Resistive Stability of 2/1 Modes Near 1/1 Resonance

D.P. Brennan
Department of Physics and Engineering Physics
The University of Tulsa

M.S. Chu, R.J. La Haye, L.L. Lao, T.H. Osborne, A.D. Turnbull,
General Atomics, San Diego, CA
S.A. Galkin
Far-Tech Inc., San Diego, CA

The NIMROD Team
http://nimrodteam.org

Presented at the 48th Meeting of the
American Physical Society Division of Plasma Physics
Philadelphia, Pennsylvania, November 2, 2006

Motivation

Recently: Equilibrium reconstructions of tokamaks become accurate enough for subtle physics to be deciphered.

Recently: Experimental attempts to access the highest β in tokamak discharges, including hybrid discharges, typically terminated by 2/1 tearing mode.

Unexplored: The complete linear stability and nonlinear behavior analyses of the onset physics of the 2/1 mode.

Needed: A thorough understanding of the equilibrium and stability in present day fusion experiments to extrapolate to a burning plasma (ITER).

Outline

Focus: approach to q=1 effect on 2/1 resistive mode stability.

- Experiment: Approaching q=1 Resonance and 2/1 Onset
    - Hybrid Discharges Defined
- Equilibrium Reconstruction at 2/1 Onset and Model Equilibria
- Linear Resistive MHD Stability of 2/1 as q=1 Approached
    - Approaching Ideal Limit and Nonresonant 1/1 Response
    - Inner Layer Model and Resistive Instability Threshold
- NIMROD Stability in Agreement
- Link to Recent Theories on Steady State Current Density
- Summary

‘Hybrid’ Scenario Occupies a Critical Strategic Position Within Tokamak Scenario Portfolio

---

High β, long pulse, well controlled, attractive scenario for ITER.

Conventional	Advanced Inductive	Hybrid	Advanced Tokamak
$\beta_N < \beta_{N,\text{no-wall}}$	$\beta_N \leq \beta_{N,\text{no-wall}}$	$\beta_N \approx \beta_{N,\text{no-wall}}$	$\beta_N \leq \beta_{N,\text{ideal-wall}}$
$q_{95} \approx 3$	$q_{95} \approx 3$	$q_{95} \geq 4$	$q_{95} \approx 5$
$q_0 \leq 1$	$q_0 \leq 1$	$q_0 \approx 1$	$q_0 > 1$
$f_{BS} < 0.3$	$f_{BS} \approx 0.3$	$f_{BS} \approx 0.5$	$f_{BS} \approx 0.8$
$H_{89} \approx 2$	$H_{89} > 2$	$H_{89} > 2$	$H_{89} > 2$

Increasing Complexity/Payoff

Highest β Tokamak Discharges, Including Hybrid Discharges, Typically Terminated by 2/1

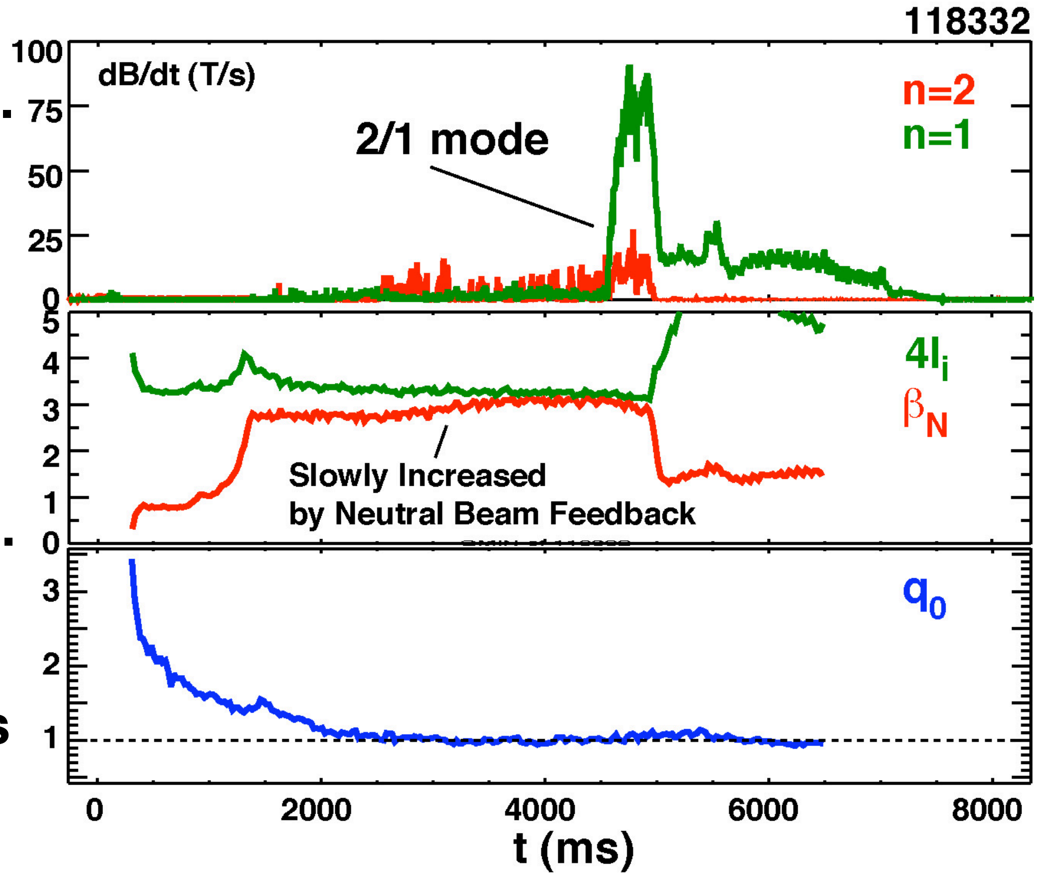
In Hybrid Discharges q_0 approaches and hovers near 1.
See: R.J. La Haye GP1.00009

Resonance induced negative current drive sustains $q_0 \geq 1$.
See: M.S.Chu IAEA 06, EX/1-5

Little to no 1/1 mode observed.

Steady state 3500ms-4500ms.

2/1 resistive mode often grows and terminates the discharge.



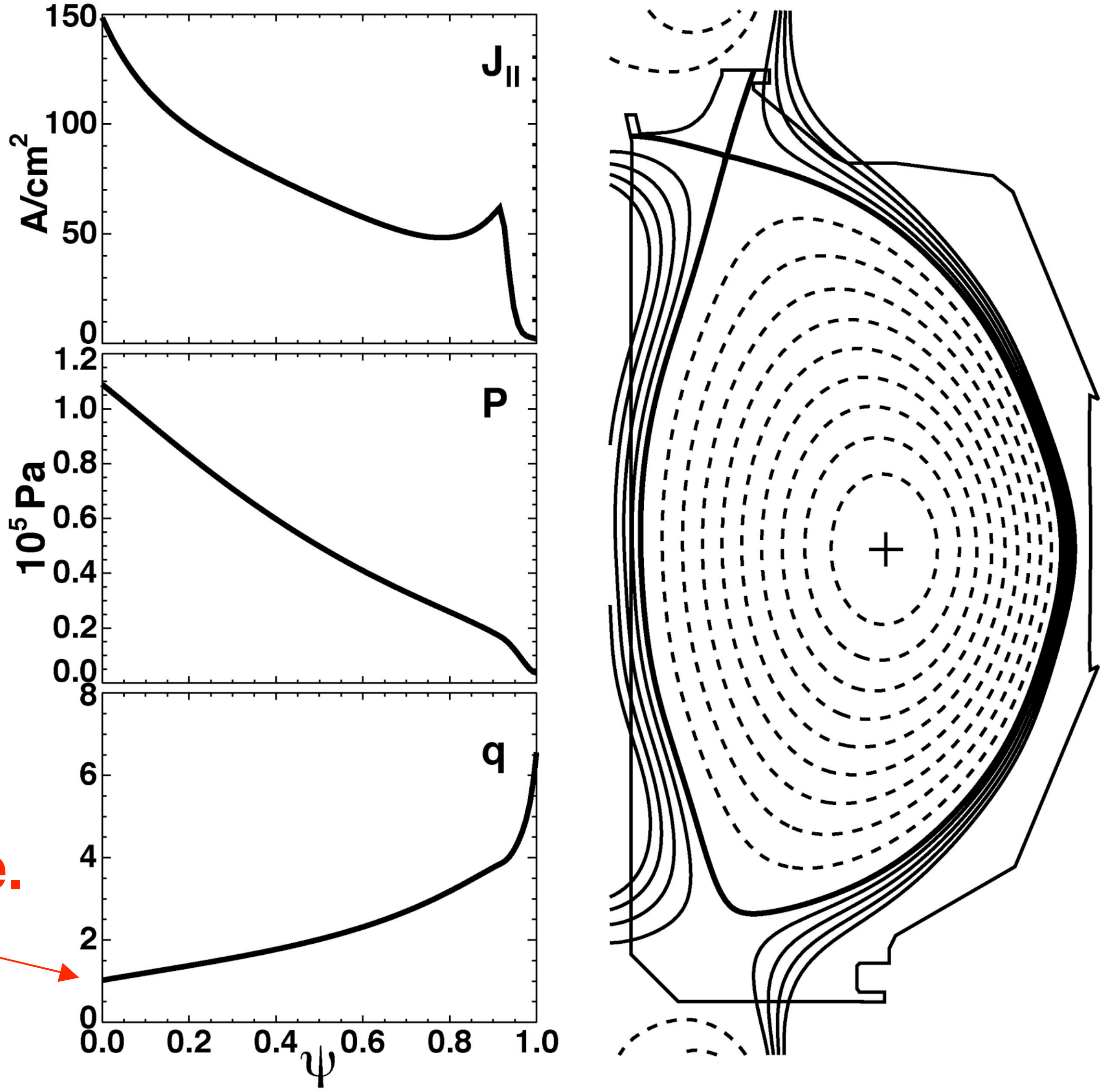
Are the q=1 resonance and the 2/1 mode onset related?

Equilibrium Reconstruction Just Before 2/1 Onset Used as Basis for “Family” of Equilibria to Examine Stability

Accurate equilibrium reconstruction uses B_z, $T_{e,i}$ and Density profile data.

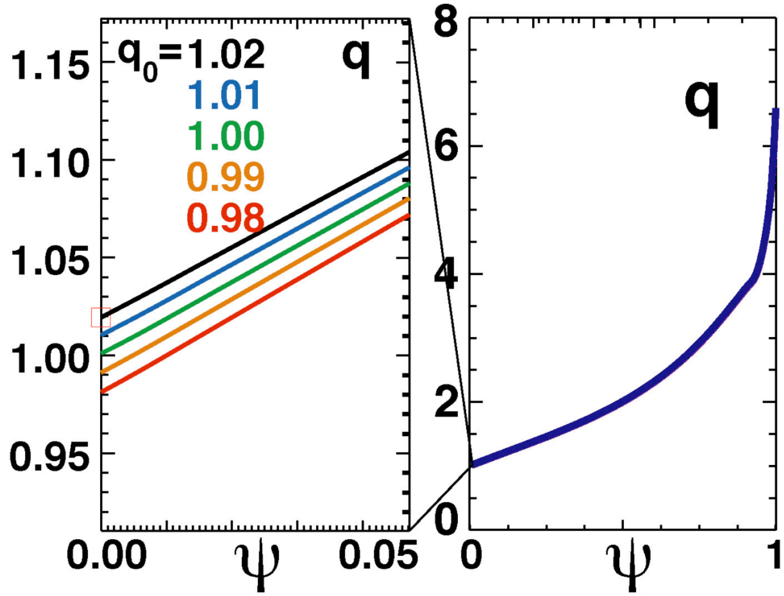
β_N=3.09
l_i=0.849
$\beta_N/4l_i$=0.91

q_0~1 has moderate shear, giving small radius of q=1 surface.



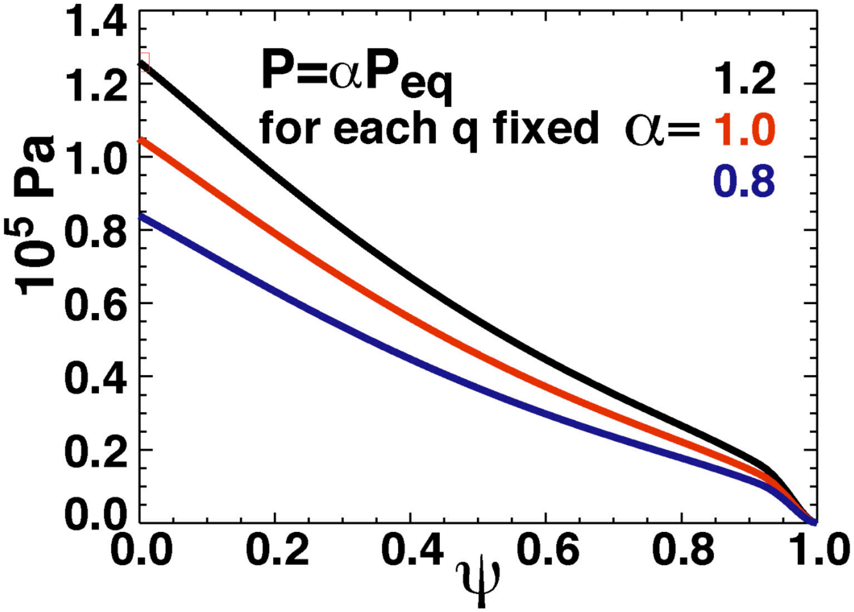
q_0 Constrained to ~2% by Data: Investigate Role in Stability by Varying P for Series of Fixed q_0

Constraint on q_0 varied within uncertainty of reconstruction, $0.98<q_0<1.02$, with little change in equilibrium near q=2 and elsewhere.



For each q_0 stability of 2/1 mode as function of β is computed.

Pressure increases with T_e at fixed density, which affects inner layer via resistivity and equilibrium pressure.



Complete Linear Stability at Rational Surfaces is Described by Matrix Dispersion Relation

$$\boxed{det[\mathbf{D}' - \mathbf{D}(Q)] = 0}$$

Solve for Q=γτ normalized growth rate.

PEST-III Outer Ideal Solution

Inner Layer Solution

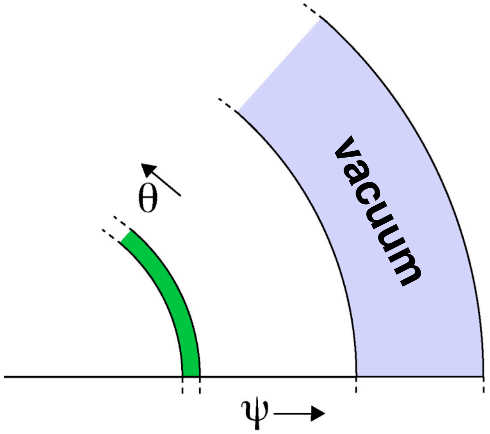
Pure Interchange Parity

$$\mathbf{D}' \equiv \frac{1}{2}\begin{bmatrix} \mathbf{A}' & \mathbf{B}' \\ \mathbf{\Gamma}' & \mathbf{\Delta}' \end{bmatrix} \qquad \mathbf{D} \equiv \frac{1}{2}\begin{bmatrix} \mathbf{A}(Q) & 0 \\ 0 & \mathbf{\Delta}(Q) \end{bmatrix}$$

Coupling

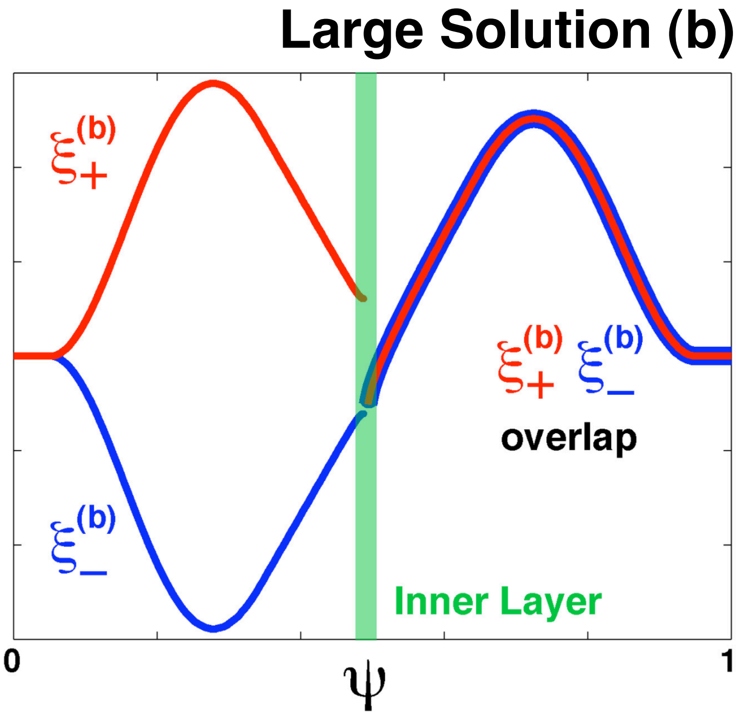
Pure Tearing Parity

We Study the single resonant surface 2/1. High flow shear between surfaces shields coupling.



Comparison Between Tearing Parity Analysis and Coupled Tearing and Interchange Clarifies Sensitivity

The Glasser, Greene and Johnson (1975) analytic inner layer is compared to the numerical result from Galkin, Turnbull, Greene and Brennan, (2002).



	parity		parity
A'	++	$A(Q)$	++
B'	+-		
Γ'	-+		
Δ'	- -	$\Delta(Q)$	- -

$$\begin{bmatrix} \mathbf{A}' & \mathbf{B}' \\ \mathbf{\Gamma}' & \mathbf{\Delta}' \end{bmatrix} - \begin{bmatrix} \mathbf{A}(Q) & 0 \\ 0 & \mathbf{\Delta}(Q) \end{bmatrix} = 0$$

Galkin 2002 solves the problem numerically finding both Δ(Q) and A(Q).

$$\mathbf{\Delta}' - \mathbf{\Delta}(Q) = 0$$

$$\mathbf{\Delta} = 2\pi \frac{V_s}{X_o} \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} Q^{5/4} (1 - \frac{\pi D_R}{4Q^{3/2}})$$

GGJ Solves the problem analytically for Δ(Q) alone, not A(Q).
Includes interchange drive through D_R.

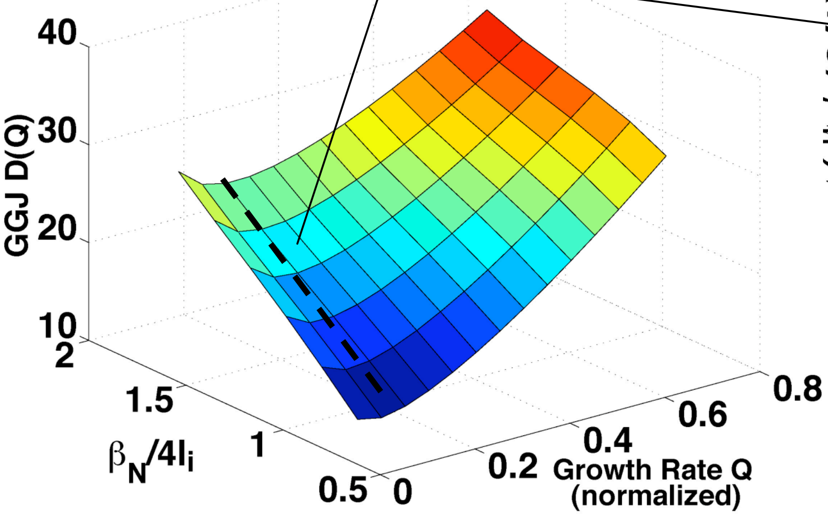
Inner Layer Analysis Indicates Δ(Q) is Large, GGJ Result Implausible

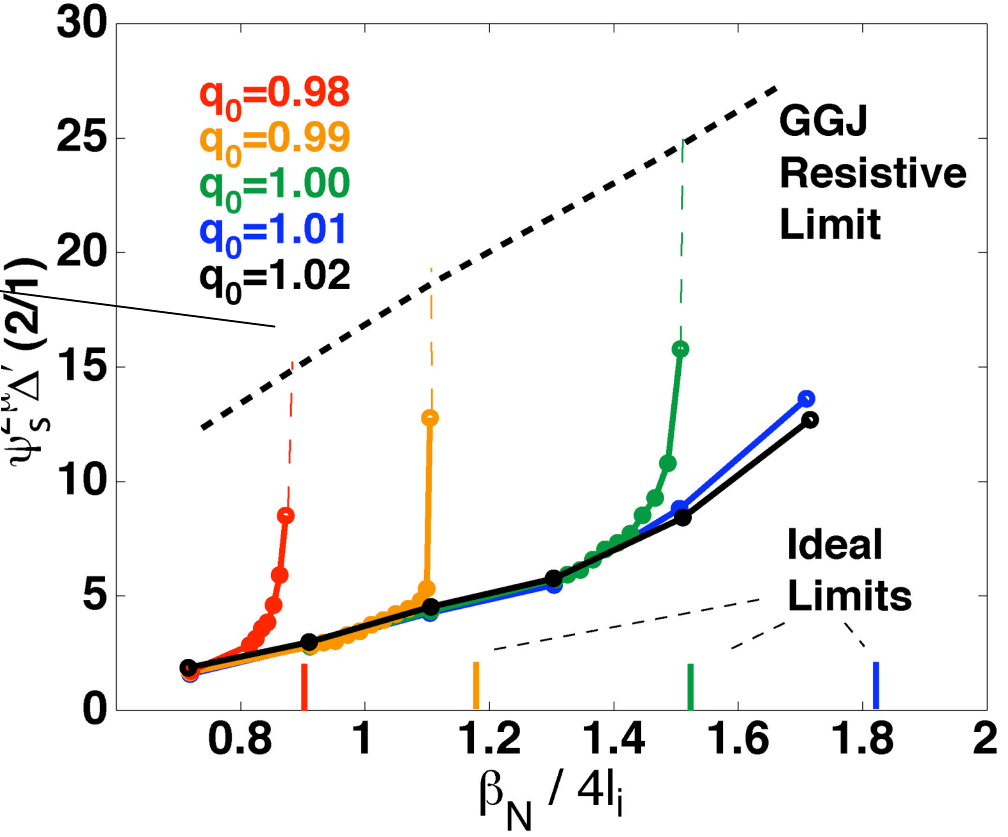
Very Large $\Delta' > \Delta(Q)$ Needed For Onset

Onset point extremely close to ideal limit, suggesting resistive instability not accessible to experiment.

$$\Delta' - \Delta(Q) = 0$$

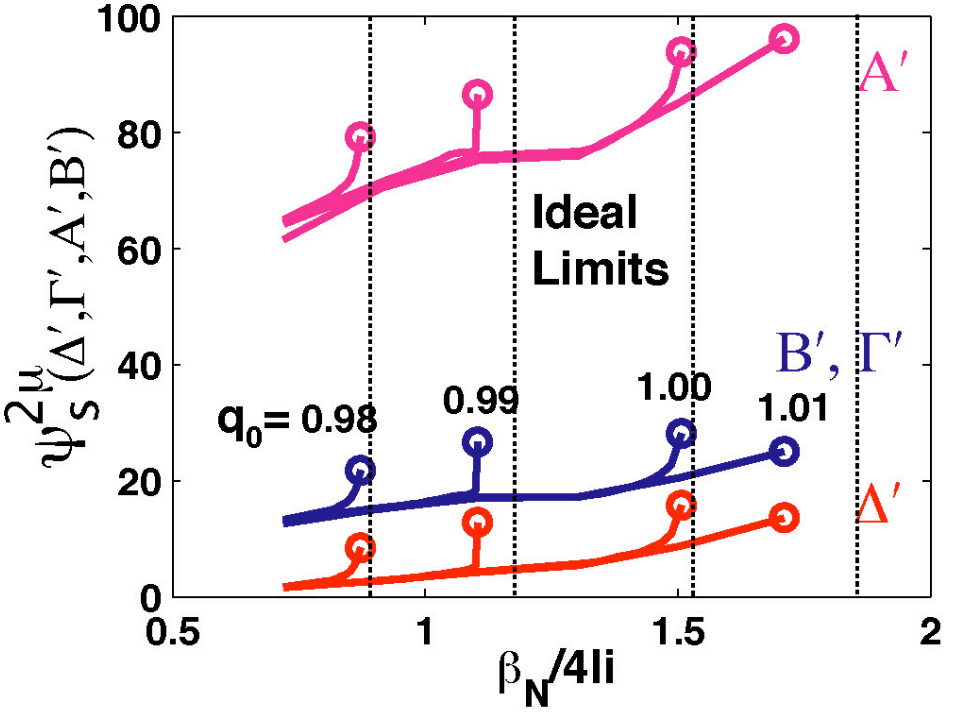
Minimum Δ' for root in Q.



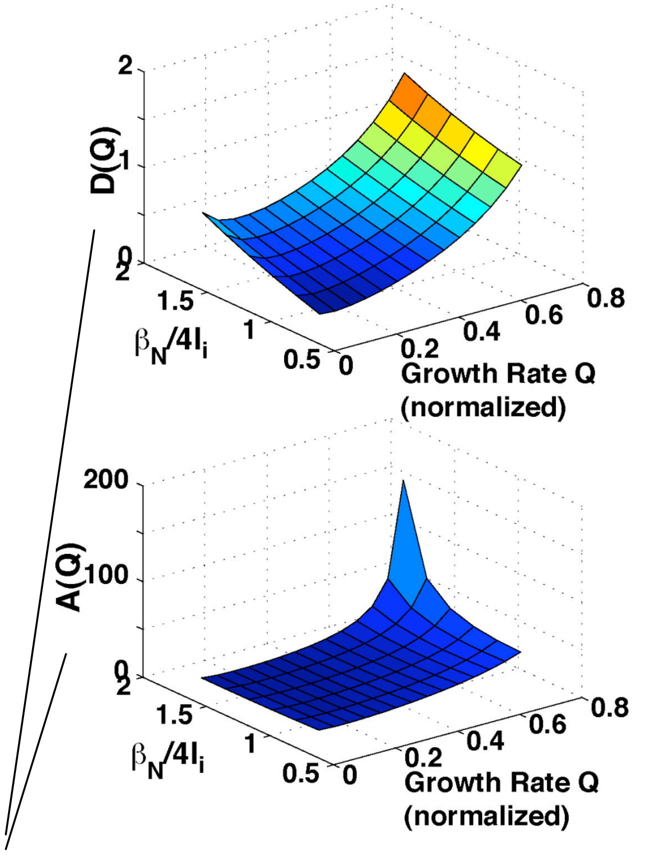


Coupled Tearing/Interchange Analysis Indicates A′,A(Q) Large, Result More Accurate

All four elements, A', B', Γ' and Δ' must be addressed for complete picture.



$$\begin{bmatrix} \mathbf{A}' & \mathbf{B}' \\ \mathbf{\Gamma}' & \mathbf{\Delta}' \end{bmatrix} - \begin{bmatrix} \mathbf{A}(Q) & 0 \\ 0 & \mathbf{\Delta}(Q) \end{bmatrix} = 0$$



Both inner layer solutions critical to analysis.

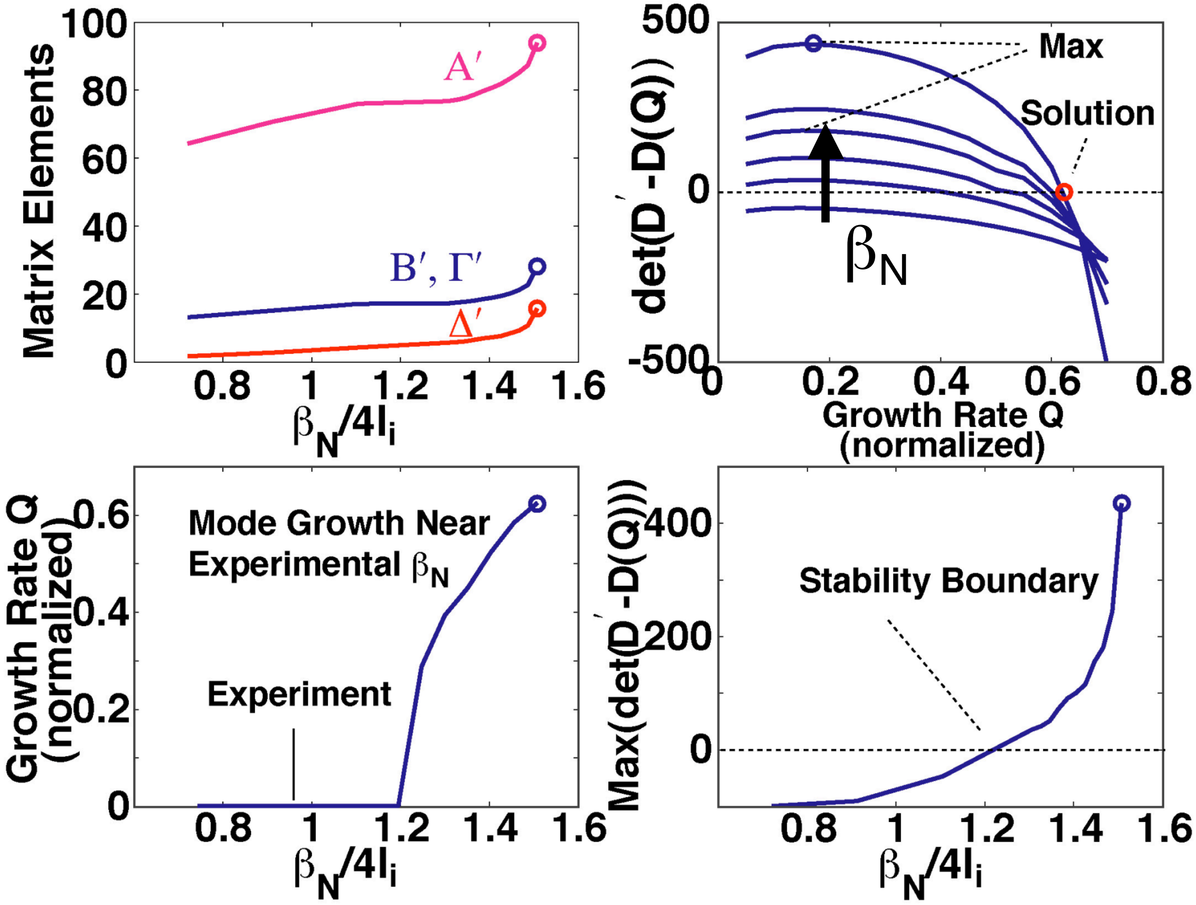
Coupled Tearing/Interchange Analysis Plausible Shows Onset At Lower β_N

Maximum of matrix determinant as a function of growth rate Q gives stability boundary.

For lower q_0 lower stability boundary in β_N

Including all four matrix elements is essential for agreement.

q_0=1.01



Coupled Tearing/Interchange Analysis Explains Onset

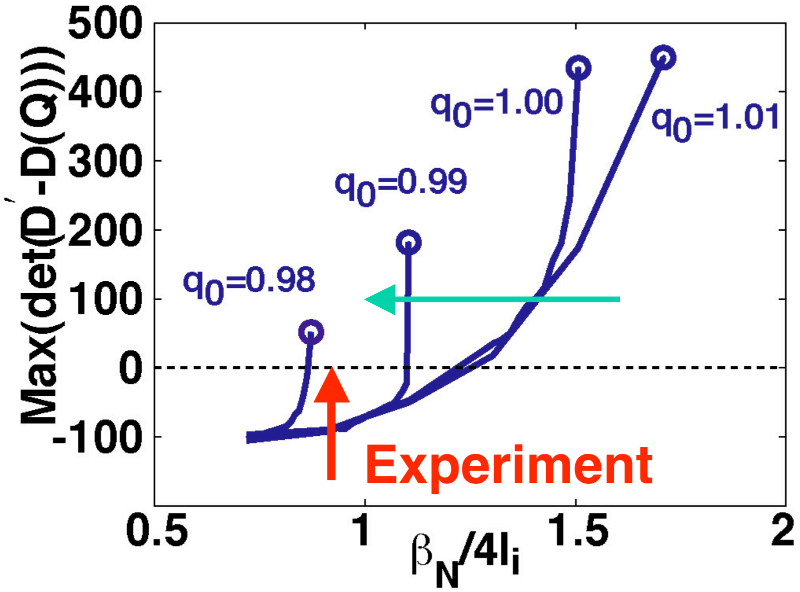
The maximum determinant crosses zero at experimental β_N, **causing onset**

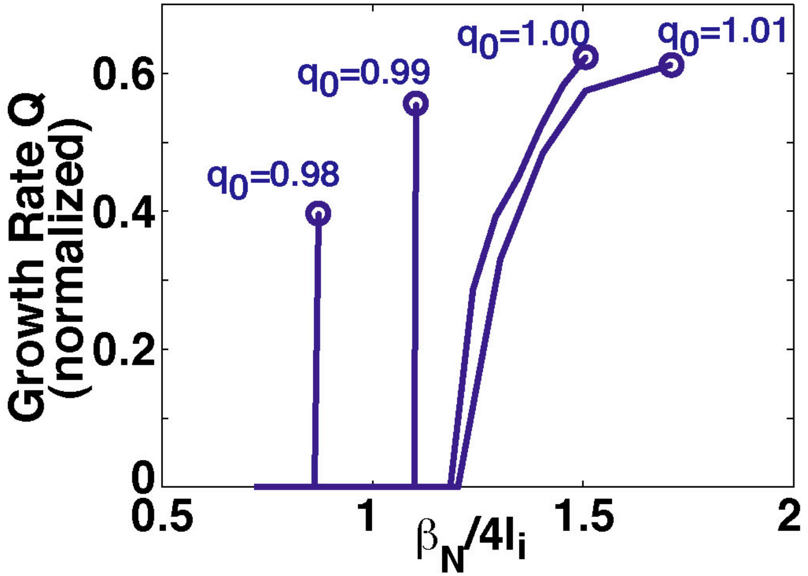
Interchange important at high β_N, and is considered.

Growth rates are within experimental observations.

$\gamma = Q/\tau$
$1/\tau \sim 60 s^{-1}$

Ideal n=1 limit crossed just above $\beta_N/4l_i$ of circle points. Internal kink unstable.



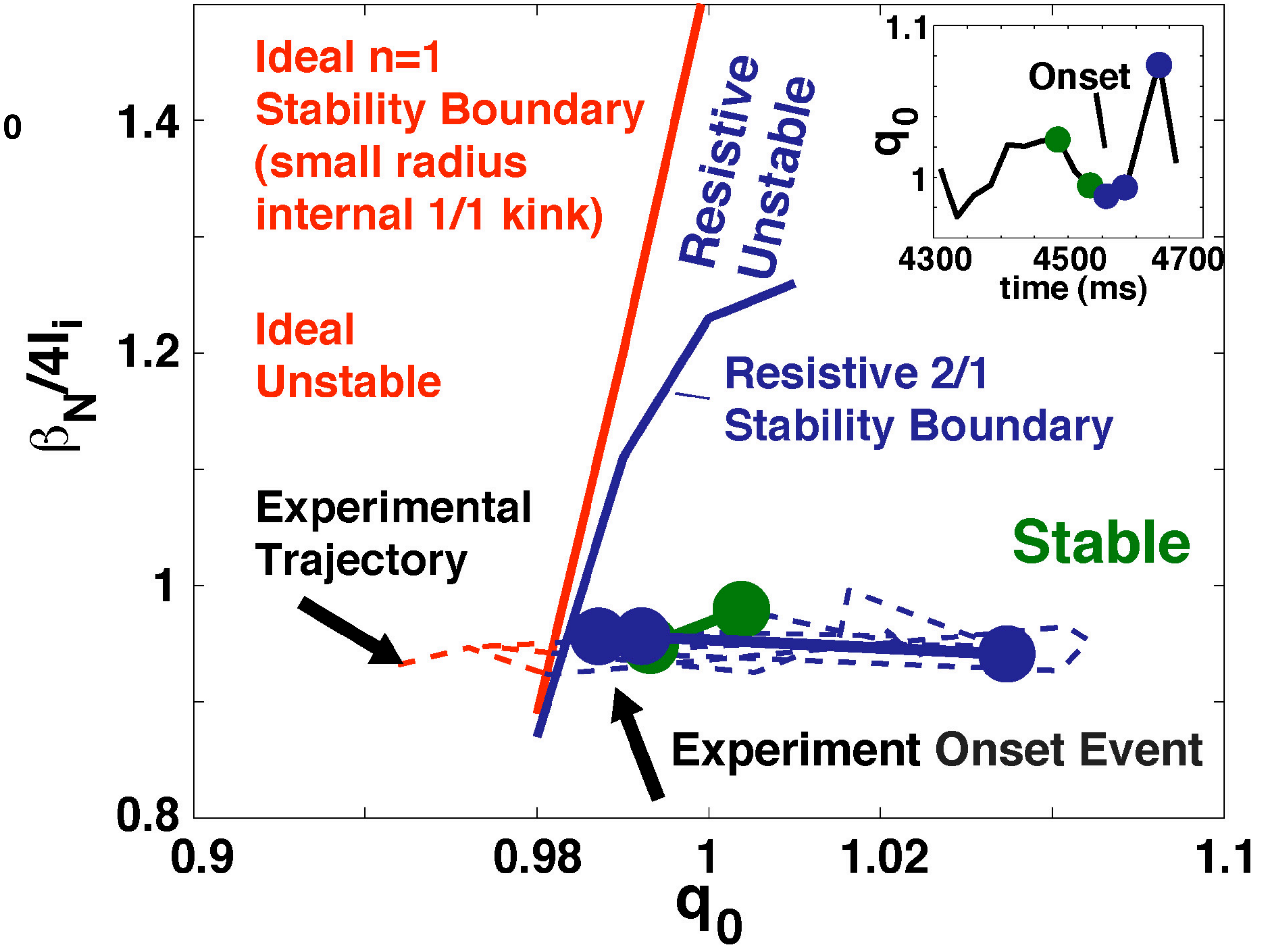


Further Evidence: Experimental Trajectory Crosses the Resistive Limit in q_0 Just Before Onset

Ideal n=1 β_N limit drops strongly as q_0 approaches 1.

2/1 resistive mode linearly unstable when trajectory crosses resistive limit.

Experiment in unstable region at onset.



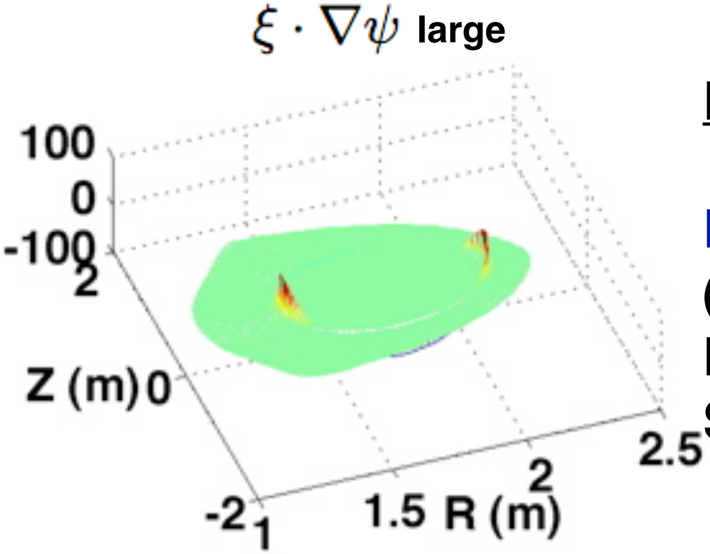
Stability map, generated in advance, could be used as real-time indicator of proximity to stability boundary.

Non-Resonant Small Solution Much Larger Outside Resonant Region Than Resonant Large Solution

Small solution (associated with Ideal instability) is very large on axis near q=1.

Instability, however, is reconnection at q=2.

$\xi \cdot \nabla\psi$ **large**



PEST-III

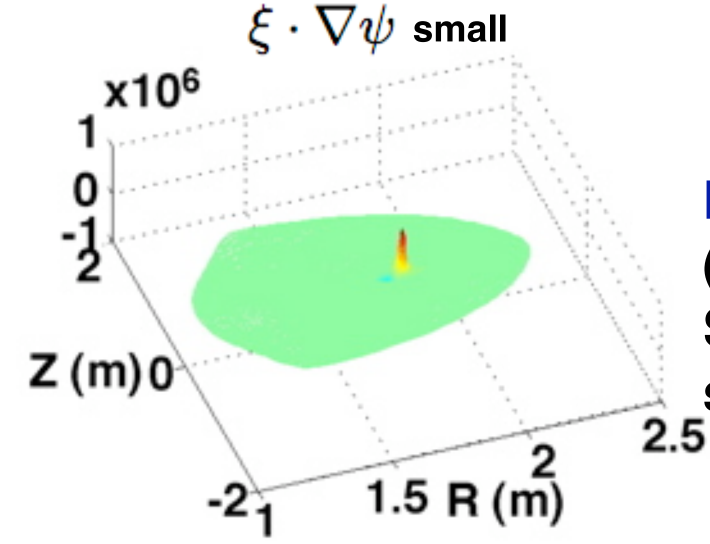
Resonant
(2/1)
Large
Solution

NIMROD calculates

$$\xi = \xi_{small} + \xi_{large}$$

dominated by small solution at axis.

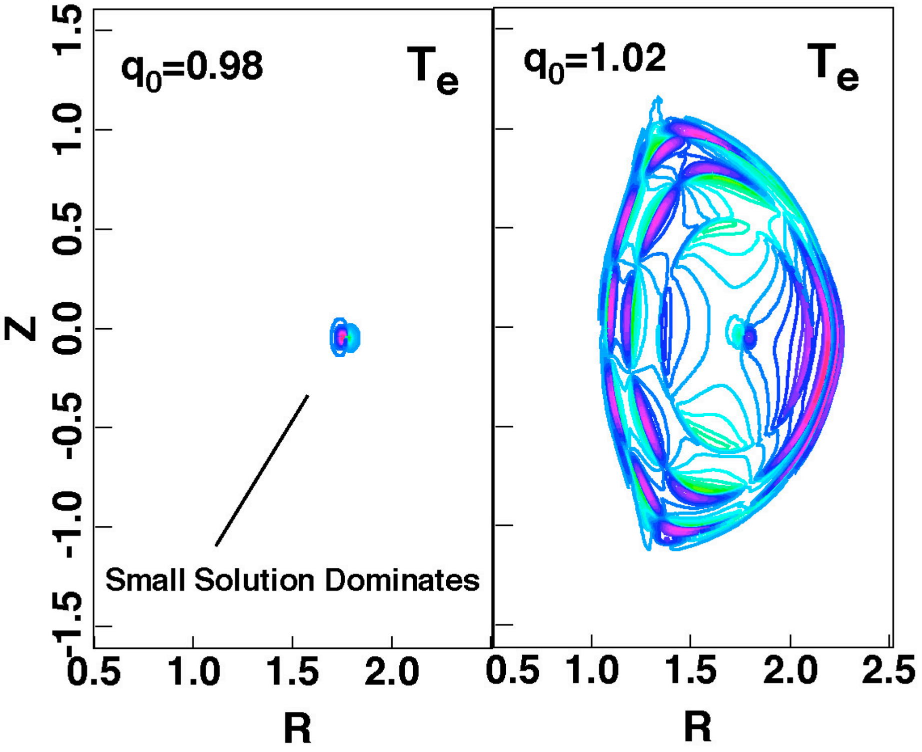
$\xi \cdot \nabla\psi$ **small**



Nonresonant
(1/1)
Small
solution

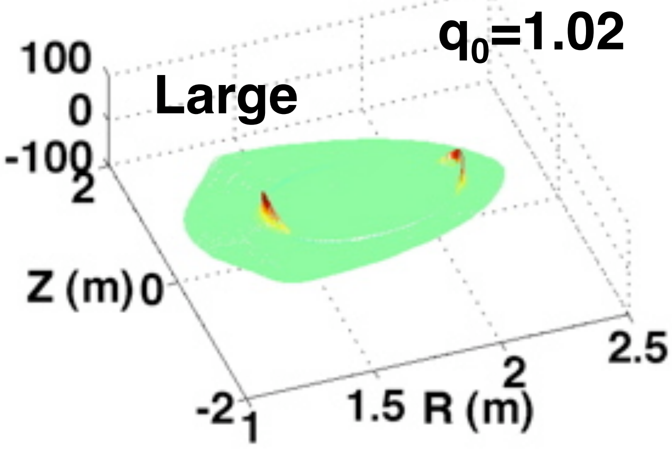
Approach to q_0=1 Causes Large Nonresonant Response on Axis.

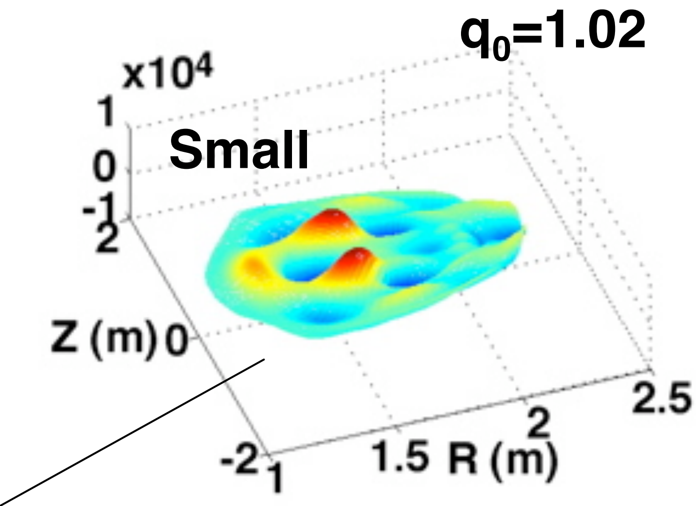
NIMROD



NIMROD and PEST-III agree that as q0 raises above 1 even slightly, small solution on axis diminishes.

PEST-III





At q0>1 small solution diminished on axis.

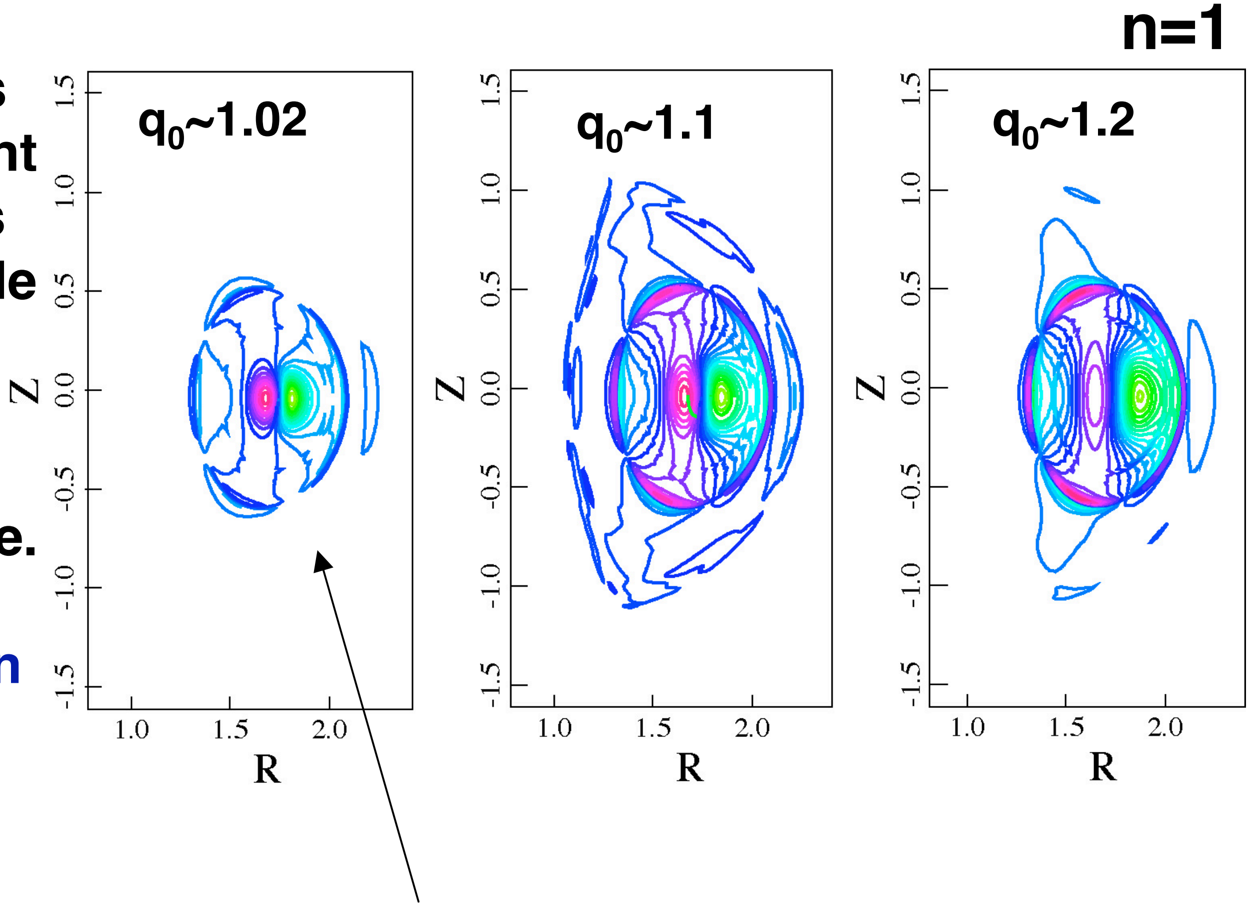
Large Nonresonant 1/1 Response on Axis Ubiquitous: Affects Linear Stability and Evolution.

As the q_0 approaches unity, the nonresonant n=1 response on axis increases in amplitude in all cases.

Hybrid discharges "hover" in this regime.

The question is: when will the 2/1 mode be affected by or set off by this resonance?

Similar hybrid case with lower q shear on axis.



Increased peak amplitudes will nonlinearly drive n=2 coupling.

Coupling to n=2 by Large n=1 Response Also Causes Counter Current Drive and Raises q_0.

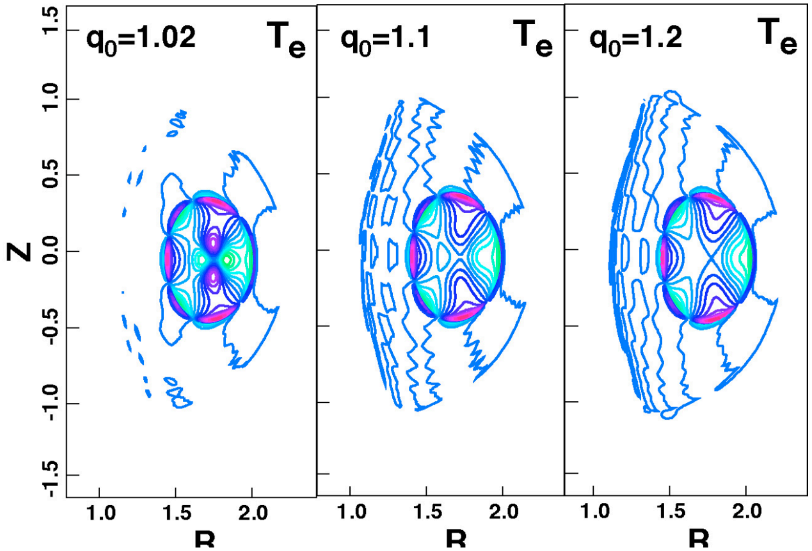
See: M.S.Chu IAEA 06, EX/1-5

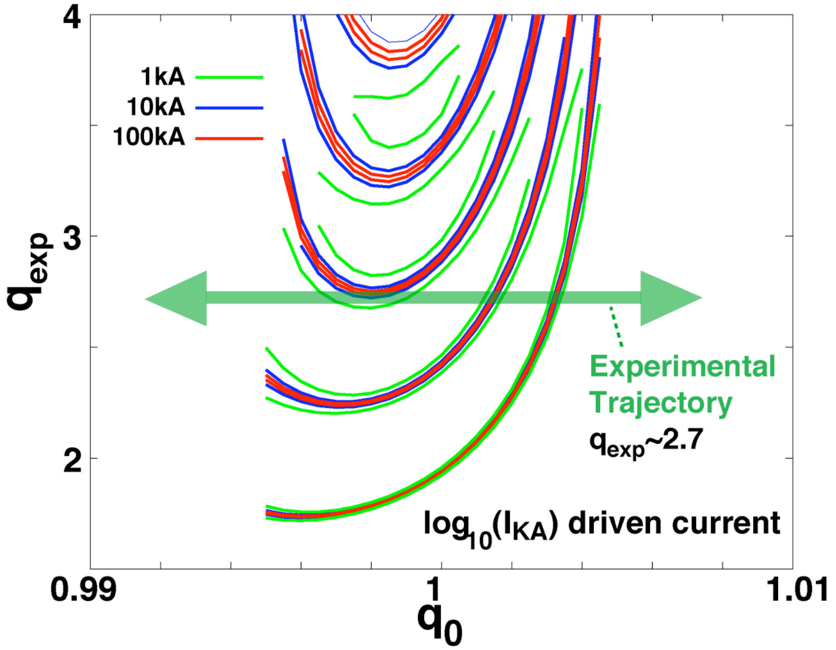
As the q_0 approaches 1, n=2 nonresonant response increases amplitude.

Nonresonant 2/2 may be responsible for current drive.

Kinetic Alfven resonance with 2/2 mode drives I_{KA} intermittently: accumulates.

This can prevent 2/1 onset by raising q_0!





Observations and Experimental Suggestions

By increasing q_0 even slightly >1, 2/1 can be avoided. Rapid increase in 2/1 β limit with q_0 can be tested experimentally.

Slowing the rate at which the q_0 approaches unity can allow current drive mechanism to prevent further q_0 reduction and resonance.

Stability map can be used for real time control of target discharges => Higher beta values accessible.

Summary

ALL matrix elements, coupling tearing and interchange, important for accurate analysis of mode onset at high β.

In DIII-D Hybrid discharges the resistive instability at q=2 is sensitive to q_0 approaching 1, as a result of the ideal β_N limit rapidly changing with $q_0 \sim 1$.

Experimental trajectory in stability map indicates this physics mechanism causes 2/1 onset => suggestions for avoidance.

Large nonresonant 2/2 component can drive current and raise q_0, playing important role in evolution to instability.