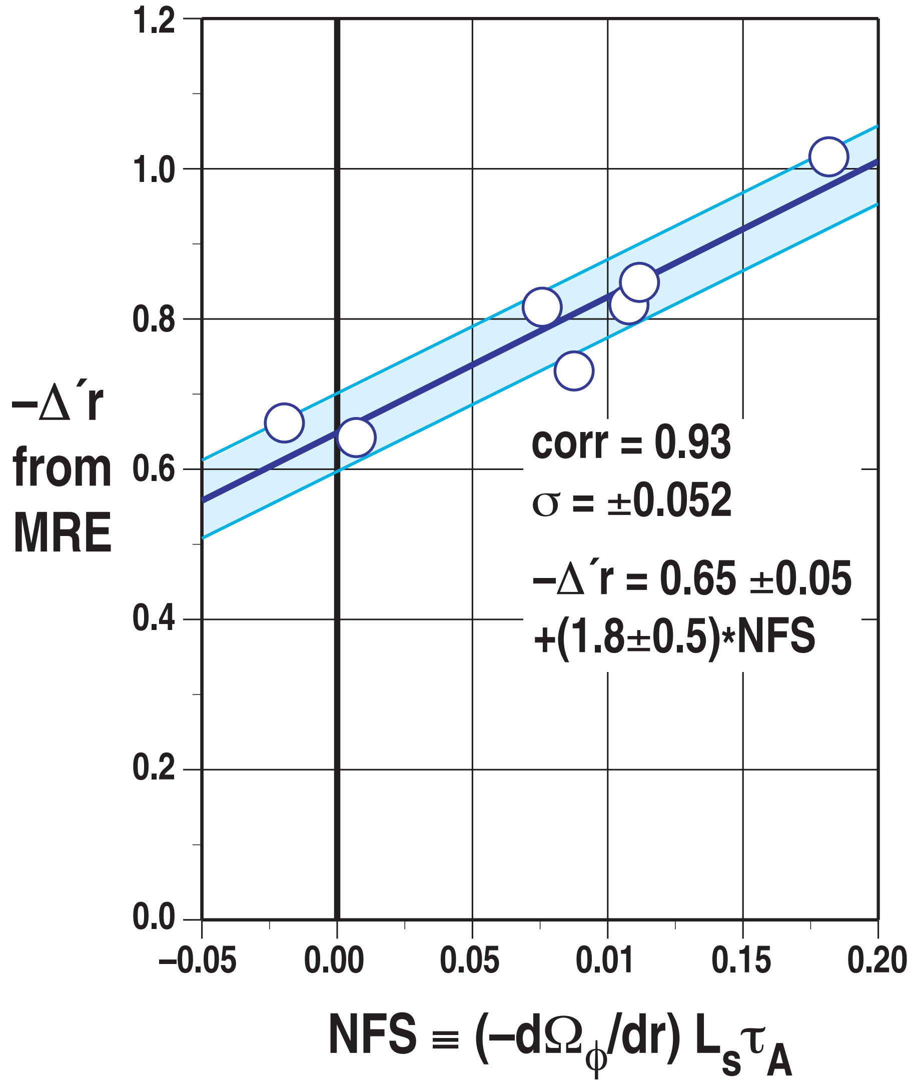
- **As flow shear is reduced . . .**
  - ★ 3/2 amplitude increases





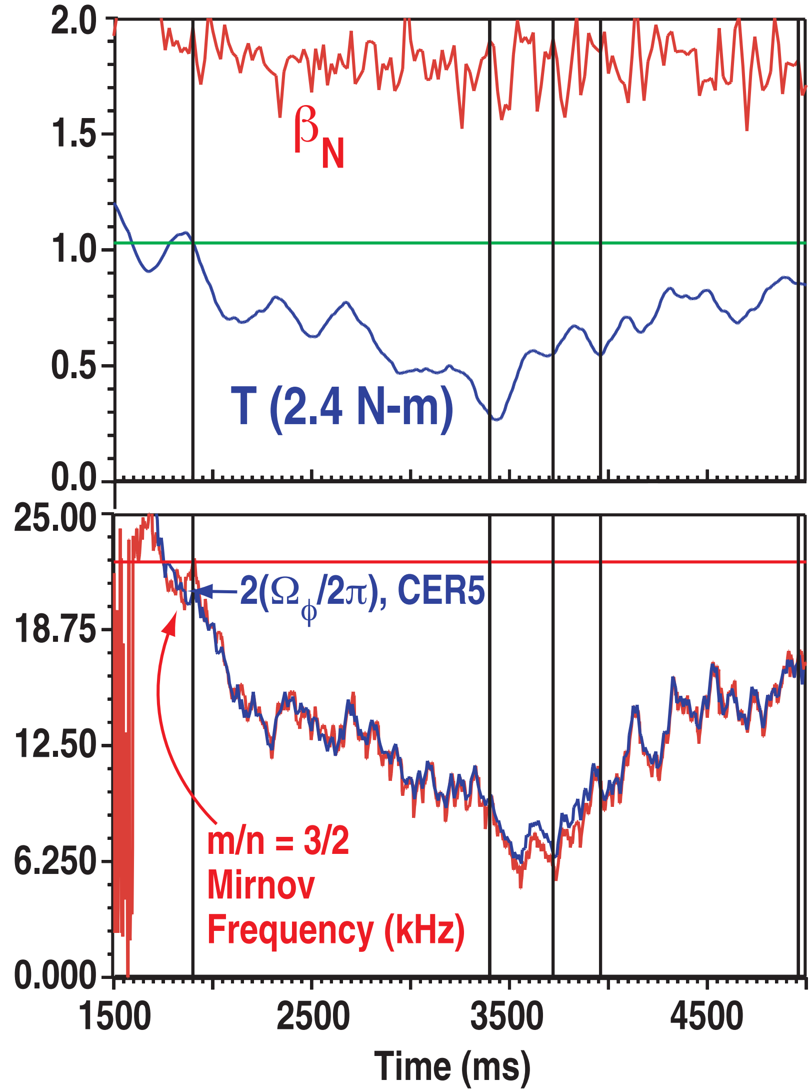
m/n = 3/2 Modes Get Larger with Lower Rotation (and Shear) as "Inferred" Δ´ Less Negative

- "Inferred" Δ´ **less negative** with reduced positive normalized flow shear
  - ★ NFS ≈ 0.4 for a factor of two increase in stability



. . . and "Inferred" Δ' Less Negative with Inverted Flow Shear with m/n = 3/2 Modes

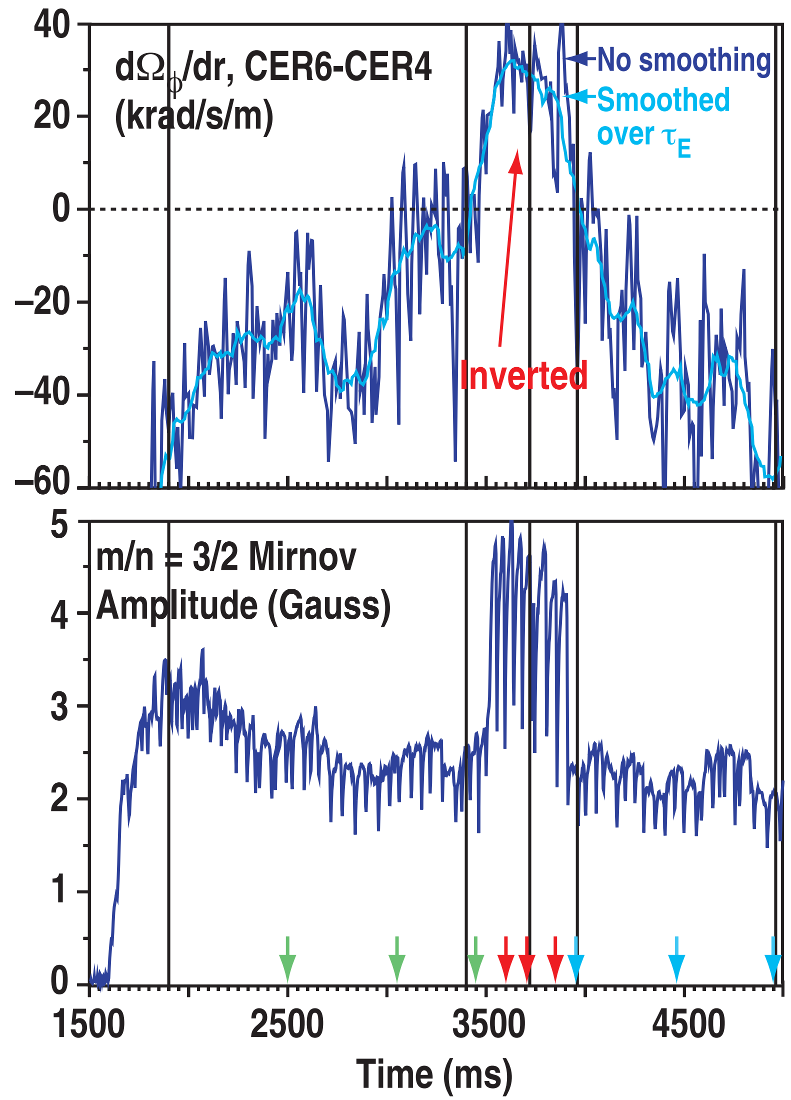
- **Reduced β_N, less initial all co-torque (#135867) to avoid m/n=2/1 mode**
  - ★ torque programmed down and back ($\sim 8\ \tau_E$ each way)



. . . and "Inferred" Δ´ Less Negative with Inverted Flow Shear with m/n = 3/2 Modes

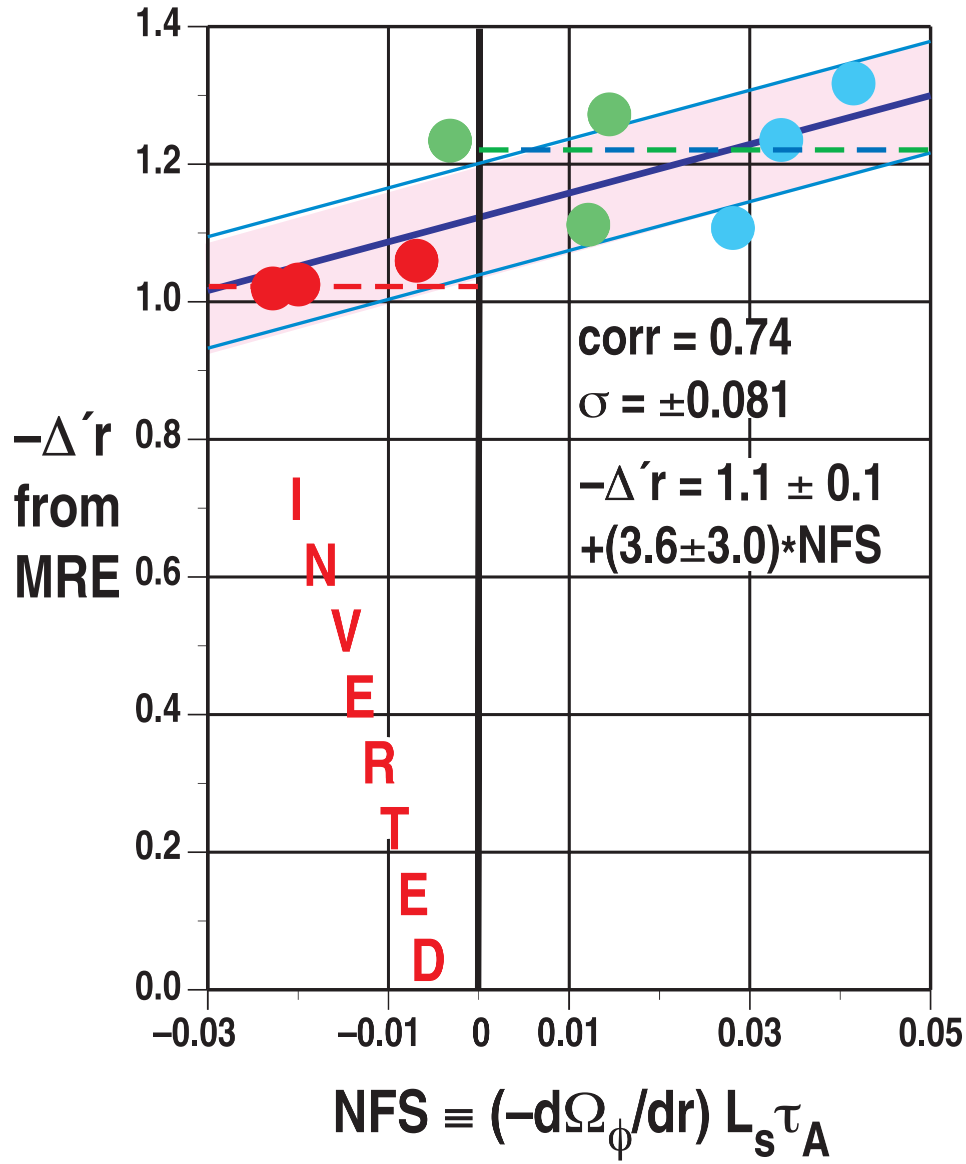
- **Initial flow shear small**
  - ★ $d\Omega_\phi/dr \approx 0$
    "forbidden" state
    . . . inverts to hollow

- **Little change in Mirnov amplitude**
  - ★ until inversion



. . . and "Inferred" Δ' Less Negative with Inverted Flow Shear with m/n = 3/2 Modes

- **"Inferred" Δ' depends on sign of $-d\Omega_\phi/dr$?**
  - ★ or shift of zero to left?
  - ★ NFS ≈ 0.3 for a factor of two increase in stability



BETA AT ONSET VERSUS FLOW SHEAR

Reduced Plasma Rotation Experimentally Destabilizes Neoclassical Tearing Modes at Lower Beta in DIII-D, NSTX and JET

- **Each device with slow ramp up in beta to instability**
  - ★ note mixing of seeding and small island effects
    . . . makes analysis of Δ´ by itself problematic

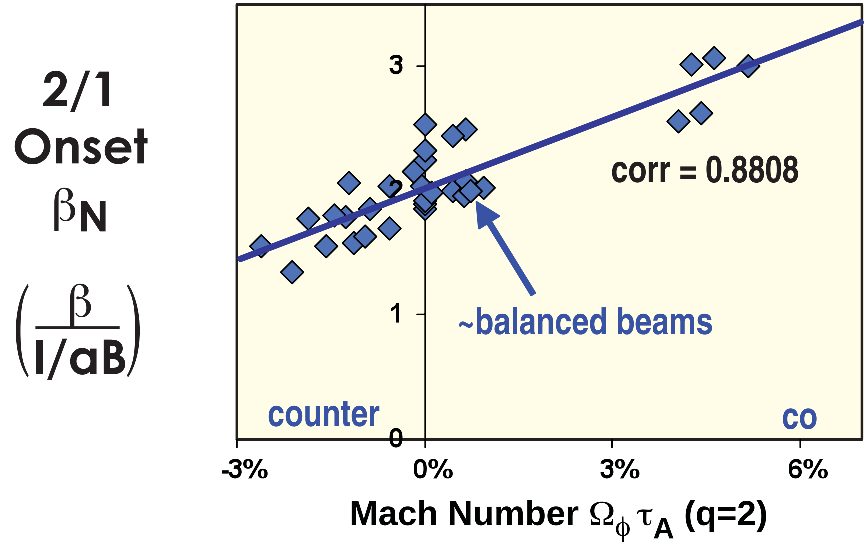
Also see R.J. Buttery PP8.00050 We PM

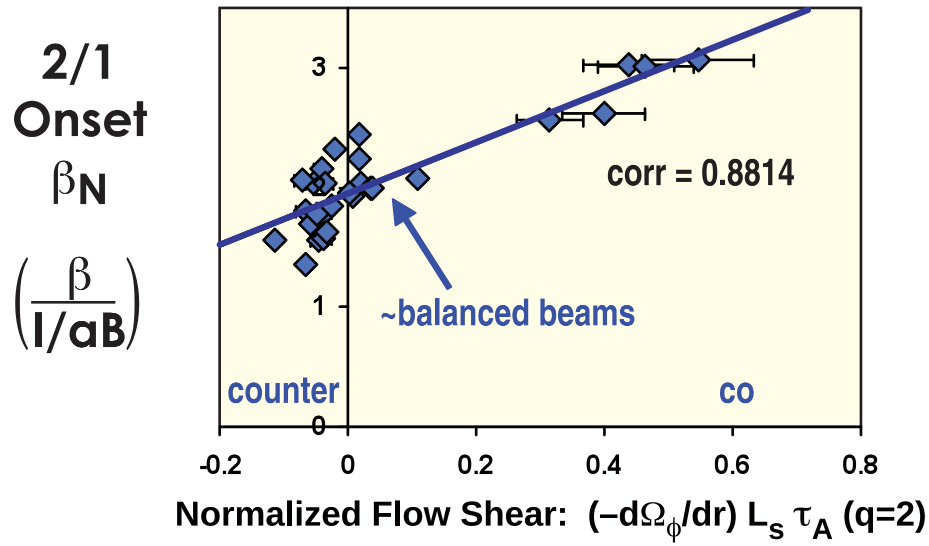
“Sensitivity of Tearing Mode Beta Limits to Rotation and Current Profile”

Onset β Continues to Fall with Counter Plasma Rotation in DIII-D

- **No systematic variations or trends in key profiles**
  - ★ rotation degenerate with flow shear
    . . . profile unchanged with co/ctr beam mix
- **For counter rotation, flow shear reverses sign wrt magnetic shear**
  - ★ relative sign important?
    . . . or "shift" in zero to left?

(R.J. Buttery et al, PoP 2008)



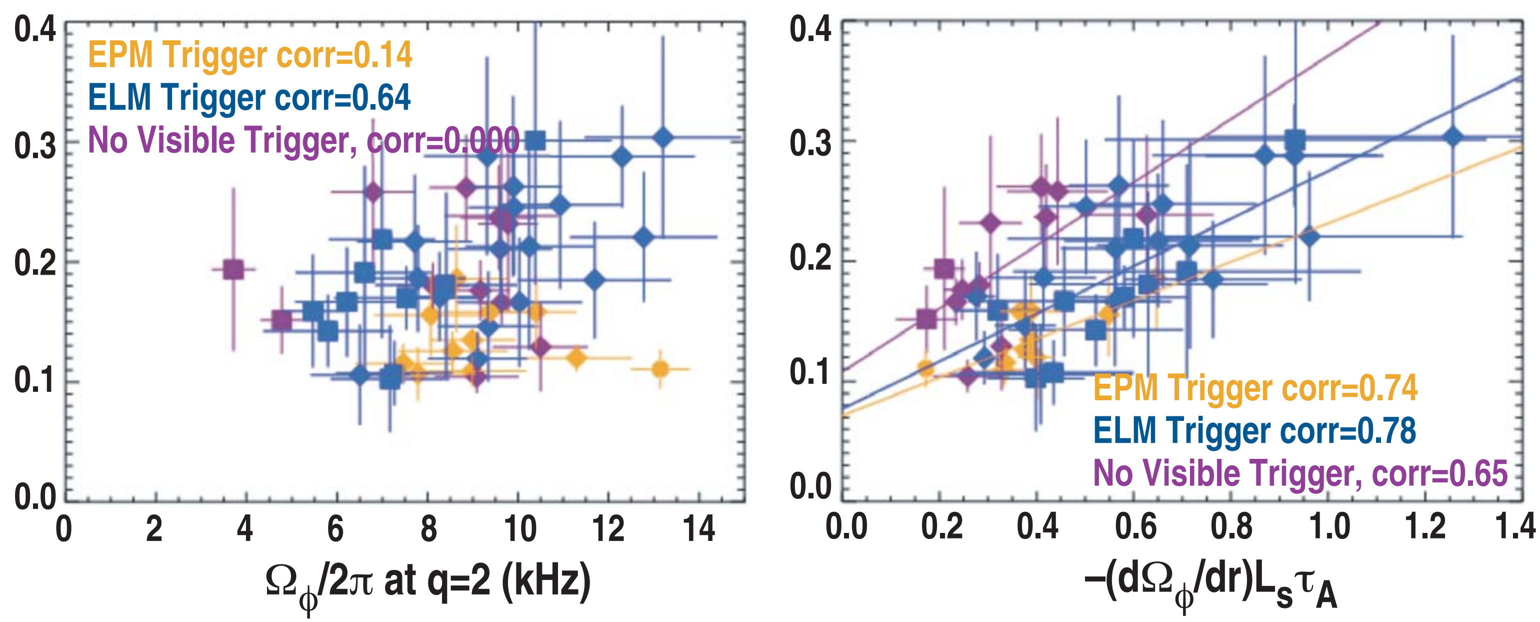


Strongest Correlation Found Between Local Mode Drive at Onset and Normalized Flow Shear in Low Aspect Ratio NSTX

- **NTM "drive" at onset only poorly correlated with q=2 (carbon) rotation, "CHERS"**
- **Essentially no correlation for Energetic Particle Modes and "Triggerless" cases**
  - ELM correlation better

- **Degeneracy between Ω_ϕ and $d\Omega_\phi/dr$ broken in NSTX with use of n = 3 magnetic braking from external coil**
- **Strongest correlation between "noise-prone" parameters (i.e., gradients)**
  - Provides confidence that this is the correct physics

$$\frac{\mu_0 L_q \delta\, j_{BS,\,Sauter}}{B_\theta} \left(\approx \frac{\varepsilon^{1/2} L_q \beta_{\theta e}}{2L_{pe}} \right)$$

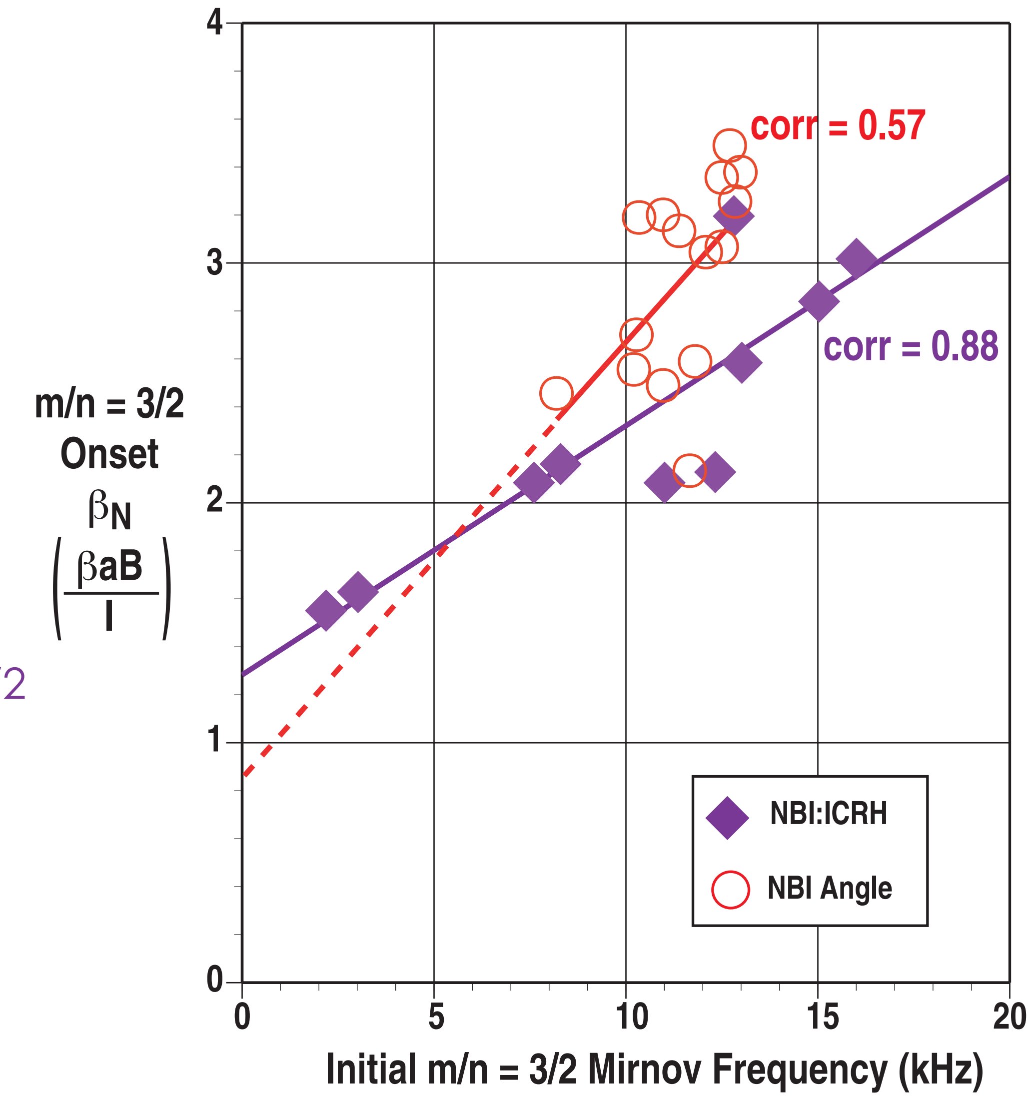


S. Gerhardt et al, NF 2009

Distinct Onset β Decline in JET with ICRH: NBI Scan or NB Only Injection Angle for Sawtooth Triggered 3/2 NTM

- **Ramp up power for different mixes of ICRH:NBI power or NBI only angle varied**
  - ★ Phase ICRH to avoid large sawteeth
- **Clear result obtained:**
  - ★ Removing NBI momentum drops NTM onset

    . . . normalized flow shear at q=3/2 $(-d\Omega_\phi/dr)\ L_S\ \tau_A \leq 0.27$



R.J. Buttery et al, IAEA08

133-09/RJL/rs

PHYSICS OF FLOW SHEAR EFFECT ON TEARING

Flow Shear Has Been Invoked for Increasing Effective Classical Tearing Stability of Total Current Profile

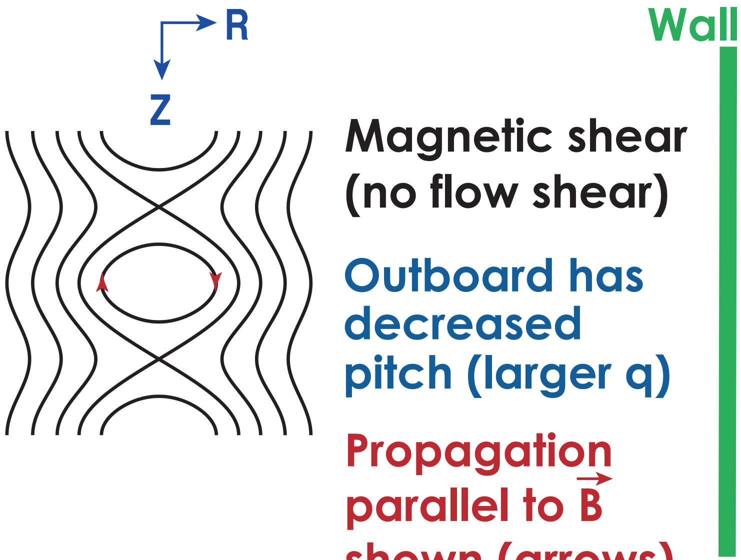
"I am afraid that I rather give myself away when I explain. Results without causes are much more impressive."

Sherlock Holmes, The Stock-Broker's Clerk

Sir Arthur Conan Doyle

- **X.L. Chen & P.J. Morrison, PF 1990, "Resistive tearing instability with equilibrium shear flow". . . but zero viscosity**
  - ★ when the flow shear is larger than the magnetic shear..., the flow freezes the magnetic field and stabilizes the tearing mode."
- **L. Ofman, P.J. Morrison & R.S. Steinolfson, PF 1993, "Nonlinear evolution of resistive tearing mode instability with shear flow and viscosity"**
  - ★ "shear flow decreases the saturated magnetic island width"
- **R. Coelho & E. Lazzaro, PoP 2007, "Effect of sheared equilbrium plasma rotation on the classical tearing mode..."**
  - ★ "above a given threshold in the rot. shear, a tearing mode, unstable in the absence of rotation, can be stabilized... the local derivative affects stability"
    - – stabilizing for $P \equiv \tau_R/\tau_V >> 1$ and $S \equiv \tau_R/\tau_A >> 1$

How Magnetic Shear Influences Tearing



Magnetic shear (no flow shear)

Outboard has decreased pitch (larger q)

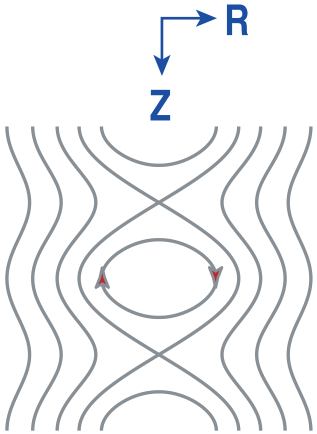
Propagation parallel to $\vec{B}$ shown (arrows)

- **Magnetic shear varies field line pitch**
  - ★ singularity at rational surface limited

$$\ldots\left(1-\frac{nq}{m}\right)=-\frac{(r-r_s)}{L_q}$$

    – makes tearing "harder" (more negative Δ')

How Flow Shear Influences Tearing

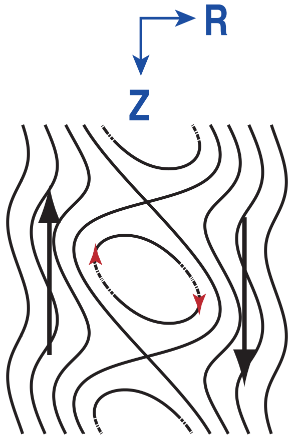


Magnetic shear (no flow shear)

Outboard has decreased pitch (larger q)

Propagation parallel to $\vec{B}$ shown (arrows)

Wall



With co to I_p flow (frame is zero flow at the O-point)

slower on outboard side, faster inboard

Wall

- **Magnetic shear varies field line pitch**
  - ★ singularity at rational surface limited

$$\ldots \left(1 - \frac{nq}{m}\right) = -\frac{(r - r_s)}{L_q}$$

  – makes tearing "harder" (more negative Δ')

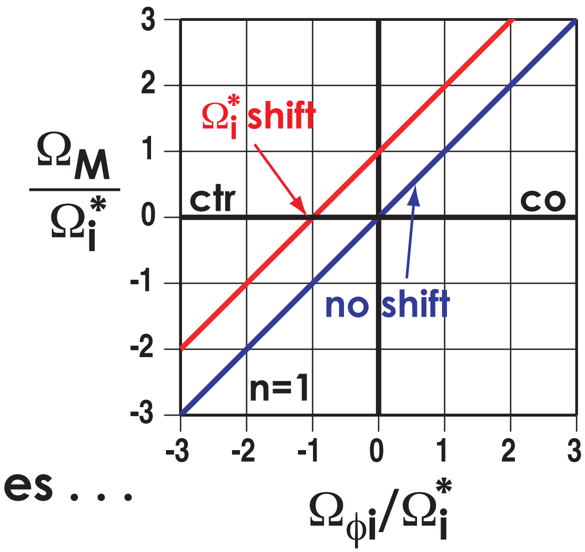
- **Flow shear & viscosity make gradient of phase $\phi = n\,(Z + V_z t)/R - mx/r$**
  - ★ tearing resonance limited radially

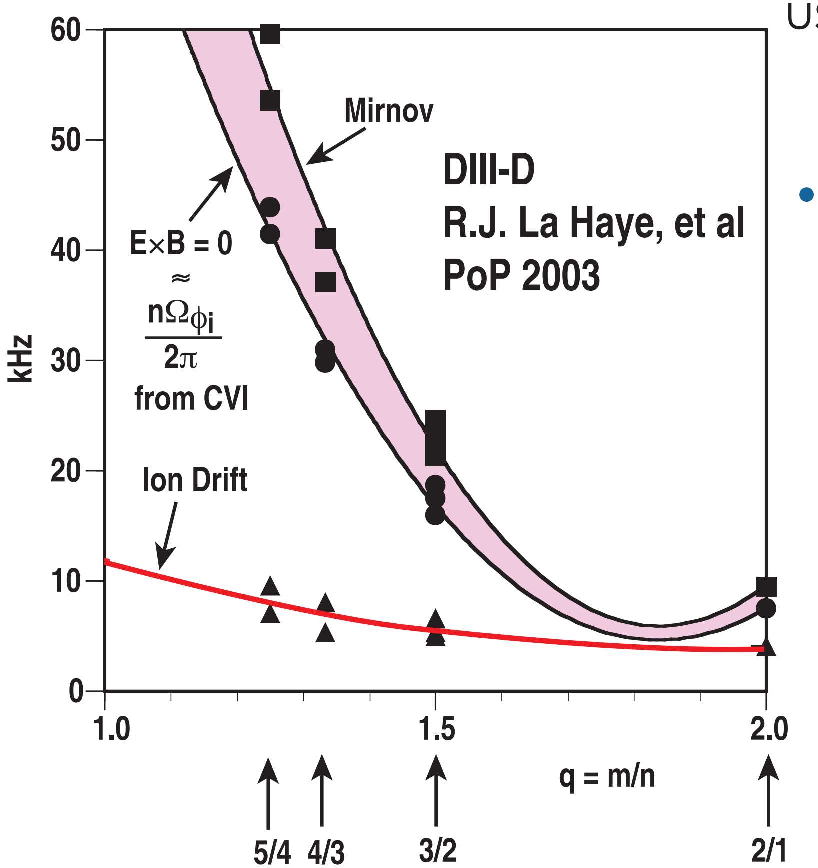
$$\ldots\ \delta\phi \propto \nu_\perp\, d(V_z/R)/dr\ (r - r_s)$$

  – makes tearing "harder" (inner layer effect dominant?)

As Islands Tend to Propagate Faster than the Plasma, Shear in This Rotation May be Mimicking a Sign Dependence

- **viscosity forces island to propagate faster than n times the plasma toroidal rotation**
  - ★ $\Omega_{Mirnov} \approx n\Omega_{\phi i} + \Omega_i^*$ with $\Omega_i^* = n\, k_B T_i\, (dp_i/d\Psi)/ep_i$ (ion diamag. drift freq.) usually in co-direction)





- **Flow shear now becomes . . .**

$$\approx \frac{d\Omega_{\phi i}}{dr} - \frac{\Omega_i^*}{nL_{Ti}}$$

can change sign **"always" negative**

★ null shifted to counter direction

. . . need deeper counter data to confirm

Conclusions and Acknowledgements

- A flow shear $-\frac{d\Omega_\phi}{dr} \approx \mathcal{O}\left(\frac{\tau_A^{-1}}{2L_s}\right)$ @ q=m/n

  ★ has a significant stabilizing effect on tearing

  – thus an advantage for all co beams

  . . . interpreted as an effect in classical Δ' of total plasma current profile

  ★ but positive flow shear (counter peaked, co inverted) destabilizing?

  – may not be a sign effect but an offset?

- Acknowledgements to Dylan Brennan (University of Tulsa),
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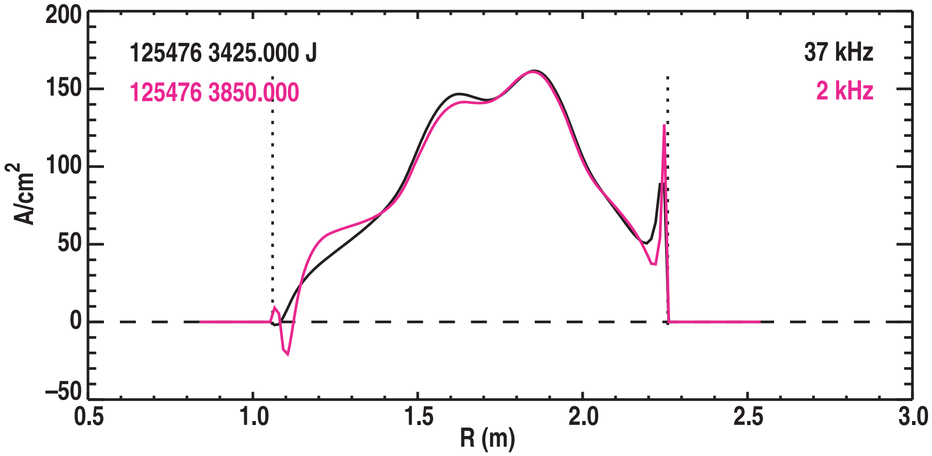
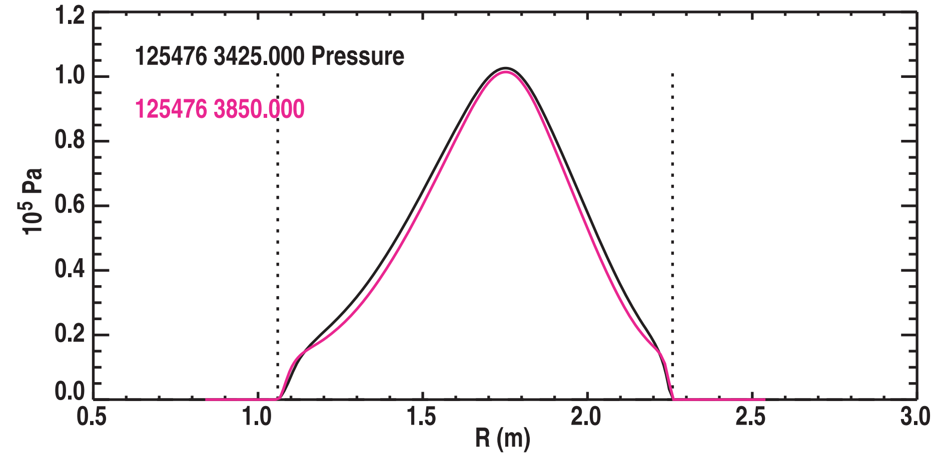
Craig Petty (GA),Peter Politzer (GA), Ted Strait (GA),
Francois Waelbroeck, (IFS Austin),
and the DIII-D, NSTX and JET experimental teams

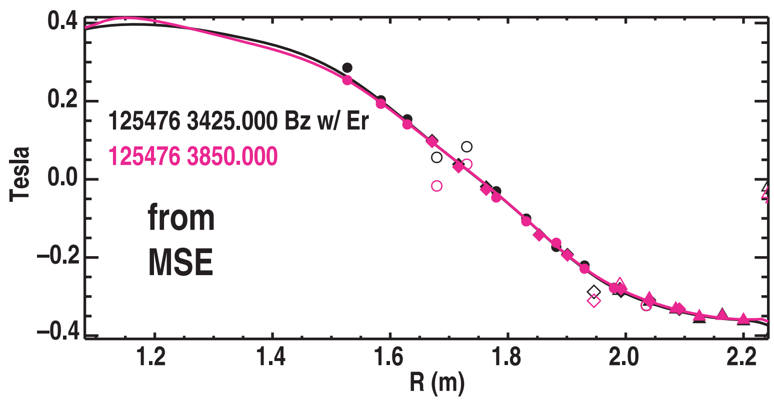
BACKUPS

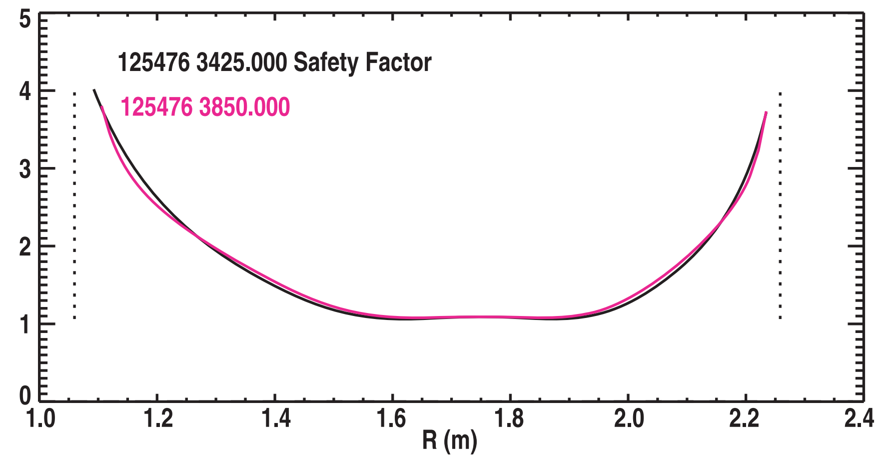
Lower Rotation in Hybrid Scenario and NBI Feedback on Beta Produce Very Similar J and q Profiles

- **Difficult to discern significant change in Δ' from change in equilibrium**
  - ★ PEST-3 runs from "kinetic" EFITs with MSE ⇒ 3/2 stable at both times
    . . . rotation is not included in the code





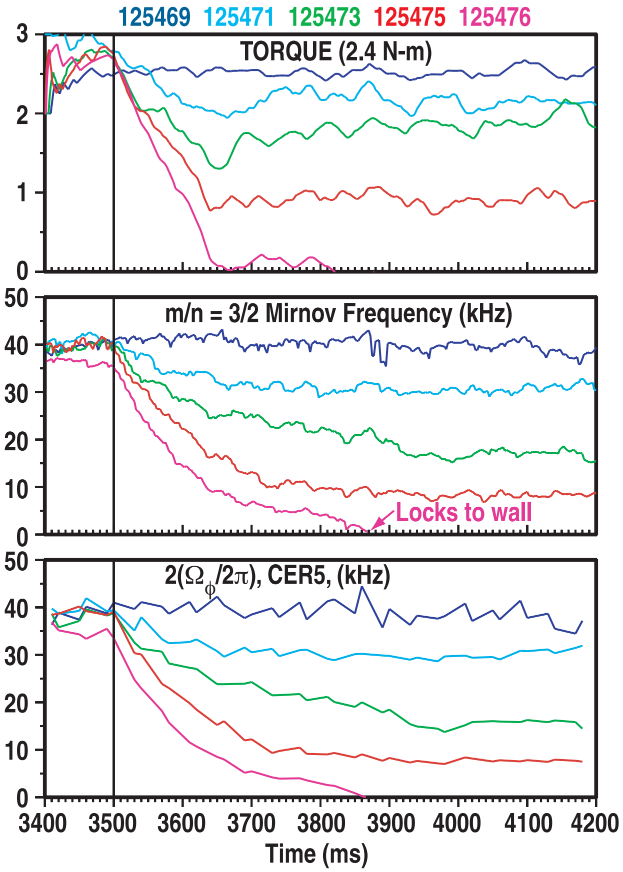




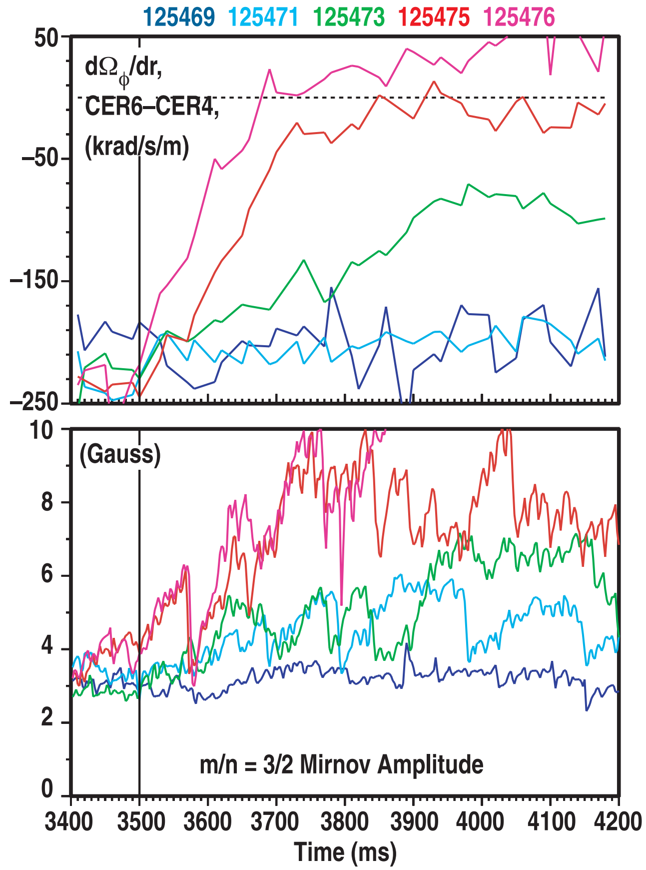
Also see F. Turco JP8.00097 Tu PM

m/n = 3/2 Modes Get Larger with Lower Rotation (and Shear) as "Inferred" Δ´ Less Negative

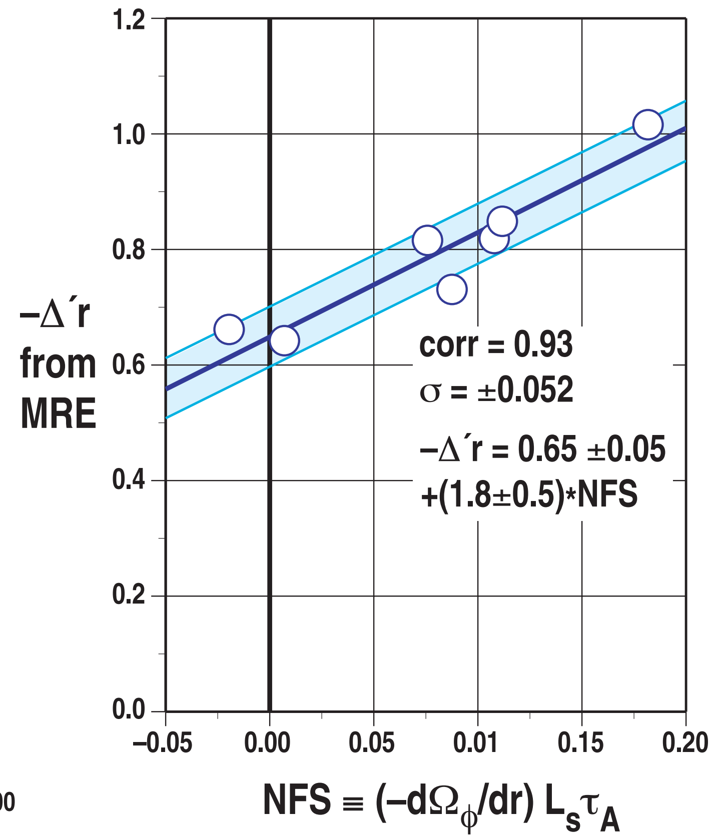
- **All co-NBI has counter beams added ($\beta_N \approx 2.6$, $\bar{n}_{13} \approx 3.5$)**
  ★ rotation reduced



- **As flow shear is reduced . . .**
  ★ 3/2 amplitude increases

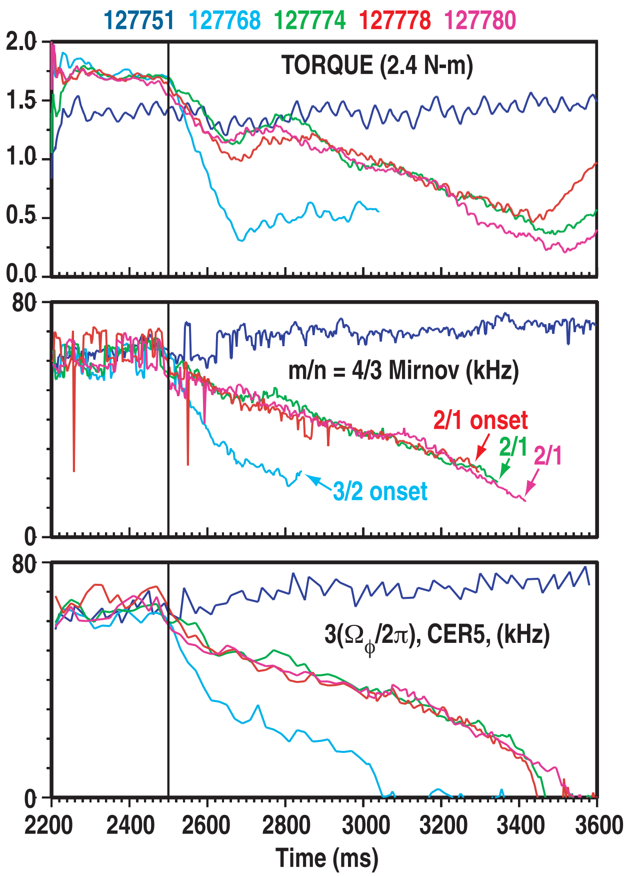


- **"Inferred" Δ´ less negative with reduced positive normalized flow shear**

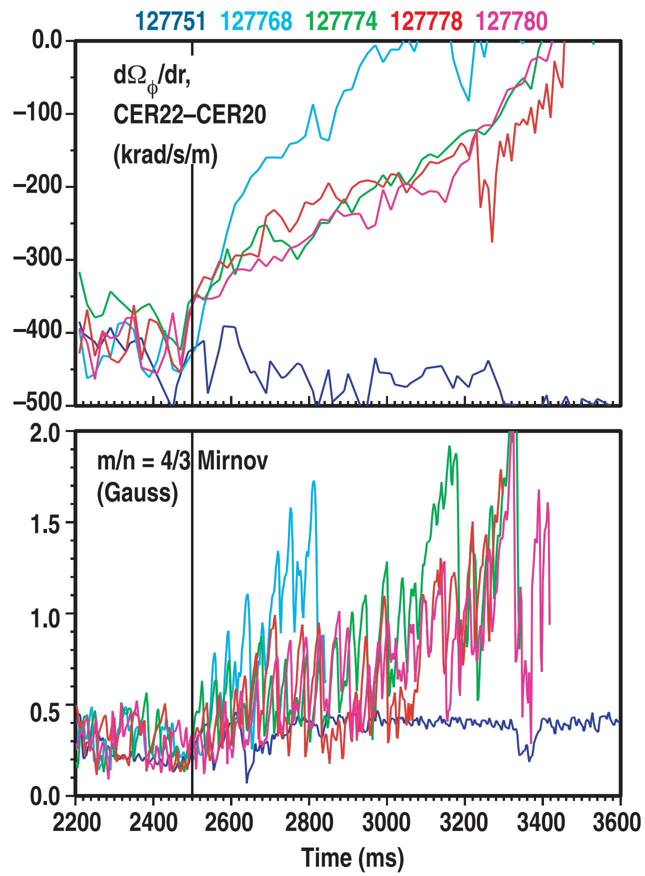


m/n = 4/3 Modes Also Larger with Lower Rotation (and Shear) as "Inferred" Δ´ Less Negative

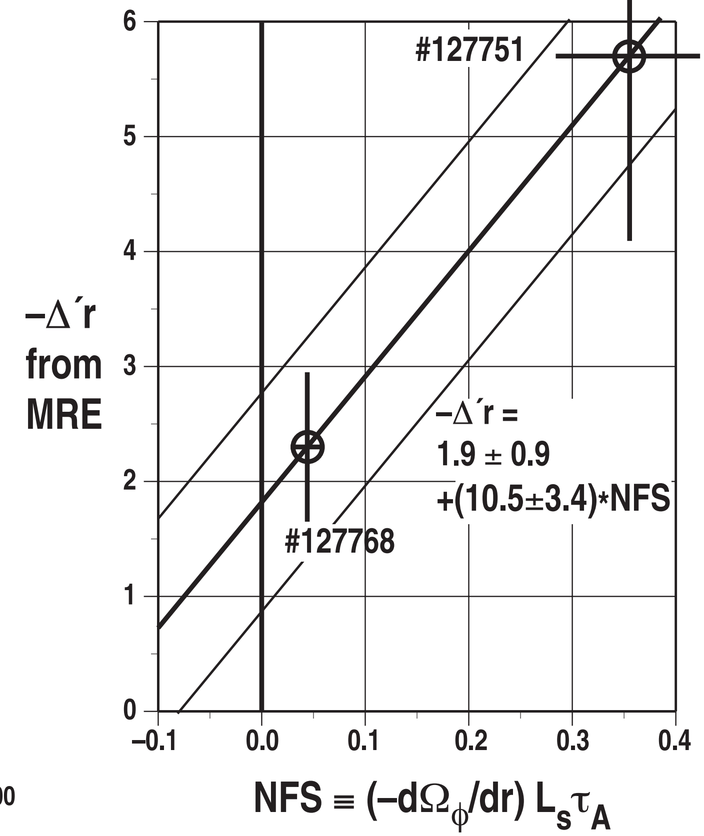
- **All co-NBI has counter beams added ($\beta_N \approx 2.7$, $\bar{n}_{13} \approx 3.7$)**
  - ★ rotation reduced



- **As flow shear is reduced . . .**
  - ★ 4/3 amplitude increases

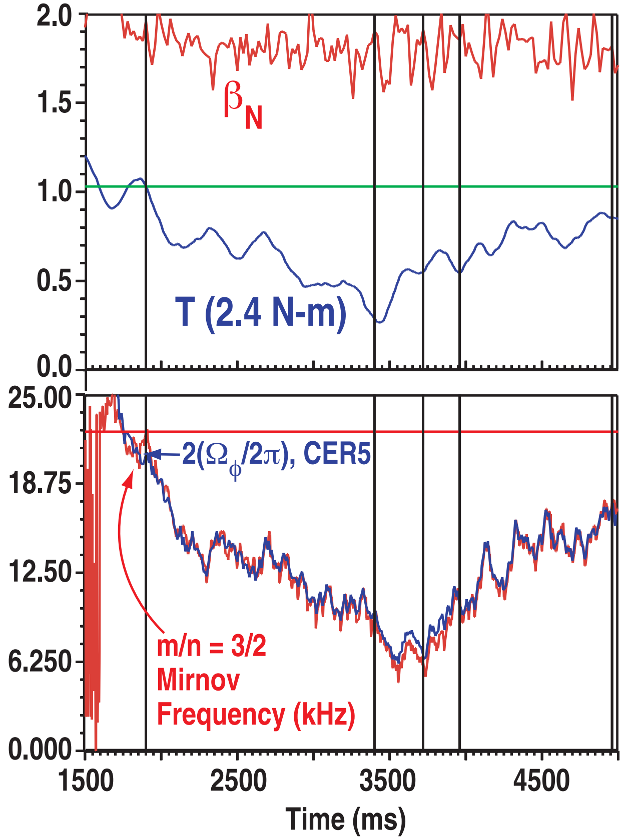


- **"Inferred" Δ´ less negative with reduced flow shear (only points with Thomson)**

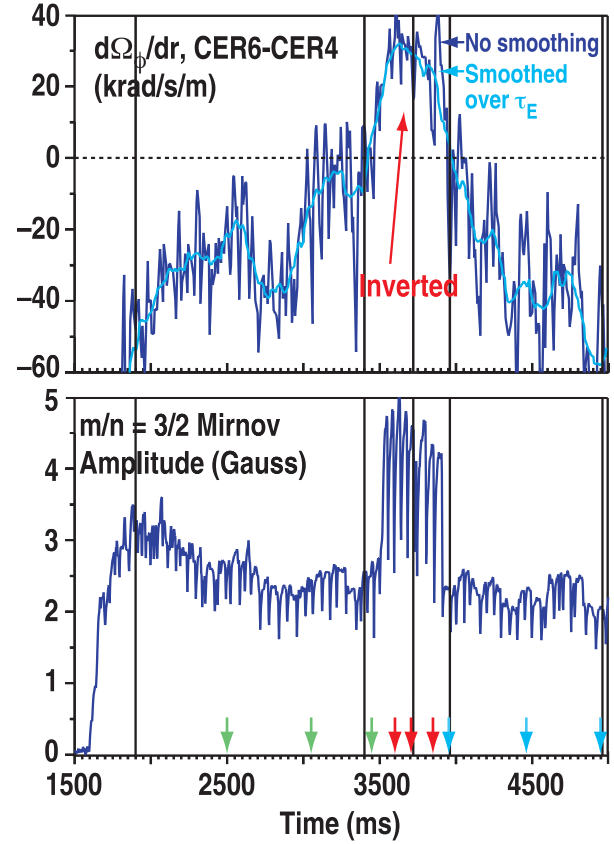


. . . and "Inferred" Δ´ Less Negative with Inverted Flow Shear with m/n = 3/2 Modes

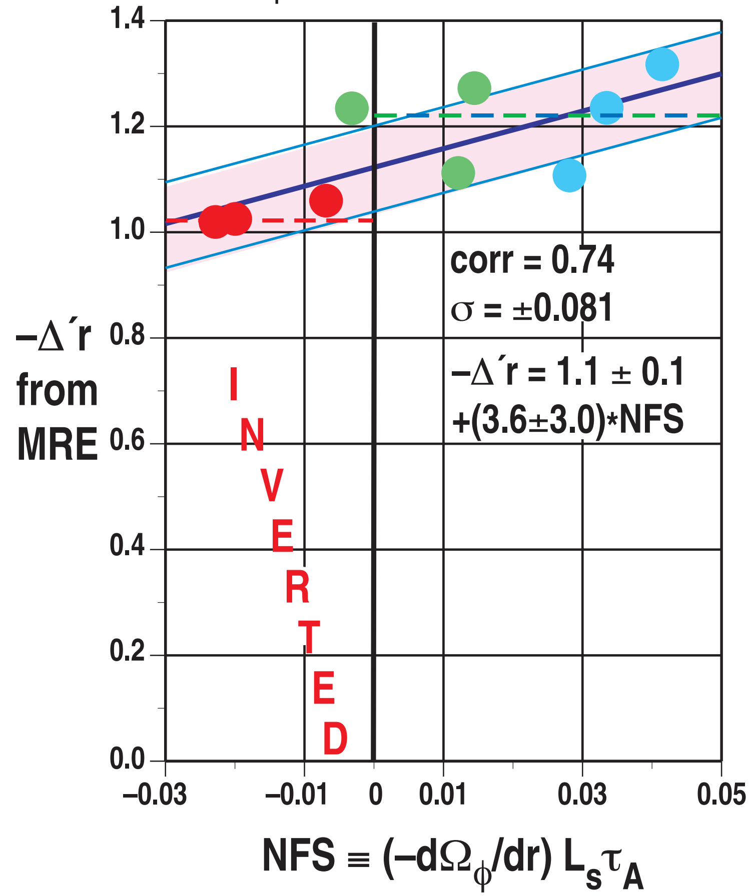
- **Reduced β_N, less initial all co-torque (#135867)**
  - ★ torque programmed down and back (~8 τ_E each way)



- **Little change in n=2 Mirnov amplitude**
  - ★ until $d\Omega_\phi/dr \approx 0$ "forbidden" state



- **"Inferred" Δ´ depends on sign of $-d\Omega_\phi/dr$?**
  - ★ Ω_ϕ co



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Flow Shear Can Be Put Into an Analytic Expression for Δ' for Guiding Comparison to Experiments: I

- **C.C. Hegna & J.D.Callen (PoP 1994) simple analytic expression for Δ'**
  - ★ large aspect ratio circular cylinder $m \geq 2$ asymptotic solution

. . . $\Delta' = -(2m/r)\,\lambda\pi \cot(\lambda\pi)$ with $\lambda = -\dfrac{rq}{m(dq/dr)}\,\dfrac{\mu_o}{B_\theta}\,\dfrac{dj_{||}}{dr}\Big|_{r=r_s}$ with $L_q = \dfrac{q}{dq/dr}$

– note $dj_{||}/dr$ (negative) destabilizing
and magnetic shear dq/dr (positive) stabilizing

. . . expanding in λ, $-\Delta' r/2m \approx 1 - (\lambda\pi)^2/2 \Rightarrow 1$ as $\lambda \Rightarrow 0$, $\Rightarrow 0$ as $\lambda \Rightarrow \dfrac{\sqrt{2}}{\pi} \approx 0.5$

- **Add radial magnetic shear and "parallel" projection of toroidal flow shear**

★ $\dfrac{1}{L_q} \rightarrow \dfrac{1}{L_q} - \left(\dfrac{q}{r/R}\right)\dfrac{d\Omega_\phi}{dr}\,\tau_A$ ★ $\dfrac{\Delta' r}{-2m} \approx 1 - \dfrac{\pi^2}{2}\,\dfrac{\lambda^2}{1-2\,(d\Omega_\phi/dr)L_s\,\tau_A + (d\Omega_\phi/dr)^2 L_s^2\,\tau_A^2}$

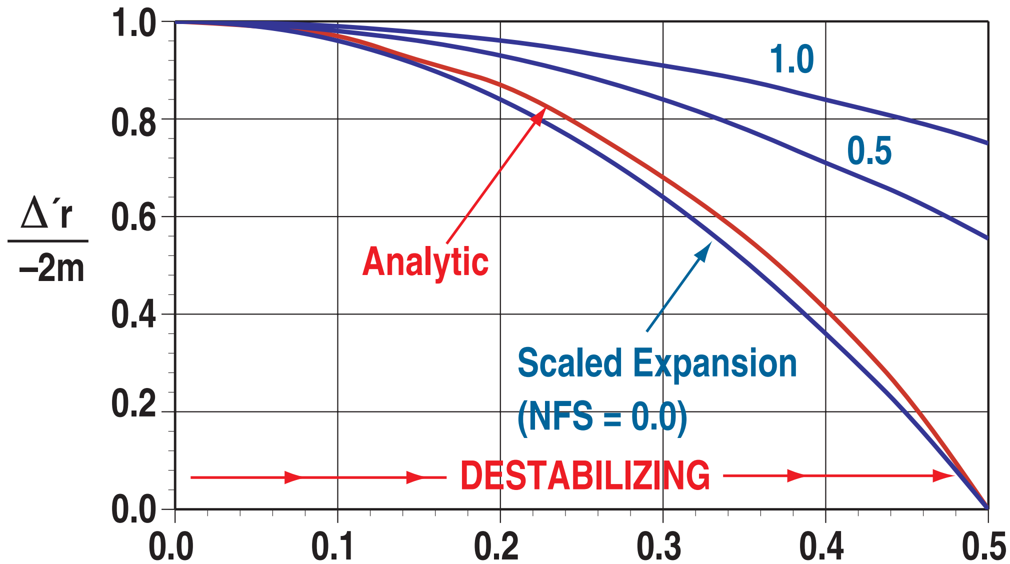
– L_s arises in parallel connection length along an island of full width w [$\lambda_{//} = L_s/(k_\theta\, w/2)$ with $k_\theta = m/r$, $L_s = q\, L_q/(r/R)$ and $L_q = q/dq/dr$]

– $\tau_A = R/V_A$ with $V_A = B_Z/\sqrt{\mu_o n_e m_i}$ [resistive MHD is akin to ideal MHD field line bending]

Flow Shear Can Be Put Into an Analytic Expression for Δ' for Guiding Comparison to Experiments: II

- **Expansion without flow shear is in good agreement with analytic expression for λ scaled by 0.901 times (λ = 0.0 to 0.5)**
  - ★ $\frac{\Delta' r}{-2m} \approx 1 - \frac{\pi^2 \lambda^2}{2} + \pi^2\lambda^2 \left(-d\Omega_\phi/dr\, L_s \tau_A\right) + \ldots$ to leading order in NFS

  and normalized flow shear NFS $\equiv -(d\Omega_\phi/dr) L_s\, \tau_A$



$$\lambda = -\frac{rq}{m(dq/dr)} \frac{\mu_o}{B_\theta} \frac{dj_{||}}{dr}\bigg|_{r = r_s}$$

- **Flow shear makes stable classical tearing yet more stable**
  - ★ and insensitive to details in profiles . . . i.e., λ

Numerical Modeling is in Process

- **NEAR code (D. Chandra, D. Raju, and A. Sen, IPR, Gujarat, India)**
  - ★ **dominant effect comes from the outer layer modification to Δ'**

    . . . scaling of Δ' to velocity shear is quite close to heuristic model
    - – clearly shows dependence on sign of flow shear
    - – but sign and magnitude in question

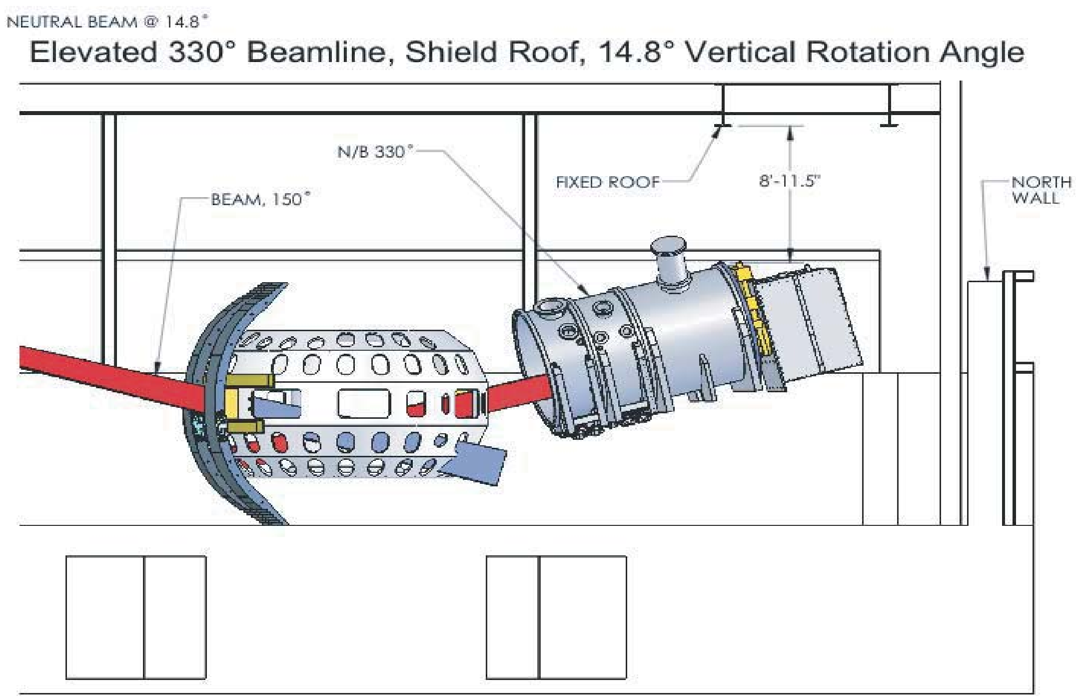
- **MARS code (M. Chu, GA)**
  - ★ **MARS has sheared flow but is not time dependent**

    . . . computes linear growth rates <u>if</u> unstable

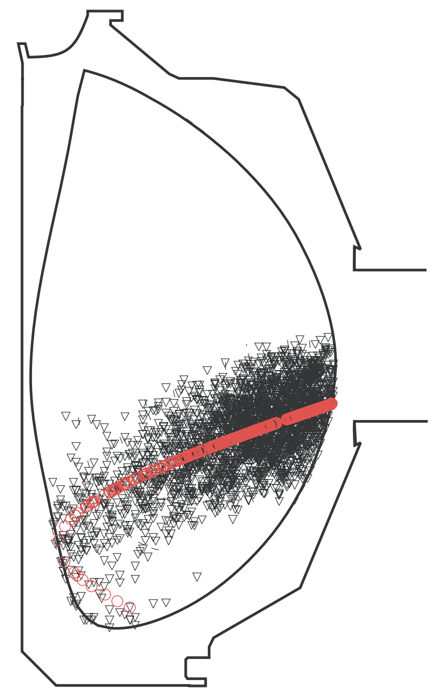
- **NIMROD code (D.Brennan, Univ. Tulsa and S. Kruger, Tech-X Corp)**
  - ★ **recently added single fluid sheared plasma rotation**

    . . . computes linear <u>and</u> nonlinear states from initial values

DIII-D 5-Year Plan Has Off-Axis Neutral Beam Injection Allowing Independent Control of Plasma Rotation Profile



Poloidal projection of fast ion birth points on a single toroidal plane



See J.M. Park: ORNL APS08 and PoP 2009