Predictive Gyrokinetic Transport Simulations and Application of Synthetic Diagnostics

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GENERAL ATOMICS

Preview: The TGYRO Project

Simulate multiple space and time scales using first-principles physics

TGYRO provides high-level control of GYRO, NEO, TGLF, etc.

▷ Complete hierarchical (ordered) solution of **Fokker-Planck** equations.

▷ Proper analytic **equilibrium** solution for **rotating** plasmas.

▷ **Neoclasical transport**: direct spectral solution of the drift-kinetic equation.

NEO code (Belli) or **NEO formula module**

▷ **Turbulent transport**: direct Eulerian solution of nonlinear gyrokinetic equations.

GYRO code or **TGLF model (Staebler)**

▷ Fully-implicit solution of transport equations in **integral form**.

▷ **Rapid convergence** for quasilinear transport models (TGLF, IFS-PPPL).

▷ Handy tool for transport model **comparison** or **development**.

GENERAL ATOMICS

Outline of this Talk

1. Review of **Fokker-Planck theory** from start to finish.

2. Summary of **TGYRO**: numerical approach and convergence tests.

3. Overview of **synthetic diagnostic** tools.

4. Sample **application of TGYRO** to DIII-D discharge.

Fokker-Planck Theory of Plasma Transport

Comprehensive series of papers by Sugama and coworkers

The Fokker-Planck (FP) equation provides the **fundamental theory** for **plasma equilibrium**, **fluctuations**, and **transport**:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{e_a}{m_a}\left((\mathbf{E}+\hat{\mathbf{E}}) + \frac{\mathbf{v}}{c}\times(\mathbf{B}+\hat{\mathbf{B}})\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right](f_a+\hat{f}_a) = C_a(f_a+\hat{f}_a) + S_a$$

$f_a \longrightarrow$ **ensemble-averaged distribution**

$\hat{f}_a \longrightarrow$ **fluctuating distribution**

$S_a \longrightarrow$ **sources (beams, RF, etc)**

$C_a = \sum_b C_{ab}(f_a+\hat{f}_a, f_b+\hat{f}_b) \longrightarrow$ **nonlinear collision operator**

Fokker-Planck theory

Comprehensive, consistent framework for equilibrium profile evolution

The general approach is to separate the FP equation into **ensemble-averaged**, $\mathcal{A}$, and **fluctuating**, $\mathcal{F}$, components:

$$\mathcal{A} = \left.\frac{d}{dt}\right|_{\rm ens} f_a - \langle C_a \rangle_{\rm ens} - D_a - S_a \ ,$$

$$\mathcal{F} = \left.\frac{d}{dt}\right|_{\rm ens} \hat{f}_a + \frac{e_a}{m_a}\left(\hat{\mathbf{E}} + \frac{\mathbf{v}}{c} \times \hat{\mathbf{B}}\right) \cdot \frac{\partial}{\partial \mathbf{v}}(f_a + \hat{f}_a) - C_a + \langle C_a \rangle_{\rm ens} + D_a \ ,$$

where

$$\left.\frac{d}{dt}\right|_{\rm ens} \doteq \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e_a}{m_a}\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right) \cdot \frac{\partial}{\partial \mathbf{v}} \ ,$$

$$D_a \doteq -\frac{e_a}{m_a}\left\langle \left(\hat{\mathbf{E}} + \frac{\mathbf{v}}{c} \times \hat{\mathbf{B}}\right) \cdot \frac{\partial \hat{f}_a}{\partial \mathbf{v}} \right\rangle_{\rm ens} \ .$$

▷ D_a is the **fluctuation-particle interaction operator**.

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Fokker-Planck theory

Space- and time-scale expansion in powers of $\rho_* = \rho_s/a$

Ensemble averages are expanded in powers of ρ_* as

$$
\begin{aligned}
f_a &= f_{a0} + f_{a1} + f_{a2} + \ldots \ , \\
S_a &= \phantom{f_{a0} + f_{a1} +} S_{a2} + \ldots \ \textbf{(transport ordering)}, \\
\mathbf{E} &= \mathbf{E}_0 + \mathbf{E}_1 + \mathbf{E}_2 + \ldots \ , \\
\mathbf{B} &= \mathbf{B}_0 \ .
\end{aligned}
$$

Fluctuations are also expanded in powers of ρ_* as

$$
\begin{aligned}
\hat{f}_a &= \hat{f}_{a1} + \hat{f}_{a2} + \ldots \ , \\
\hat{\mathbf{E}} &= \hat{\mathbf{E}}_1 + \hat{\mathbf{E}}_2 + \ldots \ , \\
\hat{\mathbf{B}} &= \hat{\mathbf{B}}_1 + \hat{\mathbf{B}}_2 + \ldots \ .
\end{aligned}
$$

Built-in assumption about scale separation **hard to escape**.

Fokker-Planck theory

Lowest-order conditions for flow and gyroangle independence

Lowest-order Constraints

The lowest-order ensemble-averaged equation gives the **constraints**

$$\mathcal{A}_{-1} = 0 : \qquad \mathbf{E}_0 + \frac{1}{c}\mathbf{V}_0 \times \mathbf{B} = 0 \qquad \text{and} \qquad \frac{\partial f_{a0}}{\partial \xi} = 0$$

where ξ is the gyroangle.

Large mean flow

The only **equilibrium flow** that persists on the fluctuation timescale is

$$\mathbf{V}_0 = R\,\omega_0(\psi)\mathbf{e}_\varphi \quad \text{where} \quad \omega_0 \doteq -c\frac{\partial \Phi_0}{\partial \psi} \ .$$

[F.L. Hinton and S.K. Wong, Phys. Fluids **28** (1985) 3082].

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Fokker-Planck theory

Equilibrium equation is a formidable nonlinear PDE

Equilibrium equation

The gyrophase average of the zeroth order ensemble-averaged equation gives the **collisional equilibrium** equation:

$$\int_0^{2\pi} \frac{d\xi}{2\pi} \mathcal{A}_0 = 0 : \qquad \left(\mathbf{V}_0 + v_\parallel' \mathbf{b}\right) \cdot \nabla f_{a0} = C_a(f_{a0})$$

where $\mathbf{v}' = \mathbf{v} - \mathbf{V}_0$ is the velocity in the rotating frame.

Equilibrium distribution function

The exact solution for f_{a0} is a **Maxwellian in the rotating frame**, such that the centrifugal force causes the density to vary on the flux surface:

$$f_{a0} = n_a(\psi, \theta) \left(\frac{m_a}{2\pi T_a}\right)^{3/2} e^{-m_a (v')^2 / 2T_a} \ .$$

Fokker-Planck theory

Equations for neoclassical transport and turbulence at $\mathcal{O}(\rho_*)$

Drift-kinetic equation

Gyroaverage of first-order $\mathcal{A}_1$ gives expressions for **gyroangle-dependent ($\tilde{f}_{a1}$)** and **gyroangle-independent ($\bar{f}_{a1}$)** distributions:

$$\int_0^{2\pi} \frac{d\xi}{2\pi} \mathcal{A}_1 = 0 : \qquad f_{a1} = \tilde{f}_{a1} + \bar{f}_{a1} \; , \quad \tilde{f}_{a1} = \frac{1}{\Omega_a} \int^{\xi} d\xi \, \widetilde{\mathcal{L} f_{a0}}$$

▷ Ensemble-averaged $\bar{f}_{a1}$ is determined by the **drift kinetic equation (NEO)**.

Gyrokinetic equation

Gyroaverage of first-order $\mathcal{F}_1$ gives an expression for **first-order fluctuating distribution ($\hat{f}_{a1}$)** in terms of the distribution of the gyrocenters, $h_a(\mathbf{R})$:

$$\int_0^{2\pi} \frac{d\xi}{2\pi} \mathcal{F}_1 = 0 : \qquad \hat{f}_{a1}(\mathbf{x}) = -\frac{e_a \hat{\phi}(\mathbf{x})}{T_a} + h_a(\mathbf{x} - \rho)$$

▷ Fluctuating $\hat{f}_{a1}$ is determined by the **gyrokinetic equation (GYRO)**.

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Drift-Kinetic Equation for Neoclassical Transport

NEO gives complete solution with full kinetic e-i-impurity coupling

$$v'_{\parallel}\mathbf{b}\cdot\nabla\bar{g}_a - C_a^L(\bar{g}_a) = \frac{f_{a0}}{T_a}\left[-\frac{1}{N_a}\frac{\partial N_a T_a}{\partial\psi}W_{a1} - \frac{\partial T_a}{\partial\psi}W_{a2} + c\frac{\partial^2\Phi_0}{\partial\psi^2}W_{aV} + \frac{\langle BE_{\parallel}^A\rangle}{\langle B^2\rangle^{1/2}}W_{aE}\right]$$

$$\bar{g}_a \doteq \bar{f}_{a1} - f_{a0}\frac{e_a}{T_a}\int^{\ell}\frac{dl}{B}\left(BE_{\parallel} - \frac{B^2}{\langle B^2\rangle}\langle BE_{\parallel}\rangle\right) \ ,$$

$$W_{a1} \doteq \frac{m_a c}{e_a}v'_{\parallel}\mathbf{b}\cdot\nabla\left(\omega_0 R + \frac{I}{B}v'_{\parallel}\right) \ ,$$

$$W_{a2} \doteq W_{a1}\left(\frac{\varepsilon}{T_a} - \frac{5}{2}\right) \ ,$$

$$W_{aV} \doteq \frac{m_a c}{2e_a}v'_{\parallel}\mathbf{b}\cdot\nabla\left[m_a\left(\omega_0 R + \frac{I}{B}v'_{\parallel}\right)^2 + \mu\frac{R^2 B_p^2}{B}\right] \ ,$$

$$W_{aE} \doteq \frac{e_a v'_{\parallel} B}{\langle B\rangle^{1/2}} \ .$$

Gyro-Kinetic Equation for Turbulent Transport

GYRO gives complete solution with full $(\phi, A_\parallel, B_\parallel)$ electromagnetic physics.

$$
\begin{aligned}
\frac{\partial h_a(\mathbf{R})}{\partial t} &+ \left(\mathbf{V}_0 + v_\parallel' \mathbf{b} + \mathbf{v}_{da} - \frac{c}{B}\nabla\hat{\Psi}_a \times \mathbf{b}\right)\cdot \nabla h_a(\mathbf{R}) - C_a^{GL}\left(\hat{f}_{a1}\right) \\
&= f_{a0}\left[-\frac{\partial \ln(N_a T_a)}{\partial\psi}\hat{W}_{a1} - \frac{\partial \ln T_a}{\partial \psi}\hat{W}_{a2} + \frac{c}{T_a}\frac{\partial^2 \Phi_0}{\partial\psi^2}\hat{W}_{aV} + \frac{1}{T_a}\hat{W}_{aT}\right]
\end{aligned}
$$

$$
\begin{aligned}
\hat{W}_{a1}(\mathbf{R}) &\doteq -\frac{c}{B}\nabla\hat{\Psi}_a \times \mathbf{b}\cdot\nabla\psi \ , \\
\hat{W}_{a2}(\mathbf{R}) &\doteq \hat{W}_{a1}\left(\frac{\varepsilon}{T_a} - \frac{5}{2}\right) \ , \\
\hat{W}_{aV}(\mathbf{R}) &\doteq -\frac{m_a R c}{B}\left\langle (\mathbf{V}_0 + \mathbf{v}')\cdot \mathbf{e}_\varphi \nabla\left(\hat{\phi} - \frac{1}{c}(\mathbf{V}_0 + \mathbf{v}')\cdot\hat{\mathbf{A}}\right)\times \mathbf{b}\cdot\nabla\psi\right\rangle_\xi \ , \\
\hat{W}_{aT}(\mathbf{R}) &\doteq e_a\left\langle\left(\frac{\partial}{\partial t} + \mathbf{V}_0\cdot\nabla\right)\left(\hat{\phi} - \frac{1}{c}(\mathbf{V}_0 + \mathbf{v}')\cdot\hat{\mathbf{A}}\right)\right\rangle_\xi \ . \\
\hat{\Psi}_a(\mathbf{R}) &\doteq \left\langle \hat{\phi}(\mathbf{R}+\boldsymbol{\rho}) - \frac{1}{c}(\mathbf{V}_0 + \mathbf{v}')\cdot\hat{\mathbf{A}}(\mathbf{R}+\boldsymbol{\rho})\right\rangle_\xi \\
&\to J_0\left(\frac{k_\perp v_\perp'}{\Omega_a}\right)\left(\hat{\phi}(\mathbf{k}_\perp) - \frac{\mathbf{V}_0}{c}\cdot\hat{\mathbf{A}}(\mathbf{k}_\perp) - \frac{v_\parallel'}{c}\hat{A}_\parallel(\mathbf{k}_\perp)\right) + J_1\left(\frac{k_\perp v_\perp'}{\Omega_a}\right)\frac{v_\perp'}{c}\frac{\hat{B}_\parallel(\mathbf{k}_\perp)}{k_\perp} \ .
\end{aligned}
$$

Gyro-Kinetic Equation for Turbulent Transport

GYRO gives complete solution with full $(\phi, A_\parallel, B_\parallel)$ electromagnetic physics.

Must also solve the electromagnetic field equations on the **fluctuation scale**:

$$\frac{1}{\lambda_D^2}\left(\hat{\phi}(\mathbf{x}) - \frac{\mathbf{V}_0}{c}\cdot\hat{\mathbf{A}}\right) = 4\pi\sum_a e_a \int d^3v\, \hat{h}_a(\mathbf{x}-\boldsymbol{\rho}) \ ,$$

$$-\nabla_\perp^2 \hat{A}_\parallel(\mathbf{x}) = \frac{4\pi}{c}\sum_a e_a \int d^3v\, \hat{h}_a(\mathbf{x}-\boldsymbol{\rho}) v_\parallel' \ ,$$

$$\nabla \hat{B}_\parallel(\mathbf{x}) \times \mathbf{b} = \frac{4\pi}{c}\sum_a e_a \int d^3v\, \hat{h}_a(\mathbf{x}-\boldsymbol{\rho}) \mathbf{v}_\perp' \ .$$

▷ Can one compute equilibrium-scale potential Φ_0 from the Poisson equation?

▷ Practically, no; need higher-order theory and extreme numerical precision.

▷ All codes must take care to avoid **nonphysical potential** at long wavelength

... F. Parra, next talk.

▷ TGYRO gets $\omega_0(\psi) = -c\partial_\psi \Phi_0$ from the **momentum transport equation**.

Transport Equations

Flux-surface-averaged moments of Fokker-Planck equation

$$\left\langle \int d^3v \, \mathcal{A} \right\rangle_\theta$$ **density**

$$\left\langle \int d^3v \, \varepsilon \mathcal{A} \right\rangle_\theta$$ **energy**

$$\sum_a \left\langle \int d^3v \, m_a v'_\varphi \mathcal{A} \right\rangle_\theta$$ **toroidal momentum**

Only terms of order ρ_*^2 survive these averages

$$\boxed{\rho_*^{-1} = 10^3 \quad \rho_*^0 = 1 \quad \rho_*^1 = 10^{-3} \quad \rho_*^2 = 10^{-6}}$$

Transport Equations

Flux-surface-averaged moments of Fokker-Planck equation to $\mathcal{O}(\rho_*^2)$

$$n_a(r): \quad \frac{\partial \langle n_a \rangle}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial r}\left(V'\Gamma_a\right) = S_{n,a}$$

$$T_a(r): \quad \frac{3}{2}\frac{\partial \langle n_a T_a \rangle}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial r}\left(V' Q_a\right) + \Pi_a \frac{\partial \omega_0}{\partial \psi} = S_{W,a}$$

$$\omega_0(r): \quad \frac{\partial}{\partial t}(\omega_0 \langle R^2 \rangle \sum_a m_a n_a) + \frac{1}{V'}\frac{\partial}{\partial r}(V' \sum_a \Pi_a) = \sum_a S_{\omega,a}$$

$$S_{n,a} = S_{n,a}^{\mathrm{beam}} + S_{n,a}^{\mathrm{wall}} \quad \text{and} \quad \Gamma_a = \Gamma_a^{\mathrm{GV}} + \Gamma_a^{\mathrm{neo}} + \Gamma_a^{\mathrm{tur}}$$

$$S_{W,a} = S_{W,a}^{\mathrm{aux}} + S_{W,a}^{\mathrm{rad}} + S_{W,a}^{\alpha} + S_{W,a}^{\mathrm{tur}} + S_{W,a}^{\mathrm{col}} \quad \text{and} \quad Q_a = Q_a^{\mathrm{GV}} + Q_a^{\mathrm{neo}} + Q_a^{\mathrm{tur}}$$

$$\Pi_a = \Pi_a^{\mathrm{GV}} + \Pi_a^{\mathrm{neo}} + \Pi_a^{\mathrm{tur}}$$

RED: TGYRO   GREEN: NEO   BLUE: GYRO

A Comment on Full-F Gyrokinetic Models

Focus on collisionless Hamiltonian substructure of model

Consider the collisionless **full-F gyrokinetic equation** (Hahm 1988):

$$\frac{dF_a}{dt} = \frac{\partial F_a}{\partial t} + [F_a, \langle H \rangle] = 0$$

▷ Symplectic averaging technique for **collisionless** d/dt.

▷ **Elegant formalism** for expansion $\langle H \rangle = \sum_n H_n \rho_*^n$.

▷ $C_a(F_a)$ and S_a are usually **ignored**, or C_a **linearized**.

▷ Unclear how to implement nonlinear $C_a(F)$.

▷ Unclear how to apply to transport timescale simulation

... M. Barnes, previous talk.

TGYRO Design Goals and Philosophy

Integrated solution of Fokker-Planck hierarchy

Goal

Solve transport equations for profiles $n_a(r), T_a(r), \omega_0(r)$ that drive fluxes that balance specified input power (from experiment or model scenario).

Essence of numerical approach

- $\triangleright$ Choose initial **profiles** and radial **pivot point**.
- $\triangleright$ Calculate fluxes using GYRO+NEO at radii $\{r_j\}$, where $0 < r_j \leq r_{\max} < a$.
- $\triangleright$ **Adjust gradients** until GYRO+NEO fluxes balance heating sources.
- $\triangleright$ Effectively a nonlinear **root-finding** problem.

Brief Description of the TGYRO Solver

Illustration: Steady-state problem for energy transport

Balance between **transport flux**, $Q_a = Q_a^{\rm tur} + Q_a^{\rm neo}$, and **target flux**, Q_a^T:

divergence form (standard) $$\frac{1}{V'}\frac{\partial}{\partial r}\left(V' Q_a\right) = S_a(r)$$

integral form (TGYRO) $$Q_a(\{z_a, T_a\}, r) = Q_a^T(r)$$

$$Q_a^T(r) \doteq \frac{1}{V'(r)} \int_0^r dx\, V'(x)\, S_a(x) \quad \text{and} \quad z_a \doteq -\frac{1}{T_a}\frac{\partial T_a}{\partial r} \ .$$

Specify the temperature at a matching radius r_* and discretize $r \longrightarrow \{r_j\}$:

$$T_a(r) = T_{a*} \exp\left(\int_r^{r_*} dx\, z_a(x) \right)$$

$$T_a(r_{j-1}) = T_a(r_j) \exp\left\{ \frac{1}{2} \left[z_a(r_j) + z_a(r_{j-1}) \right] \left[r_j - r_{j-1} \right] \right\} \ .$$

Brief Description of the TGYRO Solver

Use integrated form of the transport equations

Solve the discrete equations using gyroBohm normalization $(\widehat{Q} \doteq Q/Q_{\rm GB})$:

$$\widehat{Q}_{a,j}(\{z_{a,j}\}) = \widehat{Q}^T_{a,j}(\{z_{a,j}\})$$

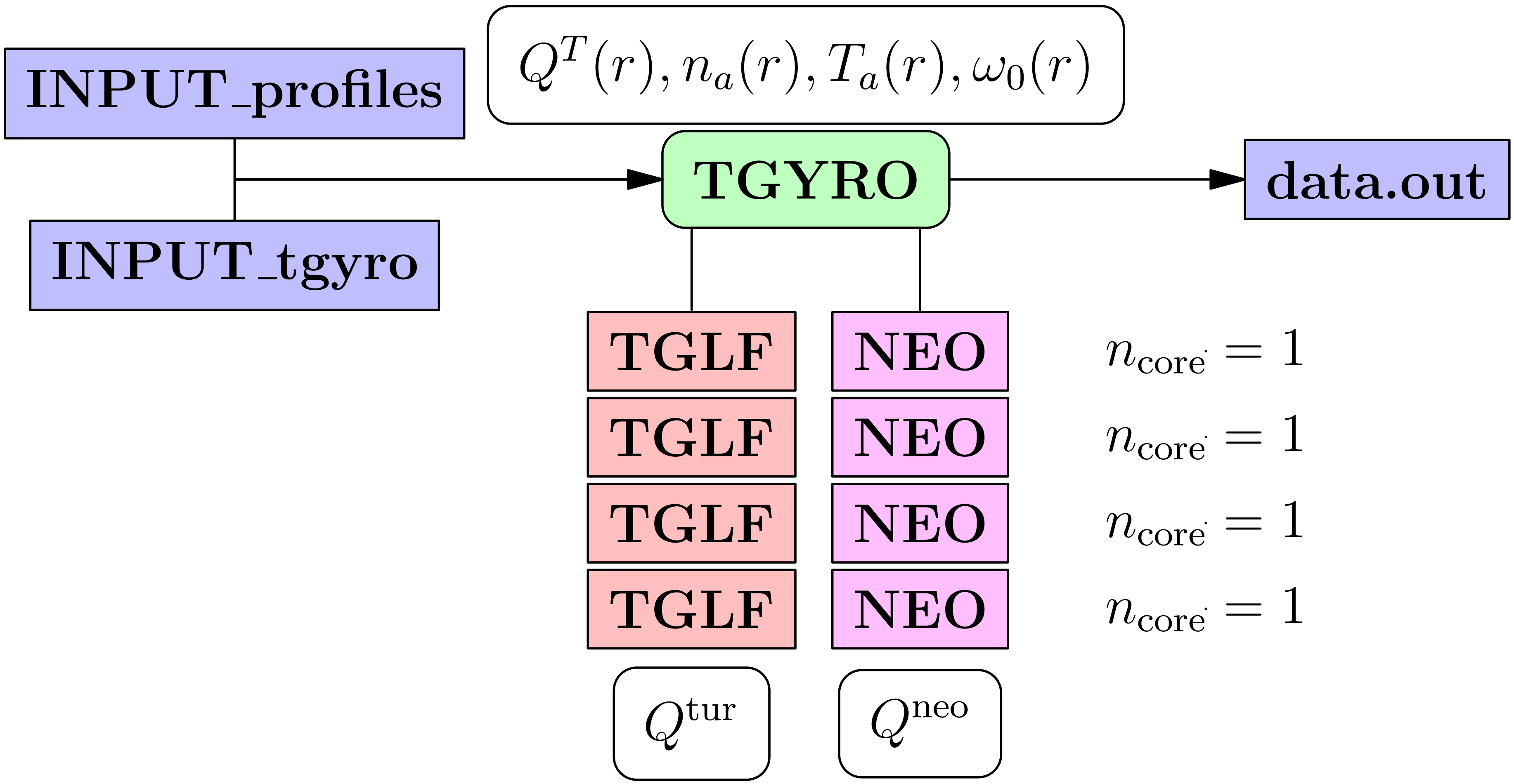
The problem is thus reduced to **n-dimensional root-finding**:

$$f_{a,j}(\{z_{a,j}\}) = 0 \quad \longrightarrow \quad \left(\frac{\partial f}{\partial z}\right)_{aa',jj'} \delta z_{a',j'} = -\eta_{a,j} f_{a,j}$$

- $\triangleright$ $\widehat{Q}_{a,j} = \widehat{Q}^{\rm tur}_{a,j} + \widehat{Q}^{\rm neo}_{a,j}$ are the **transport fluxes** (GYRO, NEO, TGYRO)
  - $\longrightarrow$ for economy, **use block-diagonal Jacobian**: $\dfrac{\partial \widehat{Q}_{a,j}}{\partial z_{a',j}} \delta_{jj'}$.
  - $\longrightarrow$ statistical steady state and large $\Delta z_{a,j}$ required for Jacobian calculation.
- $\triangleright$ $\widehat{Q}^T_{a,j}$ are the **target fluxes** (INEXPENSIVE)
  - $\longrightarrow$ for accuracy, **use dense Jacobian**: $\dfrac{\partial \widehat{Q}^T_{a,j}}{\partial z_{a',j'}}$.

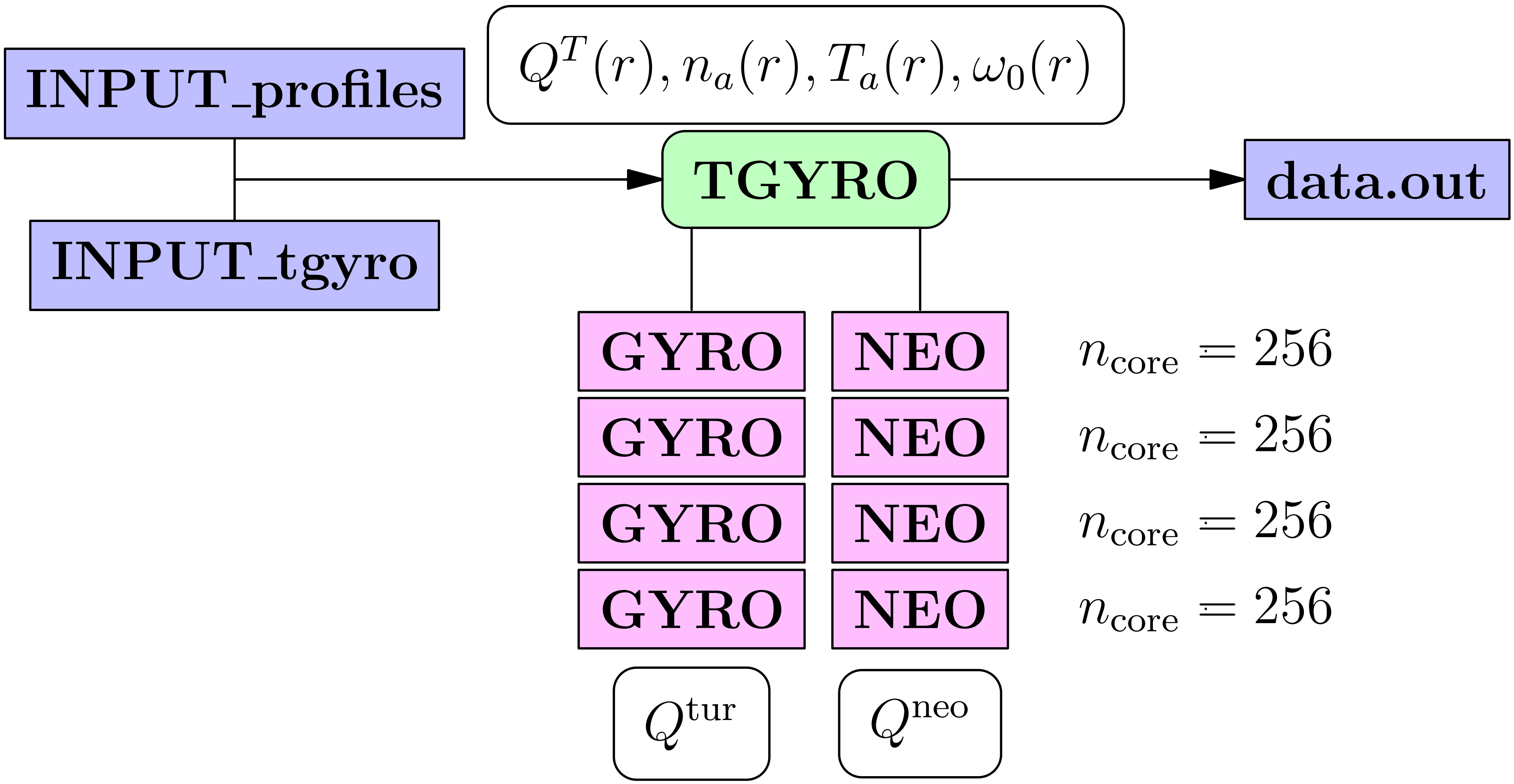
TGYRO Structure

TGYRO can use TGLF for turbulent fluxes, $Q^{\rm tur}$



TGYRO Structure

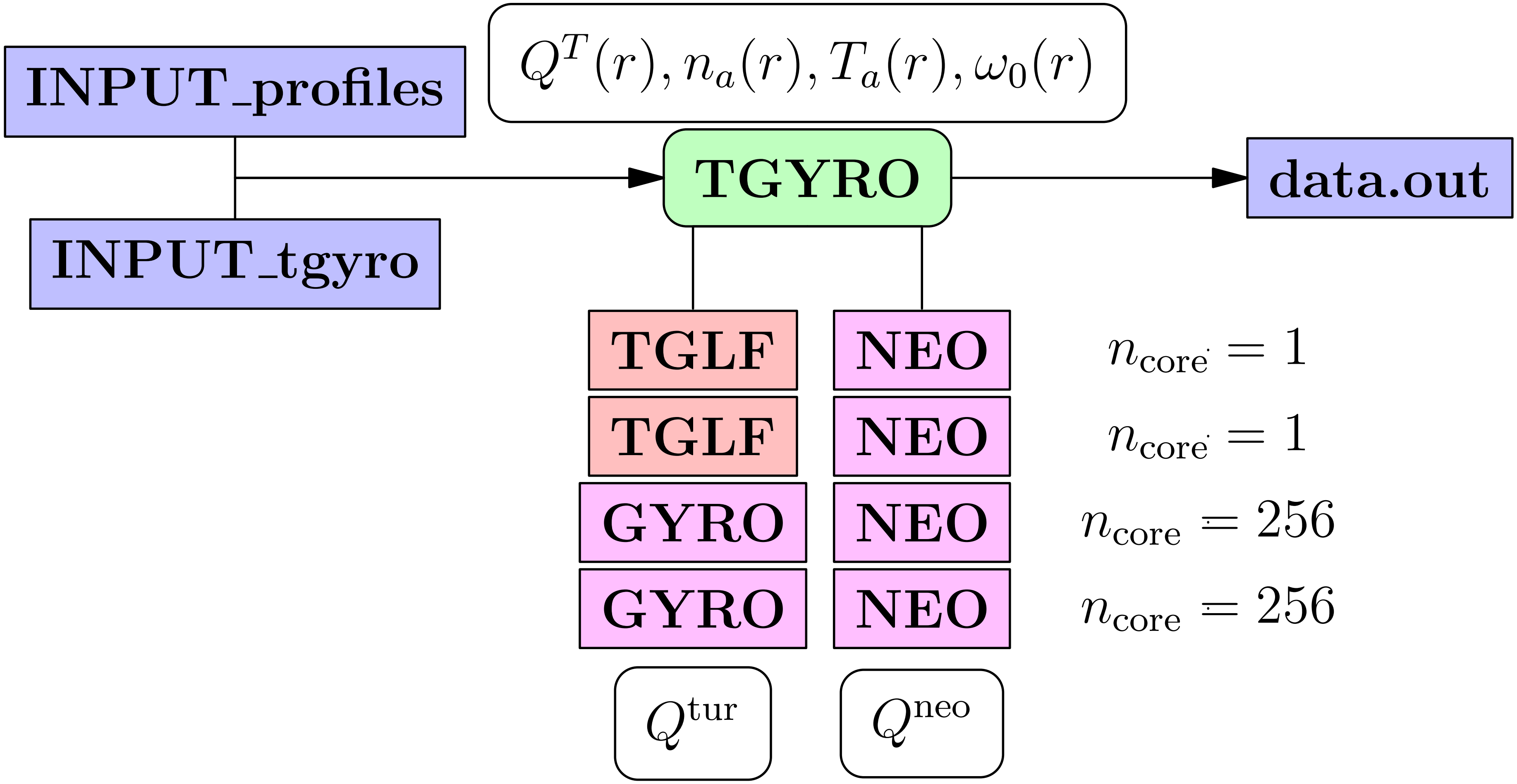
TGYRO can use full nonlinear GYRO runs for turbulent fluxes, $Q^{\rm tur}$



GENERAL ATOMICS

TGYRO Structure

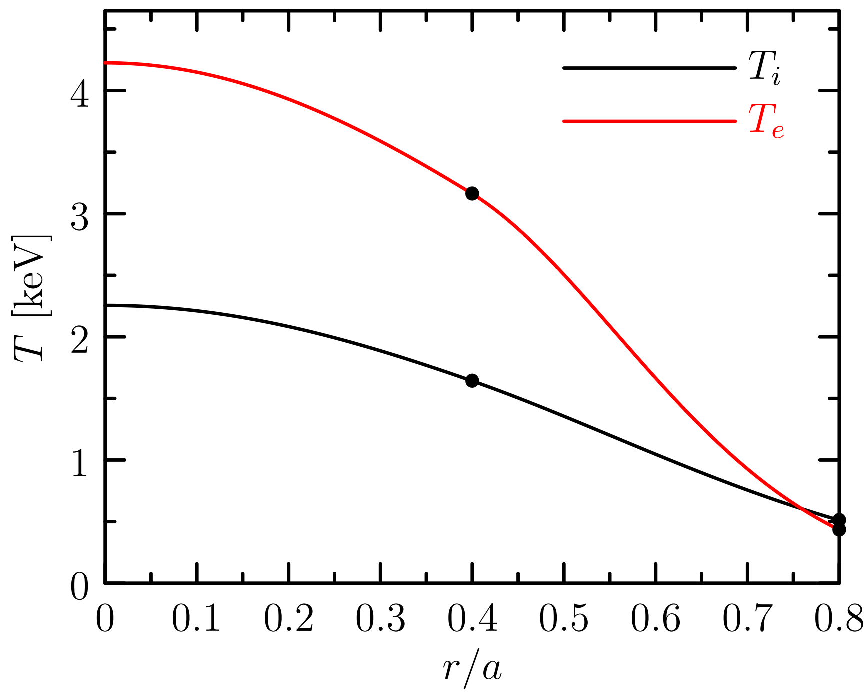
We can also mix turbulence models

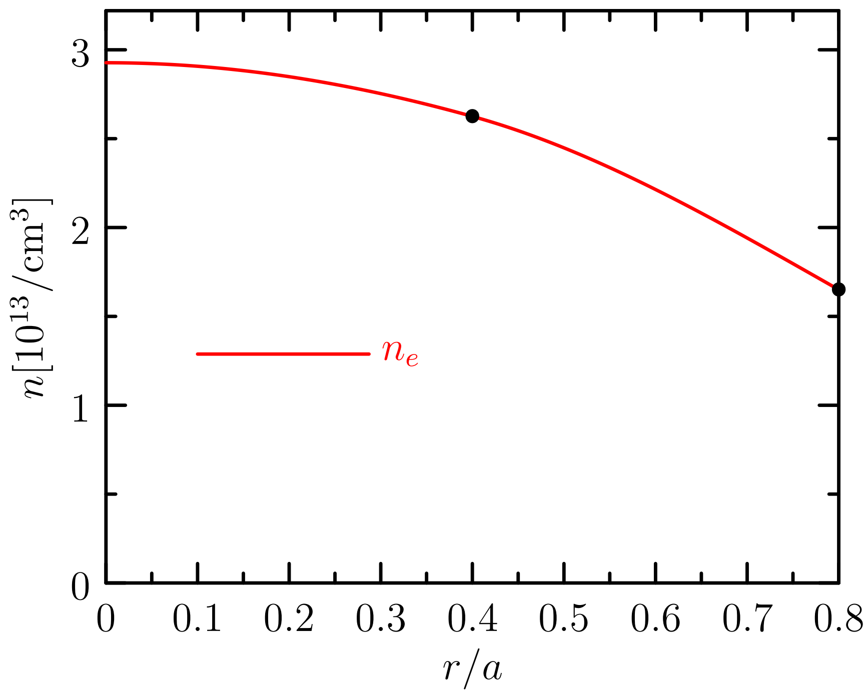


TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

Rapid profile convergence with number of radial nodes

2 radial nodes





$$T_a(r) = T_{a*} \exp\left(\int_r^{r_*} dx\, z_a(x) \right)$$

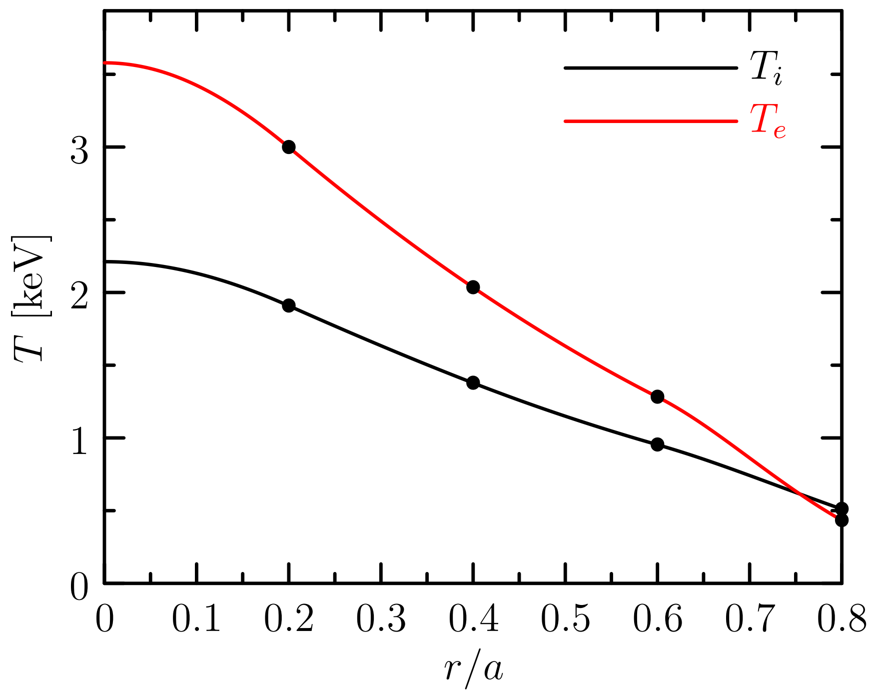
$z_a(x)$ taken to be **piecewise-linear**

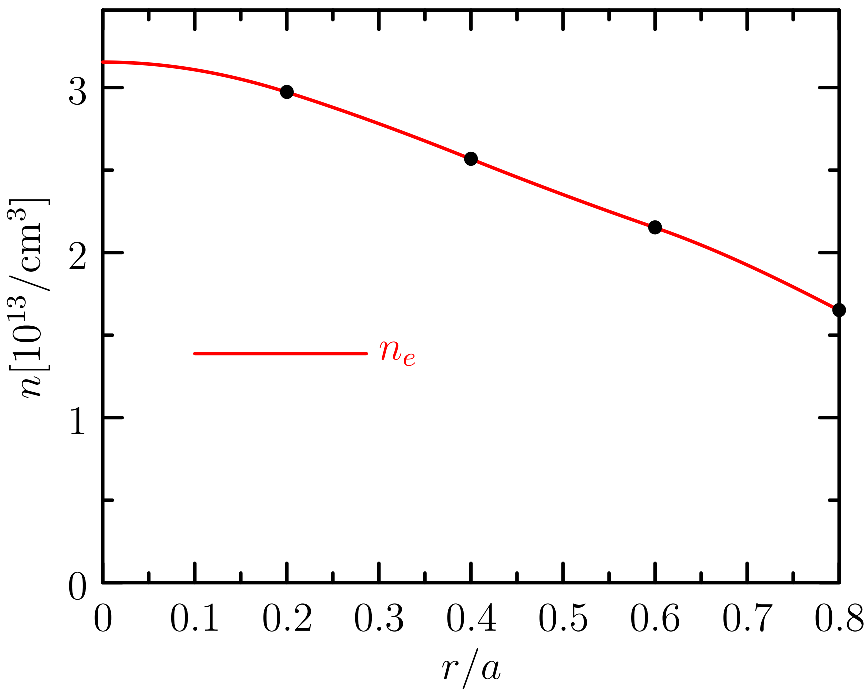
GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

Rapid profile convergence with number of radial nodes

4 radial nodes





$$T_a(r) = T_{a*} \exp\left(\int_r^{r_*} dx\, z_a(x)\right)$$

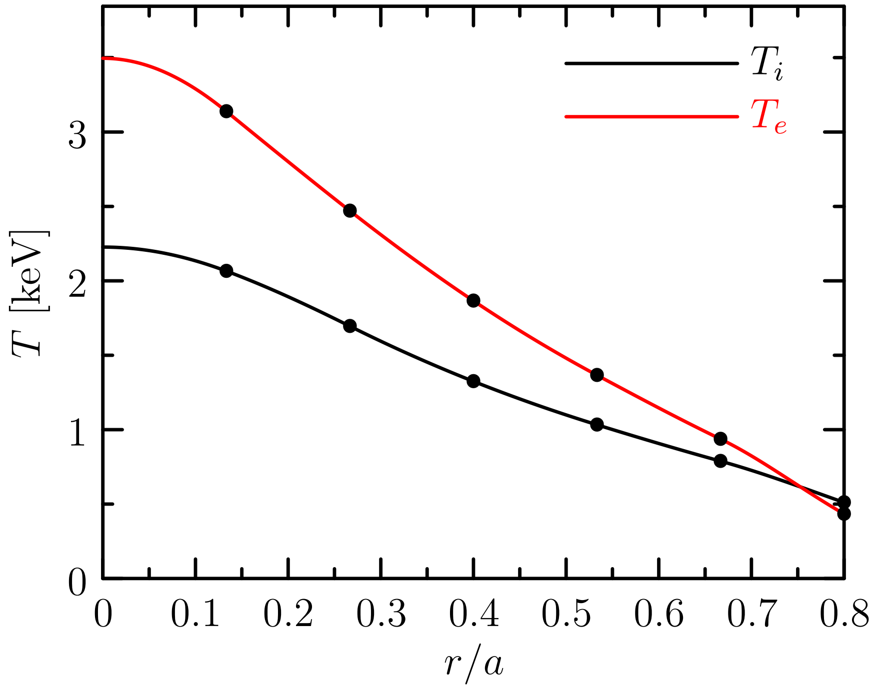
$z_a(x)$ taken to be **piecewise-linear**

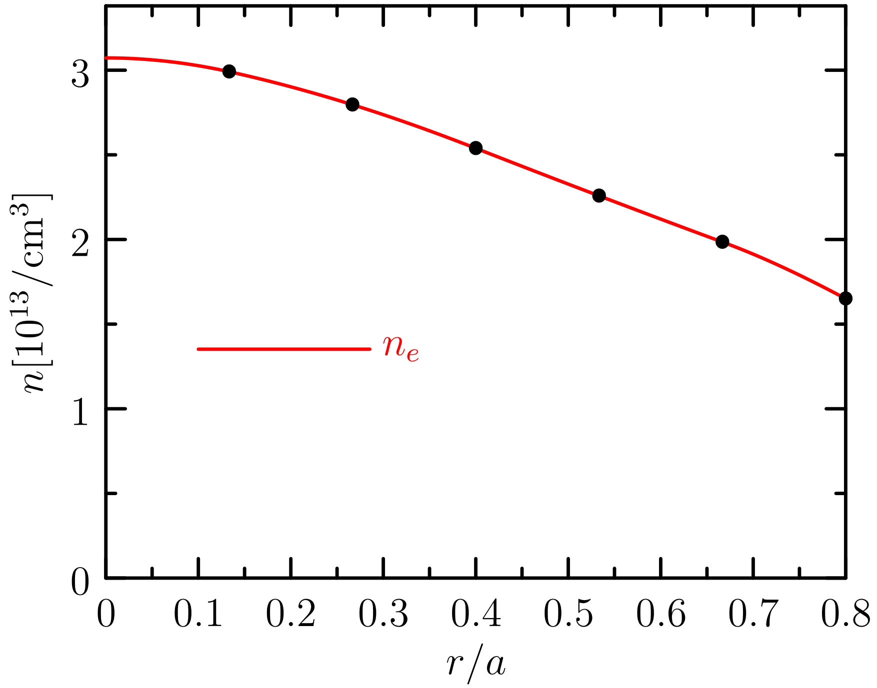
GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

Rapid profile convergence with number of radial nodes

6 radial nodes





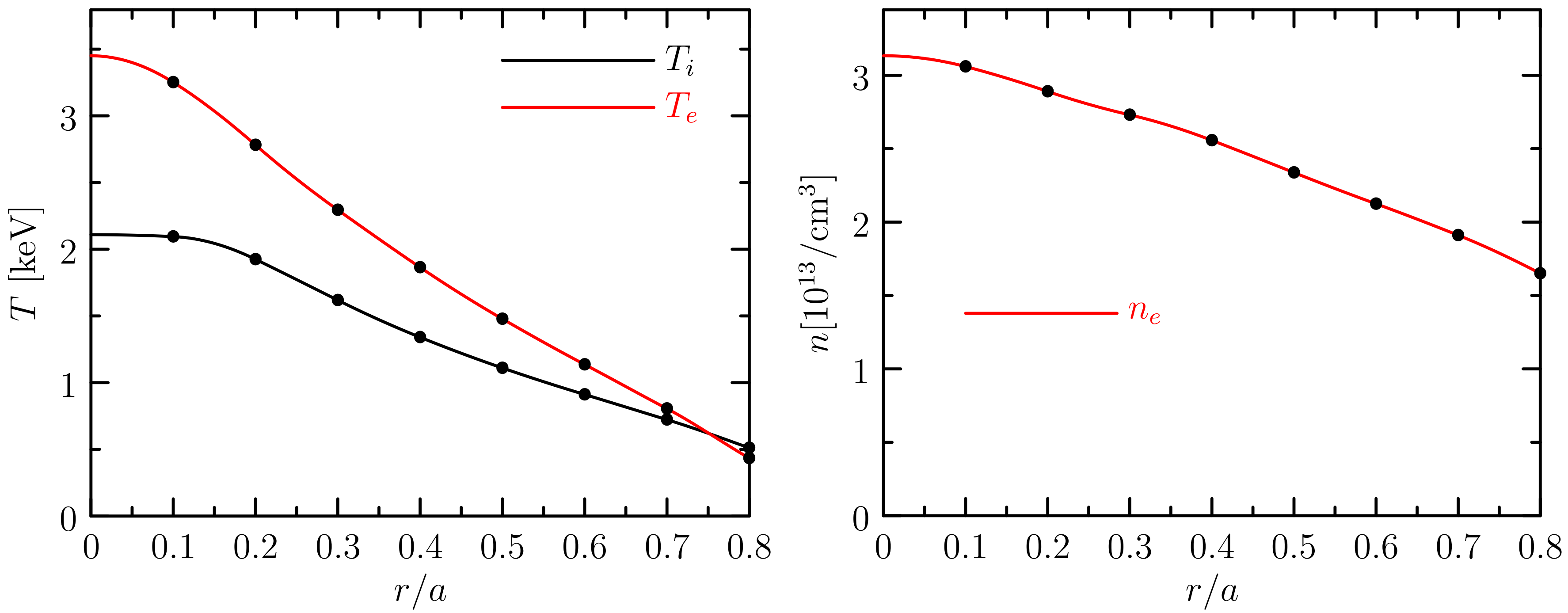
$$T_a(r) = T_{a*} \exp\left(\int_r^{r_*} dx\, z_a(x)\right)$$

$z_a(x)$ taken to be **piecewise-linear**

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

Rapid profile convergence with number of radial nodes

8 radial nodes



$$T_a(r) = T_{a*} \exp\left(\int_r^{r_*} dx\, z_a(x)\right)$$

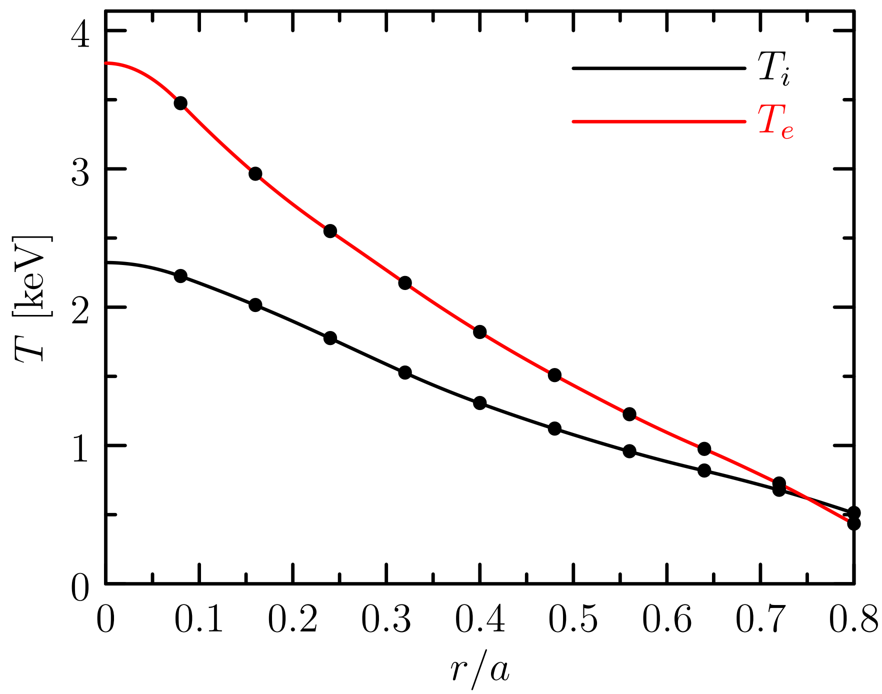
$z_a(x)$ taken to be **piecewise-linear**

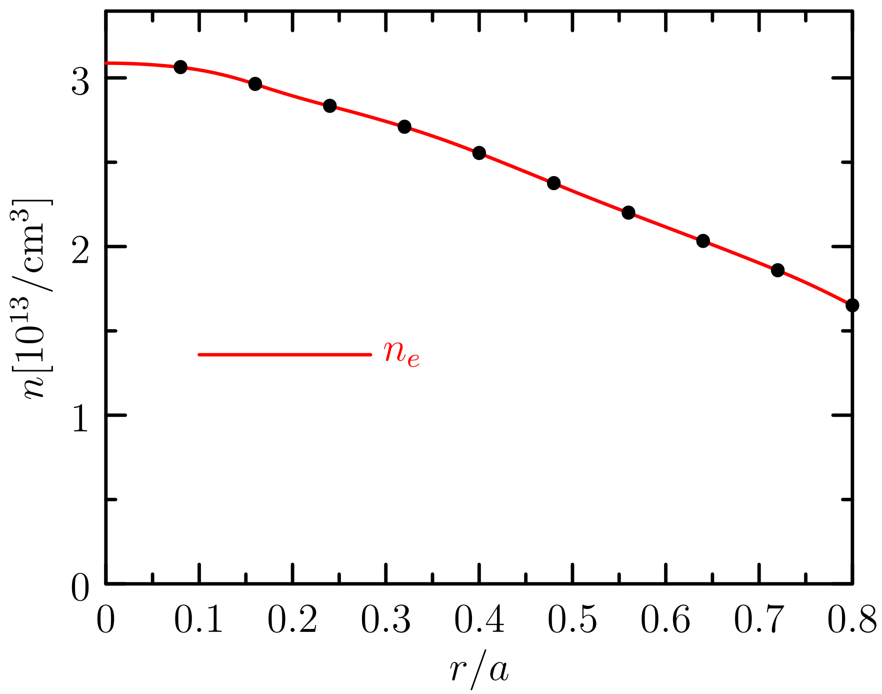
GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

Rapid profile convergence with number of radial nodes

10 radial nodes





$$T_a(r) = T_{a*} \exp\left(\int_r^{r_*} dx\, z_a(x)\right)$$

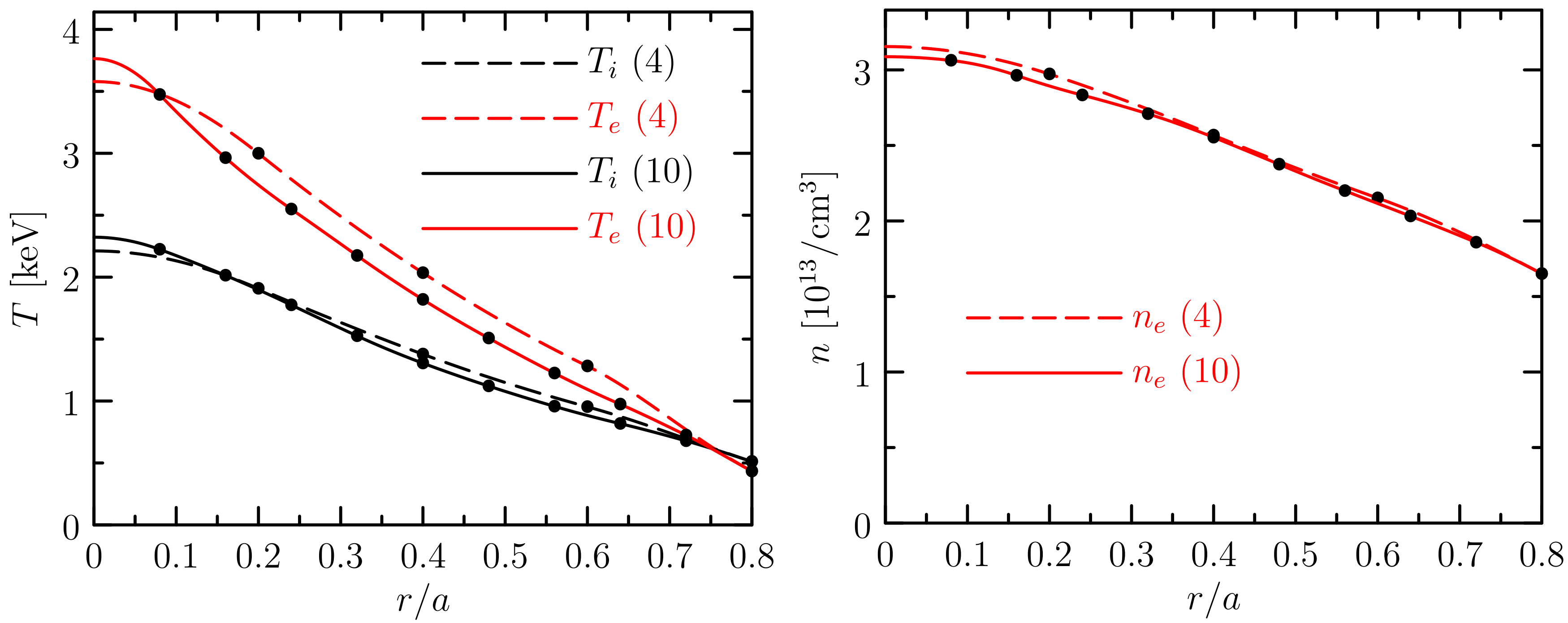
$z_a(x)$ taken to be **piecewise-linear**

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

Rapid profile convergence with number of radial nodes

4 vs 10 radial nodes

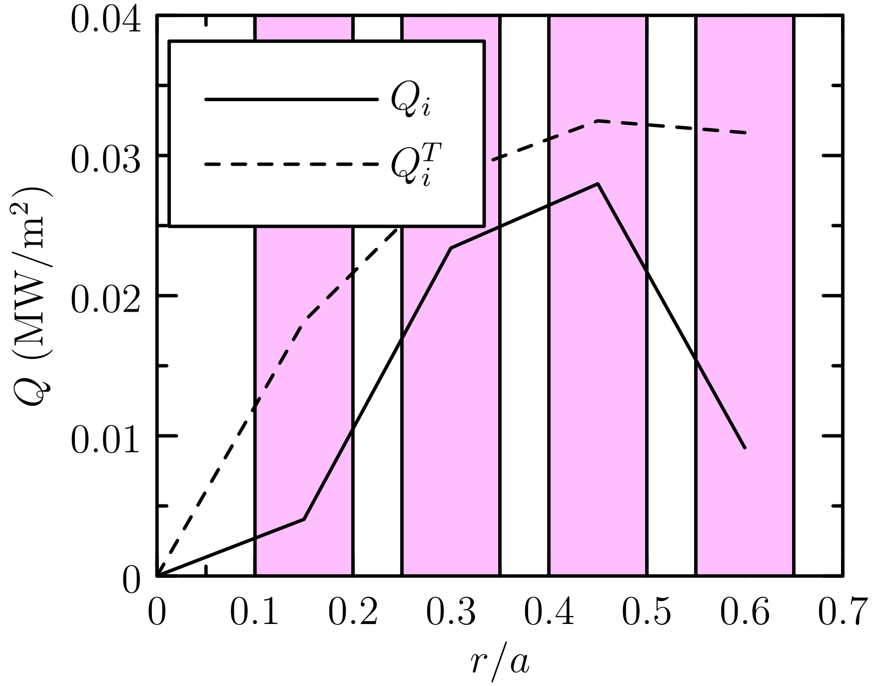


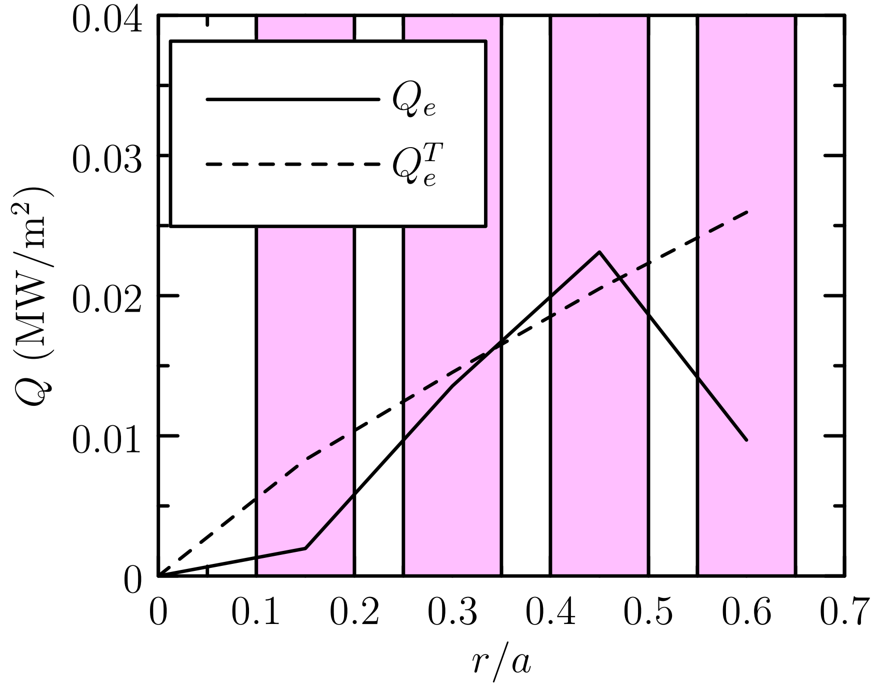
- ▷ Prediction for typical DIII-D L-mode plasma converged with 10 radial nodes.
- ▷ Very good result with **only 4 nodes**!

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 0





Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

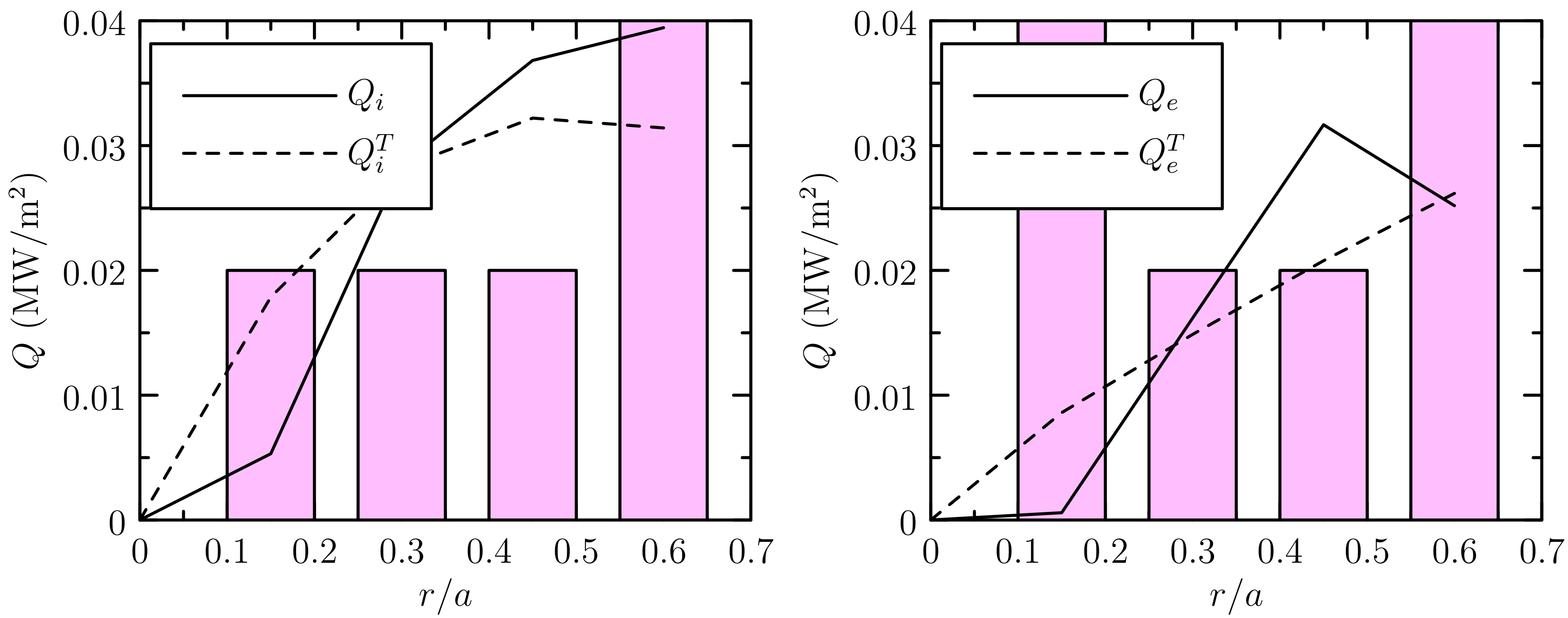
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 1



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

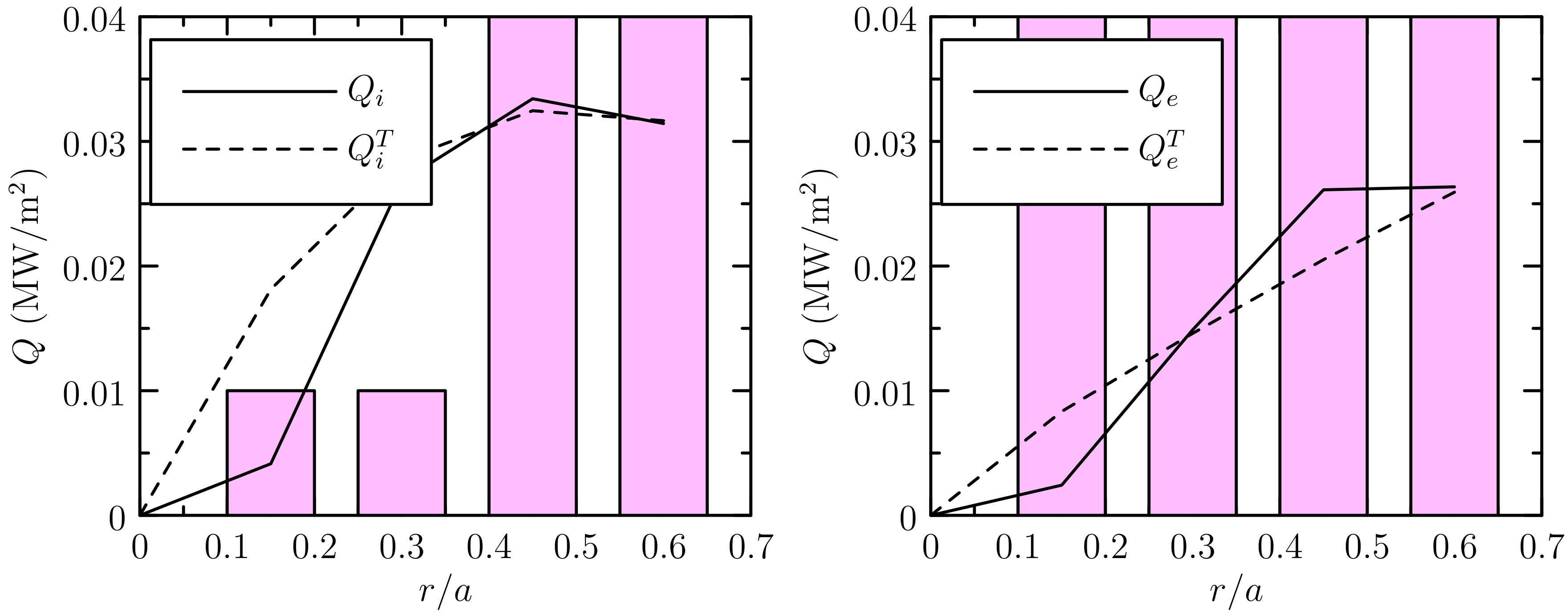
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 2



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

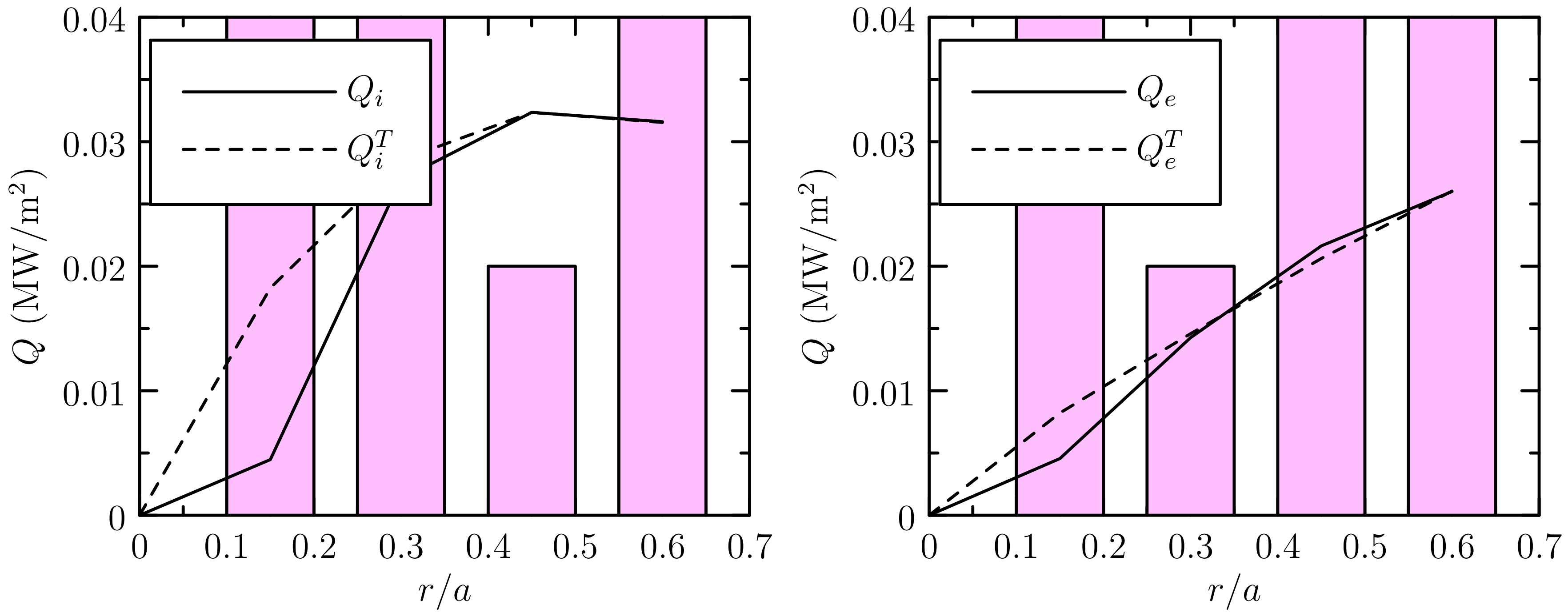
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 3



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

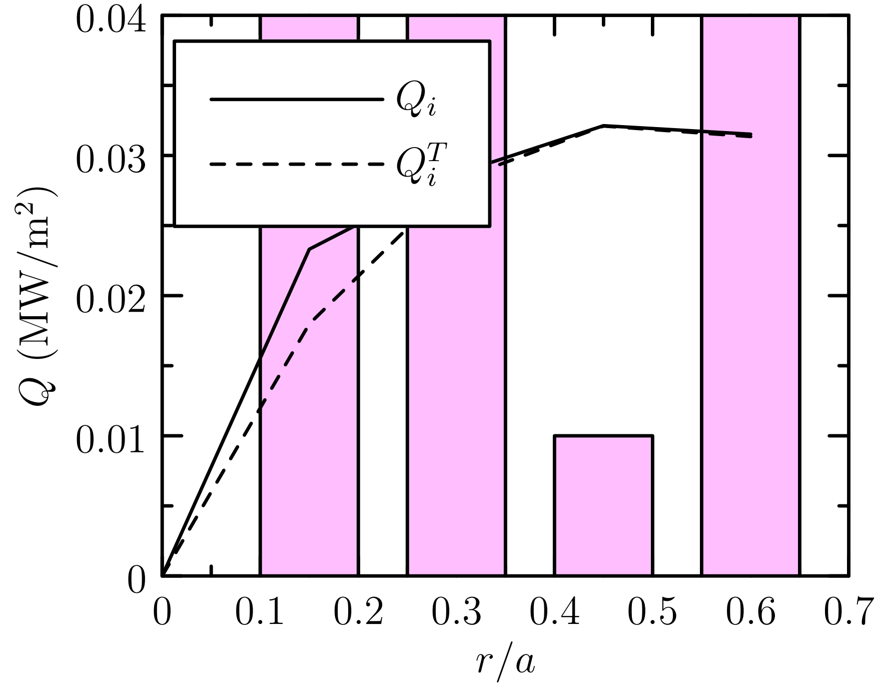
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

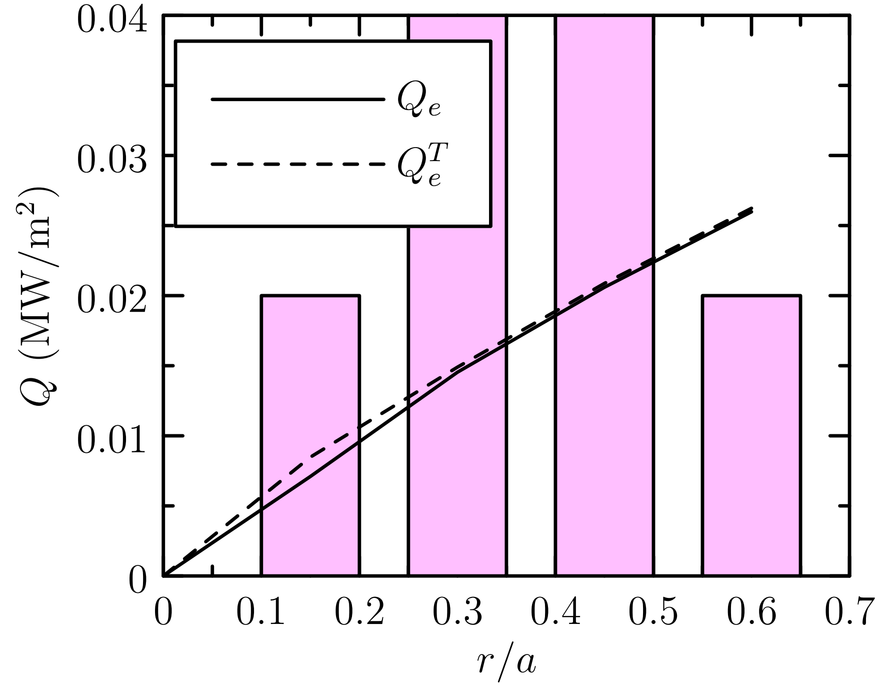
GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 4





Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

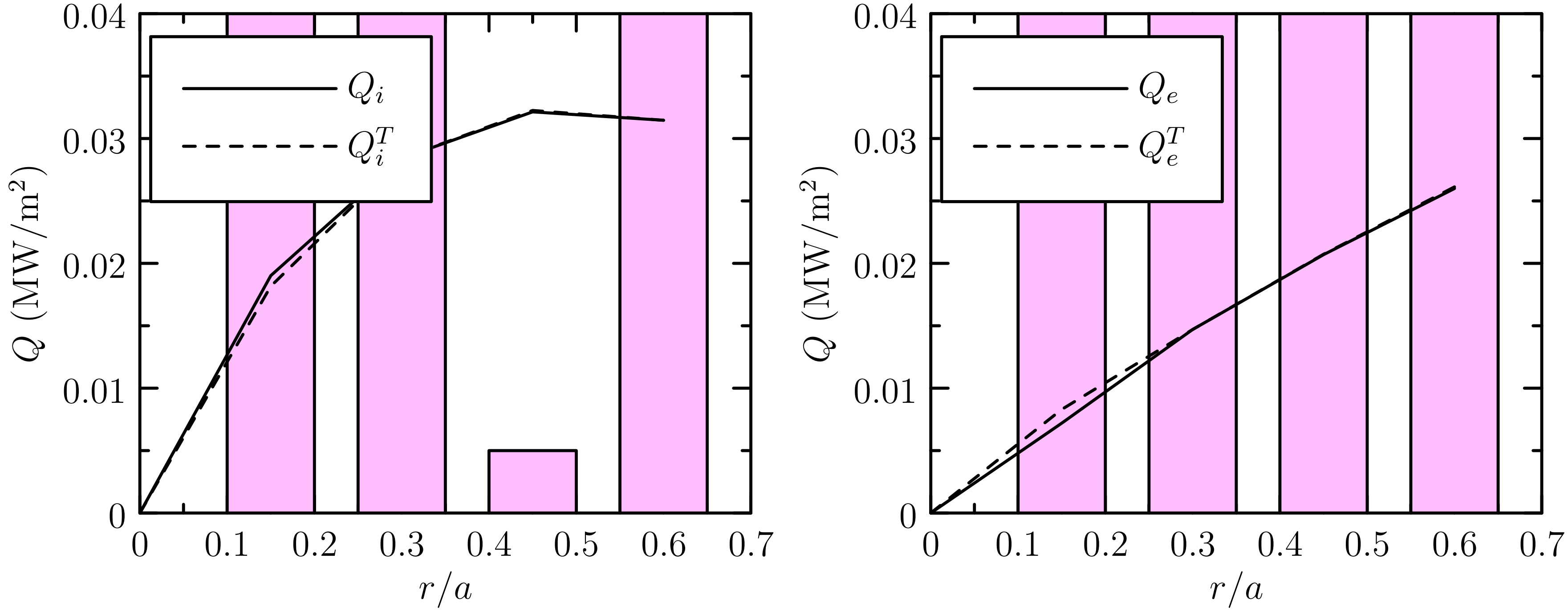
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 5



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

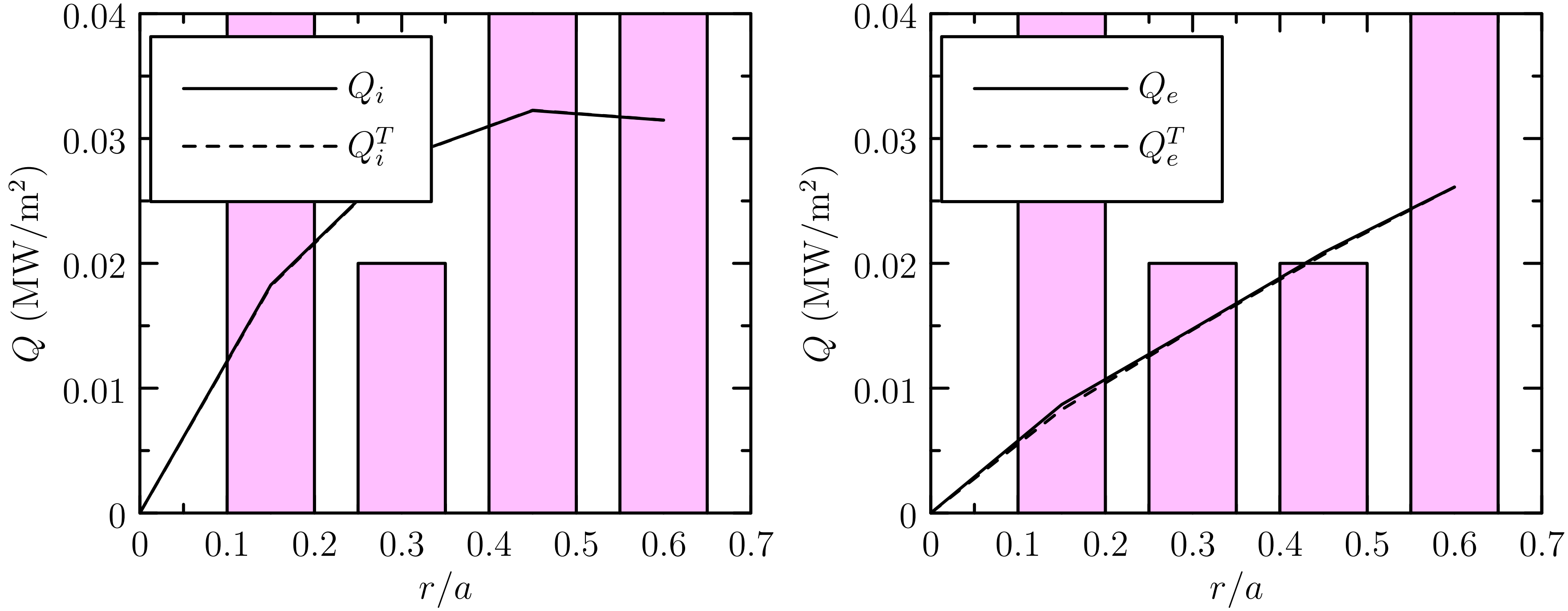
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 6



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

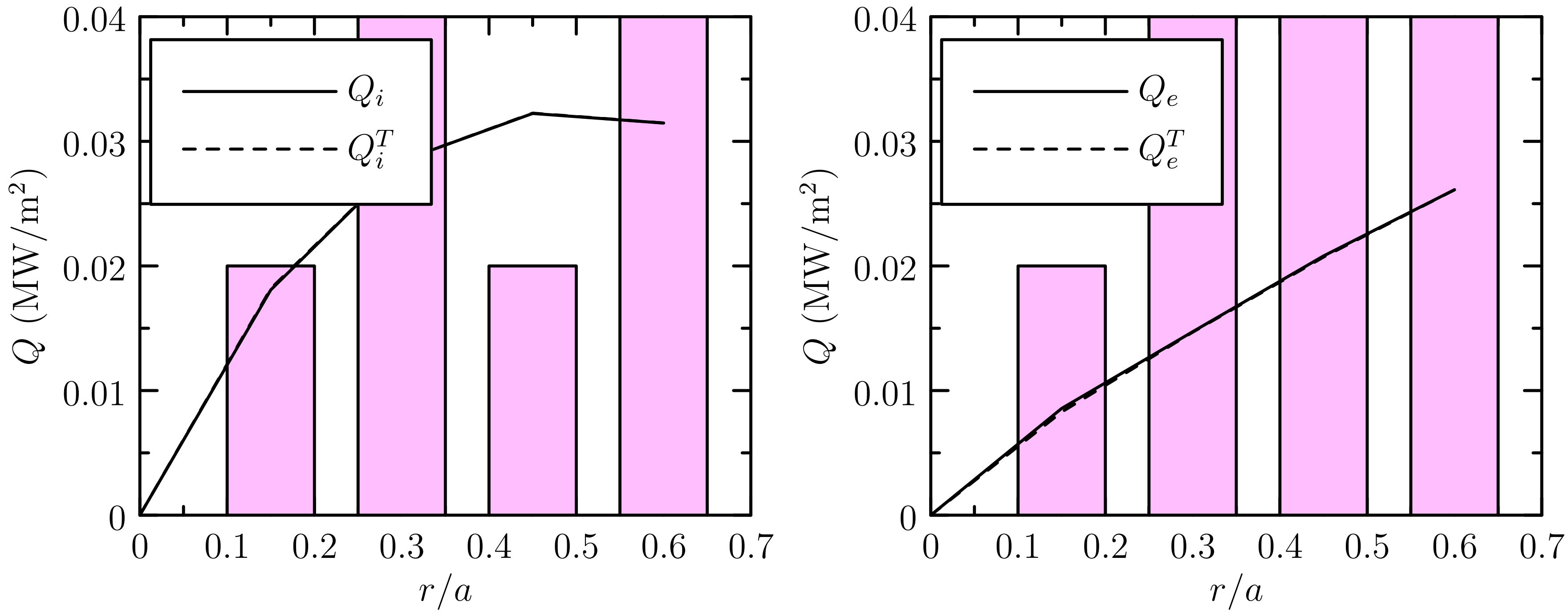
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 7



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

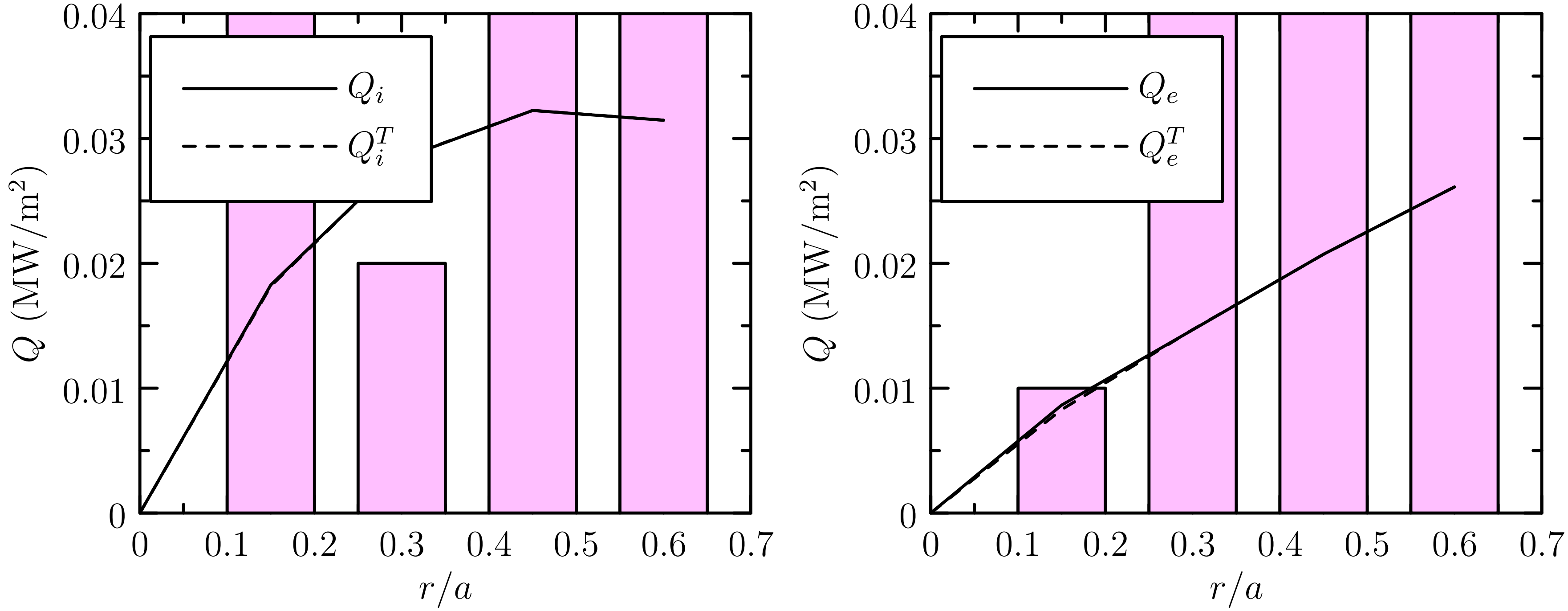
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 8



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

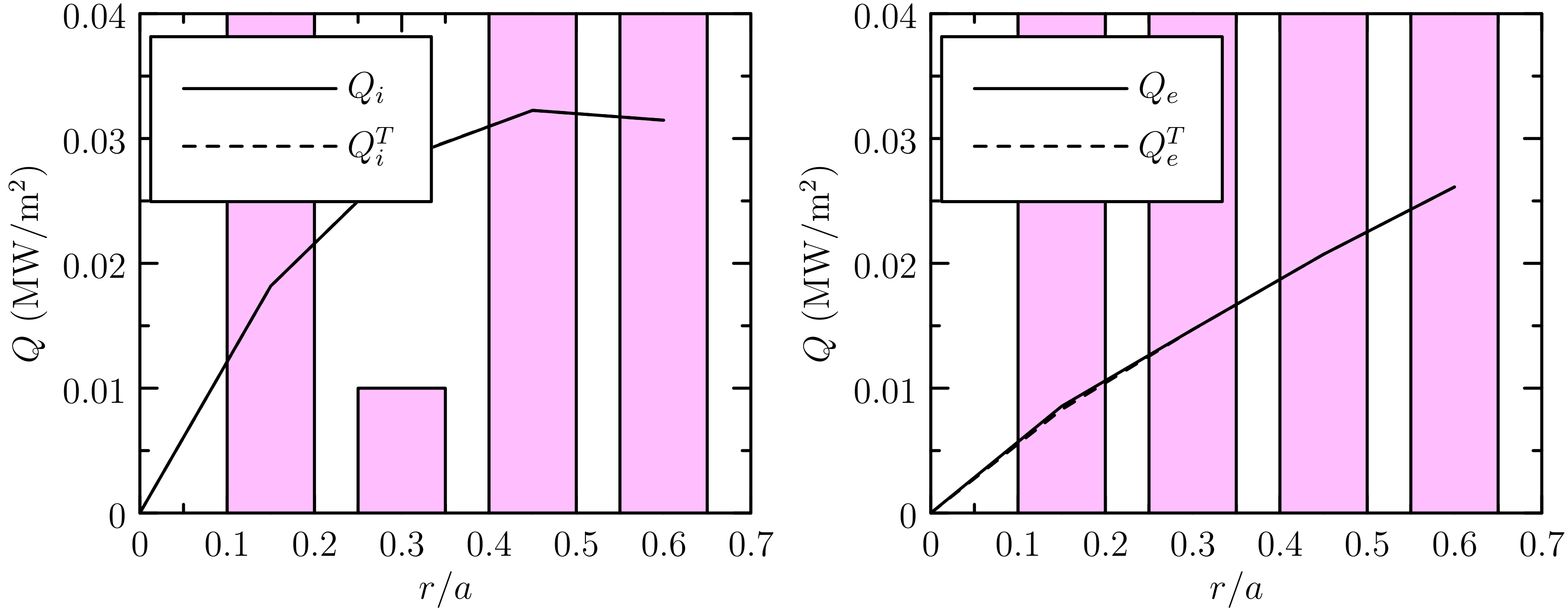
$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

TGYRO-TGLF Test Case (DIII-D L-mode discharge 128913)

4-node case, no density evolution

Iteration 9



Adjust relaxation parameter $\eta_{a,j}$ based on **local residual**

$$R_{a,j} = \sum_{a,j} \left| \widehat{Q}_{a,j} - \widehat{Q}^T_{a,j} \right|$$

GENERAL ATOMICS

Application of Synthetic Diagnostics

Essential component of code-experiment comparisons

- ▷ Experimental diagnostic output is often not directly comparable with simulation data (for example, frequency/wavenumber sensitivity).
- ▷ One solution is to mimic the effect of the diagnostic via a **transfer function** which acts on the raw simulation data to produce **filtered** simulation data.
- ▷ Work still underway to define statistical metrics to quantify the **goodness of agreement** (poster by C. Holland, Thurs. 9:30am, DIII-D section, ID TP8.00012).
- ▷ For complete details:

  C. Holland, A.E. White, G.R. McKee, M.W. Shafer, J. Candy, R.E. Waltz, et al., Phys. Plasmas **16** (2009) 052301.

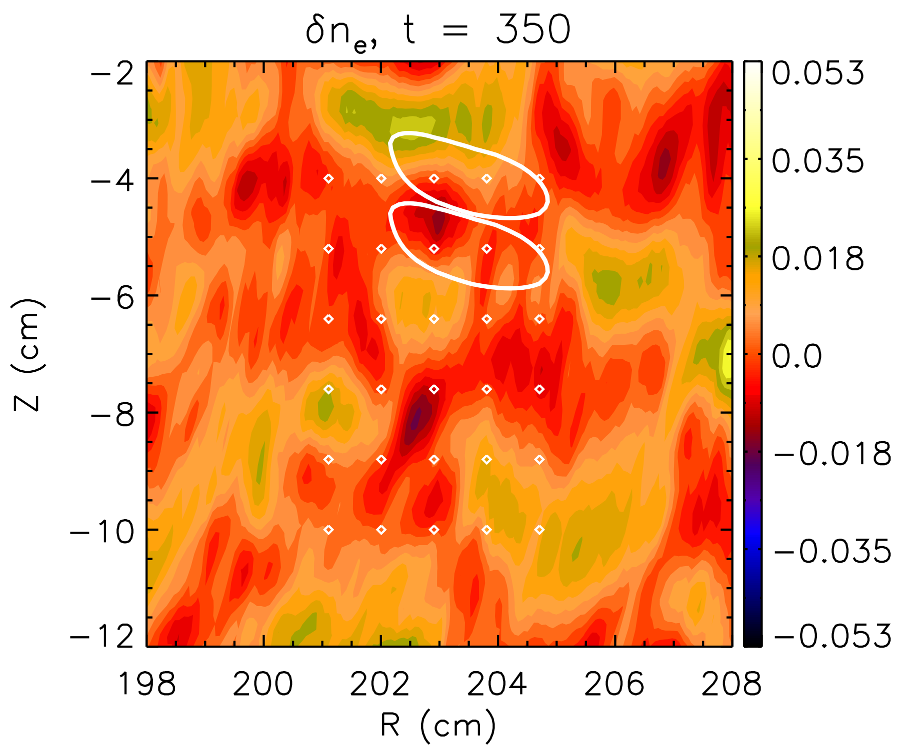
- ▷ In order of complexity, we can perform synthetic diagnostic analysis on

  1. GYRO simulation using **pure experimental input data**.
  2. GYRO simulation based on converged **TGYRO[TGLF+NEO] prediction**.
  3. GYRO simulation based on converged **TGYRO[GYRO+NEO] prediction**.

Synthetic Beam Emission Spectroscopy (BES)

From C. Holland *et al.*, Phys. Plasmas 16 (2009) 052301.

BES measures **local density fluctuations** via 2D array (5×6);
Poloidal channels cross-correlated to improve S/N ratio;
50% **point-spread function** contours shown.



G.R. McKee, R.J. Fonck, D.K. Gupta, D.J. Schlossberg, *et al.*, Plasma Fusion Res. **2**, S1025 2007.

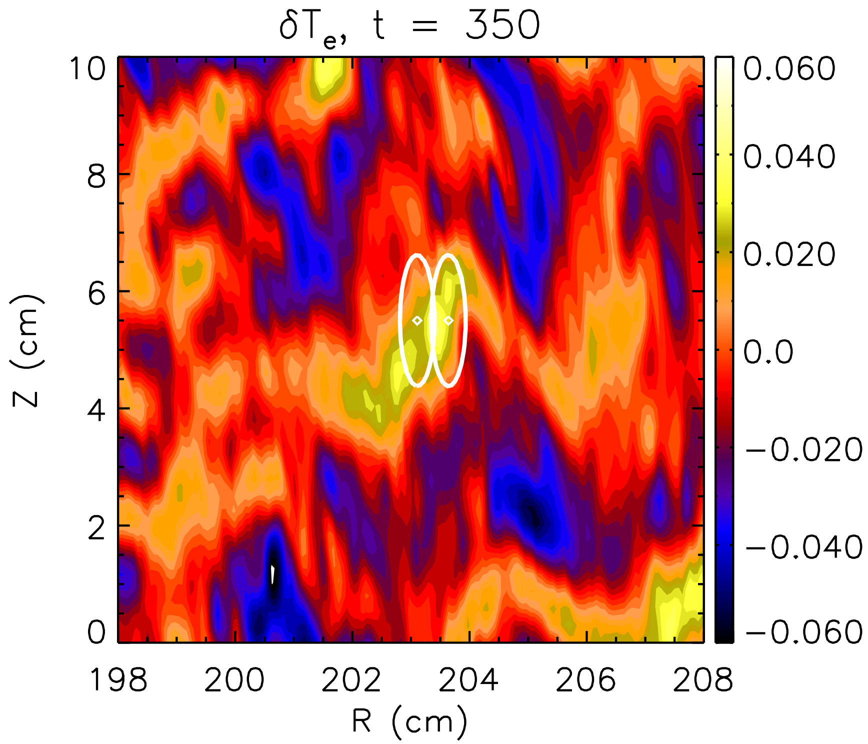
GENERAL ATOMICS

Synthetic Correlation Electron Cyclotron Emission (CECE)

From C. Holland *et al.*, Phys. Plasmas 16 (2009) 052301.

Measures local T_e **fluctuations** via radial cross-correlation;

50% **point-spread function** contours shown.



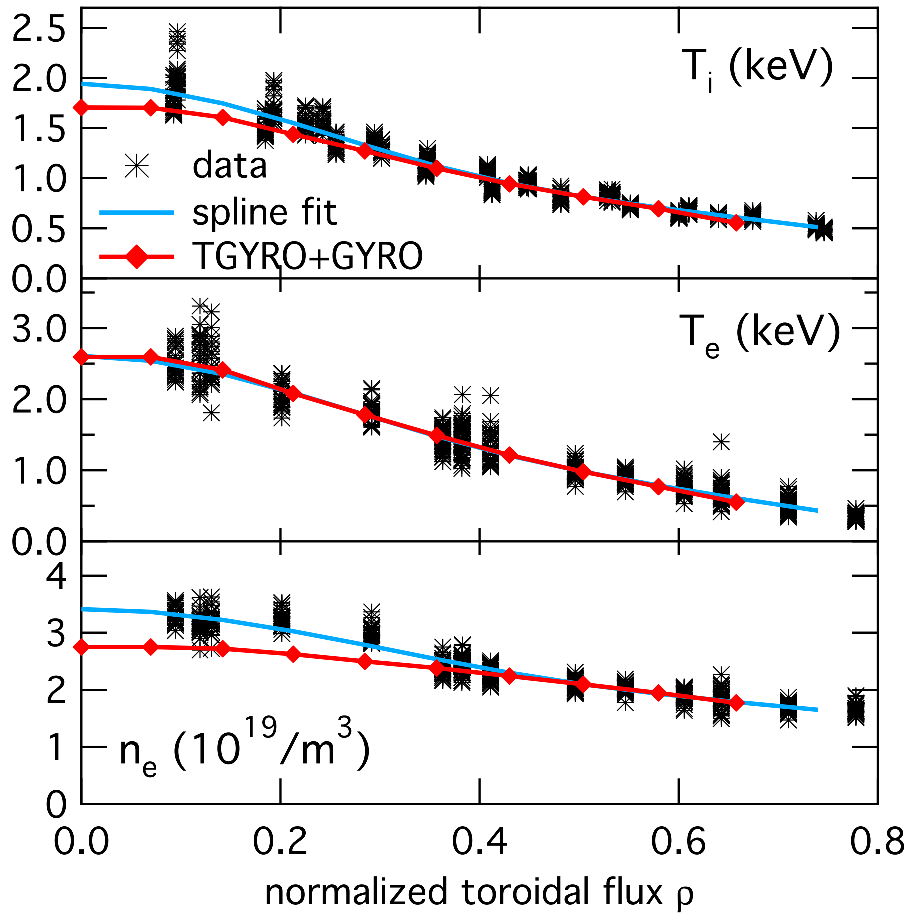
A.E. White, L. Schmitz, G.R. McKee, C. Holland, *et al.*, Phys. Plasmas **15**, 056116 2008.

GENERAL ATOMICS

Measured Versus Simulated Profiles (DIII-D 128913 1500ms)

Plasma core is non-stationary.

GYRO simulation near top of pedestal **does not converge**; **strong ETG regime**

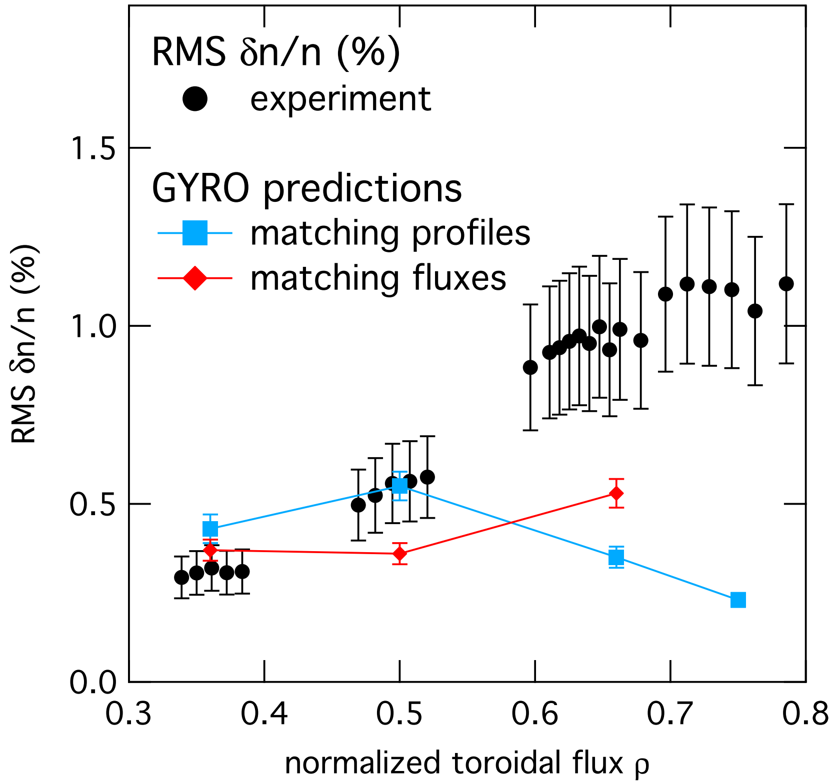


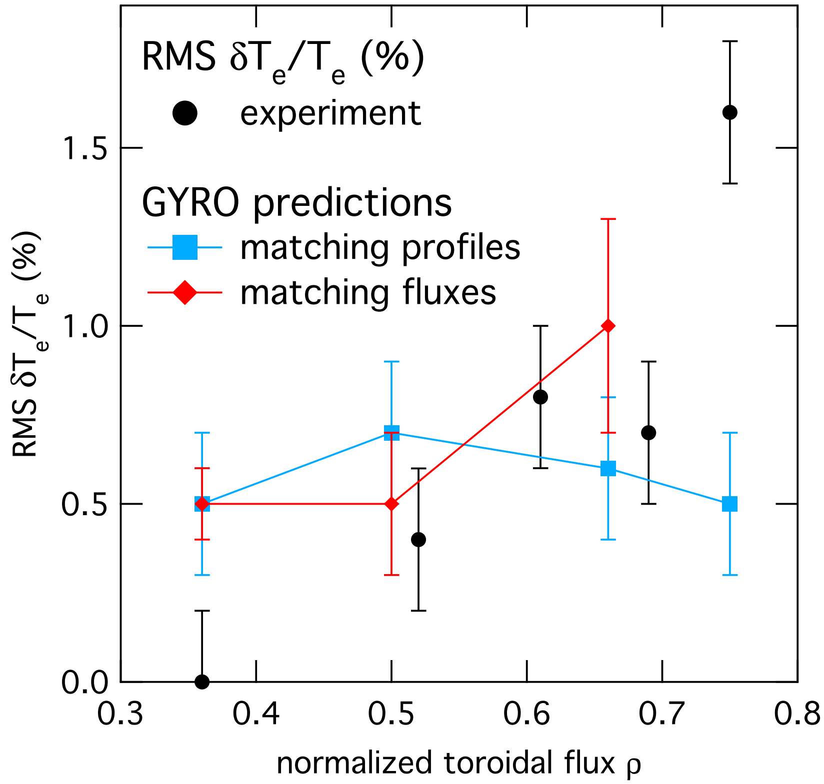
DIII-D 128913 1300-1700 ms

Measured Versus Simulated Fluctuations (DIII-D 128913 1500m

Underprediction previously reported by White, Holland APS07

Fluctuation level sensitive to instability threshold





DIII-D 128913-23 1300-1700 ms

Red diamonds: MATCH POWER **Blue squares: MATCH PROFILES**

Summary

1. TGYRO solves **Fokker-Planck hierarchy** through **4 orders in ρ_***.
2. We have implemented a **novel approach** to solve the transport problem:
   - Alternative to fixed-gradient gyrokinetic/neoclassical simulations.
   - New **integral method** for solving transport equations.
3. Results can be routinely analyzed with **synthetic diagnostic** tools:
   - Critical for in-depth code **validation**.
4. Demonstrated **application of TGYRO** to DIII-D discharge:
   - Observe improved agreement with experimental trends.
   - Perhaps **multi-scale ITG-ETG simulations** are needed in pedestal region.
5. Coming soon: **ITER** core performance projections:
   - Computational cost roughly the same as for DIII-D.

Extra Slides

Fokker-Planck Equation with COLLISIONS

Complete theory derived by Sugama and coworkers

1. H. Sugama and W. Horton, **Neoclassical and anomalous transport in axisymmetric toroidal plasmas with electrostatic turbulence**, Phys. Plasmas **2** (1995) 2989.

2. H. Sugama, M. Okamoto, W. Horton and M. Wakatani, **Transport processes and entropy production in toroidal plasmas with gyrokinetic electromagnetic turbulence**, Phys. Plasmas **3** (1996) 2379.

3. H. Sugama and W. Horton, **Transport processes and entropy production in toroidally rotating plasmas with electrostatic turbulence**, Phys. Plasmas **4** (1997) 405.

4. H. Sugama and W. Horton, **Neoclassical electron and ion transport in toroidally rotating plasmas**, Phys. Plasmas **4** (1997) 2215.

5. H. Sugama and W. Horton, **Nonlinear electromagnetic gyrokinetic equation for plasmas with large mean flows**, Phys. Plasmas **5** (1998) 2560.

GENERAL ATOMICS

Fokker-Planck Transport Theory

Transport of Density on $\mathcal{O}(\rho_*^2)$ timescale

Γ_a^{GV}	Gyroviscous particle flux
$\Gamma_a^{\mathrm{neo}} = \left\langle \int d^3v \, \bar{g}_a W_{a1} \right\rangle$	Neoclassical particle flux
$\Gamma_a^{\mathrm{tur}} = \left\langle \left\langle \int d^3v \, \hat{h}_a \hat{W}_{a1} \right\rangle \right\rangle$	Turbulent particle flux density
$S_{n,a}^{\mathrm{beam}}$	Beam density source rate
$S_{n,a}^{\mathrm{wall}}$	Wall density source rate

Fokker-Planck Transport Theory

Transport of Energy on $\mathcal{O}(\rho_*^2)$ Timescale

$Q_a^{\rm GV}$	Gyroviscous energy flux
$Q_a^{\rm neo} = \left\langle \int d^3v \, \bar{g}_a \, \frac{\varepsilon}{T_a} W_{a1} \right\rangle$	Neoclassical energy flux
$Q_a^{\rm tur} = \left\langle \left\langle \int d^3v \, \hat{h}_a \, \frac{\varepsilon}{T_a} \hat{W}_{a1} \right\rangle \right\rangle$	Turbulent energy flux
$S_{W,a}^{\rm aux}$	Auxiliary heating power density
$S_{W,a}^{\rm rad}$	Radiation heating power density
$S_{W,a}^{\alpha}$	Alpha heating power density
$S_{W,a}^{\rm tur} = \left\langle \left\langle \int d^3v \, \hat{h}_a W_{aT} \right\rangle \right\rangle$	Turbulent exchange power density
$S_{W,a}^{\rm col}$	Collisional exchange power density

Fokker-Planck Transport Theory

Transport of Toroidal Angular Momentum on $\mathcal{O}(\rho_*^2)$ Timescale

Π_a^{GV}	Gyroviscous angular momentum flux
$\Pi_a^{\mathrm{neo}} = \left\langle \int d^3v \, \bar{g}_a W_{aV} \right\rangle$	Neoclassical angular momentum flux
$\Pi_a^{\mathrm{tur}} = \left\langle \left\langle \int d^3v \, \hat{h}_a \hat{W}_{aV} \right\rangle \right\rangle$	Turbulent angular momentum flux
$S_{\omega,a}$	Angular momentum density source rate