Prediction of Sawtooth Periods in Fast-Wave Heated DIII-D Experiments Using Extensions of the Porcelli Model

by
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with
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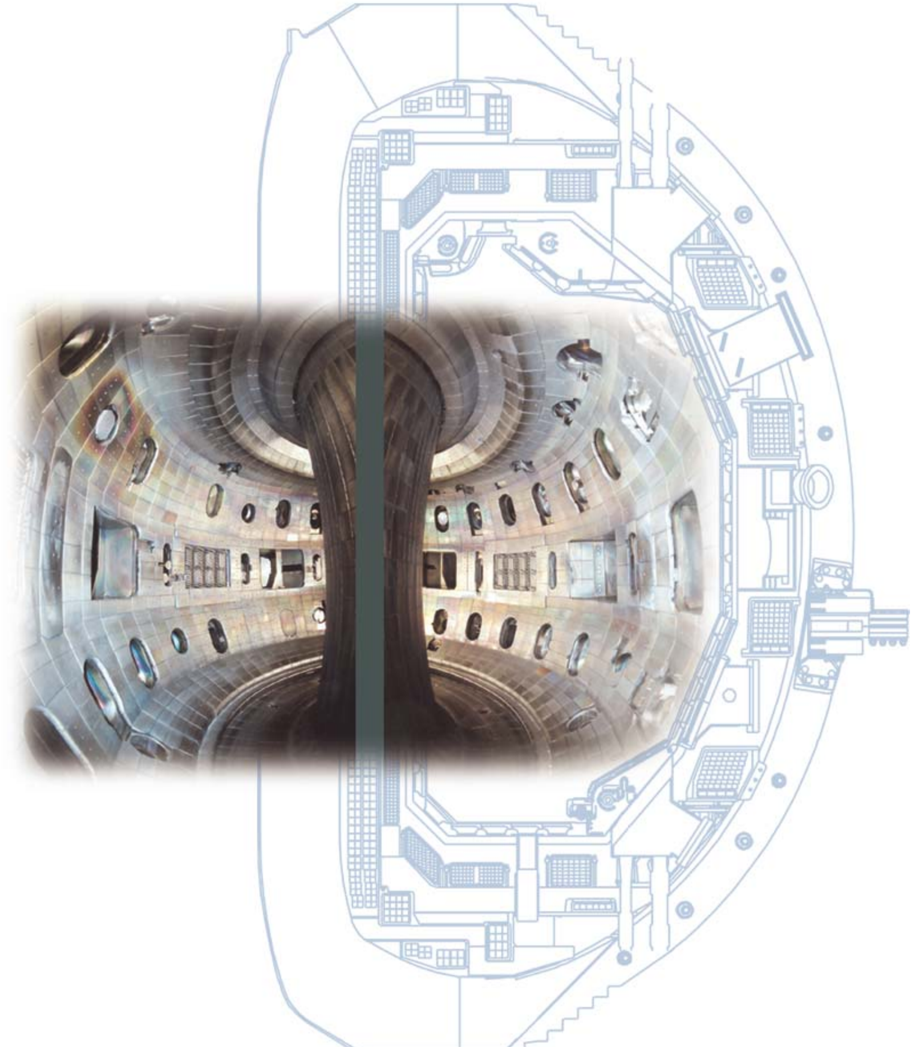

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Kinetic Fast Ion Pressure Can Stabilize Ideal Internal Kink Mode Involved in Sawteeth

- **Experiments with large fast ion pressure exhibit long sawtooth periods followed by a giant sawtooth:**
  ⇒ Interpreted as stabilization of ideal internal kink by fast ion kinetic effects

Porcelli model developed for deciding when a sawtooth is triggered

– Contributions from ideal and kinetic effects from fast and thermal ions

– Intended to be used in conjunction with 1 1/2 D transport models:
  - Model is semi-phenomenological and ad-hoc
  - Original form utilizes simplified models for the individual contributions

- **Porcelli model successfully predicts qualitative sawtooth behavior in JET, TCV, and ASDEX experiments:**
  – Average sawtooth periods in transport simulations
  – Changes in sawtooth period with evolving discharge conditions

Can the Porcelli Model be used quantitatively to predict a specific sawtooth crash in an actual discharge?

Porcelli Model Can Account For Change in Sawtooth Behavior and Predict the Giant Sawtooth Period

- **We quantitatively tested the Porcelli model in DIII-D using:**
  - Reconstructed DIII-D discharge equilibria from several times within a sawtooth cycle
  - More realistic numerical models for the individual contributions

- **Porcelli sawtooth crash criteria evaluated over first giant sawtooth cycle reproduces the time of crash quantitatively:**
  - Discharge equilibria reconstructed from EFIT
  - Numerically calculated γ_{ideal} from GATO for the reconstructed equilibria
  - Numerically calculated pressure from:
    - o ORBIT-RF using non-Maxwellian ion distribution with finite orbits (**ORBIT** : R.White, Phys. Plasma 2 (1995)) and
    - o RF wave field from TORIC (M. Brambilla, Plasma Phys. Control. Fusion 41 (1999))

Key is accurate evaluation of the discharge equilibria

The Porcelli Sawtooth Trigger Model Is Based on the Linear Stability Threshold Against the 1/1 MHD Mode

Basic Model: unstable ideal kink stabilized by kinetic effects

Condition for instability of the ideal internal kink mode: $\delta\hat{W}_{MHD} < 0$

Ideal internal kink mode stabilized by $\delta\hat{W}_{Th} > 0 \ and$ $\delta\hat{W}_{fast} > 0$

Sawtooth instability is triggered when one of three criteria is satisfied:

((Porcelli, Plasma Phys. Control. Fusion 38 (1996))

$$\delta\hat{W} = \delta\hat{W}_{MHD} + \delta\hat{W}_{Th} + \delta\hat{W}_{fast}$$

Ideal mode not stabilized by fast ions → $-(\delta\hat{W}_{MHD} + \delta\hat{W}_{Th}) > c_h \omega_{Dh} \tau_A$ **Fast trapped ion effect**

Ideal mode not stabilized by fast ion+ion diamagnetic frequency → $-\delta\hat{W} > 0.5\omega_{*i}\tau_A$ **Diamagnetic effect**

Modified Ideal Mode

Non-ideal resistive or ion kinetic effects → $\left\{ \begin{array}{l} -c_\rho \hat{\rho} < -\delta\hat{W} < 0.5\omega_{*i}\tau_A \\ s_1 \equiv r_1 \left.\dfrac{dq}{dr}\right|_{r_1} > s_{crit} \end{array} \right\}$ **Nonideal layer effect**

Resistive/ Ion-kinetic Mode

Porcelli Model Evaluated Using Ideal Contribution to Perturbed Energy and Numerical Fast Ion Pressure

$$\delta\hat{W} = \delta\hat{W}_{MHD} + \delta\hat{W}_{Th} + \delta\hat{W}_{fast}$$

Ideal MHD:

GATO

$$\delta\hat{W}_{MHD} = -\gamma_I \tau_A$$

γ_I = Numerically calculated growth rate for n=1

Trapped Thermal ion:

$$\delta\hat{W}_{Thermal} = \delta\hat{W}_{KO}$$
$$= 0.6 c_p \varepsilon_1^{1/2} \beta_{i0} / s_1$$
$$c_p = (5/2)\int_0^1 x^{3/2} p_i(x)/p_{i0}$$
$$x = r/x_1$$

Kruskal-Oberman

Valid if $\delta\hat{W}_{KO} > \omega_{*i}\tau_A$

Trapped Fast ion :

ORBIT-RF

$$\delta\hat{W}_{fast} = c_f \varepsilon_1^{3/2} \beta_{ph} / s_1$$

$$\beta_{ph} = -\frac{2\mu_0}{B_p^2}\int_0^1 x^{\frac{3}{2}} \frac{dp_h}{dx} dx$$

$$= -\frac{2\mu_0}{B_p^2}\left[p(\psi_1) - \frac{\int_0^{\psi_1} \psi^{-\frac{1}{4}} p_h d\psi}{\int_0^{\psi_1} \psi^{-\frac{1}{4}} d\psi} \right]$$

$$p_h = \frac{2}{3}\sum_i (E_i - e\phi_i) \qquad c_f \approx 1$$

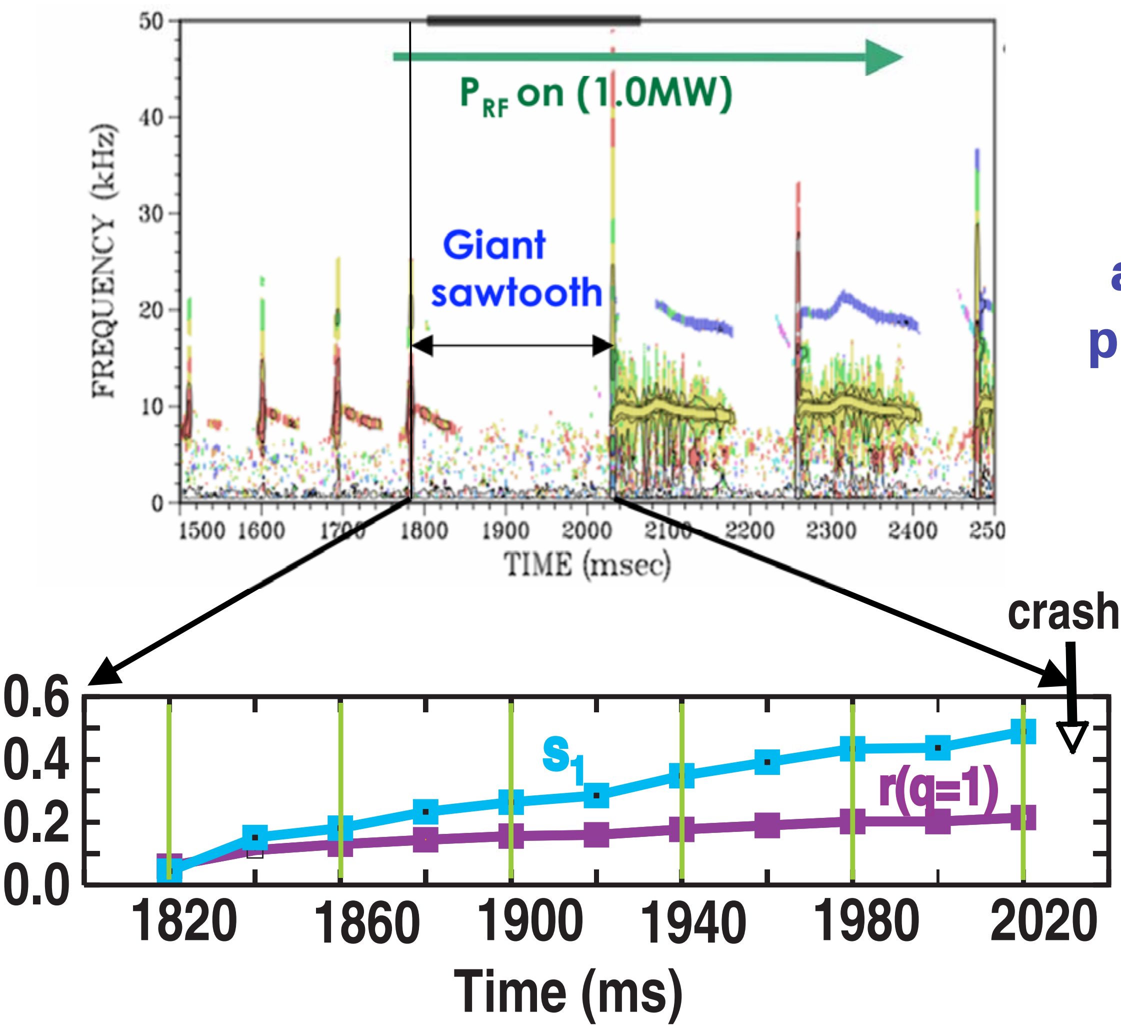
$$\varepsilon_1 \equiv r_1/R \qquad s_1 \equiv r_1 \left.\frac{dq}{dr}\right|_{r_1}$$

Original Porcelli model used:

$$\begin{cases} \delta\hat{W}_{MHD} & \text{from Bussac analytic internal kink model} \\ \delta\hat{W}_{fast} & \text{from model for fast ion pressure } p_h \end{cases}$$

Fast Wave Accelerates Beam Ions In DIII-D Discharge #96043 Leading to Modified Sawtooth Behavior

DIII-D 96043: 60 MHz Fast Wave at $4\Omega_D$



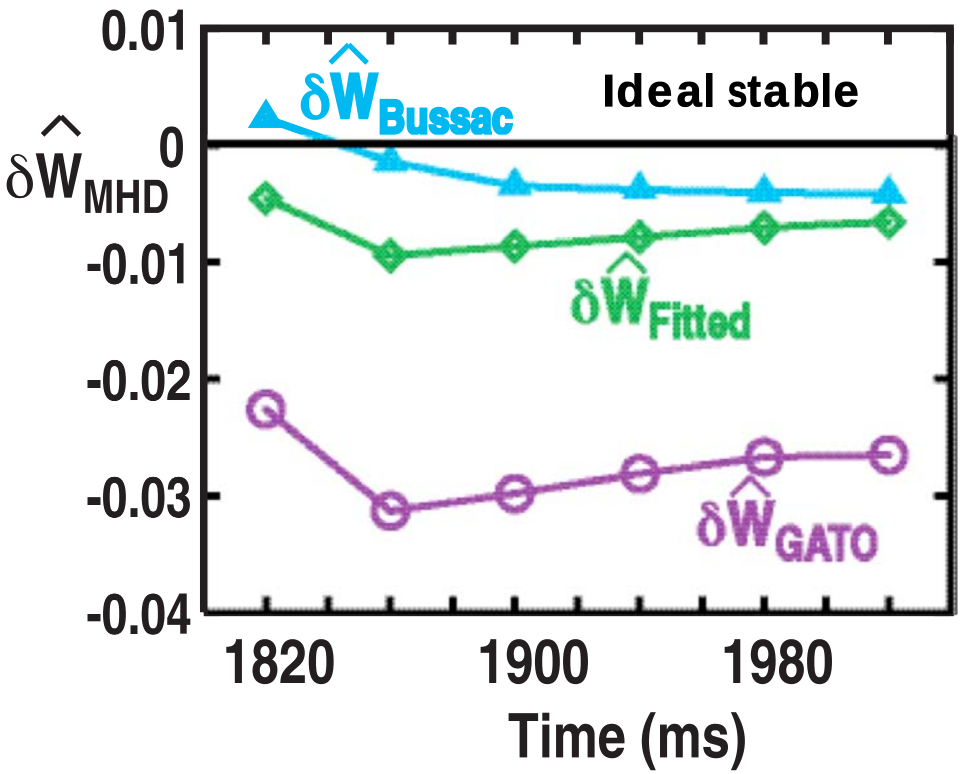
Conventional interpretation:

Fast ions stabilize ideal kink
Sawtooth is delayed and occurs after internal energy has built up, producing a delayed, much larger "giant sawtooth"

Porcelli sawtooth crash criteria are evaluated for discharge equilibria every 40 msec during the first giant sawtooth cycle

Ideal and Fast Particle Contributions to Porcelli Model Numerically Calculated for Reconstructed Equilibria

- GATO ideal result more unstable than analytic or numerically fitted formulas normally used in Porcelli model



- **Fast particle contribution from numerically computed fast ion pressure:**
- **TORIC code calculates**

$$E_{+}^{m}(\rho) \quad k_{//}^{m} \quad k_{\perp}^{m}$$

- **Resonant kicks from RF wave are modeled In ORBIT-RF using stochastic quasi-linear diffusion operator:**

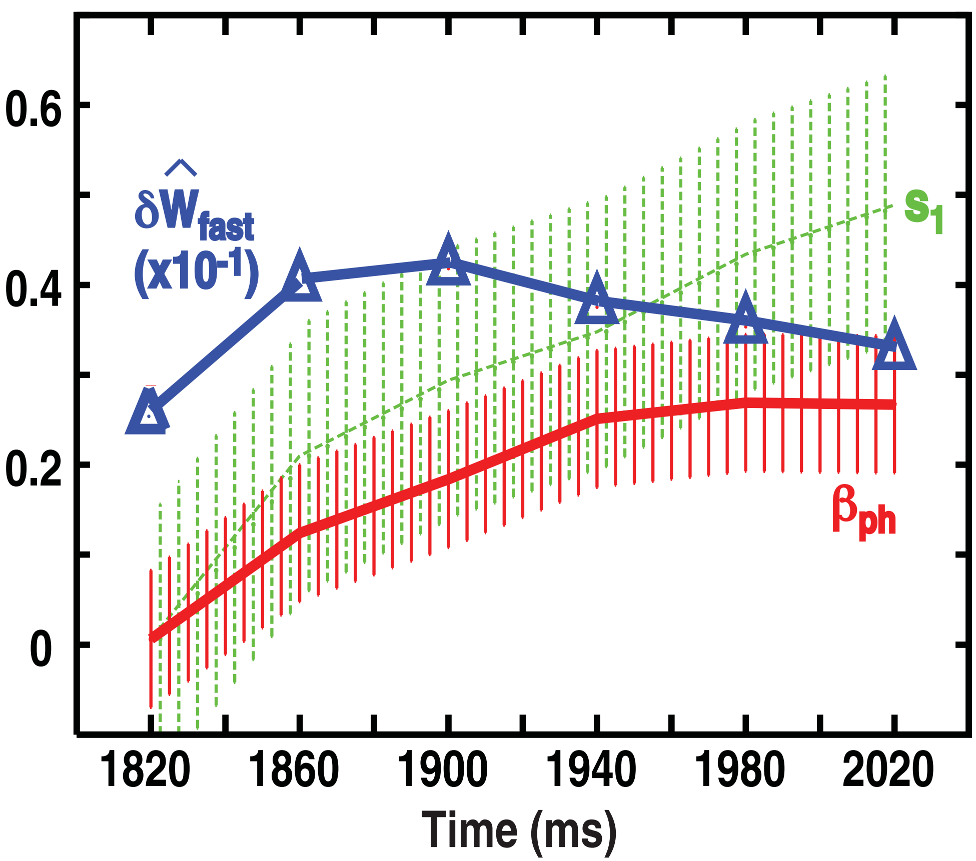
$$D_{\perp l}(k_{//}) = \mu\, \delta(\omega_l)\, \frac{\pi B}{m_i} K \times \sum_{m'} \left[E_{+}^{m'} J_{l-1}\left(k_{\perp}^{m'} \rho_i\right) \right]^{*} \sum_{m} \left[E_{+}^{m} J_{l-1}\left(k_{\perp}^{m} \rho_i\right) \right]$$

- **Diffusion in velocity space treated as random walk in μ:**
  - kicks from resonant interaction between ions and wave

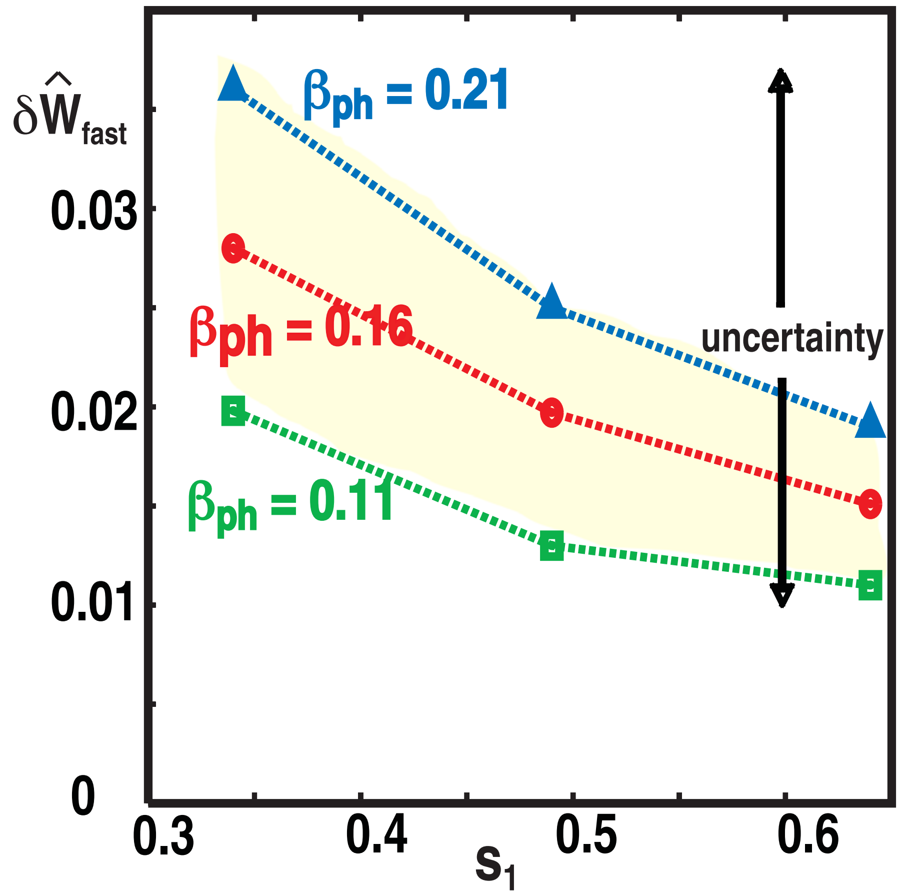
$\delta\hat{W}_{fast}$ is Sensitive to Calculated β_{ph} due to Monte-Carlo Noise and Uncertainty in Reconstructed s_1

- s_1 increases as β_{ph} saturates
  $\Rightarrow \delta W_{fast}$ **decreases**

Estimated 30% uncertainty in s_1 and β_{ph}



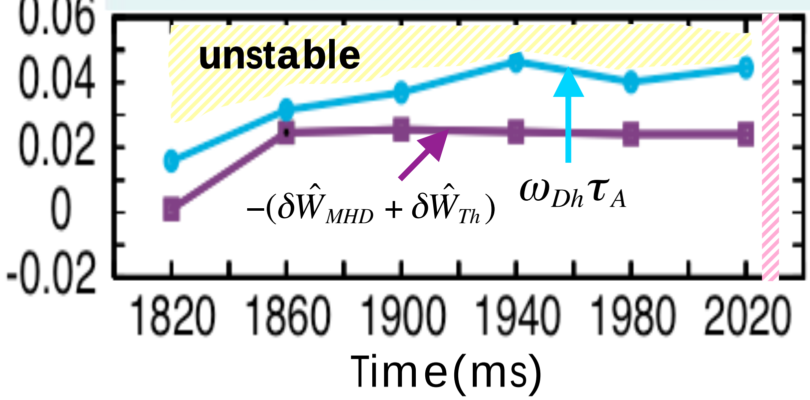
- Uncertainty in s_1 and β_{ph} translates into uncertainty in δW_{fast}



Stability Evaluation Using ORBIT-RF/GATO For First Giant Sawtooth In Agreement With Experimental Crash Time

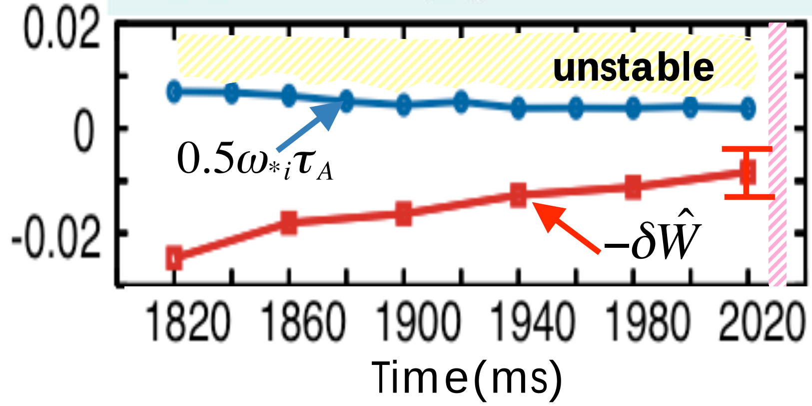
Fast ion effect: Never satisfied

$$-(\delta\hat{W}_{MHD} + \delta\hat{W}_{Th}) > c_h \omega_{Dh} \tau_A$$



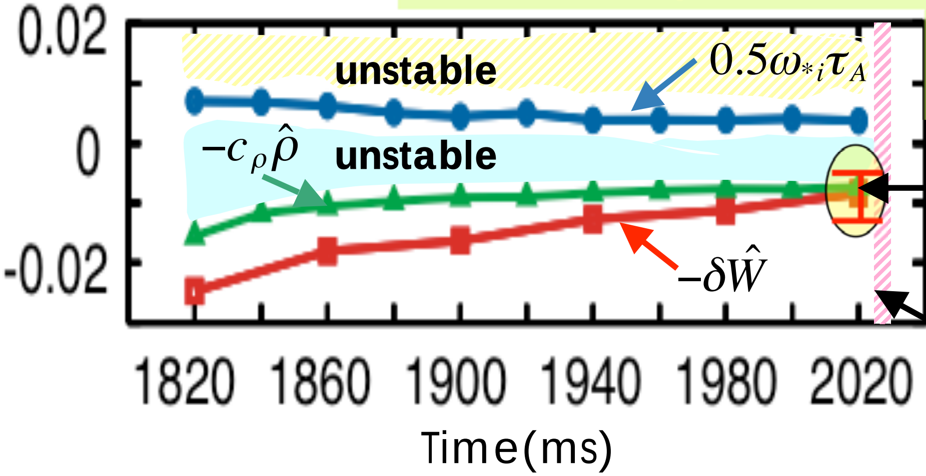
Diamagnetic effect: Never satisfied

$$-\delta\hat{W} > 0.5\omega_{*i}\tau_A$$



Nonideal layer effect:

$$\begin{cases} -c_\rho \hat{\rho} < -\delta\hat{W} < 0.5\omega_{*i}\tau_A & \text{Satisfied at 2020 ms} \\ s_1 \equiv r_1 \left.\dfrac{dq}{dr}\right|_{r_1} > s_{crit} & \text{Satisfied each time} \end{cases}$$



Sawtooth crash predicted from ion kinetic regime mode just after 2020 ms

Actual sawtooth crash at 2035 ms

Porcelli Model Applied to Sequence of Reconstructed DIII-D Discharge Equilibria Works Remarkably Well

- **Model using ORBIT-RF/TORIC/GATO numerical input yields results quantitatively in agreement with experimental sawtooth crash:**
  - Trigger quantitatively agrees with crash time for first giant sawtooth
- **Sawtooth trigger is the ion kinetic regime mode:**
  - From decreasing fast ion stabilization as s_1 increases faster than β_{ph}:
  - Kinetic stabilizing contribution of fast ions is sensitive to:
    - o Magnetic shear at q = 1 surface
    - o Poloidal beta of trapped fast ions inside q=1 surface

$$\left.\begin{array}{l} \\ \end{array}\right\} \delta\hat{W}_{fast} \propto \beta_{ph} / s_1$$

- **Predictions using analytic formula or formula fitted to ideal kink numerical stability calculation database do not yield quantitative agreement**
- **Stability evaluation for subsequent sawtooth crash is also consistent with experimental sawtooth crash time:**

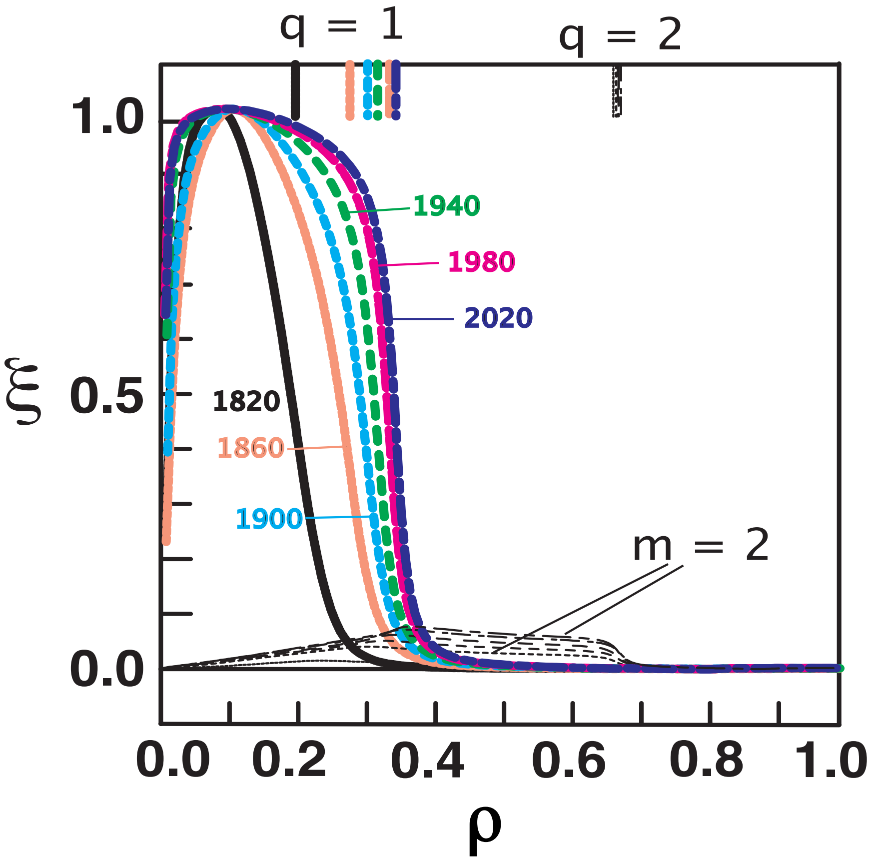
$-\delta\hat{W} \sim 0.5\omega_{*i}\tau_A$ through most of the cycle within 30% nominal uncertainty

  - Analysis is ongoing to evaluate uncertainties and quantitatively check crash criteria

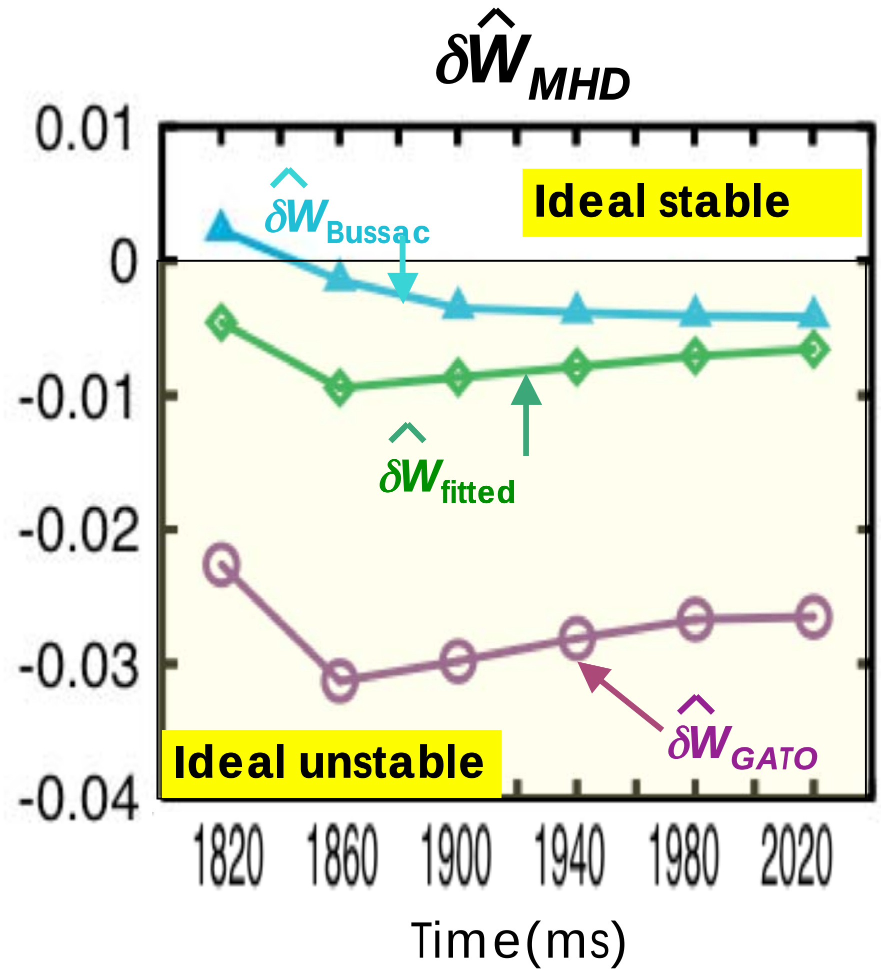
NOVA-K simulations planned to verify conclusions from Porcelli model

Ideal Kink Mode Unstable Through Sawtooth Cycle With Mode Structure Only Approximately "Top-Hat"

"Top-Hat" structure assumed in Bussac model



GATO result more unstable than analytic or numerically fitted formulas normally used in Porcelli model



Resonant Kicks From RF Are Modeled In ORBIT-RF Using Stochastic Quasi-Linear Diffusion Operator

- **TORIC calculates wave fields** $E_+^m(\rho) \quad k_{//}^m \quad k_\perp^m$ **from antenna carrying unit current:**
  - Single toroidal mode
  - $-15 \le m \le +15$
  - $E_+^m(\rho)$ from TORIC scaled to match experimental input power:
    - $\Rightarrow$ Complete absorption of input power according to linear theory

$$|E_+|^2(\rho,\theta) = \sqrt{\frac{P_{exp}}{R_{Toric}}} \times \mathrm{Re}\left[\sum_{m'} (E^{m'}(\rho)e^{im'\theta})^* \sum_m E^m(\rho)e^{im\theta}\right]$$

- **Quasi-linear RF Diffusion operator in ORBIT-RF due to wave field:**

Parallel wavenumber

Left-hand polarized field

Perpendicular wavenumber

$$D_{\perp l}(k_{//}) = \frac{\pi B \mu}{m_i} K \delta(\omega_l) \sum_{m'} \left[E_+^{m'} J_{l-1}\left(k_\perp^{m'} \rho_i\right)\right]^* \sum_m \left[E_+^m J_{l-1}\left(k_\perp^m \rho_i\right)\right]$$

$$K = K(\varsigma, \dot{\omega}_l, \ddot{\omega}_l, \dot{\Omega}, \ddot{\Omega})$$

Resonance condition: Single wave harmonic l

Bessel Function

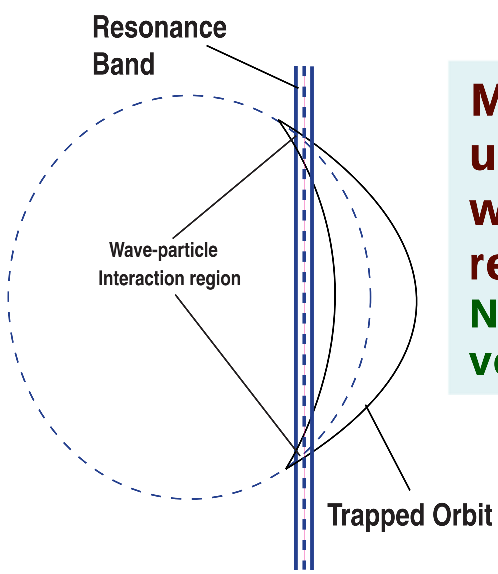
Larmor radius

$$\omega_l = \omega - l\Omega - k_{//}v_{//} \qquad \Omega = qB/m$$

$$\dot{\omega}_l = 0 - l\mathrm{v}.\nabla\Omega - \dot{k}_{//}v_{//} - k_{//}\dot{v}_{//}$$

Magnetic Moment Undergoes Quasi-Linear Diffusion on Each Interaction With Resonant Wave Field

- **Quasi-linear diffusion in velocity space modeled as random walk with kicks from resonant interaction between ions and wave**
  - Kicks are random and small at typical experimental power levels



Magnetic moment undergoes random walk in interaction region:
No spatial or parallel velocity diffusion

"kick" in magnetic moment

Random number

$$\Delta\mu_{rf} = \overline{\Delta\mu_{rf}} + R_s\sqrt{\left\langle \overline{\overline{\Delta\mu_{rf}}}^{\,2} \right\rangle}$$

mean change

Fluctuating change

$$\overline{\Delta\mu_{rf}} = \int_{\Delta t} dt \frac{q^2 l^2 \Omega^2}{\omega^2 B^2} \frac{\partial D_l(k_{//})}{\partial \mu}$$

$$\left\langle \overline{\overline{\Delta\mu_{rf}}}^{\,2} \right\rangle = \int_{\Delta t} dt \frac{2q^2 l^2 \Omega^2}{\omega^2 B^2} D_l(k_{//})$$

Δt = interaction time in resonance region