Sizing Up Plasmas Using Dimensionless Parameters

by
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Presented at
48th Annual Meeting of the
Division of Plasma Physics
Philadelphia, Pennsylvania

October 30 – November 3, 2006

GENERAL ATOMICS

Dimensional Analysis Plays a Vital Role in Experimental Investigations of Physical Processes

- **Since plasmas are complex physical systems, dimensional analysis is a valuable tool for**
  - Improving our physical understanding
  - Providing scalings (trends) that are compatible with the governing equations

- **This talk reviews what the scaling of phenomena with dimensionless parameters has taught us about the**
  - Physics of magnetically-confined fusion plasmas
  - Extrapolation of present-day experiments to burning plasma devices

Outline

I. **Introduction to Dimensional Analysis**
   A. Buckingham Π Theorem
   B. Connor-Taylor Scale Invariance

II. **Validation of Similarity**
   A. Confinement Time
   B. Edge Plasma Characteristics

III. **Dependences on Dimensionless Parameters**
   A. Collisionality
   B. Beta
   C. Relative Gyroradius

IV. **Extrapolation to ITER**

V. **Conclusions**

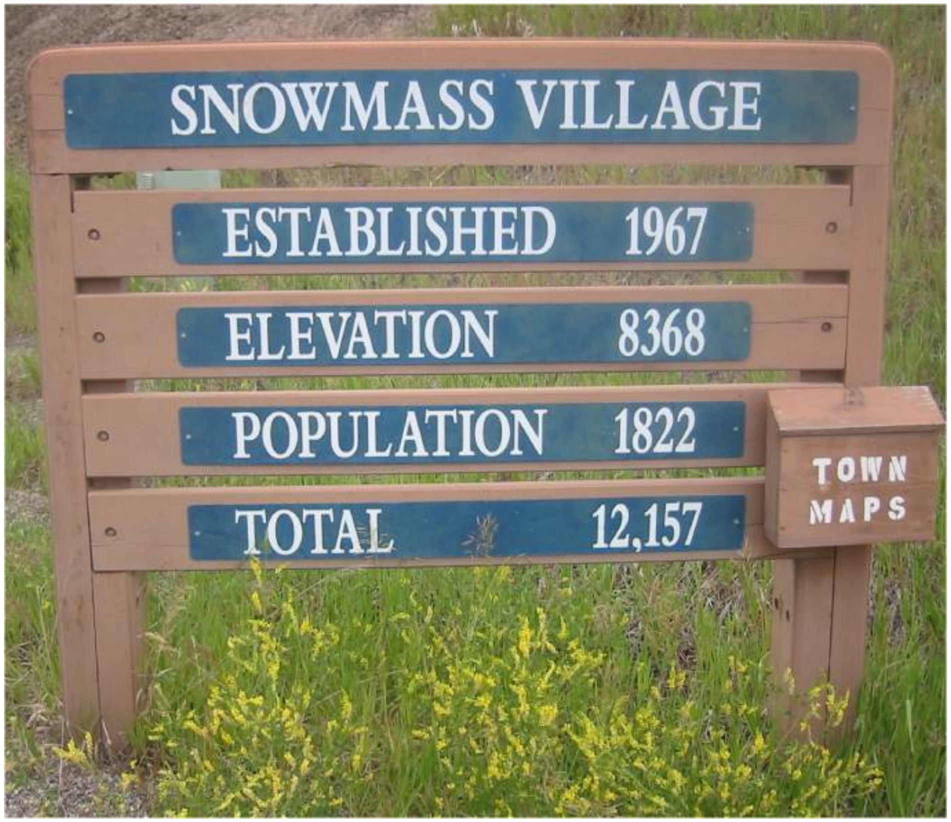
Every Correct Physical Equation is Dimensionally Homogeneous

Jean-Baptiste Joseph Fourier
1768 - 1830

- Joseph Fourier seems to have been the first to state that all the terms of a physical equation must have the same dimensions
- Therefore, all physically correct equations can be recast in terms of dimensionless combinations of the physical variables

Despite Increasing and Widespread Use of Dimensional Analysis, Mistakes Still Happen

- **Physical quantities $Q_1 \ldots Q_M$ may only appear in specific forms in physical equations**
  - Expressions such as $\log Q_j$, $\sin Q_k$, and $(Q_j + Q_k)$ do not occur in physical systems
  - Rather, physical quantities only appear in combination as products of powers, i.e., $Q_1^a Q_2^b \ldots Q_M^n$



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Edgar Buckingham was First Author to Formalize Technique of Dimensional Analysis (Phys. Rev. 1914)

Buckingham Π Theorem

- **Any physical system can be described by an equation of the general form**

  $F(Q_1, Q_2, \ldots, Q_M) = 0$

  where Q_i have physical dimensions

Application

- **The state of a simple hydrogen plasma can be described by seven dimensional parameters**

  B_T toroidal magnetic field
  B_p poloidal magnetic field
  T plasma temperature
  n plasma density
  m_i ion mass
  R torus major radius
  a plasma width

  and three fundamental constants of nature (cgs units)

  m_e electron mass
  e electron charge
  c speed of light

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Application of Buckingham Π Theorem to Magnetically Confined Plasmas

Buckingham Π Theorem

- **Quantities with the same dimensional units can be treated as ratios r_i**

  $F(Q_1, Q_2, \ldots, Q_{N;}\ r_1, r_2, \ldots, r_{M-N}) = 0$

- **The N dimensional quantities can be divided into two groups:**
  - "fundamental" set of K quantities $\{Q_1, Q_2, \ldots, Q_K\}$ where K is determined by the number of physical units
  - "derived" set of N-K quantities $P = \{Q_{K+1}, Q_{K+2}, \ldots, Q_N\}$

Application

- **Identify ratios of like quantities**

  $$\frac{B_p}{B_T},\ \frac{R}{a},\ \frac{m_e}{m_i}$$

  leaving seven dimensional quantities

  $Q = \{B_T, T, n, e, a, m_i, c\}$

- **Divide Q into two sets:**
  - "fundamental" parameters $\{a, m_i, c\}$ representing length, mass, time
  - "derived" parameters $P = \{B_T, T, n, e\}$

Derivation of Independent Dimensionless Parameters for Magnetically Confined Plasmas

Buckingham Π Theorem

- Buckingham showed that any equation describing a physical system can be expressed in the form

$$F(\Pi_1, \Pi_2, \ldots, \Pi_{N-K}; r_1, r_2, \ldots, r_{M-N}) = 0$$

where the products Π_i are the complete set of independent dimensionless variables

$$\Pi_i = Q_1^a Q_2^b \ldots Q_K^k P_i$$

Application

- Using "derived" parameters $P = \{B_T, T, n, e\}$, Buckingham's dimensionless Π products are

$$\Pi_1 = a^{3/2} m_i^{-1/2} c^{-1} B_T$$
$$\Pi_2 = m_i^{-1} c^{-2} T$$
$$\Pi_3 = a^3 n$$
$$\Pi_4 = a^{-1/2} m_i^{-1/2} c^{-1} e$$

- These dimensionless parameters, along with the ratio of like quantities, fully describe all plasma physics (but not atomic physics) in a simple hydrogen plasma

Definition of Dimensionless Parameters Used in this Review Talk

- **Boris Kadomtsev** [Sov. J. Plasma Phys. 1975] **first derived the set of independent dimensionless parameters for toroidal plasmas in standard usage today**
  - Ratio of like quantities

    Inverse aspect ratio: $\varepsilon = a/R$

    Safety factor: $q = \varepsilon\kappa B_T/B_p$

    Mass ratio: m_e/m_i
  - Derived plasma physics quantities

    Relative gyroradius: $\rho_* = \rho_i/a$ (where ρ_i is the Larmor radius)

    Ratio of kinetic to magnetic pressure: $\beta = 16\pi nT/B_T^2$

    Collisionality: $\nu_* = e^4 anq/\varepsilon^{5/2}T^2$

    Number of particles in Debye sphere: $N_D = T^{3/2}/e^3 n^{1/2}$

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    Ratio of kinetic to magnetic pressure: $\beta = 16\pi\;\Pi_1^{-2}\,\Pi_2\,\Pi_3$

    Collisionality: $\nu_* = \Pi_2^{-2}\,\Pi_3\,\Pi_4^{4}\,q\,\varepsilon^{-5/2}$

    Number of particles in Debye sphere: $N_D = \Pi_2^{3/2}\,\Pi_3^{-1/2}\,\Pi_4^{-3}$

Scale Invariance is Complementary to Dimensional Analysis

- **Jack Connor and Brian Taylor** [Nucl. Fusion 1977] **showed that if the basic physical equations are invariant under certain scale transformations, then any result derived from these equations must exhibit the same scale invariance**
- **Consider a plasma model consisting of the Vlasov equation including collisions, the Maxwell equations, and charge neutrality:**

$$\frac{\partial f_i}{\partial t} + \vec{v} \bullet \vec{\nabla} f_i + \frac{e_i}{m_i} \left(\vec{E} + \vec{v} \times \vec{B}\right) \bullet \frac{\partial f_i}{\partial \vec{v}} = C\,(f, f)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c^2}\vec{j}$$

$$\sum_i e_i \int f_i\,(x,v)\, d^3v = 0$$

  – There is only one scale transformation that leaves this set of equations invariant:

$$f \rightarrow \alpha^5 f,\ v \rightarrow \alpha v,\ x \rightarrow \alpha^{-4} x,\ B \rightarrow \alpha^5 B,\ t \rightarrow \alpha^{-5} t,\ E \rightarrow \alpha^6 E,\ j \rightarrow \alpha^9 j$$

Application of Scale Invariance to Confinement Scaling

- **Under this scale transformation the confinement time transforms as $\tau \rightarrow \alpha^{-5}\tau$, and since $B \rightarrow \alpha^{5}B$, the quantity $B\tau$ is scale invariant**
- **Now consider a scaling law for the confinement time given by**

$$B\tau \propto n^p T^q B^r a^s \qquad (1)$$

- **The requirement that this confinement scaling law remains invariant under this scale transformation imposes the constraint**

$$2p + q/2 + 5r/4 - s = 0 \qquad (2)$$

- **Consequently, the confinement scaling law must be of the form**

$$B\tau \propto (na^2)^p \, (Ta^{1/2})^q \, (Ba^{5/4})^r \qquad (3)$$

- **Finally, this relation can be made dimensionally homogeneous and recast in terms of Kadomtsev's dimensionless parameters**

$$(eB/cm_i)\,\tau = \Omega_i \tau \sim (\rho_*)^{-2p-q-3r/2} \, (\nu_*)^{-q/2-r/4} \, (\beta)^{p+q/2+r/4} \qquad (4)$$

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$$(eB/cm_{i})\, \tau = \Omega_{i}\, \tau = F(\, \rho_{*} \,,\, \nu_{*} \,,\, \beta\,) \qquad (4)$$

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$$(eB/cm_i)\, \tau = \Omega_i \tau = F(\, \rho_* \, , \, \nu_* \, , \, \beta \,) \qquad (4)$$

- **Because the charge neutrality assumption neglects physical phenomena at the scale of the Debye length, N_D is absent above**

Scale Invariance Approach Shows a Reduced Physics Model Yields Fewer Dimensionless Variables

- **In electrostatic limit, the magnetic fields are fixed rather than determined self-consistently from the Maxwell equations**
  - There are now two scale transformations that leave the plasma model unchanged, so the scaling law ($B\tau \propto n^p T^q B^r a^s$) is subject to two constraints

    $f \rightarrow \beta f,\ v \rightarrow \beta v,\ B \rightarrow \beta B,\ t \rightarrow \beta^{-1} t,\ E \rightarrow \beta^2 E$ $\quad$ $4p + 2q + r = 0$

    $f \rightarrow \gamma^{-1} f,\ x \rightarrow \gamma x,\ B \rightarrow \gamma^{-1} B,\ t \rightarrow \gamma t,\ E \rightarrow \gamma^{-1} E$ $\quad$ $s - p - r = 0$
  - Consequently, the confinement scaling law must be of the form

    $$B\tau \propto (T/a^2B^2)^q\,(n/a^3B^4)^p$$

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    $$\Omega_i \tau = F(\rho_* , \nu_*)$$

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- **For the collisionless Vlasov equation in the electrostatic limit, there are three scale transformations, and thus three constraints**
  - In this case, the confinement scaling law must be of the form

    $$B\tau \propto (T/a^2B^2)^q$$

Scale Invariance Approach Shows a Reduced Physics Model Yields Fewer Dimensionless Variables

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  - In this case, the confinement scaling law must be of the form

    $$\Omega_i \tau = F(\rho_*)$$

Outline

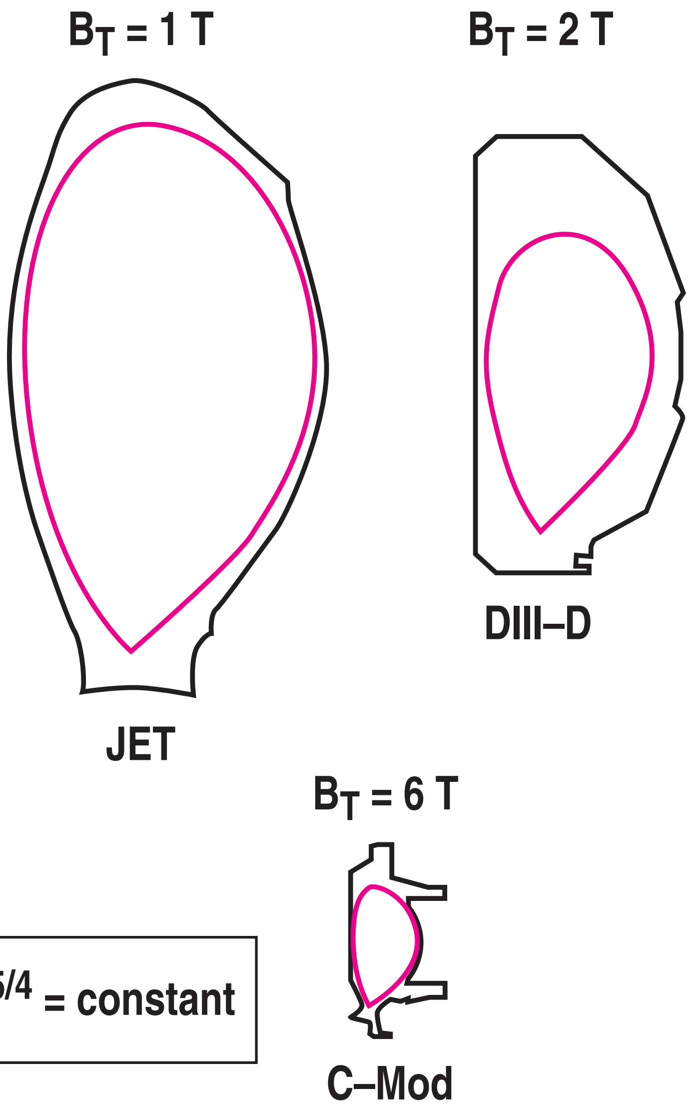
Verifying the Principle of Similarity is an Essential Step in Experimentally Applying Dimensional Analysis

- **Similarity dictates that plasmas with the same dimensionless parameters, but different dimensional parameters, exhibit the same physical behavior**
  - Analogous to wind tunnel tests using scale models
  - Sometimes similarity goes by the name of "identity"

- **Similarity can be tested by comparing heat transport on tokamaks with different physical sizes**
  - Match ρ_*, ν_*, β and plasma geometry
  - If turbulent transport is dependent only upon dimensionless parameters, then normalized thermal diffusivities should be the same
    - Thermal diffusivities normalized by Bohm diffusion coefficient

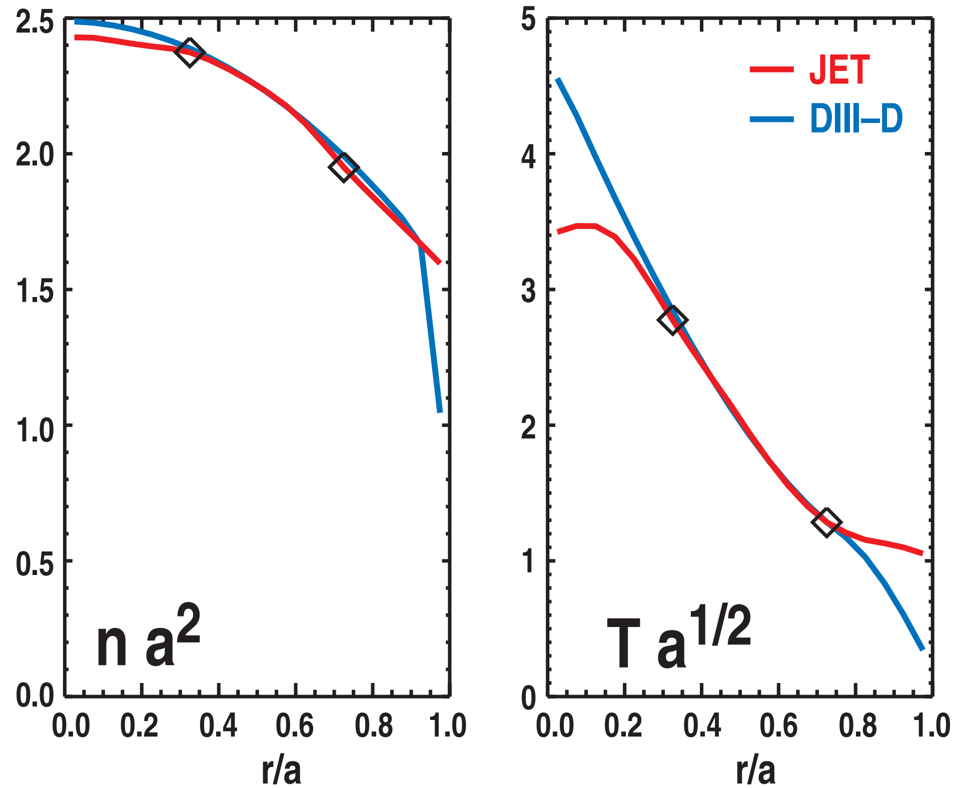
$$\chi_B \sim cT/eB$$

Devices With Different Physical Size Can Operate With Same Dimensionless Parameters by Varying the Magnetic Field



$Ba^{5/4}$ = constant

Normalized density and temperature profiles are well matched:



T.C. Luce, Nucl. Fusion 2002

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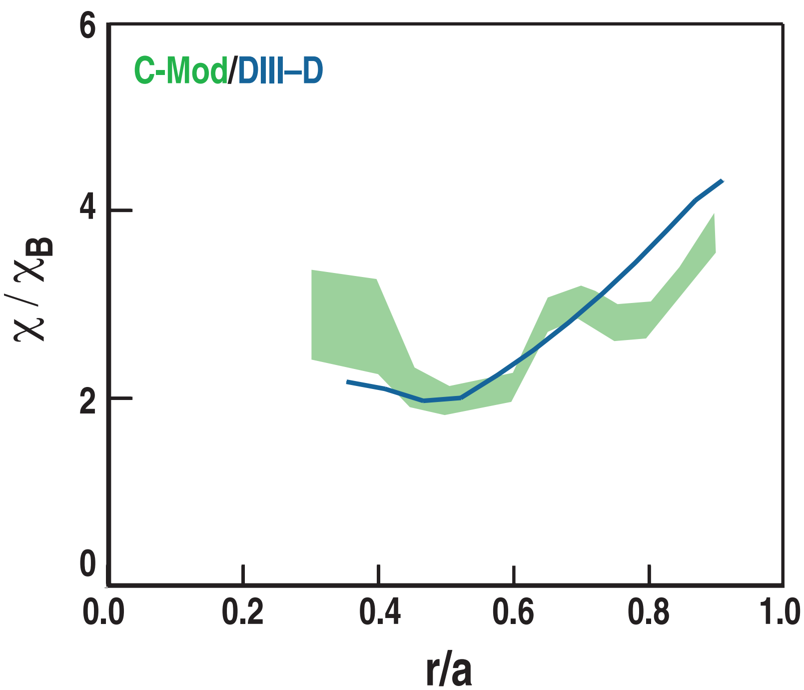
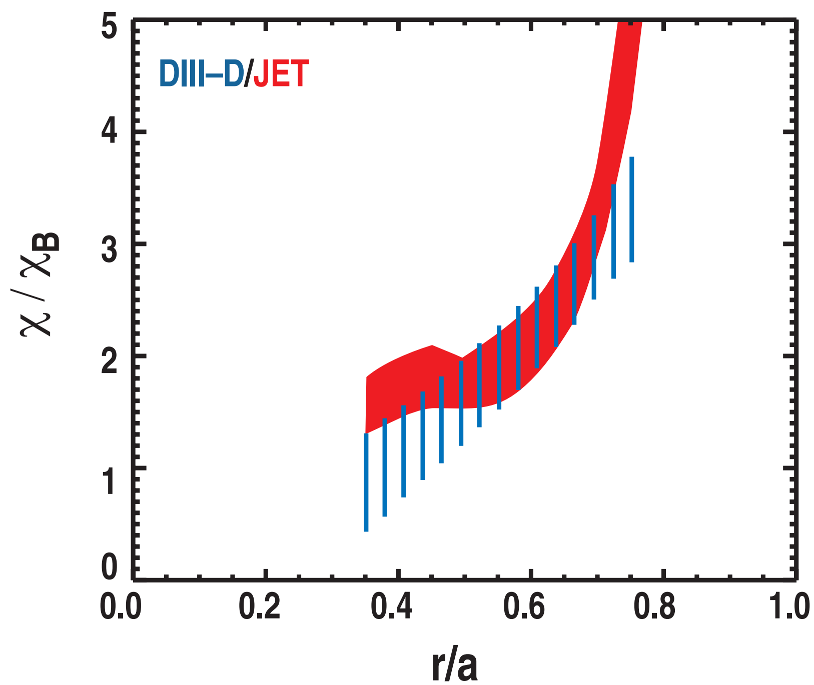
Normalized Thermal Diffusivities Agree Within Experimental Errors, Validating Similarity in High Temperature Plasmas

- Thermal diffusivity $\chi = -\frac{q}{n\nabla T}$ where q = heat flux

Low-Confinement Mode



High-Confinement Mode



T.C. Luce, Nucl. Fusion 2002

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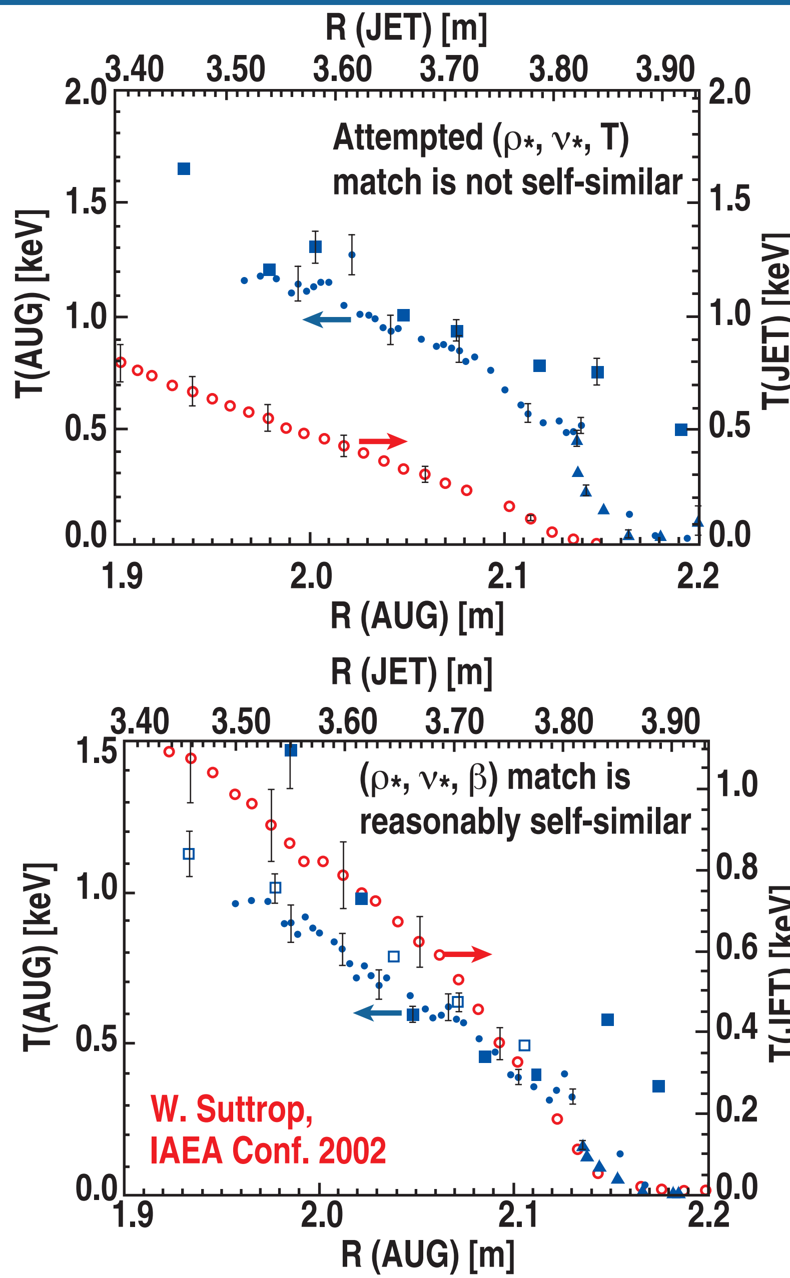
Similarity Can Determine Which Dimensionless Parameters Govern Edge Plasma Characteristics

- **ASDEX Upgrade and JET compared transition from low (L) to high (H) confinement mode**
- **Experiment tested importance of (ρ_*, ν_*, β) along with dimensionless atomic physics variable T/ξ_o**
- **Self-similar edge profiles only observed when (ρ_*, ν_*, β) are matched at time of L-H transition**
  - T/ξ_o is not critical variable



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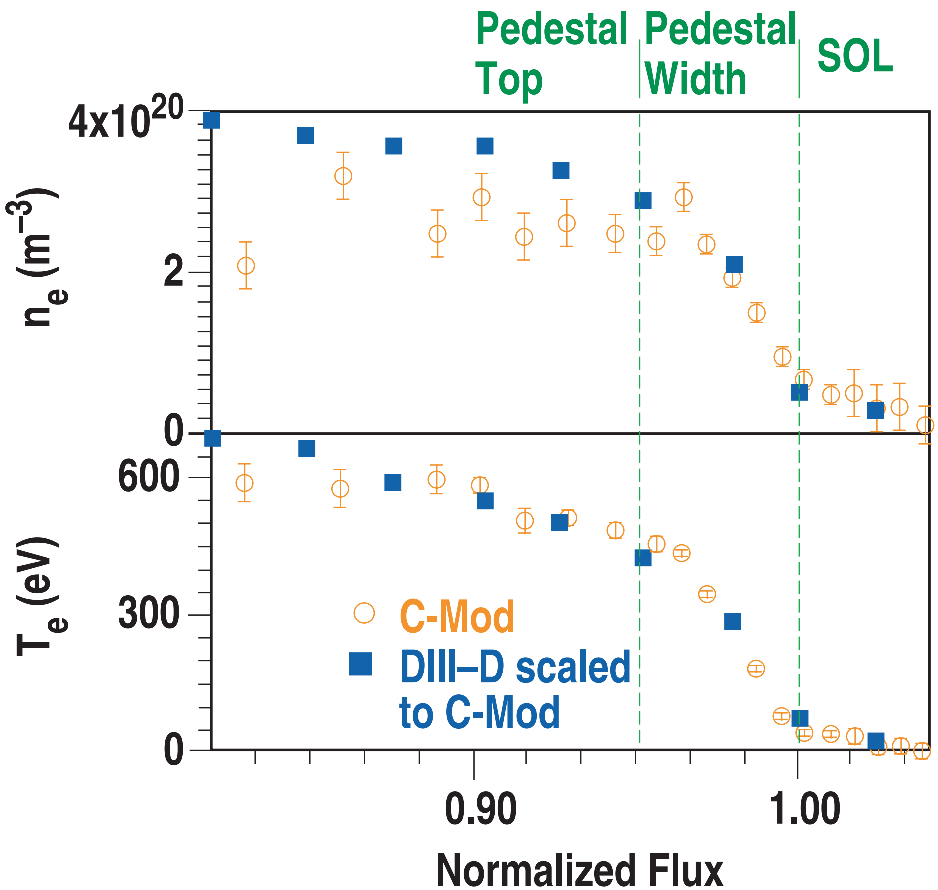
H-mode Pedestal Region Behaves Similarly When Edge Dimensionless Parameters Are Matched

- **Alcator C-Mod and DIII–D compared H-mode pedestal profiles for matched (ρ_*, ν_*, β)**
  - Found that width of H-mode pedestal scales with machine size
- **Self-similar edge profiles appear to argue against atomic physics processes being dominant**



D.A. Mossessian, Phys. Plasmas 2003

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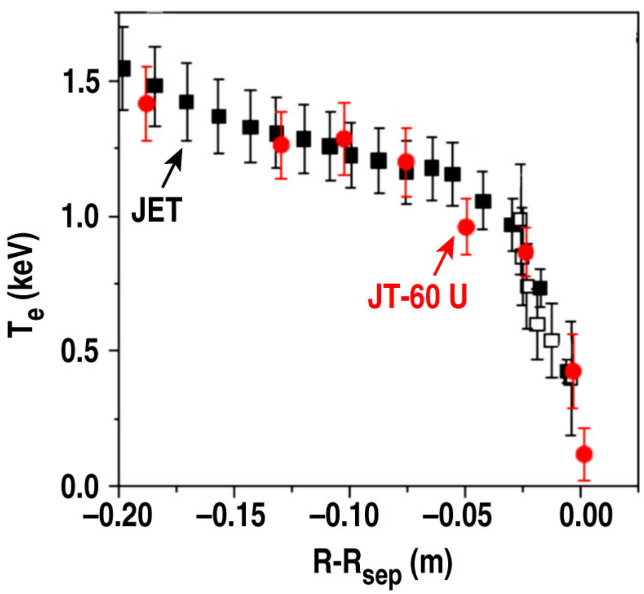
Failure to Achieve Self-Similar Profiles Can Help Identify "Hidden" Physics

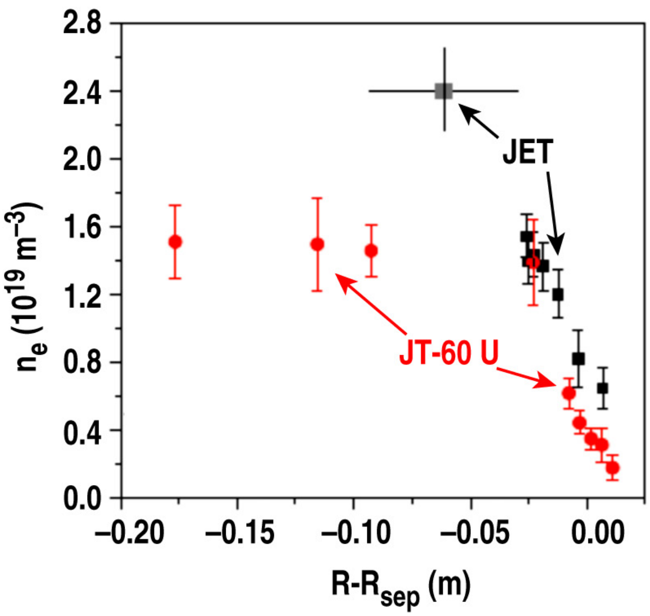
- **Comparison of H-mode pedestal profiles in JT-60U and JET with same size plasmas**

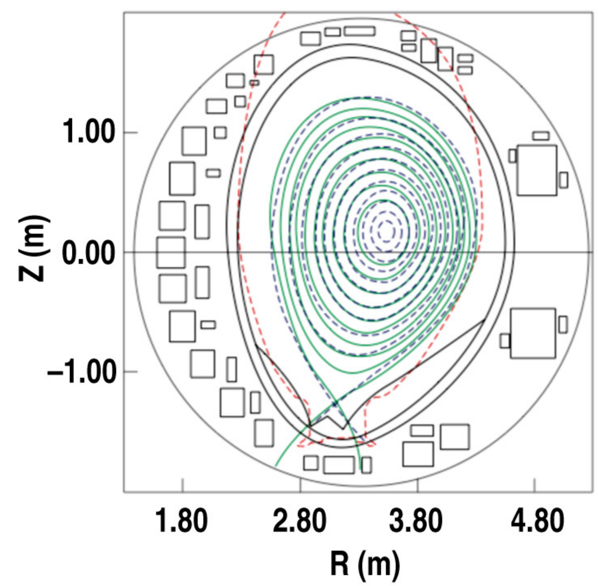
  – Electron temperature profiles are well matched near plasma edge

  – Electron density at pedestal top is much lower in JT-60U, however

  – Ripple-induced fast ion losses are ≈10× larger in JT-60U







G. Saibene, IAEA Conf. 2004

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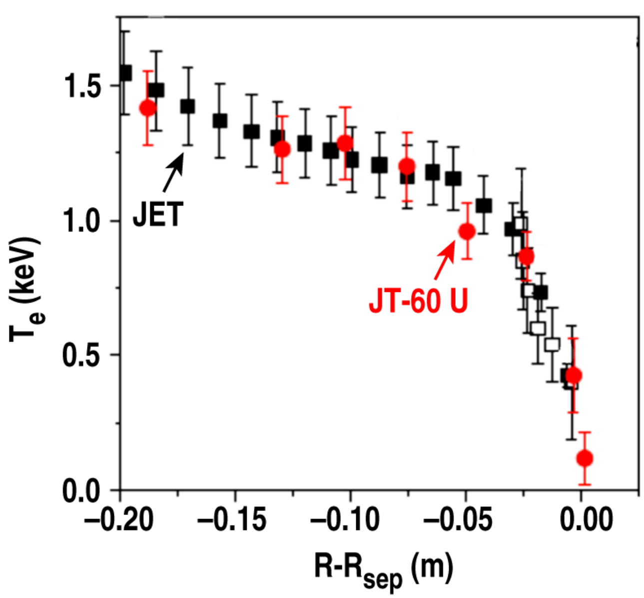
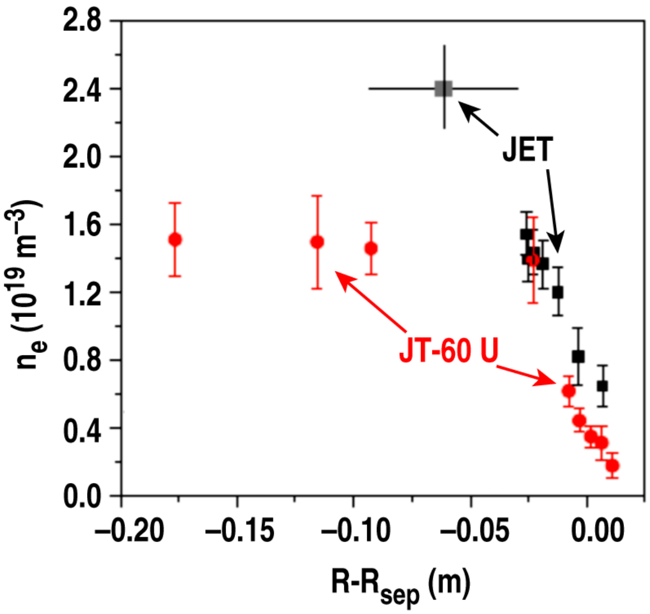
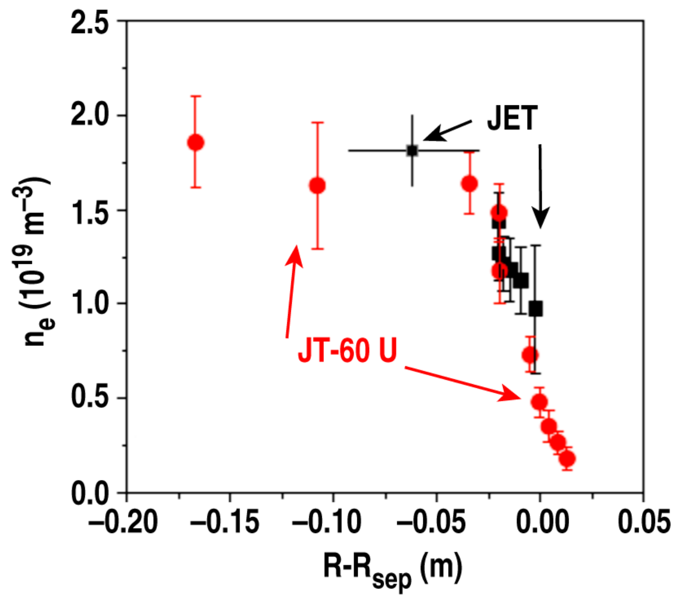
- **Comparison of H-mode pedestal profiles in JT-60U and JET with same size plasmas**
  - Electron temperature profiles are well matched near plasma edge
  - Electron density at pedestal top is much lower in JT-60U, however
  - Reducing fast ion losses by ~50% in JT-60U improves density match







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The Dependence on Dimensionless Parameters Reveals the Physical Processes Involved

- **For example, there is a straightforward relationship between transport and the dimensionless "plasma physics" variables**

  ν_* ⇒ effect of collisions and mirror trapping
  β ⇒ electrostatic vs electromagnetic turbulence
  ρ_* ⇒ short wavelength vs long wavelength turbulence

- **The ratio of "like quantities" also influence the plasma properties and should be kept fixed during (ν_*, β, ρ_*) scans**

  ε = (minor radius)/(major radius)
  κ = (plasma height)/(plasma width)
  $q = \varepsilon\kappa B_T/B_p$
  $M = m_e/m_i$
  T_e/T_i
  Z_{eff}
  V_{tor}/C_s (Mach number)

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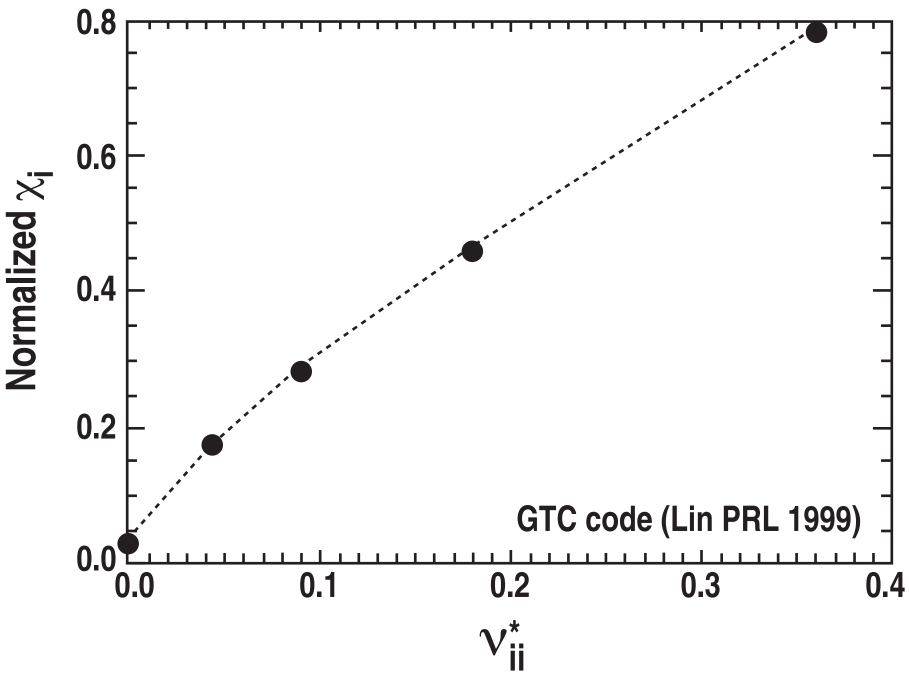
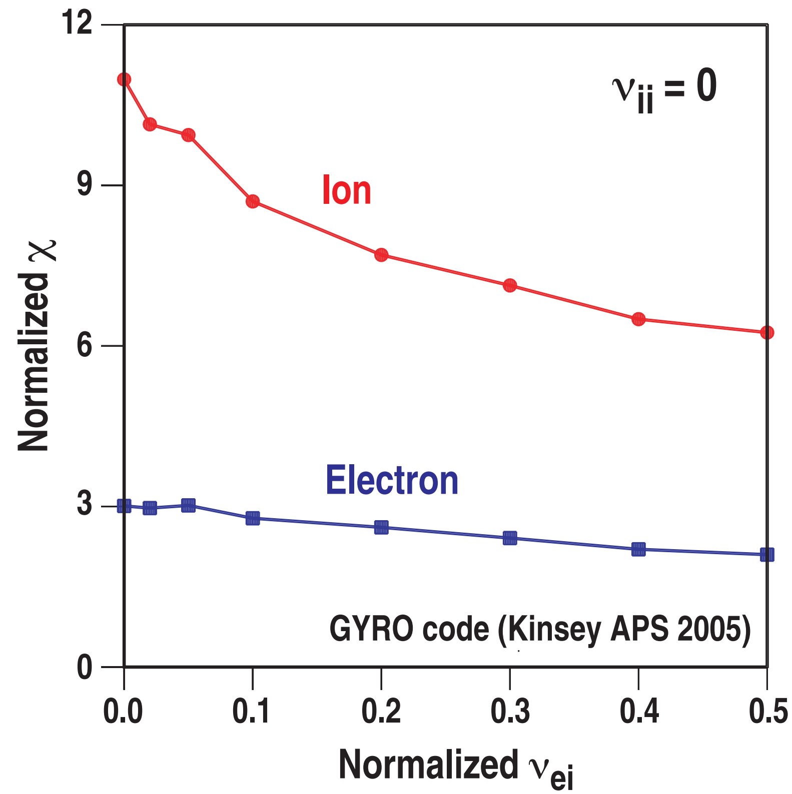
Scaling of Transport With Collisionality (ν_*) Helps to Distinguish Between Different Mechanisms of Turbulence

- Ion-ion collisions damp beneficial zonal flows, increasing transport



- However, electron-ion collisions stabilize trapped electron mode, decreasing transport



- Resistive ballooning mode also has unfavorable ν_* dependence

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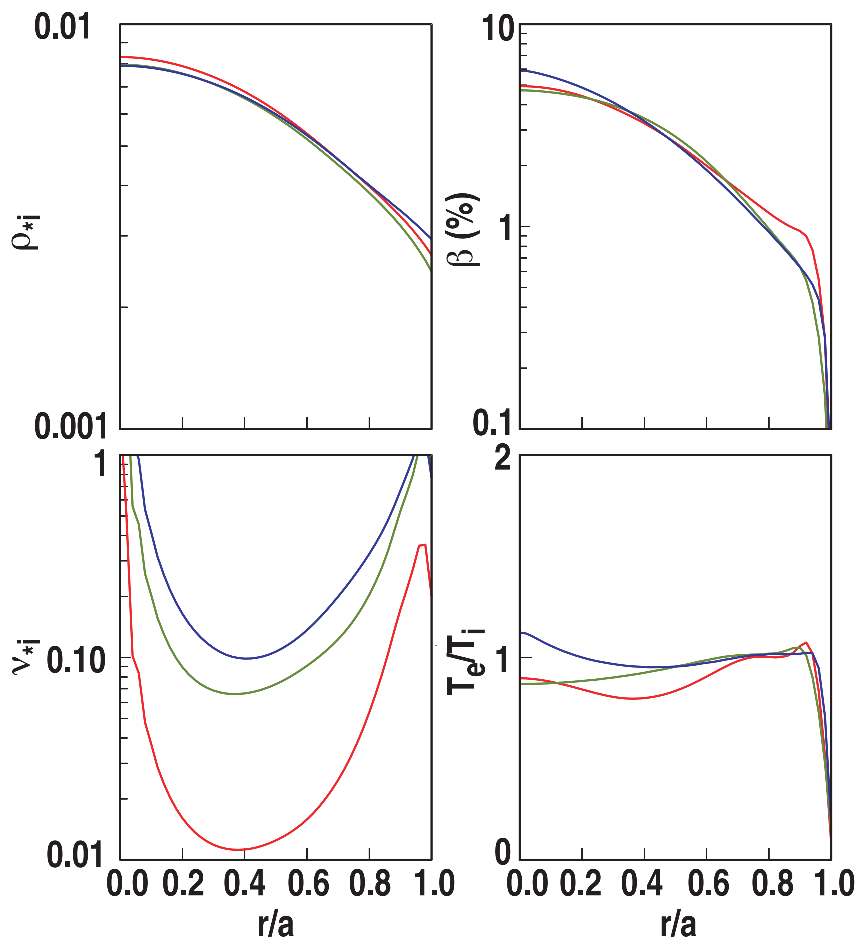
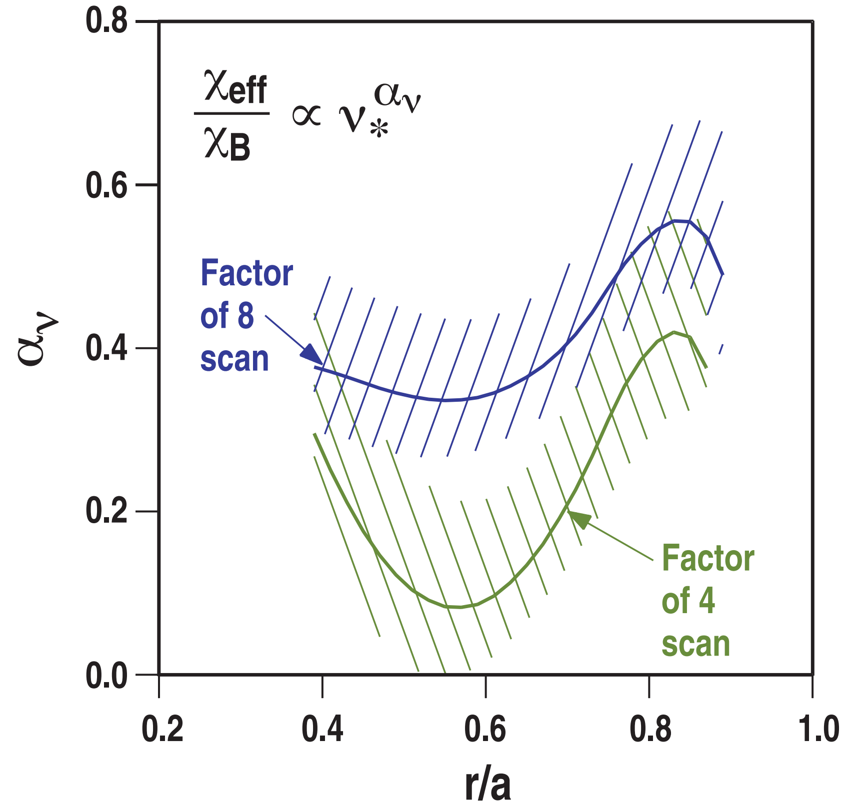
Heat Transport Increases With Collisionality in High Confinement Mode, Especially at Higher ν_*

- **DIII–D scanned ν_* by factor of 8 while keeping (ρ_*, β, q) constant**
  ⇒ Requires $n \propto B^0 a^{-2}$, $T \propto B^2 a^2$, $I \propto B\,a$





C.C. Petty, Phys. Plasmas 1999

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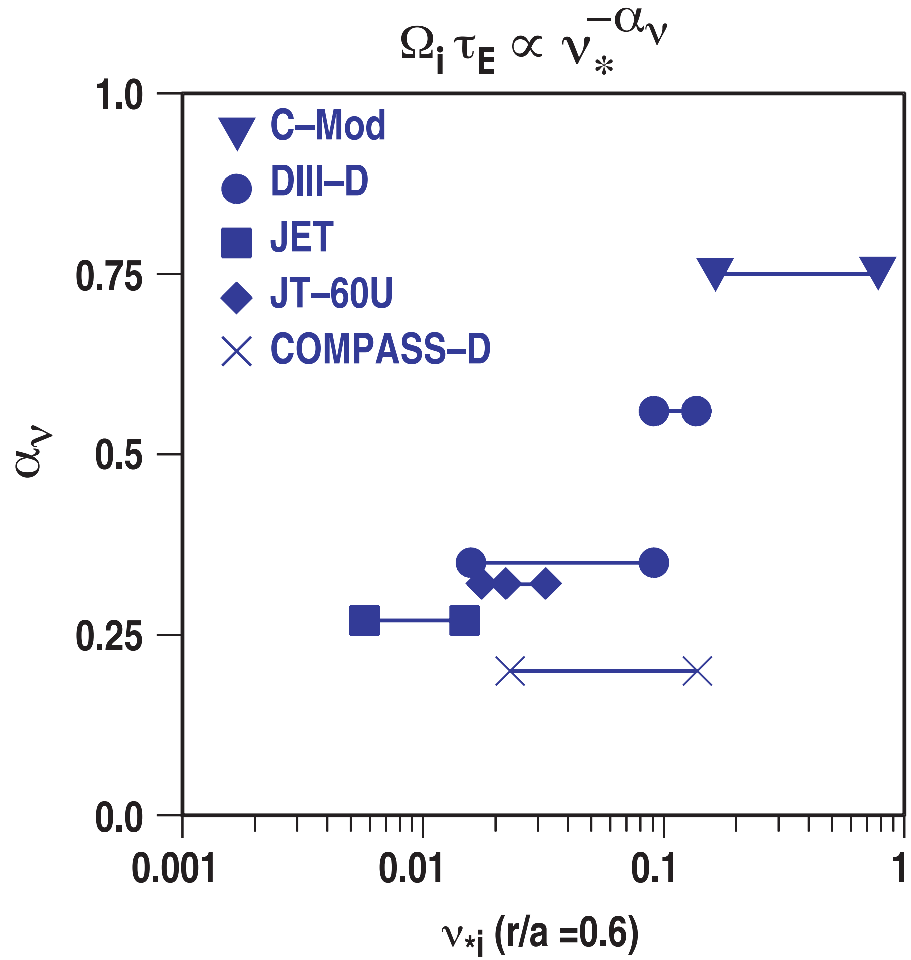
Varying ν_* by Changing Different Dimensional Parameters Gives the Same Transport Scaling

$$\nu_* = \frac{e^4 \, a \, n \, q}{\varepsilon^{5/2} \, T^2}$$

- **Experiments on single tokamak varied ν_* by changing temperature (fixed size)**



M. Greenwald, PPCF 1998
C.C. Petty, Phys. Plasmas 1999
J.G. Cordey, IAEA Conf. 1996
H. Shirai, PPCF 2000
M. Valovic, EPS. Conf. 1999

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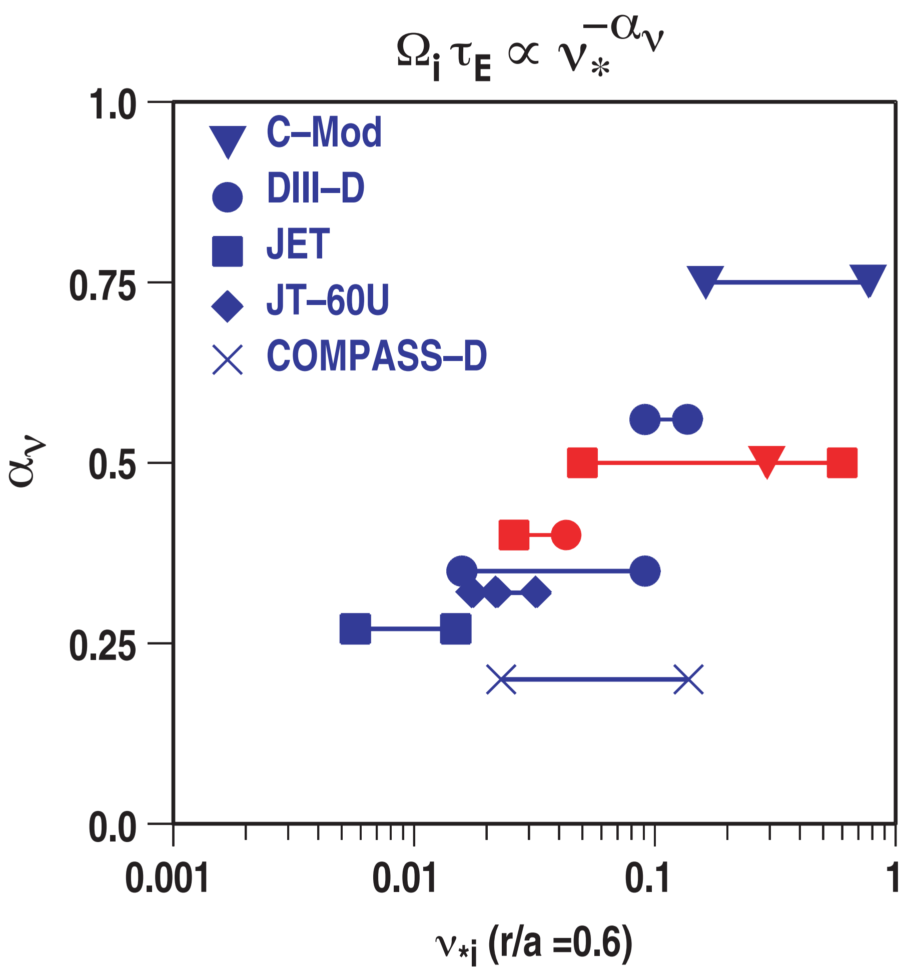
Varying ν_* by Changing Different Dimensional Parameters Gives the Same Transport Scaling

$$\nu_* = \frac{e^4\, a\, n\, q}{\varepsilon^{5/2}\, T^2}$$

- Experiments on single tokamak varied ν_* by changing temperature (fixed size)

- New experiments across multiple tokamaks varied ν_* by changing size (fixed temperature)



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C.C. Petty, PPCF 2004
H. Leggate, EPS Conf. 2005

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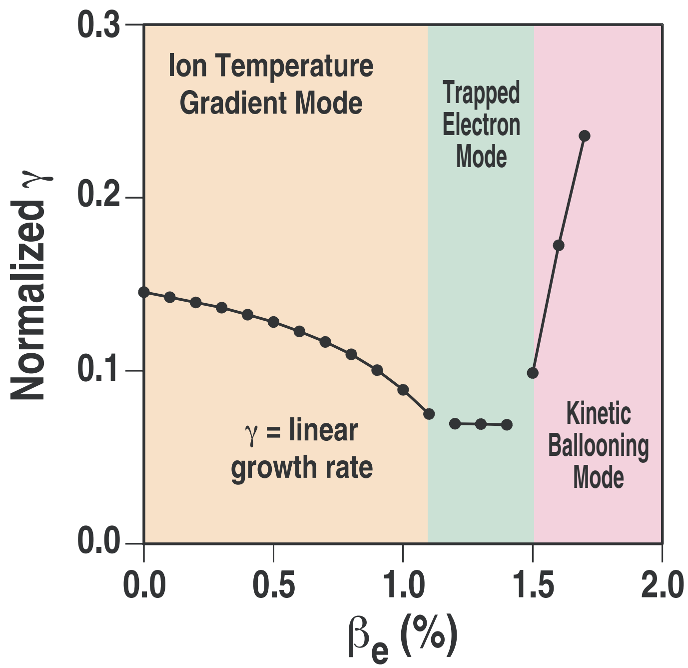
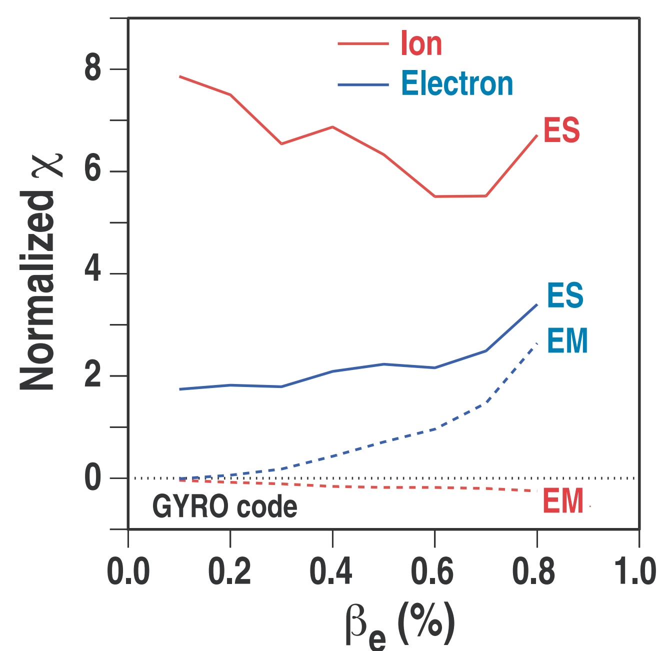
Scaling of Transport With β (= Plasma Pressure/ Magnetic Pressure)

- **Experimentally determining the β scaling differentiates between various mechanisms of turbulent transport:**
  - ExB transport (primarily electrostatic)
  - Magnetic flutter transport (primarily electromagnetic)

Different microinstabilities have different β dependences:



Nonlinear gyrokinetic simulations predict magnetic flutter transport is significant at high β:



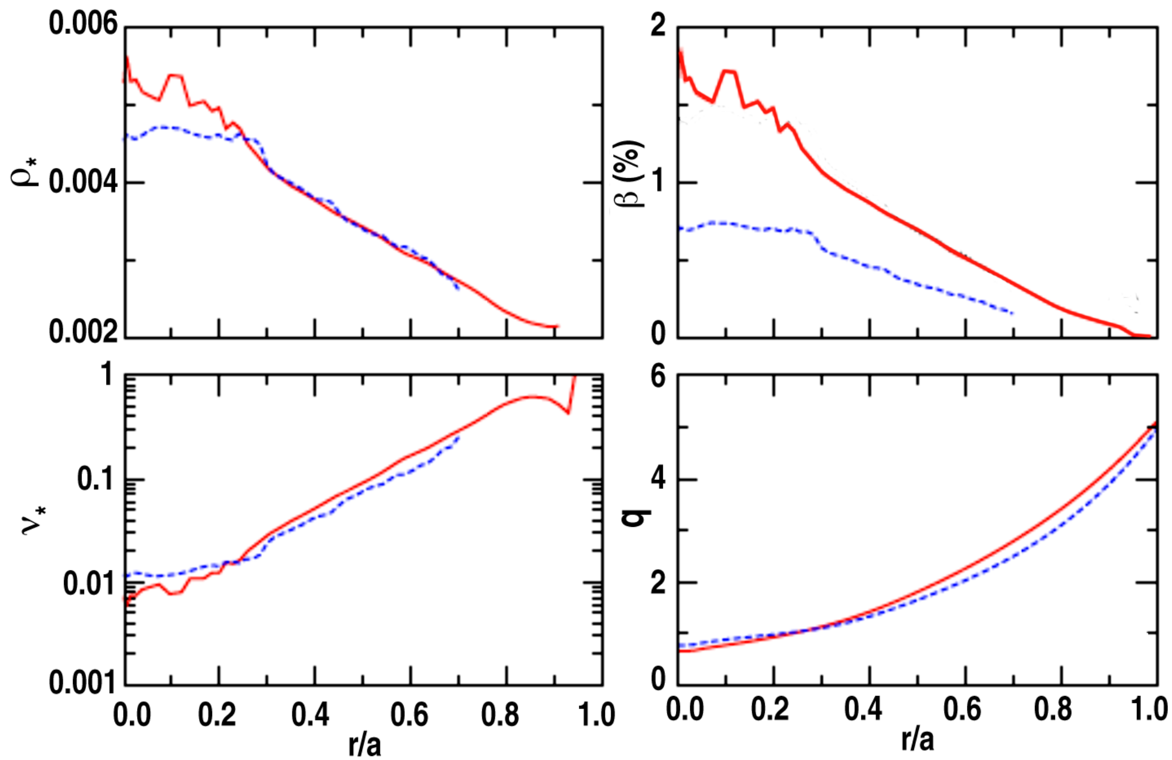
J. Candy, Phys. Plasmas 2005

278-06/CCP/rs

If Transport has Strong, Unfavorable β Dependence, Then Electromagnetic Effects are Essential

- **Tore Supra varied β by factor of 2 while keeping (ρ_*,ν_*,q) constant in low confinement mode**

$\Rightarrow$ Requires $n \propto B^4 a^3$, $T \propto B^2 a^2$, $I \propto B a$



A. Sirinelli, EPS Conf. 2006

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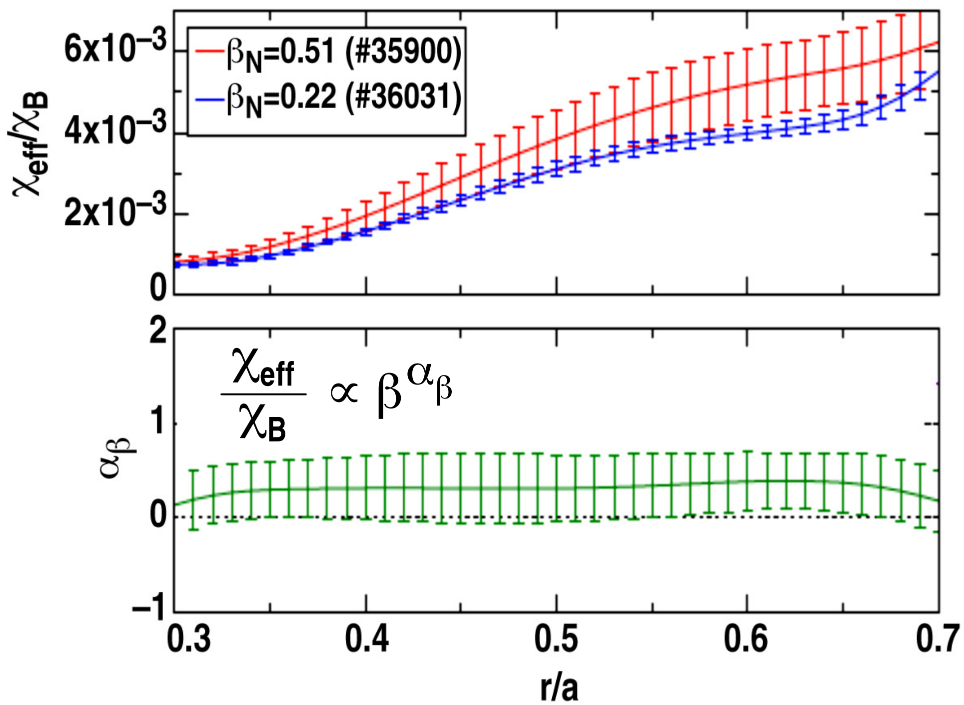
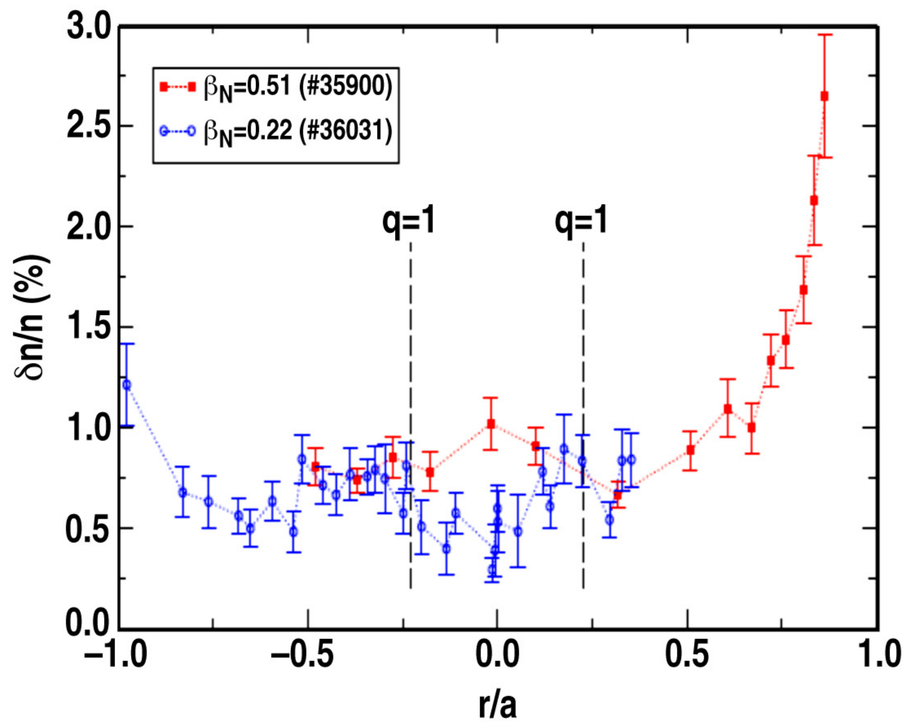
Thermal Diffusivity and Turbulence Vary Little During β Scan

- On Tore Supra, measured change in effective thermal diffusivity is within error bars

- Normalized density fluctuation level does not change outside q=1





A. Sirinelli, EPS Conf. 2006

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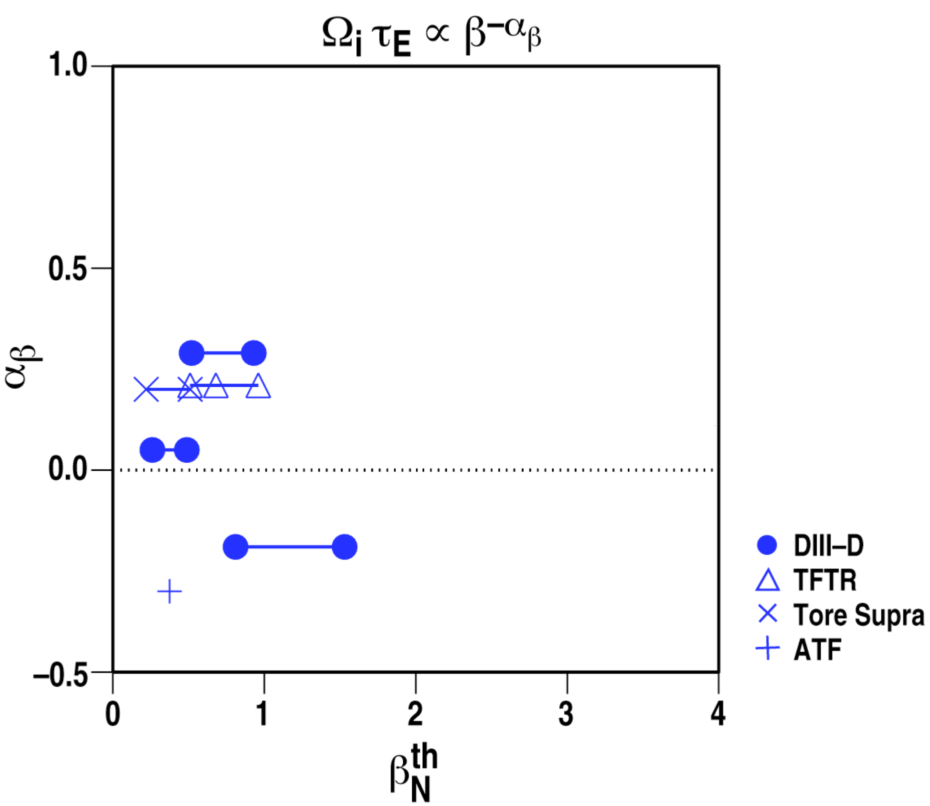
Experiments Consistently Show a Weak, Possibly Nonexistent, β Scaling of Confinement

- **For low (L) confinement mode, β scaling exponent confined to range $-0.3 \leq \alpha_\beta \leq 0.3$**



S.D. Scott, IAEA Conf. 1992

J.B. Wilgen, Phys. Fluids B 1993

C.C. Petty, Nucl. Fusion 1998

A. Sirinelli, EPS Conf. 2006

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Experiments Consistently Show a Weak, Possibly Nonexistent, β Scaling of Confinement

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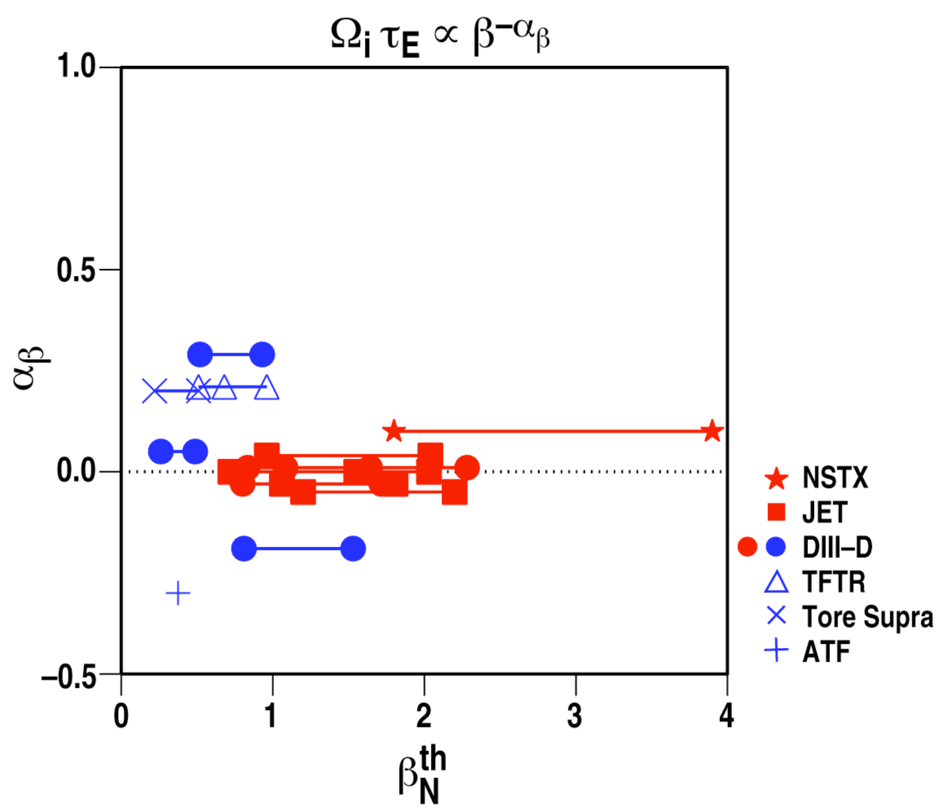
- **At higher β in high (H) confinement mode, initial results tightly clustered around $\alpha_\beta = 0$**

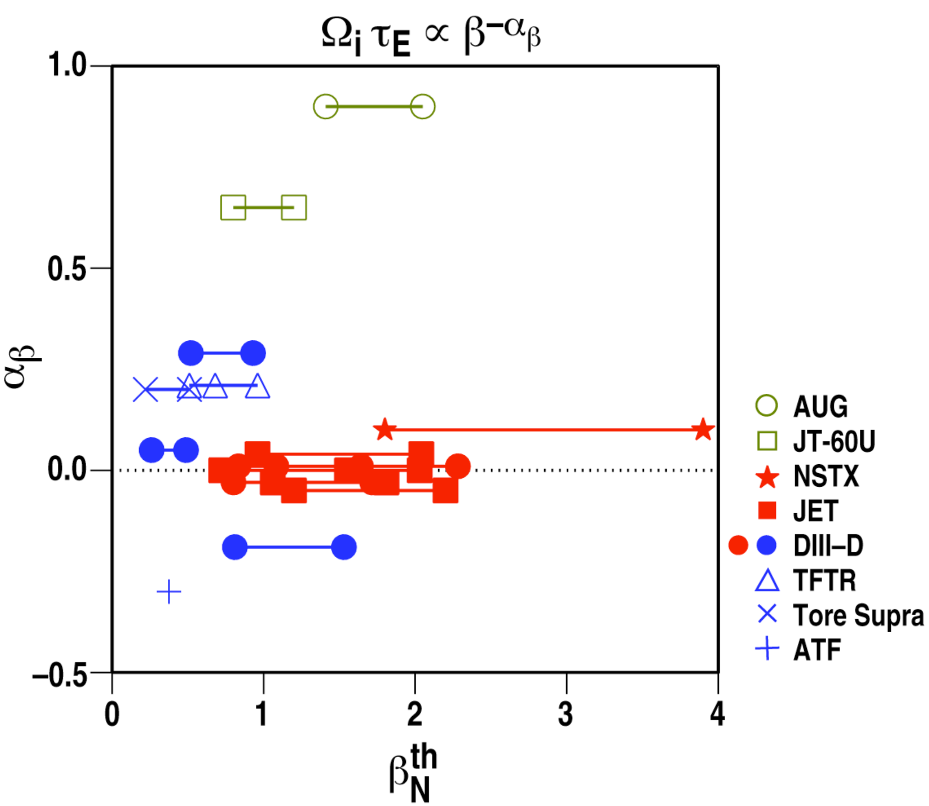


S.D. Scott, IAEA Conf. 1992
J.B. Wilgen, Phys. Fluids B 1993
C.C. Petty, Nucl. Fusion 1998
A. Sirinelli, EPS Conf. 2006
J.G. Cordey, IAEA Conf. 1996
C.C. Petty, Phys. Plasmas 2004
D.C. McDonald, PPCF 2004

278-06/CCP/rs

Experiments Consistently Show a Weak, Possibly Nonexistent, β Scaling of Confinement, Or Do They?

- For low (L) confinement mode, β scaling exponent confined to range $-0.3 \leq \alpha_\beta \leq 0.3$
- At higher β in high (H) confinement mode, initial results tightly clustered around $\alpha_\beta = 0$
- However, recent experiments find strong, unfavorable β scaling of H-mode confinement



S.D. Scott, IAEA Conf. 1992
J.B. Wilgen, Phys. Fluids B 1993
C.C. Petty, Nucl. Fusion 1998
A. Sirinelli, EPS Conf. 2006

J.G. Cordey, IAEA Conf. 1996
C.C. Petty, Phys. Plasmas 2004
D.C. McDonald, PPCF 2004

H. Urano, Nucl. Fusion 2006
L. Vermare, EPS Conf. 2006

278-06/CCP/rs

Scaling of Transport With Relative Gyroradius (ρ_*)

- Strength of ρ_* scaling indicates whether the step or eddy size (Δ) scales like the gyroradius or the device size

$\Rightarrow \dfrac{\chi}{\chi_B} \propto \rho_*^1$ implies $\Delta \sim \rho_*$ "gyro-reduced-Bohm"

$\Rightarrow \dfrac{\chi}{\chi_B} \propto \rho_*^0$ implies $\Delta \sim a$ "Bohm" where $\chi_B \sim cT/eB_T$

- Turbulence generated by small-scale instabilities naturally has gyroBohm transport scaling

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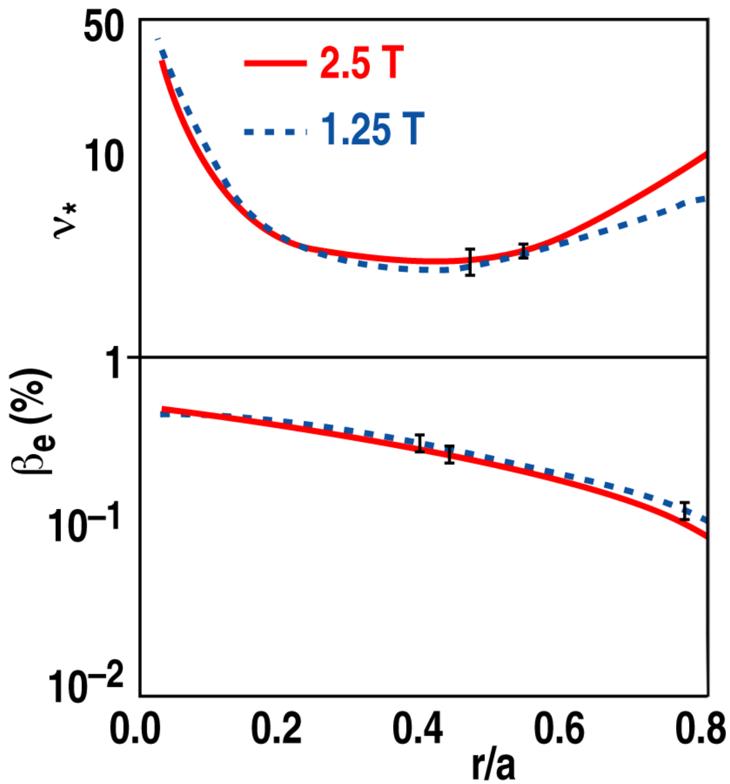
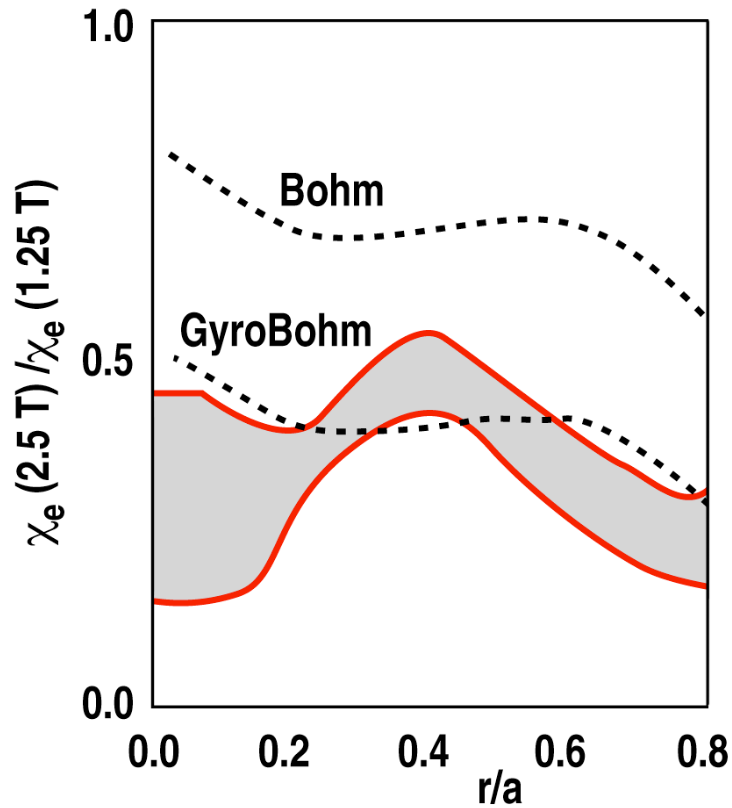
GyroBohm Scaling of Heat Transport Originally Observed in Stellarators

- Collisionality and beta kept constant for ρ_* scan on W7-AS



- Electron thermal diffusivity scales gyroBohm



U. Stroth, Phys. Rev. Lett. 1993

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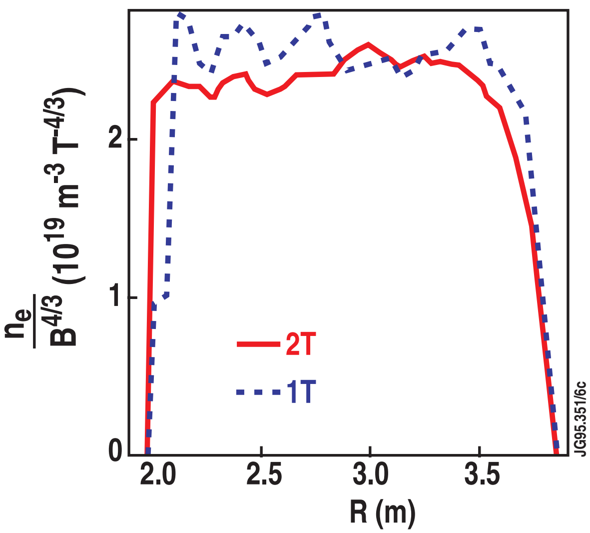
Heat Transport Follows GyroBohm Scaling in High Confinement Mode Plasmas

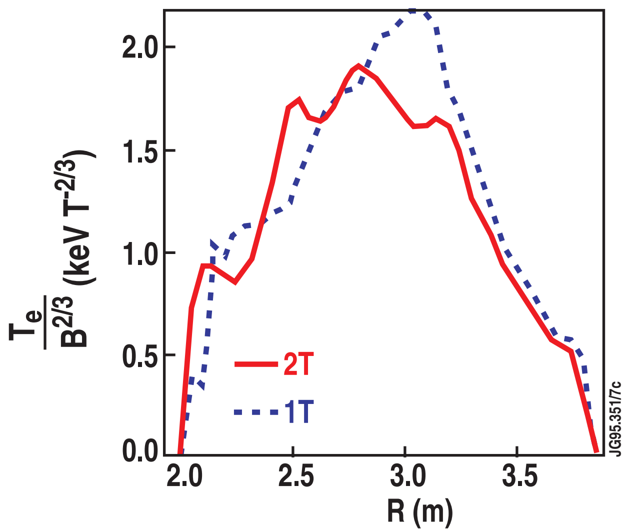
- **JET varied ρ_* by factor of 1.6 while keeping (ν_*, β, q) constant**

  $\Rightarrow$ Requires $n \propto B^{4/3} a^{-1/3}$, $T \propto B^{2/3} a^{1/3}$, $I \propto B\ a$

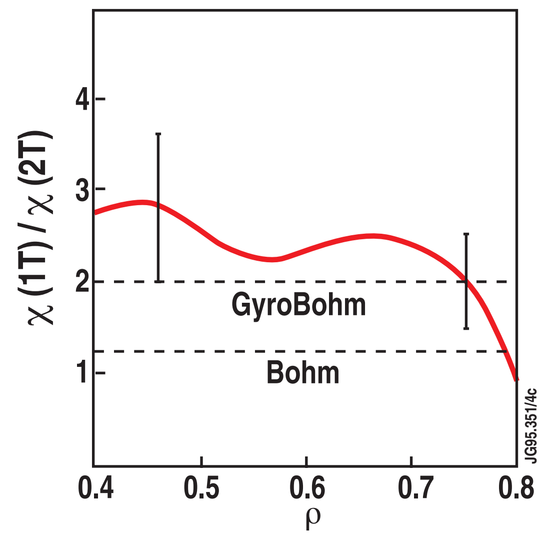
- **Scaled densities**



- **Scaled temperatures**



- **Ratio of effective thermal diffusivities**

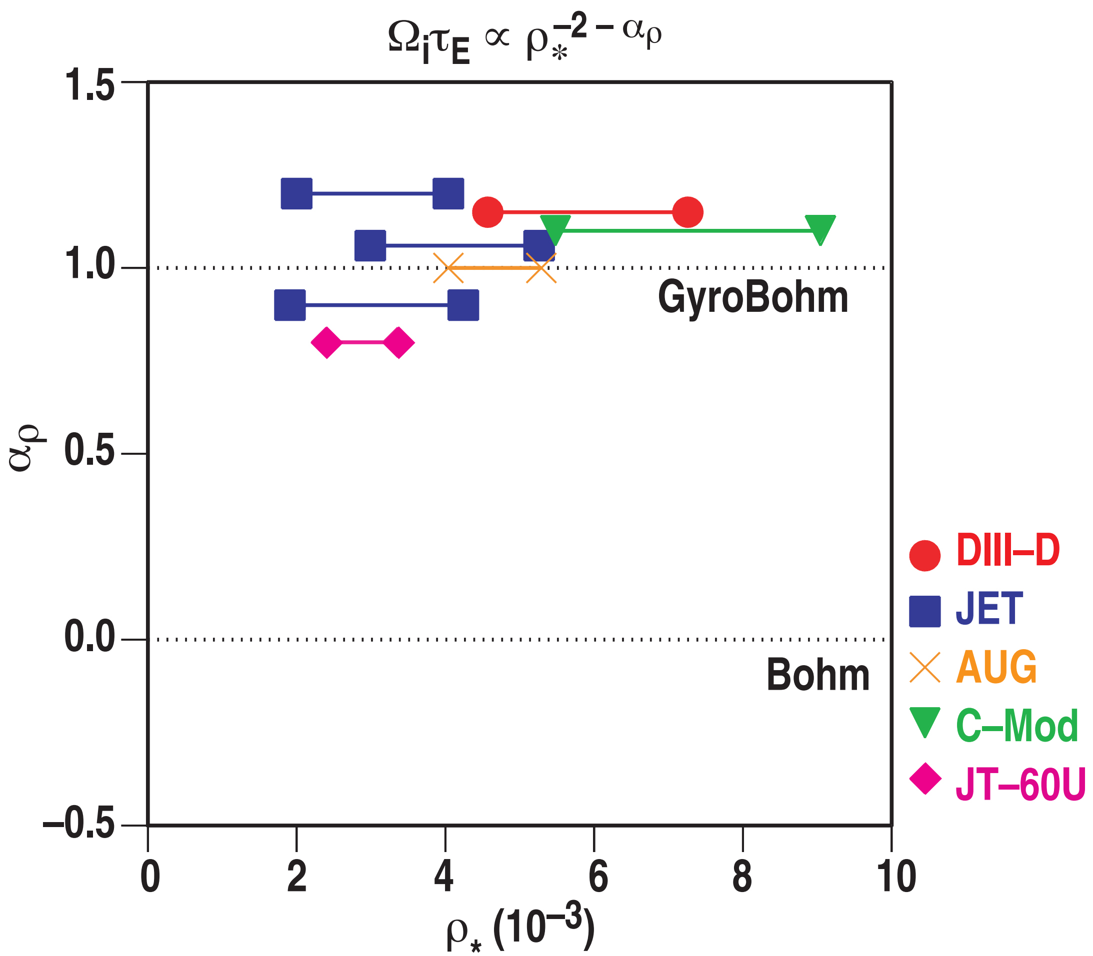


B. Balet, EPS Conf. 1995

278-06/CCP/rs

Confinement Scaling is Consistently GyroBohm for Conditions Relevant to Burning Plasmas

- **For high (H) confinement mode, ρ_* scaling exponent is typically between $0.8 \leq \alpha_\rho \leq 1.2$**

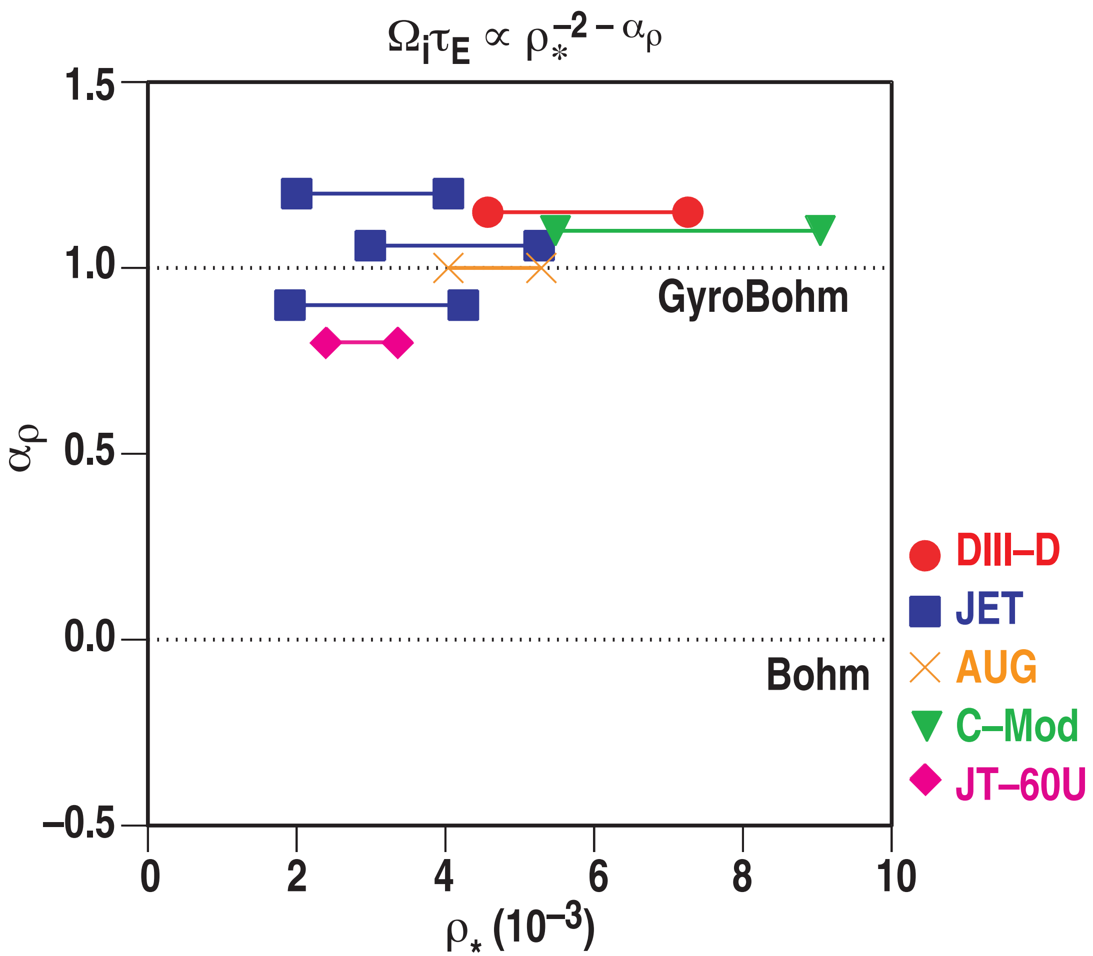


C.C. Petty, Phys. Plasmas 1995
J.G. Cordey, IAEA Conf. 1996
F. Ryter, IAEA Conf. 1996
M. Greenwald, PPCF 1998
H. Shirai, PPCF 2000
J.G. Cordey, EPS Conf. 2004

278-06/CCP/rs

Confinement Scaling is Consistently GyroBohm for Conditions Relevant to Burning Plasmas

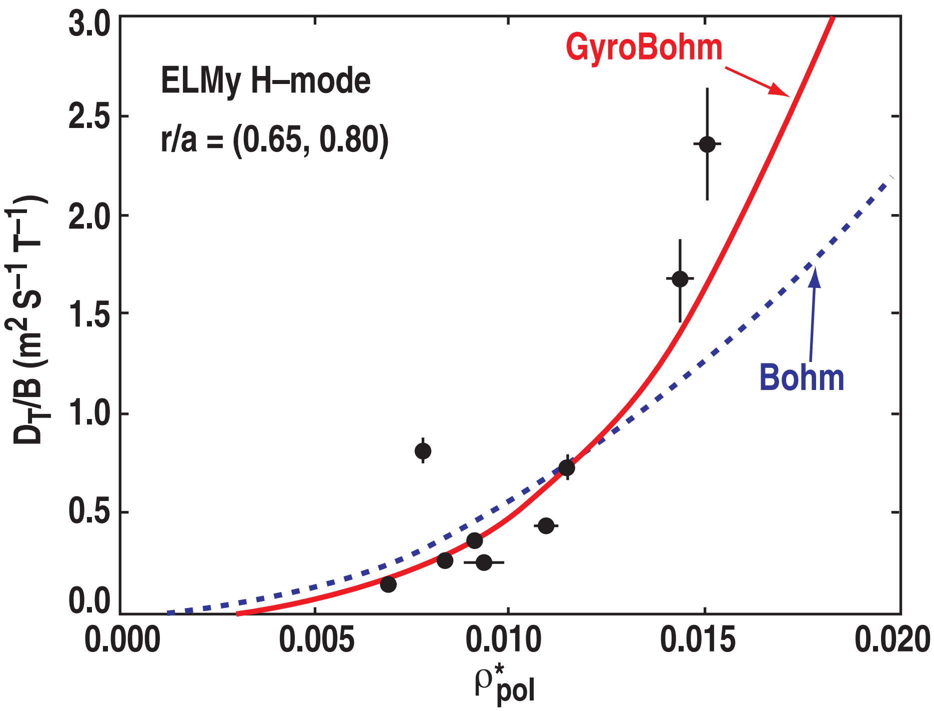
- **For high (H) confinement mode, ρ_* scaling exponent is typically between $0.8 \leq \alpha_\rho \leq 1.2$**
- **However, ρ_* scaling is more Bohm-like for:**
  - ⇒ **High q_{95} (>4)**
  - ⇒ **Heating power close to L-H threshold power**
  - ⇒ **Low (L) confinement mode** [F.W. Perkins, Phys. Fluids B 1993]



C.C. Petty, Phys. Plasmas 1995
J.G. Cordey, IAEA Conf. 1996
F. Ryter, IAEA Conf. 1996
M. Greenwald, PPCF 1998
H. Shirai, PPCF 2000
J.G. Cordey, EPS Conf. 2004

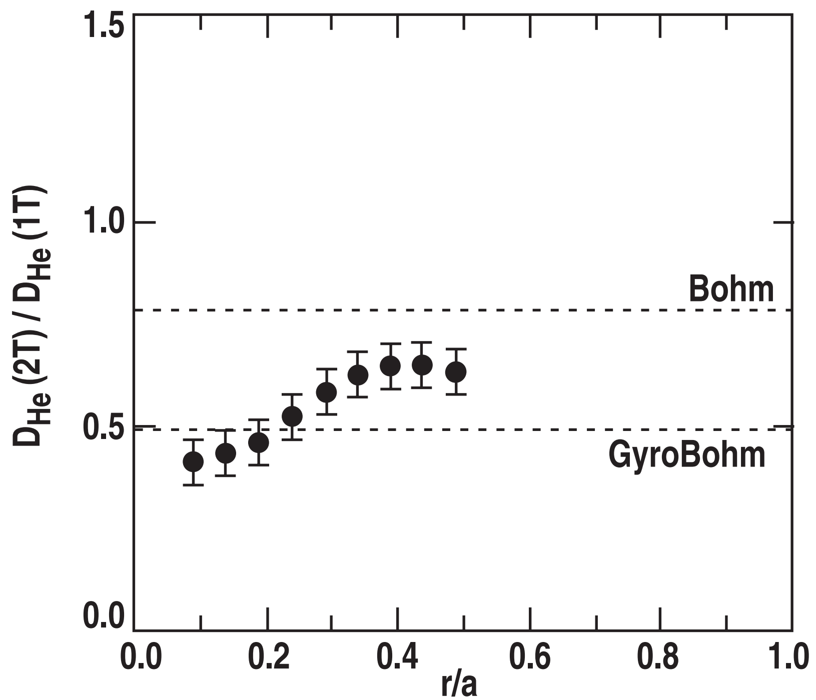
Particle Diffusivity Scales GyroBohm in High Confinement Mode (Follows Ion Thermal Diffusivity)

- **Scaling of tritium "fuel" diffusivity with ρ_* on JET**



K-D Zastrow, Plasma Phys. Control. Fusion 2004

- **Scaling of helium "ash" diffusivity with ρ_* on DIII–D**

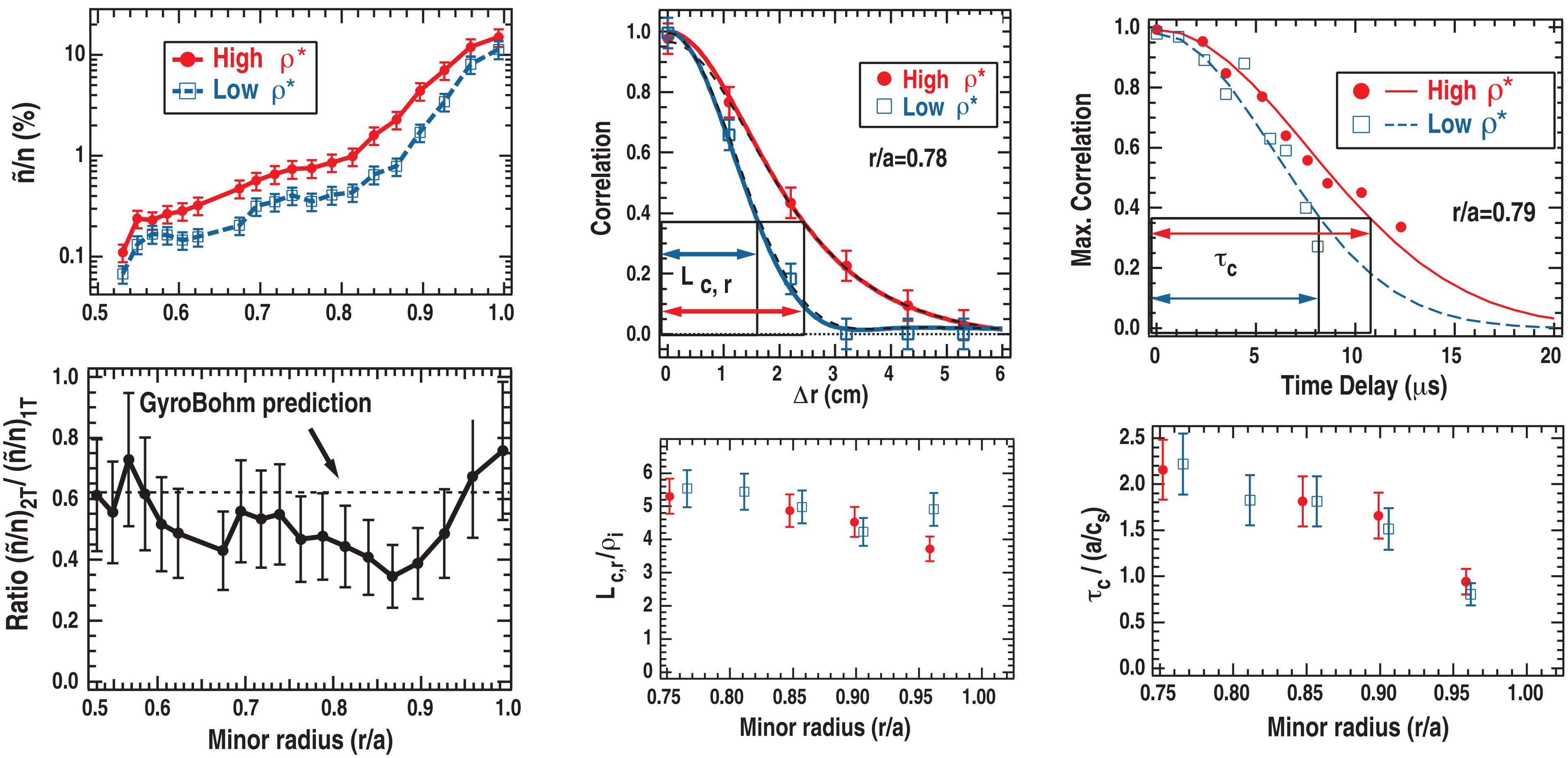


M.R. Wade, Phys. Rev. Lett. 1997

Turbulence Characteristics Scale as Expected for GyroBohm Models of Turbulent Transport

- **Beam emission spectroscopy measurements of density fluctuations during L-mode ρ_* scan on DIII–D** [G.R. McKee, Nucl. Fusion 2001]

- **Fluctuation amplitudes**
- **Radial correlation lengths**
- **Decorrelation times**



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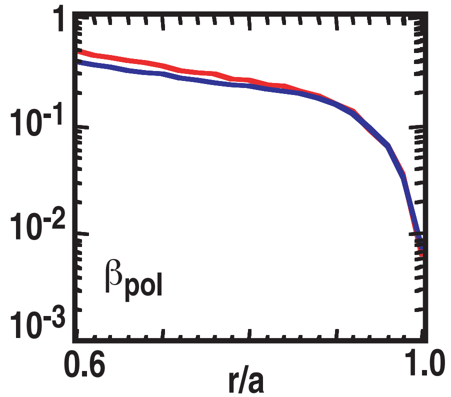
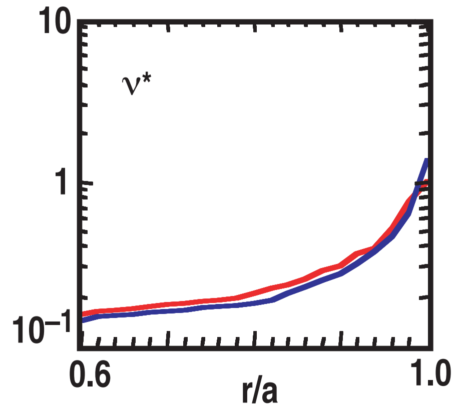
Width of H-Mode Pedestal Has Weak ρ_* Dependence

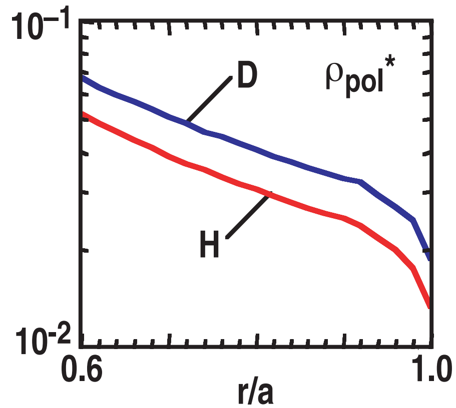
- **Comparison of hydrogen and deuterium high (H) confinement mode plasmas on JT-60U** [H. Urano, EPS Conf. 2006]

- **Beta and collisionality held fixed**



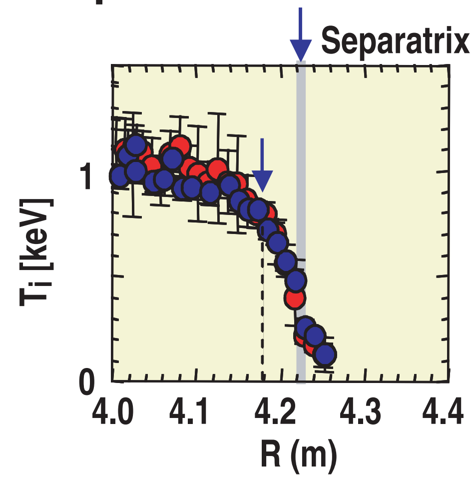


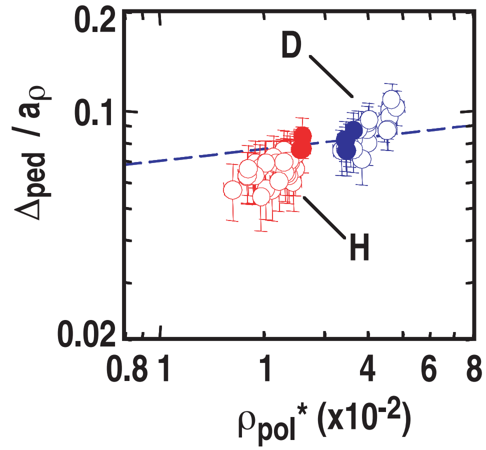
- **Scan $\rho_* \propto m_i^{1/2}$**



$$\frac{\Delta_{ped}}{a} \propto \rho_*^{0.1\pm0.15}$$

- **Ion temperature pedestal width**





Outline

I. **Introduction to Dimensional Analysis**
   A. Buckingham Π Theorem
   B. Connor-Taylor Scale Invariance
II. **Validation of Similarity**
   A. Confinement Time
   B. Edge Plasma Characteristics
III. **Dependences on Dimensionless Parameters**
   A. Collisionality
   B. Beta
   C. Relative Gyroradius
IV. **Extrapolation to ITER**
V. **Conclusions**

Scaling Plasmas to Fusion Reactors at Fixed Dimensionless Parameters, as in a Wind Tunnel, is Not Practical

- Fusion gain $Q = \frac{P_{fus}}{P_{heat}} \propto nT\tau_E$

  $\propto B_T$ (at fixed ρ_*, β, ν_*, etc.)

- Based on highest fusion performance to date on DIII–D, need to increase B_T by factor of 60 for reactor at fixed dimensionless parameters

	Q	B_T (T)	a (cm)
DIII–D	$\frac{1}{3}$	2.15	61
Reactor	20	130	2.3

- **Waltz** [Phys. Rev. Lett. 1990] showed that a practical path to a burning plasma requires an extrapolation to smaller ρ_*

$$Q \propto B_T^3\, a^{5/2} \text{ for gyroBohm scaling}$$

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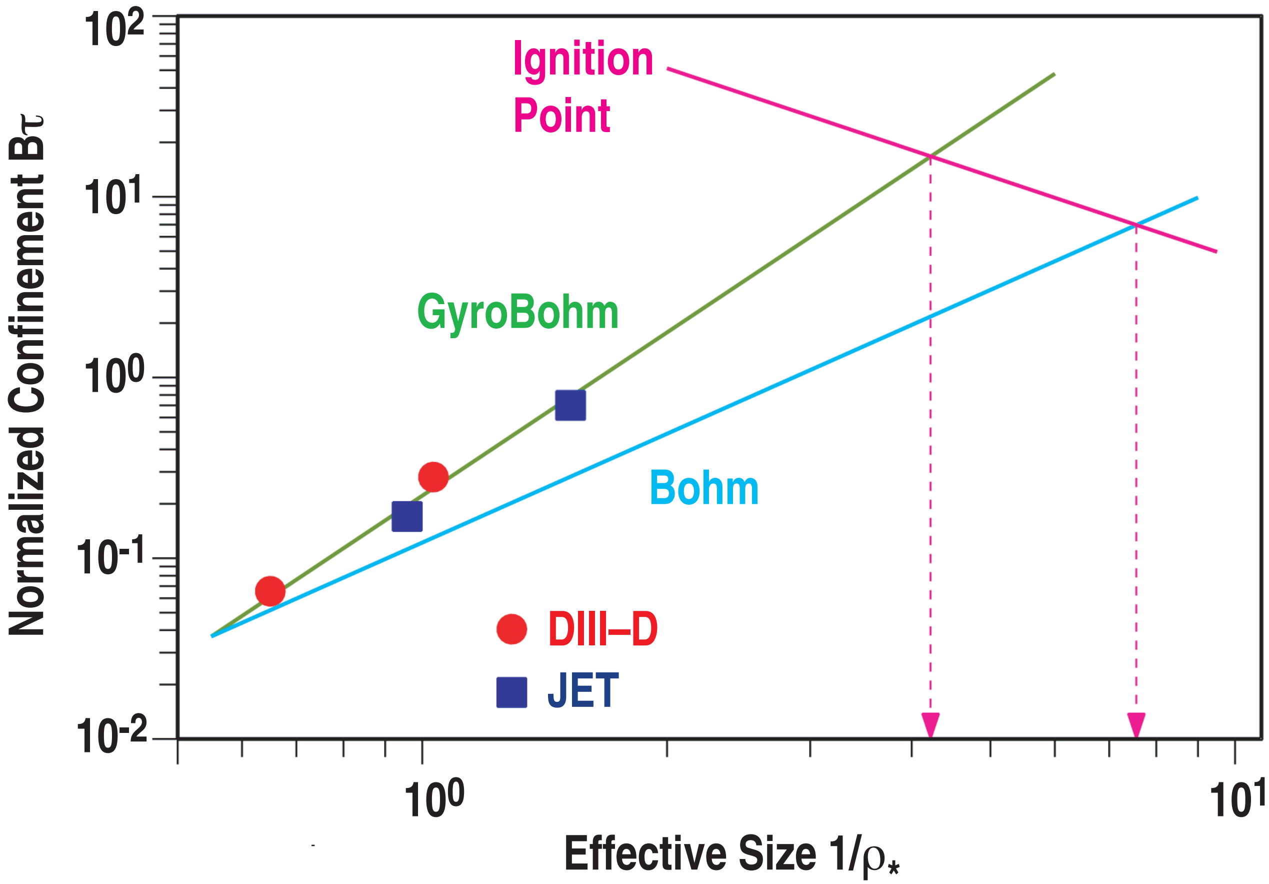
The ρ_* Scaling of Confinement Determines the Minimum Size of a Burning Plasma Device

- **Present day devices can match all of the dimensionless parameters for a burning plasma experiment except ρ_***



C.C. Petty, Fusion Sci. Technol. 2003

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Empirical Confinement Scaling Relations Contain Testable Dimensionless Parameter Scalings

- **Regression analysis of large confinement databases is typically used to derive confinement scaling relations in terms of physical quantities**
- **Important example is H-mode confinement scaling relation produced by ITER project in 1998:**

$$\tau_{98y2} = 0.0562 \; I^{0.93} B^{0.15} n^{0.41} P^{-0.69} R^{1.97} \kappa^{0.78} \varepsilon^{0.58} M^{0.19}$$

- **This relation can be recast in terms of dimensionless variables:**

$$\Omega_i \tau_{98y2} \propto \rho_*^{-2.70} \beta^{-0.90} \nu_*^{-0.01} q^{-3.0} \kappa^{3.3} \varepsilon^{0.73} M^{0.96}$$

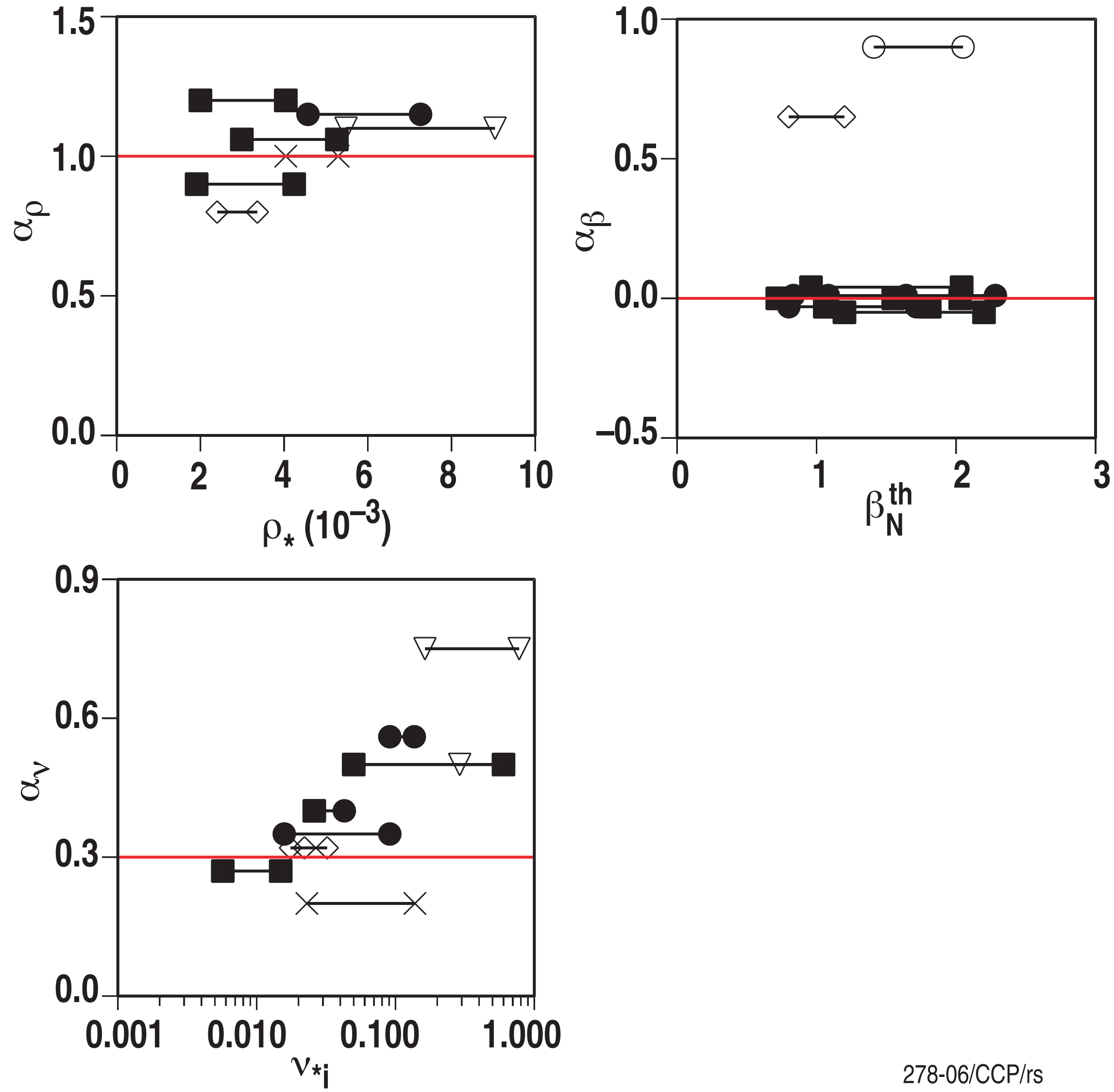
- **While the ρ_* scaling is close to gyroBohm, the β and ν_* scalings do not agree with the majority of dimensionless parameter scaling experiments**
  - This is a topic of active research in the International Tokamak Physics Activity [J.G. Cordey, Nucl. Fusion 2005]

Dimensional Analysis Puts Scaling Relations on a Firm Physics Basis

- A scaling relation for the energy confinement time can be derived from dimensionless parameter scans:

$$\Omega_i \tau_E \sim \rho_*^{-3} \beta^0 \nu^{-0.3} \times q^{-1.4} \kappa^{2.2} \varepsilon^{-0.8} M^{0.5}$$

- Worldwide H-mode dimensionless parameter scaling results



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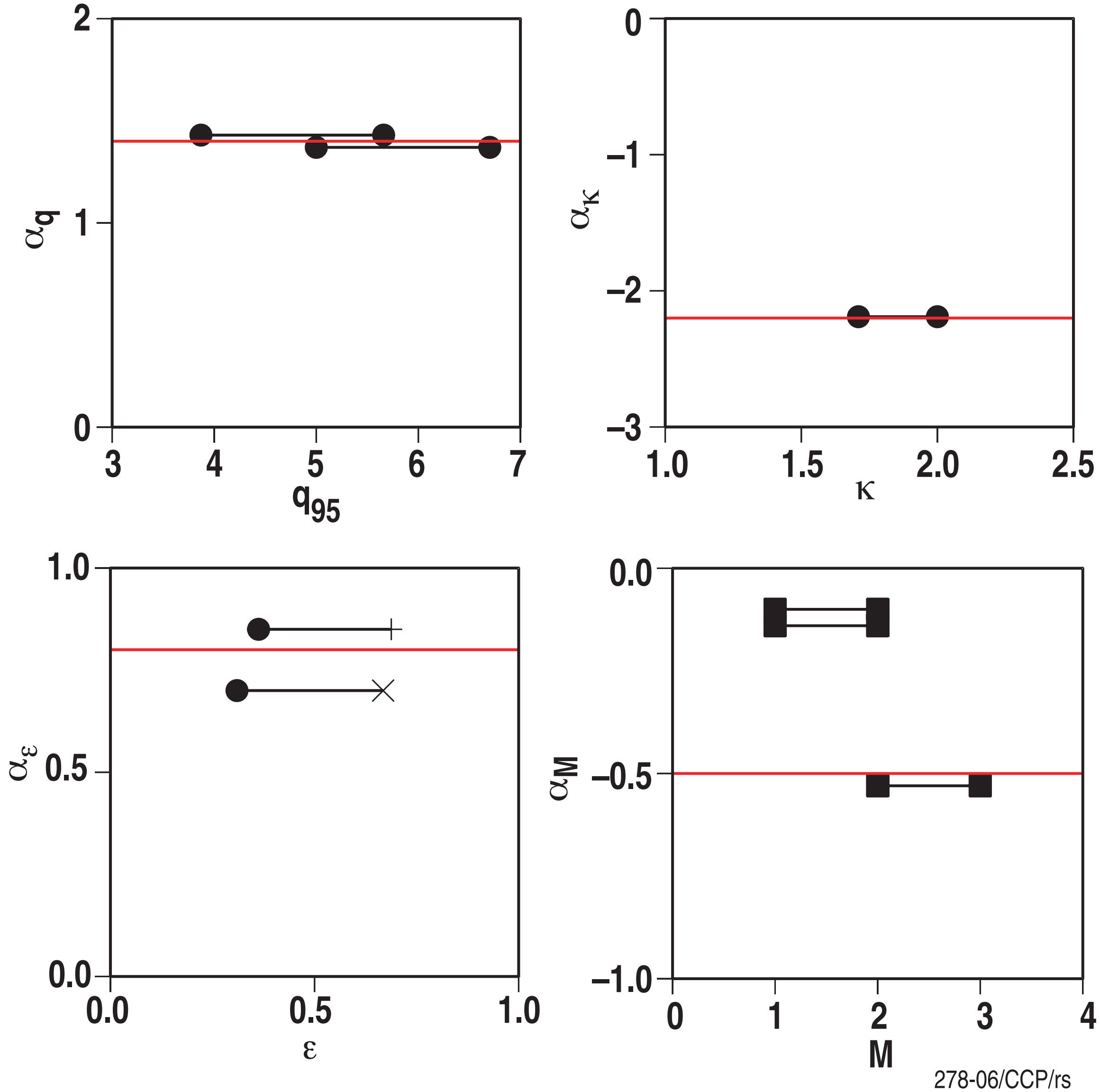
- **Worldwide H-mode dimensionless parameter scaling results**

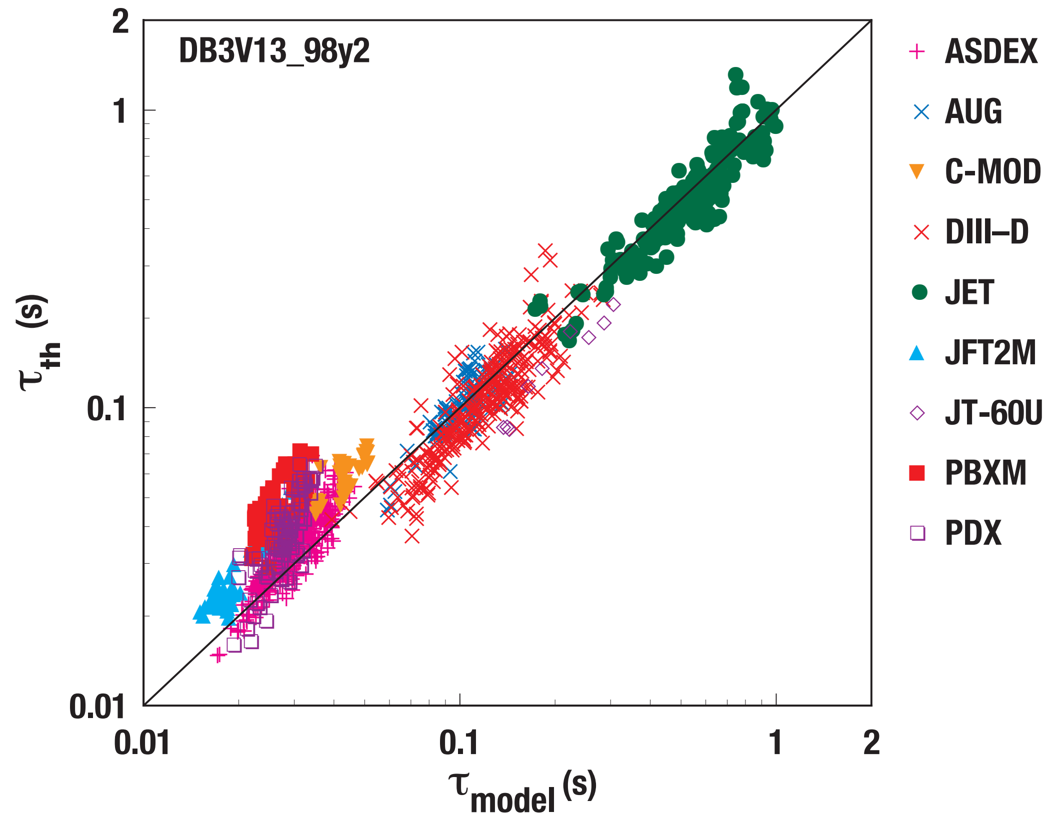


Converting to Physical Variables Gives a Confinement Scaling Relation That is Dimensionally Homogeneous

- **Normalizing the magnitude of τ_E to an International H-mode Confinement Database gives:**

$$\tau_E = 0.052\, I^{0.8}\, B^{0.3}\, n^{0.3}\, P^{-0.5}\, R^{2.1}\, \kappa^{0.9}\, \varepsilon^{0.8}\, M^{0.0}$$

- **Predicted confinement time on ITER:**
  - Empirical scaling [IPB98(y,2)]
    $\tau_E = 3.7$ s
  - Dimensionless parameter scaling
    $\tau_E = 4.8$ s



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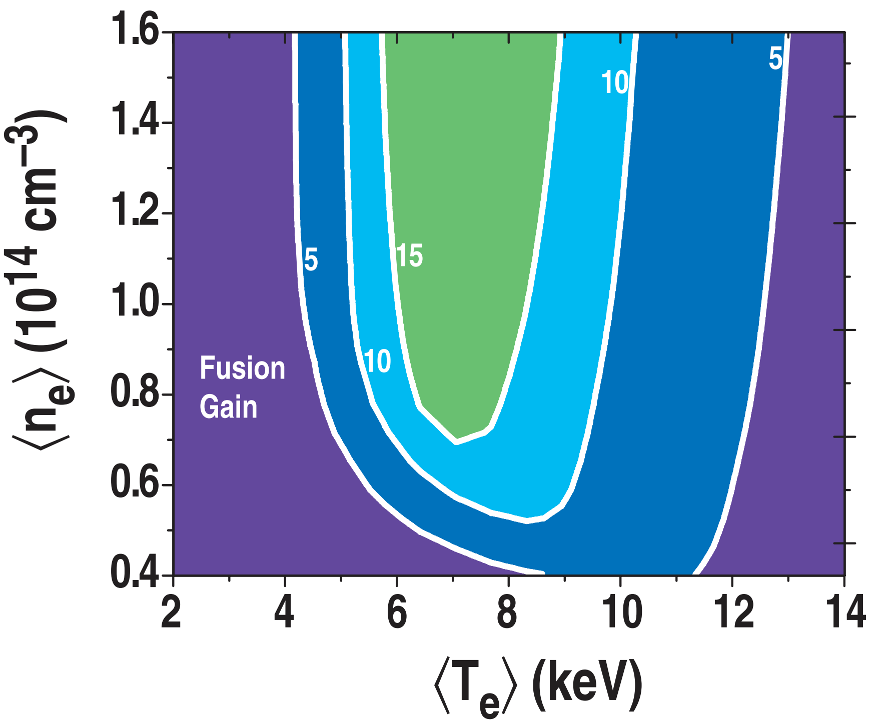
Primarily Electrostatic Transport Results in Fusion Gain Increasing Strongly with β

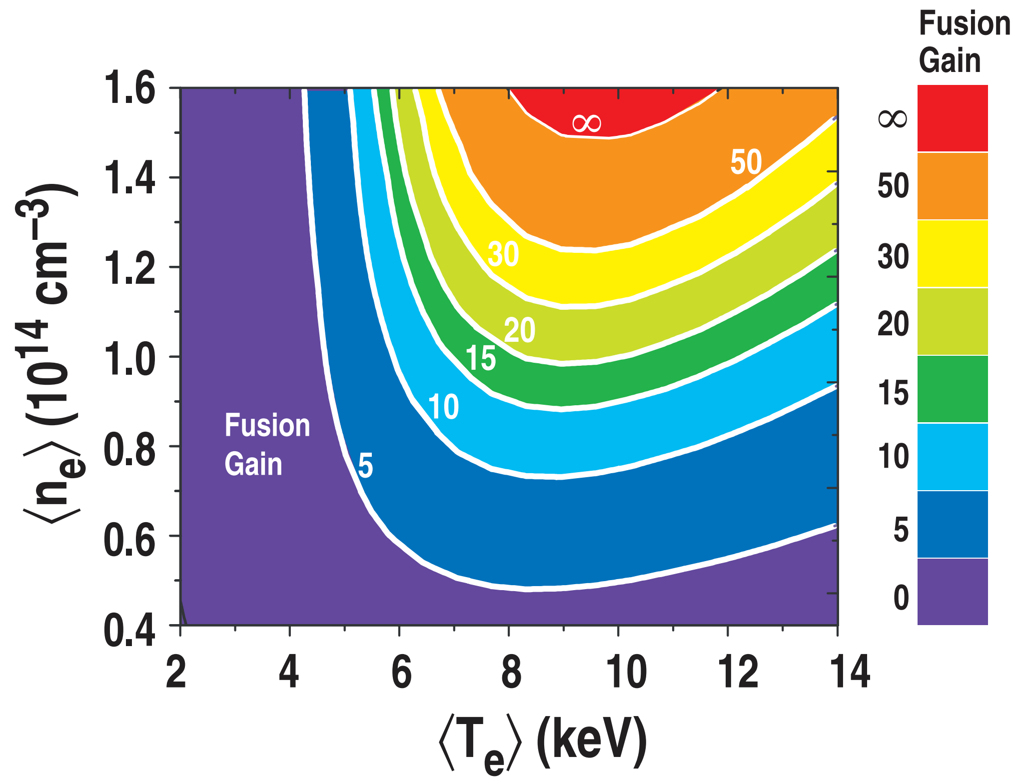
Empirical scaling [IPB98(y, 2)]:

$\Omega_i \tau_{98y2} \propto \rho_*^{-2.7} \beta^{-0.9}$

Electrostatic, gyroBohm scaling:

$\Omega_i \tau_E \propto \rho_*^{-3} \beta^{0}$





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Conclusions

- Applying the methods of dimensional analysis and scale invariance to plasma experiments helps to illuminate the underlying physical processes
- Principle of similarity has been validated in high temperature plasmas
- Dimensionless parameter scans in magnetic-confinement fusion experiments generally support drift-wave turbulent transport models
- Edge plasma characteristics also seem governed by dimensionless plasma physics variables
- Knowing the dimensionless parameter scalings gives a method for extrapolating phenomena from present-day experiments to burning plasma devices that has a firm physical basis

Comparison of Dimensional Analysis and Scale Invariance

- **Dimensional analysis starts with a set of physical variables, whereas scale invariance begins with the governing equations**

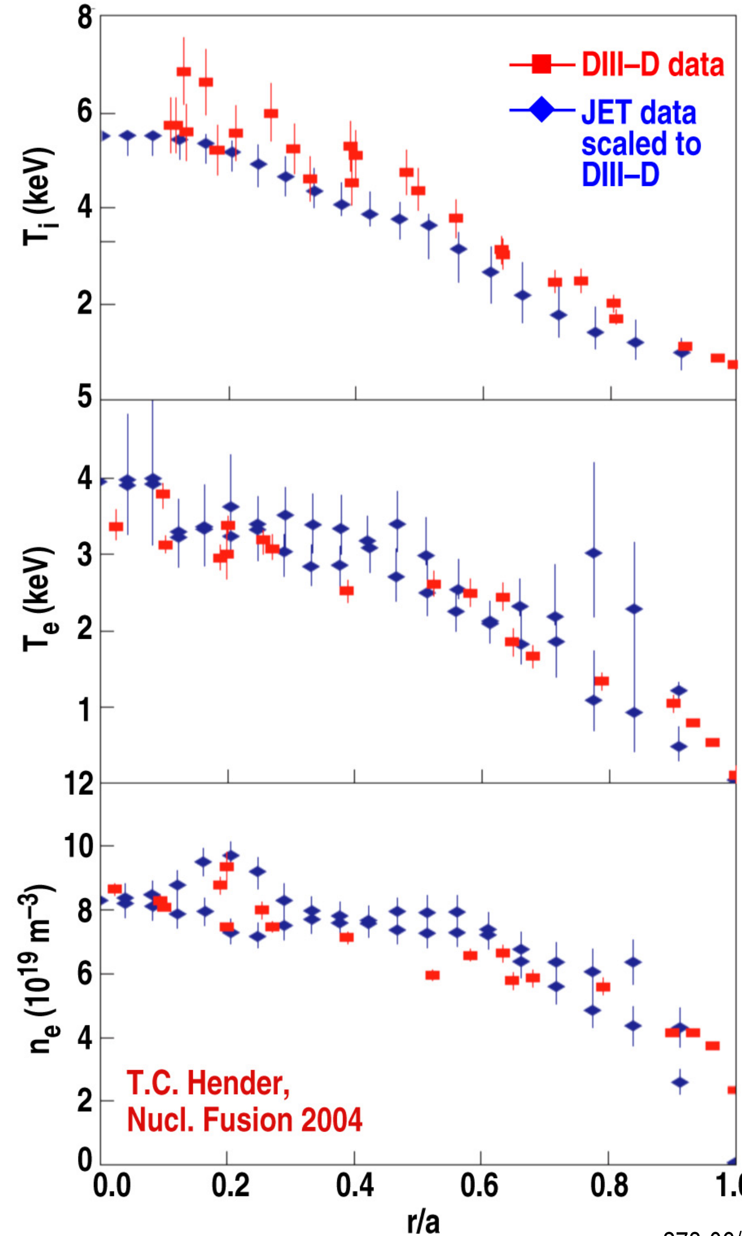
  ⇒ Some authors use a hybrid method where the Buckingham Π products are derived from the variables found in the governing equations [Beiser, Phys. Fluids 1961] [Lacina, Plasma Phys. 1971]

- **Weeding out insignificant dimensionless parameters**
  - Scale invariance naturally leaves out dimensionless variables related to neglected physical processes
  - Dimensional analysis relies on users to recognize that dimensionless variables << 1 (including reciprocals) are usually not important
    - Note that the ratio of tiny dimensionless variables can form an important dimensionless parameter that must be retained
    - Scale invariance gives same result by explicitly showing when dimensionless variables only appear in combination

- **A subtle point of scale invariance is that some essential dimensionless variables may appear only in the boundary conditions of the equations**

Validation of Similarity Has Been Extended to the Onset Physics of MHD Phenomena

- **JET and DIII-D measured β limit for m/n=2/1 neoclassical tearing mode**
- **First verified that scaled profiles ($n \propto a^{-2}$ and $T \propto a^{-0.5}$) are equivalent on the two different-sized devices**



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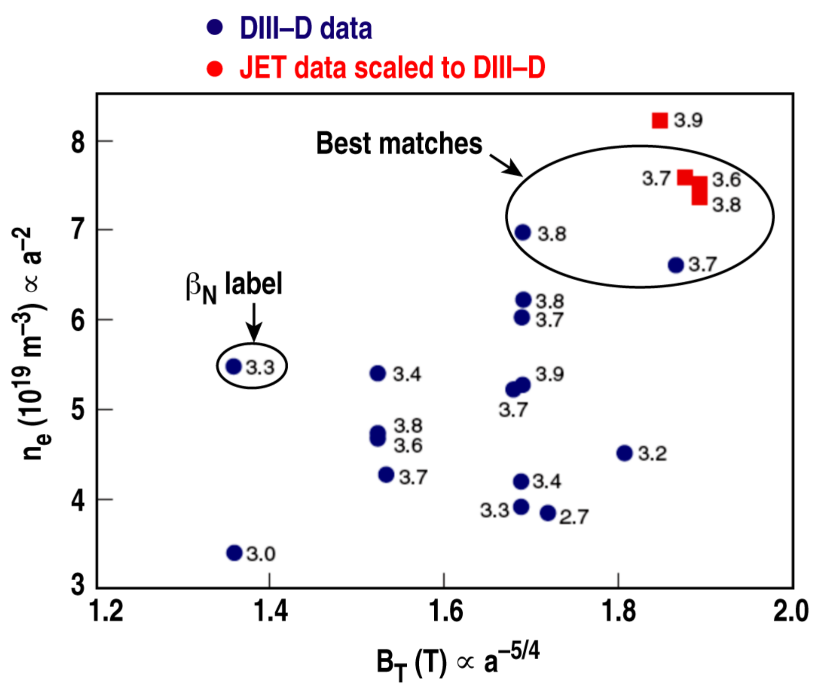
- JET and DIII-D measured β limit for m/n=2/1 neoclassical tearing mode
- First verified that scaled profiles ($n \propto a^{-2}$ and $T \propto a^{-0.5}$) are equivalent on the two different-sized devices
- Experiment confirmed JET and DIII-D have similar β limits for the 2/1 tearing mode when the other dimensionless parameters (ρ_*, ν_*, geometry) are closely matched



T. C. Hender, Nucl. Fusion 2004

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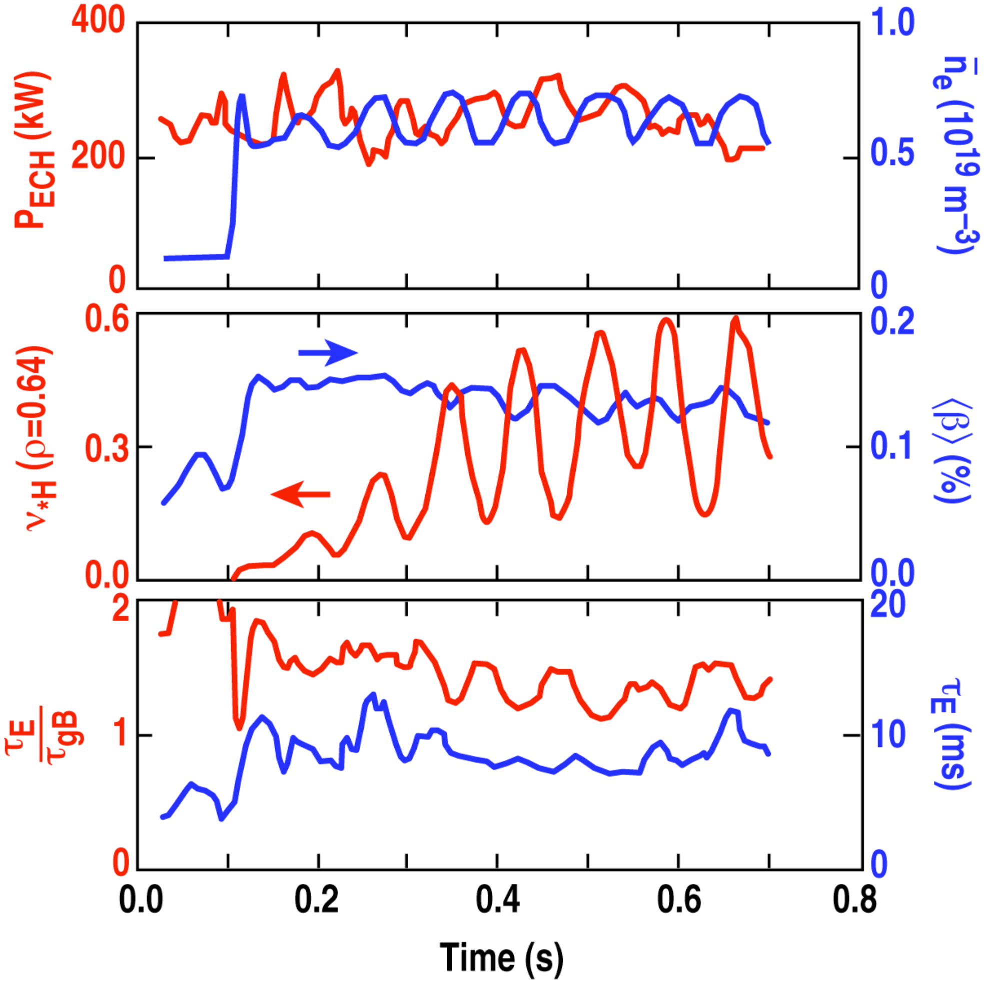
Stellarators Also Observe Relatively Weak β and ν_* Scaling of Transport, Similar to Tokamaks

- **Novel experiment on ATF torsatron modulated β and ν_* separately**
- **Analysis of resulting perturbation in confinement time found**

$$\Omega_i \, \tau_E \propto \beta^{0.3} \, \nu_*^{-0.2}$$



J.B. Wilgen, Phys. Fluids B 1993

278-06/CCP/rs

Most Unstable Toroidal Mode Number for TAE Increases With Smaller ($\rho_* q$)

- Energetic ions can drive the toroidicity-induced Alfven eigenmode (TAE) unstable
- Most unstable toroidal mode number is predicted to scale like either:
  - $n_{max} \propto (\rho_* q)^{-1}$
  - $n_{max} \propto (\rho_* q^2)^{-1}$
- Comparison of TAE on NSTX and DIII-D found n_{max} correlated well with both $(\rho_* q)^{-1}$ and $(\rho_* q^2)^{-1}$
- This scaling gives:

$n_{max} \sim 1$ for NSTX
$n_{max} \sim 5$ for DIII-D
$n_{max} \sim 10\text{-}20$ for ITER