# Comparison of RMW Stabilization Strategies in DIII-D

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## Introduction

One of the major non-axisymmetric instabilities under study in the DIII-D tokamak is the resistive wall mode (RWM), a form of plasma kink instability whose growth rate is moderated by the influence of a resistive wall. The General Atomics/Far-Tech DIII-D/RWM dynamic model represents the plasma surface as a toroidal current sheet and represents the wall using an eigenmode approach. This dynamic model is intended to be used for the design of model-based controllers that have the potential of outperforming present PD (proportional-derivative) controllers. A required step previous to the potential implementation in the PCS (Plasma Control System) of any model-based controller is the experimental validation and reconciliation of the proposed dynamic model, which is reported in this study. In addition, simulation results are presented comparing the performance of advanced controllers synthesized using the validated dynamic model and present non-model-based PD controllers.

## RWM Dynamic Model

The RWM model is given in state space form with all parameters known except the growth rate ($\gamma$) related through $c_{pp}$.

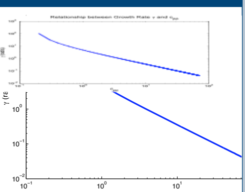
$$\dot{x} = Ax + Bu \qquad y = Cx$$
$$A = \left(M_{ss} - M_{sp}c_{pp}M_{ps}\right)^{-1} R_{ss} \qquad C = C_{ss} - C_{yp}c_{pp}M_{ps}$$
$$B = \left(M_{ss} - M_{sp}c_{pp}M_{ps}\right)^{-1}$$

| | | | |
|---|---|---|---|
| $x$ | Wall/Coil Currents | $M_{sp}$ | Mutual Inductance Wall-Coils/Plasma |
| $u$ | Control Coil Voltages | $M_{ps}$ | $M_{sp}$ transpose |
| $y$ | Sensor Outputs | $C_{ss}$ | Coupling b/t Sensor and Wall/Coils |
| $M_{ss}$ | Mutual Inductance Vessel/ Coils | $C_{yp}$ | Coupling b/t Sensor and Plasma |
| $R_{ss}$ | Wall Modes Coupling/ Coil Resistance | | |

- Model assumes mode rigidity as mode grows (far from ideal-wall limit).
- The plasma is represented by the sine and cosine phase components of a "single" mode, which results in many stable system eigenvalues and two unstable system eigenvalues associated with the mode phase components.
- The coupling of the two components of the plasma mode with the conductors is assumed identical and the diagonal values of the $C_{pp}$ matrix are the same ☒ model finally parameterized by a single scalar value $c_{pp}$.
- System is inherently unstable for all growth rates. More unstable for larger growth rates (smaller $c_{pp}$).

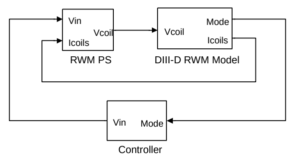
## Design Goal

- To design a controller that stabilizes the system over a large range of growth rate values.
- Model validation is a key first step.
- Controller must achieve performance requirements over a large range of growth rate values.
- Physical growth rate ($\gamma$) range: 10-5,000 rad/sec
- Corresponding $c_{pp}$ range: 0.3325-71




- Magnetic sensors. Radially directed flux: 6 midplane and 6 upper and lower magnetic loops (or saddle loops). Poloidally directed field ($B_p$): 4 midplane and 6 upper and lower magnetic probes. Signals ☒ matched filter ☒ sine and cosine components of the mode (2 outputs).
- Actuators. 12 internal feedback control coils (I-coils) in quartet configuration, i.e., locking the phase of I-coils in sets of four (3 inputs).
- The growth rate ($c_{pp}$) can be treated as an uncertain parameter that acts as a perturbation to a nominal system. Advanced control techniques may be used to synthesize a single controller that achieves stability over a pre-defined range of $c_{pp}$.

## Simulink Model Validation

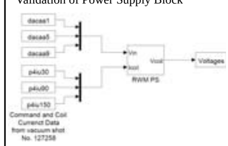



The simulation model is basically composed by three parts: Power Supply, DIII-D RWM Model and Controller.

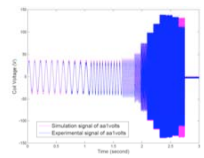
The validation results for the Power Supplies and DIII-D RWM Model shown below were obtained with data from a vacuum shot, while the results for the PD controller were obtained using data from a plasma shot.

## Model Validation Results

### Validation of Power Supply Block

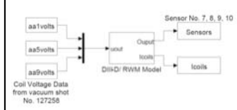



Validation of power supply with vacuum shot No. 127258. The "RWM PS" block represents the power supplies. Vin represents the command from the controller and Icoil is the coil current feedback coming from the DIII-D/ RWM Model. Vcoil is the output port for coil voltage.

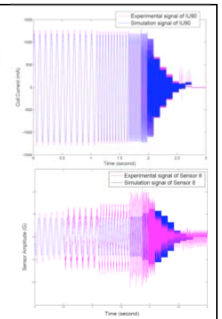



The figure above shows the comparison between simulation results and experimental data from vacuum shot No. 127258 for aa1volts (one of the three coil voltages).

### Validation of DIII-D/ RWM Model Block

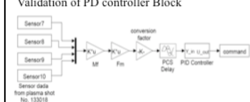



Validation of DIII-D/ RWM model with vacuum shot No. 127258. Coil voltage signals from shot No. 127258 are used. Port "Output" represents a set of 23 sensor signals. Port "Icoils" represents the last three of the system states.

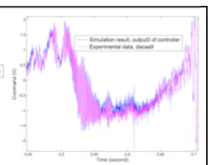



The two figures on the right show the comparison between simulation results and experimental data from vacuum shot No. 127258. The top figure shows the comparison for coil current IU90 (one of the four coil currents). The bottom figure shows the comparison for sensors No. 8.

### Validation of PD controller Block




Validation of PD controller with Plasma Shot No. 133018. The "PID Controller" block represents the controller. The inputs are the signals of sensors 7, 8, 9, 10 obtained from shot No. 133018. The output of the controller block is the command for the power supply.




The figure above shows the comparison for command dacca9 (one of the three commands).

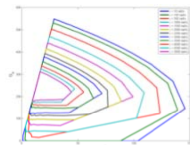
## PD Controller Design

A proportional-derivative (PD) controller is synthesized (integral action is not required for this system) to maximize the stability range as a function of growth rate ☒. The PD controller is of the form

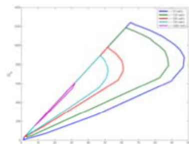
$$K = \frac{1}{1+\tau_P \cdot s}\left(G_P + G_D\frac{\tau_D \cdot s}{1+\tau_D \cdot s}\right)$$

where $G_P$ is the proportional gain, $G_D$ is the derivative gain. The relationship between $G_P$ and $G_D$ is $G_D$ = N ☒ $G_P$ where the integer N may varies between 1 and 20. The variables ☒$_P$ and ☒$_D$ represent time constants for the proportional and derivative filters (the validation of this controller model with experimental plasma shot is demonstrated at the bottom of the "Model Validation Results" column).

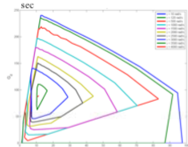
## Simulation Results for PD Control



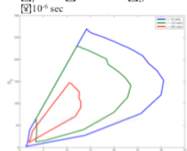
Contour of $G_P$ and $G_D$ stabilizing ☒ for ☒$_P$ = 50 ☒10$^{-6}$ sec and ☒$_D$ = 50 ☒10$^{-6}$ sec



Contour of $G_P$ and $G_D$ stabilizing ☒ for ☒$_P$ = 500 ☒10$^{-6}$ sec and ☒$_D$ = 50 ☒10$^{-6}$ sec



Contour of $G_P$ and $G_D$ stabilizing ☒ for ☒$_P$ = 50 ☒10$^{-6}$ sec and ☒$_D$ = 150 ☒10$^{-6}$ sec



Contour of $G_P$ and $G_D$ stabilizing ☒ for ☒$_P$ = 500 ☒10$^{-6}$ sec and ☒$_D$ = 150 ☒10$^{-6}$ sec

Stabilization criteria: the sensor outputs can be stabilized to lower than 0.25 G within 10 ms (initial condition is set according to plasma shot No. 133021). When ☒$_P$ and ☒$_D$ increase, the range of ☒ that can be stabilized becomes narrow and the stability window gets smaller.

## Advanced Optimal Control Design Based on Validated Model

The time-domain representation of our system is given by

$$\dot{x} = Ax + Bu + w_1$$
$$y = Cx + Du + w_2$$

where the noise effect is included; $w_1$ represents the process noise and $w_2$ represents the measurement noise. The noises are assumed to be zero-mean with covariances $Q=E(w_1 w_1^T)$ and $R=E(w_2 w_2^T)$ where E denotes the expectation operator.

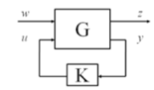
Besides achieving closed-loop stability, we are interested in designing a control law $u = Ky$ that minimizes the mode amplitude $y$ and the control power $u$ (contributing to actuator saturation avoidance), i.e.,

$$\min_K J = \frac{1}{2}\int_0^\infty \left(y^T Q_y y + u^T R_u u\right)dt$$

where $Q_y$ and $R_u$ are semi-positive and positive definite matrices defined by the designer. This a well known problem in the field of controls, and its solution is provided by Optimal Control Theory.

## Advanced Robust Control Design Based on Validated Model

By using the Laplace transform, we obtain a frequency-domain representation of our system given by




$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s)\begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix}$$

Noise signals $w_1$ and $w_2$ are grouped into $w = [w_1^T\ w_2^T]^T$. As shown in the figure, the output $y$ is used by the controller $K(s)$ to calculate the input $u$. The performance output $z$ represents a weighted function of the control power $u$ and the mode amplitude $y$ that we want to minimize. We are interested in synthesizing a stabilizing controller $K$ such that the H☒ norm (maximum energy amplification) of the transfer function $T_{zw}(G;K)$ between input $w$ and output $z$ is minimized (the mode amplitude $y$ and control power $u$ are minimized for any input $w$), i.e.,

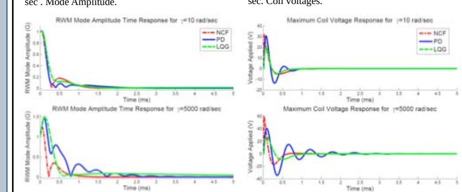
$$\min_K \left\| T_{zw}(G,K) \right\|_\infty = \min_K \left( \sup_\omega \bar{\sigma}\left(T_{zw}(G,K)(j\omega)\right) \right)$$

where $\bar{\sigma}$ denotes the maximum singular value and ☒ the frequency. This a well known problem in the field of controls, and its solution is provided by Robust Control Theory.

## Comparison of Simulation Results: LQG, NCF and PD Controllers

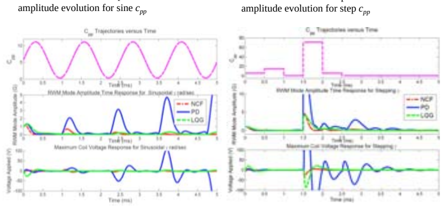
Initial condition response: RWM mode amplitude evolution for ☒=10 rad/sec and ☒=5,000 rad/sec . Mode Amplitude.

Initial condition response: RWM mode amplitude evolution for ☒=10 rad/sec and ☒=5,000 rad/sec. Coil voltages.




Initial condition response: RWM mode amplitude evolution for sine $c_{pp}$

Initial condition response: RWM mode amplitude evolution for step $c_{pp}$




## Conclusions

- Excellent agreement between vacuum shot ($c_{pp}$=0) data and model-based simulation results.
- The model-based prediction of the PD controller performance for plasmas shots ($c_{pp}$☒0) shows similar trend to what observed in recent experiments. Further comparison and validation is necessary.
- Initial results on advanced control design based on the validated model are promising. More work is necessary, particularly on the handling of the delays imposed by the power supplies and the plasma control system.