

# A Comprehensive Study of the Parametric Dependencies of Transport Using Gyrokinetic Simulations

Presented by

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# Over 320 Nonlinear Gyrokinetic Simulations Have Been Performed Using The GYRO Code

- A nonlinear simulation database has been created for benchmarking and transport model development

<http://fusion.gat.com/comp/parallel>

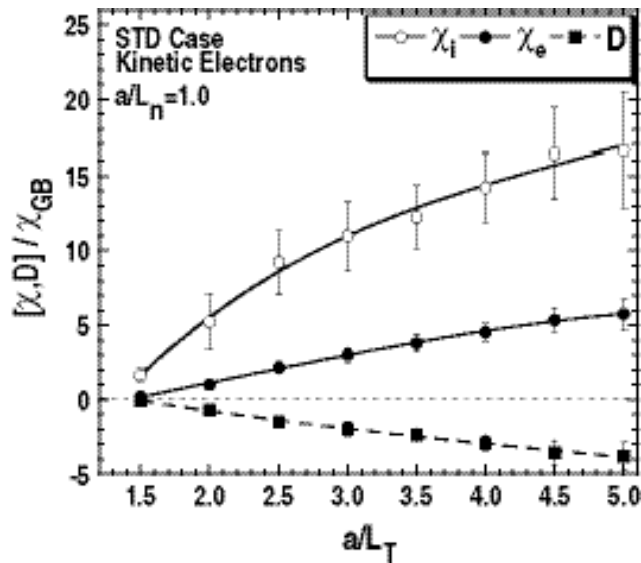
- Scans in  $R/a$ ,  $r/a$ ,  $q$ ,  $\hat{s}$ ,  $\alpha$ ,  $a/L_n$ ,  $a/L_T$ ,  $\nu$ ,  $\beta$ ,  $T_i/T_e$ ,  $\kappa$ ,  $\delta$ , dilution, and ExB shear with and without kinetic electrons (most runs w/ kinetic electrons)
- Simulations around several reference cases assuming  $\hat{s}$ - $\alpha$  geometry, electrostatic (except for  $\beta$  scan), and flat profiles across annulus, zero boundary conditions
  - GA Standard Case (STD):  $R/a=3$ ,  $r/a=0.5$ ,  $q=2$ ,  $\hat{s}=1$ ,  $\alpha=0$ ,  $a/L_T=3$ ,  $a/L_n=1$ ,  $T_i/T_e=1$ ,  $\nu=0$ ,  $\beta=0$
  - TEM1 Case: STD w/  $a/L_n=2$ ,  $a/L_T=2$
  - TEM2 Case: STD w/  $a/L_n=3$ ,  $a/L_T=1$
- Miller geometry used for  $\kappa$  and  $\delta$  scans
- Diffusivities shown are time-averaged values and are normalized to the gyro-Bohm diffusivity,  $\chi_{GB} = c_s \rho_s^2 / a$

# Nonlinear Database Comprised of Parameter Scans

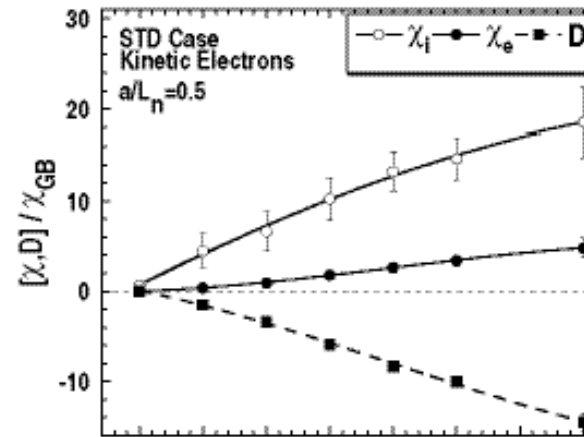
- **Temperature and density gradient scans : 25 kinetic electron simulations (plus 6 kinetic electron Cyclone simulations, J. Candy)**
- **Safety factor and magnetic shear scans :123 kinetic electron simulations** *J.E. Kinsey, R.E. Waltz, J. Candy, submitted to Phys. Plasmas*
- **Aspect ratio and minor radius scans : 12 kinetic electron simulations**
- **Collisionality and Ti/Te scans :10 kinetic electron simulations**
- **Elongation and triangularity scans : 5 adia. electron + 34 kinetic electron simulations**
- **ExB velocity shear scans: 84 adia. electron and kinetic electron simulations**  
*J.E. Kinsey, R.E. Waltz, J. Candy, Phys. Plasmas 12, 022305 (2005).*
- **Beta scans (J. Candy) : 14 kinetic electron EM simulations**  
*J. Candy, Phys. Plasmas 12, 072307 (2005).*
- **He dilution scans (C. Estrada) : 19 kinetic electron simulations**  
*C. Estrada-Mila, J. Candy, R.E. Waltz, Phys. Plasmas 12, 022305 (2005).*

# Temperature Gradient Scan for STD Case Including Kinetic Electrons

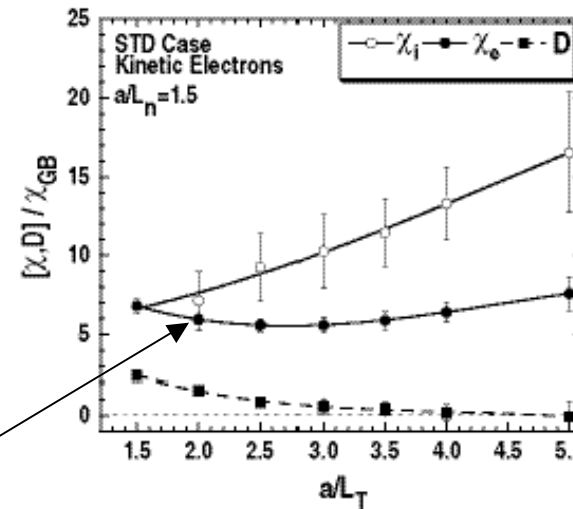
- STD case scans in  $a/L_T$  w/  $a/L_n=0.5, 1.0, \text{ and } 1.5$
- Transport not as stiff at low gradients for peaked density ( $a/L_n=1.5$ )



STD Case



Flat Density



Peaked Density

TEM

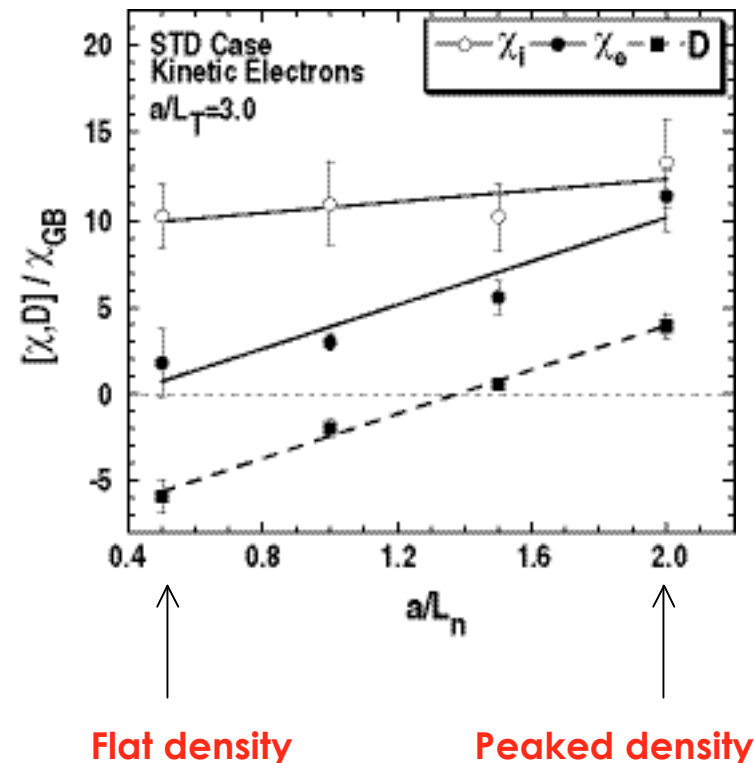
\* Note: error bars on plots are a measure of intermittency and NOT uncertainty

$$\chi_{GB} = c_s \rho_s^2 / a$$

# Density Gradient Scan for STD Case

- **STD case  $a/L_n$  scan with  $a/L_T=1.0$** 
  - Ion and electron density gradient varied together
- **Particle diffusivity changes from negative (inward) to positive (outward) as density gradient is increased**
  - $D=D_e=D_i$  due to ambipolarity
- **Electron energy and particle transport show stronger dependence on  $a/L_n$  than ion energy transport**

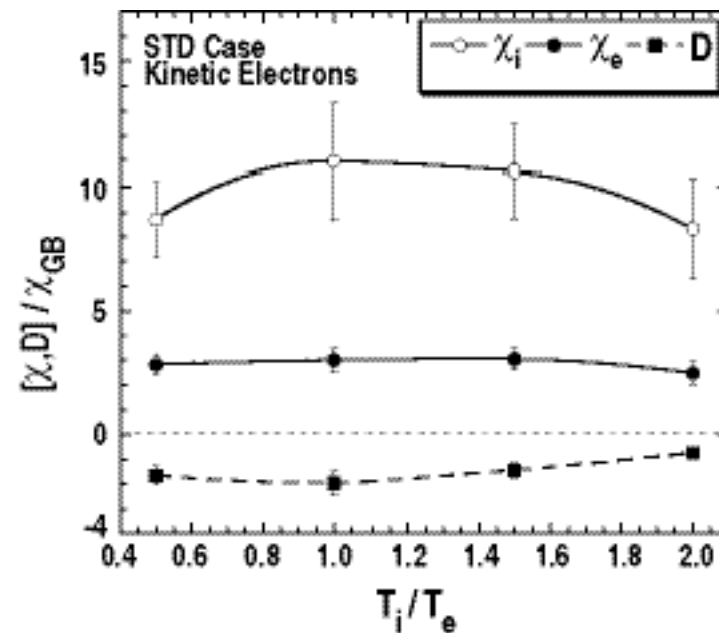
## STD Case



# Temperature Ratio Scan for STD Case

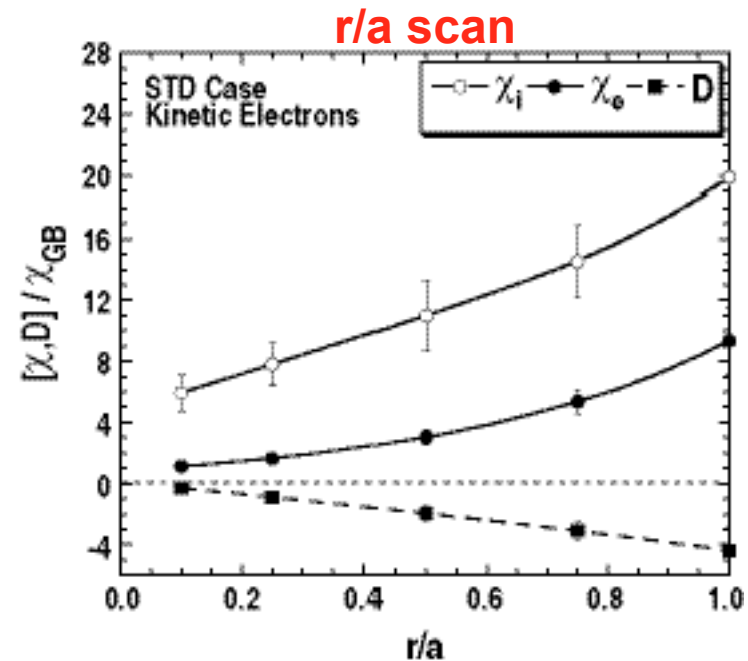
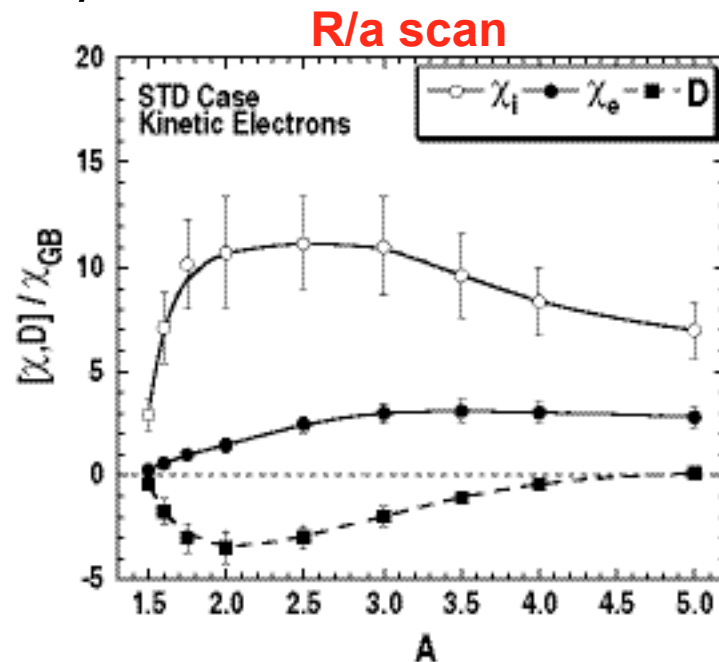
- STD case  $T_i/T_e$  scan at fixed  $a/L_T$
- Ion energy diffusivity modestly reduced going from  $T_i/T_e = 1$  to  $T_i/T_e = 2$
- Electron energy transport fairly insensitive to  $T_i/T_e$
- Particle pinch somewhat reduced as  $T_i/T_e$  is increased
- Linear stability of spectrum for  $T_i/T_e = 2$  shows ITG modes below  $k_\theta \rho_s = 0.3$  and TEM modes above  $k_\theta \rho_s = 0.3$

## STD Case



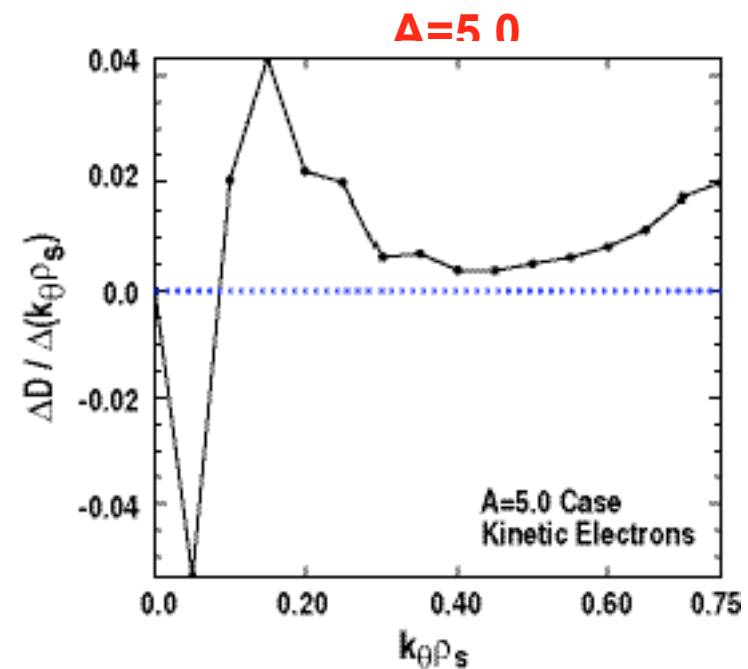
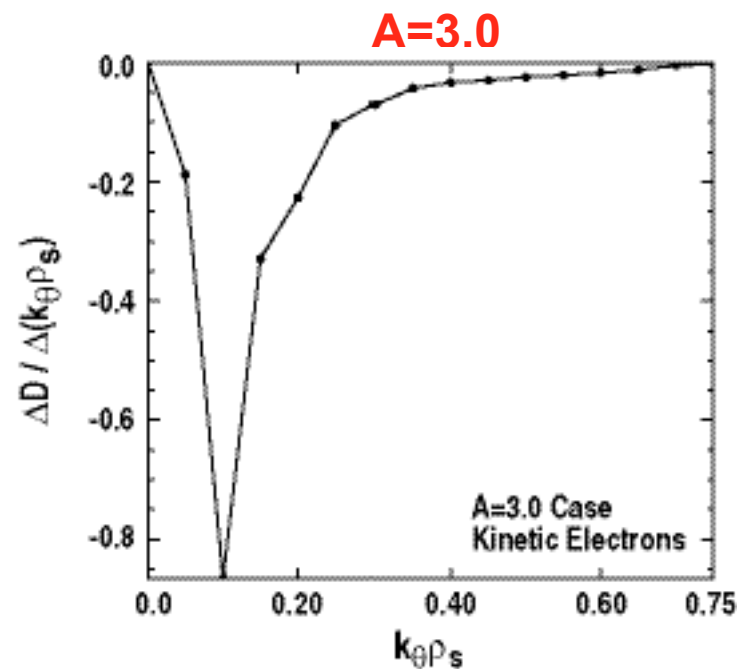
# Aspect Ratio and Minor Radius Scans for STD Case

- Scans in  $R/a$  and  $r/a$  for STD parameters with kinetic electrons
- Strong reduction in transport below  $A=2$  which is consistent with previous linear gyrokinetic studies (e.g. Rewoldt)
- Nearly linear increase in transport with increasing  $r/a$  at fixed  $R/a$



# Shorter Wavelength Modes Drive Outward Flow at High Aspect Ratio

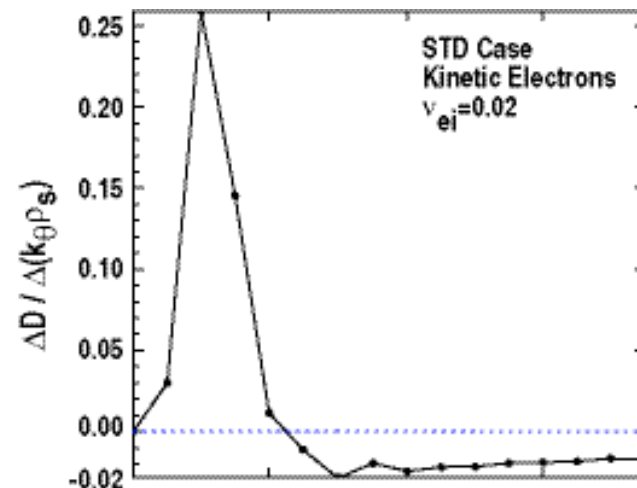
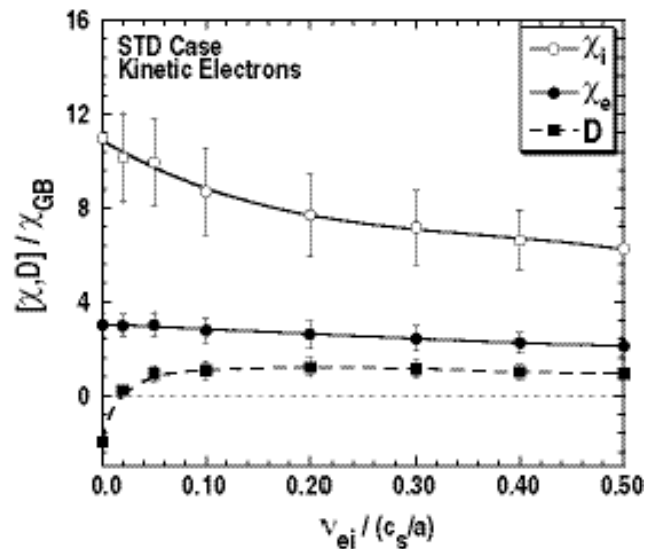
- Particle pinch predicted at low-medium aspect ratio
  - At  $A=R/a=3.0$ , all modes in spectrum drive a pinch
- Null flow point near  $A=5.0$ 
  - Modes above  $k_{\theta}\rho_s=0.15$  drive an outward flow



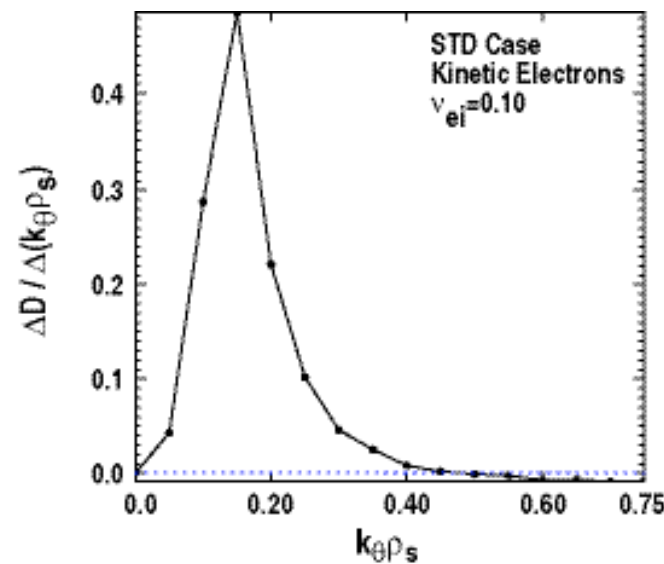


# Particle Pinch Strongly Impacted By Collisions

- Particle pinch predicted for STD case w/o collisions
- Addition of collisions increasingly eliminates particle pinch driven by low  $k_y$  modes
- Electron energy transport insensitive to collisions



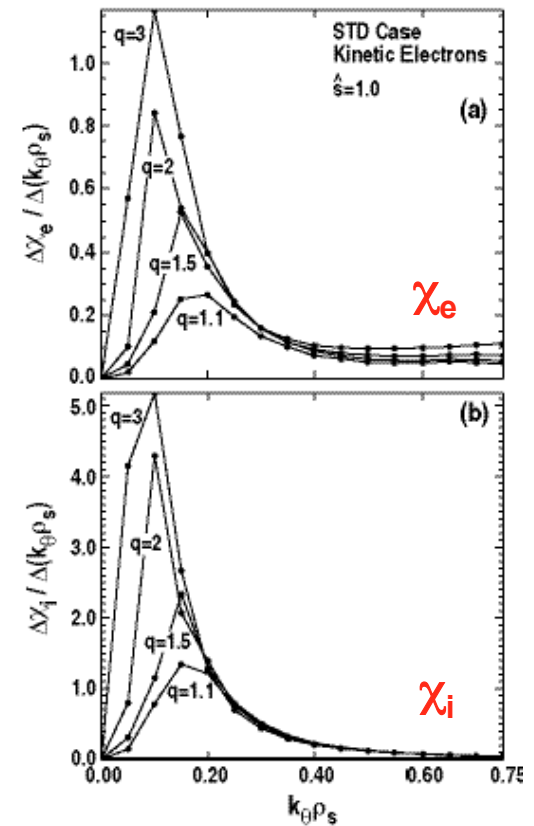
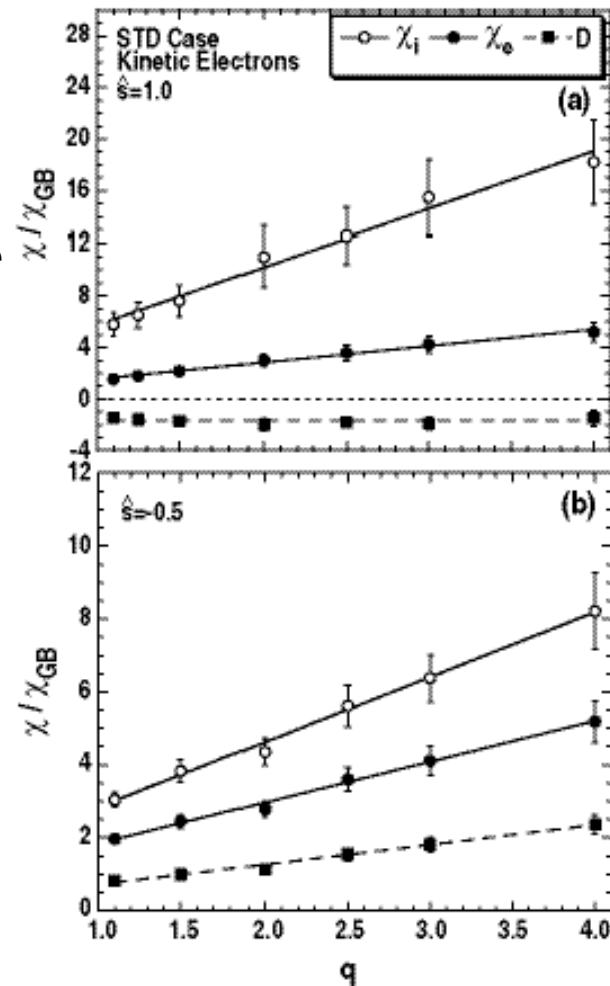
$\nu=0.02$



$\nu=0.10$

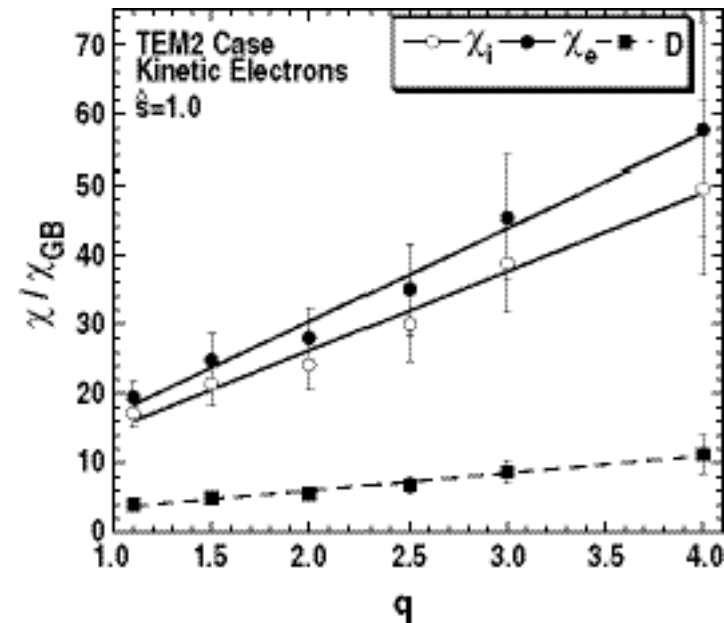
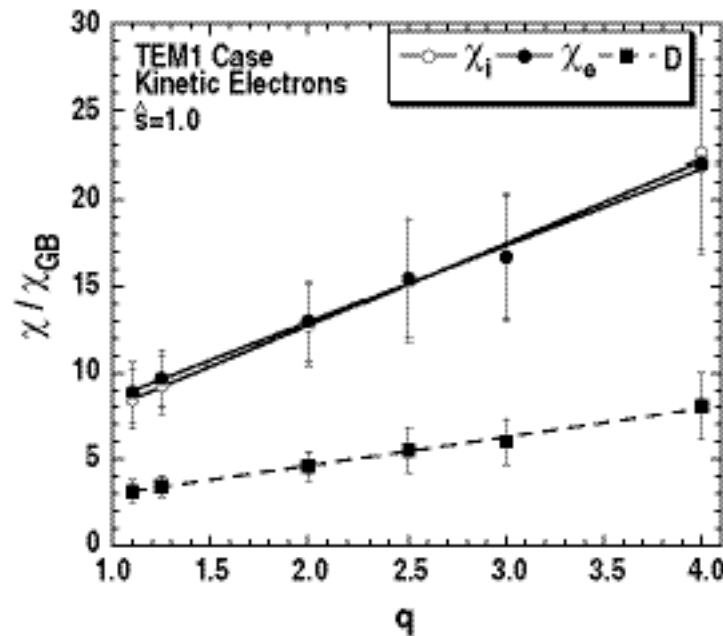
# Ion and Electron Transport Exhibits Linear $q$ Dependence in STD Case Kinetic Electron Simulations

- Energy transport exhibits linear  $q$  dependence for both positive and negative shear
  - $D$  also follows linear  $q$  dependence if  $D$  is positive
  - Spectral downshift w/ decreasing  $q$  is evident
  - Small contribution to  $\chi_e$  from higher  $k_{\theta}\rho_s$  modes
- Particle transport insensitive to  $q$  for  $s=1.0$  case where  $D$  is negative
  - Passing electron contribution to  $D$  changes sign near  $q=2$
  - $D_{\text{pass}}/D_{\text{total}}$  small (0.1-0.2) but large enough to impact  $q$  scaling



# TEM Cases Also Exhibit Linear $q$ Dependence

- TEM1: STD parameters w/  $a/L_T=2$ ,  $a/L_n=2$   
(half of modes in ion direction, others in electron direction)
- TEM2: STD parameters w/  $a/L_T=3$ ,  $a/L_n=1$   
(All modes in electron direction)



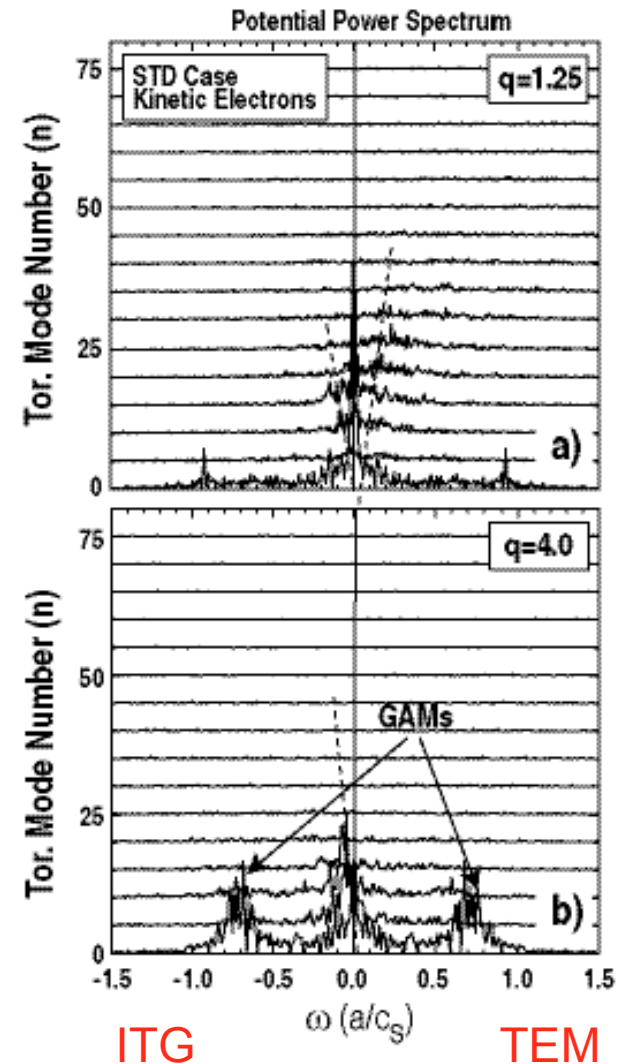
# GAM Amplitude Increases With Increasing q

- Geodesic Acoustic Modes evident at  $n=0$  at  $\omega(a/c_s) = \pm 0.75$
- Amplitude in simulations is consistent with  $q$  scaling of GAM damping rate:

$$\gamma_G = -\omega_G \exp(-q^2)$$

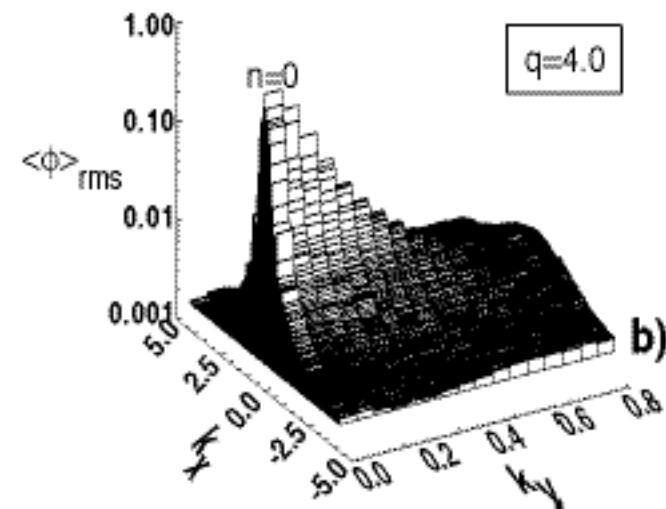
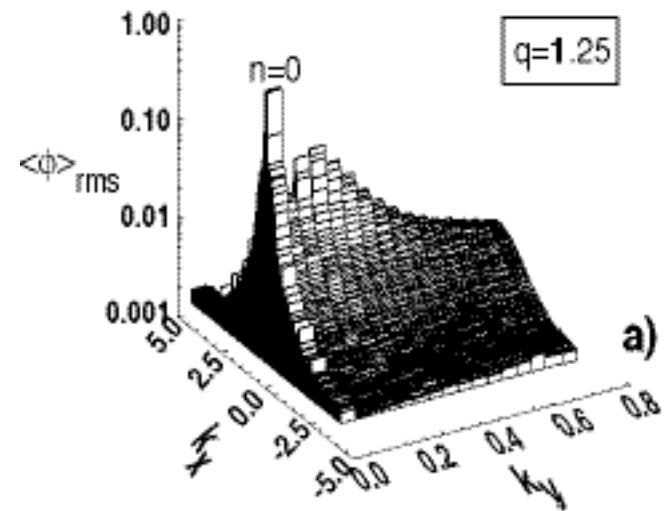
where  $\omega_G$  is the GAM frequency (Hinton, 1999)

- Fluctuations ITG dominated in  $q=4$  case with 1 peak at negative  $\omega(a/c_s)$
- Spectrum shows both ITG and TEM peaks for  $q=1.25$  case



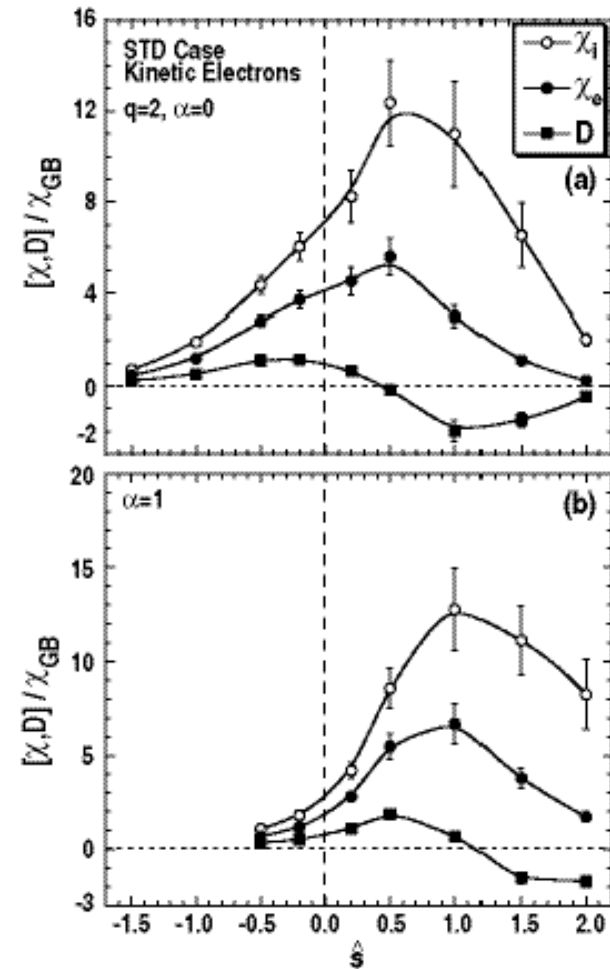
# At Low $q$ , $n=0$ Amplitude Larger Than Neighboring $n > 0$ Mode Amplitudes

- Time-averaged RMS spectral intensity of the potential fluctuations compared for STD case w/  $q=1.25$  and  $q=4.0$
- Spectral intensity shown in  $(k_x, k_y)$  plane where  $k_x = k_r \rho_s$  and  $k_y = k_\theta \rho_s$
- Also evident that higher  $k_y$  modes contribute more to transport at low  $q$  compared to high  $q$



# Particle Diffusivity Can Change Sign As Magnetic Shear Is Varied

- Magnetic shear  $\hat{s}$  varied at fixed  $q$  for  $\alpha=0$  and  $\alpha=1$
- For STD case, null flow point found at  $\hat{s}=0.5$  for  $\alpha=0$ 
  - $D$  negative for  $\hat{s} > 0.5$
  - $D$  positive for  $\hat{s} < 0.5$
- Alignment of null point with maximum in  $\chi$  particular to STD parameters
- Alpha stabilizing for negative shear (e.g.  $\hat{s}=-0.5$ ), and destabilizing for positive shear (e.g.  $\hat{s}=2$ )

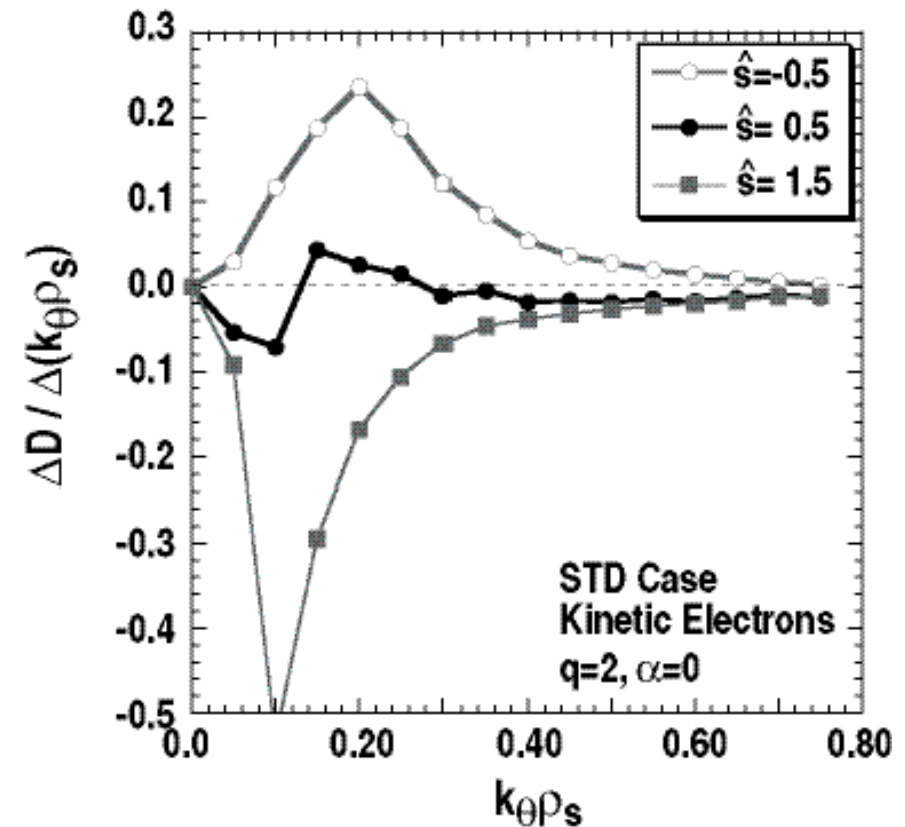


$\alpha=0$

$\alpha=1$

# Spectral Analysis at Null Point Shows Some Modes Drive A Negative Flow With Other Modes Drive A Positive Flow

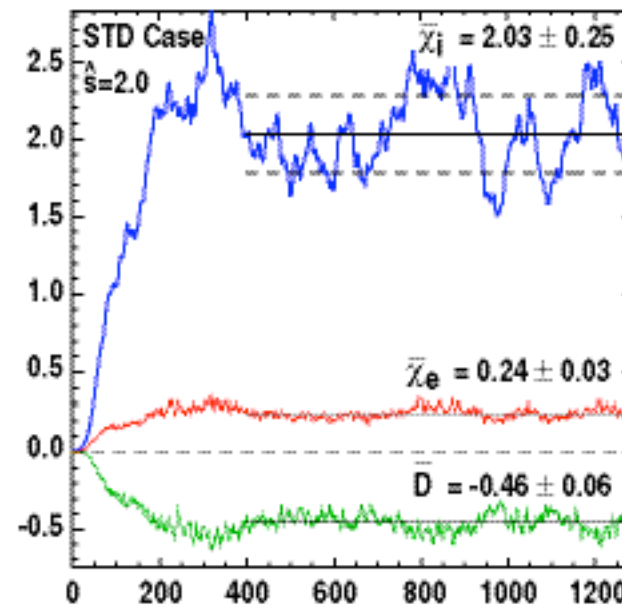
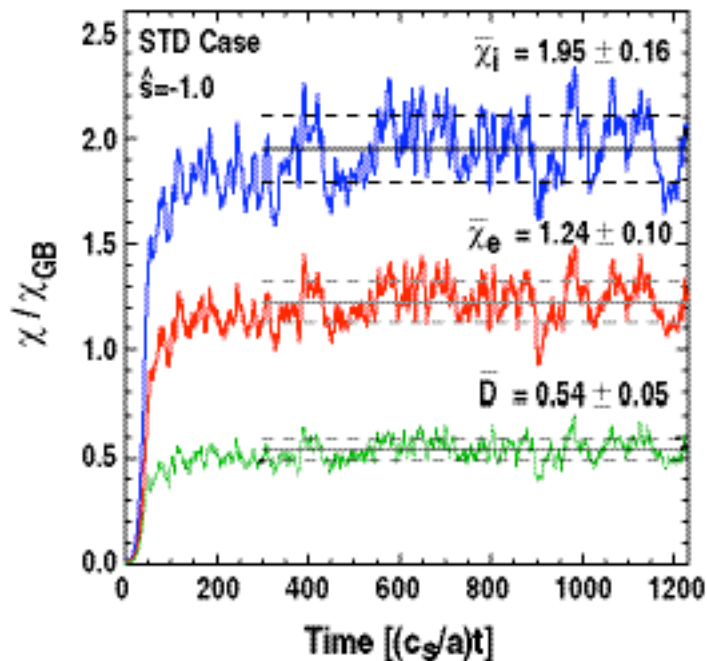
- Spectrum includes 16 toroidal mode numbers with a maximum  $k_{\theta}\rho_s=0.75$
- Null flow point at  $\hat{s}=0.5$ 
  - Linear stability shows transition from ITG to TEM above  $k_{\theta}\rho_s=0.65$
- Time-average particle diffusivity is negative for  $\hat{s}=1.5$  and positive for  $\hat{s}=-0.5$ 
  - Linear stability shows TEM above  $k_{\theta}\rho_s=0.50$  for  $\hat{s}=1.5$
  - Linear stability shows TEM above  $k_{\theta}\rho_s=0.65$  for  $\hat{s}=-0.5$





# Ratio Of Ion to Electron Energy Transport Changes Dramatically With Magnetic Shear

- For STD case, ratio of  $\chi_i/\chi_e$  changes significantly with shear
  - At  $\hat{s}=2$ ,  $\chi_i/\chi_e=9.7$
  - At  $\hat{s}=1$ ,  $\chi_i/\chi_e=3.4$
  - At  $\hat{s}=-1$ ,  $\chi_i/\chi_e=1.6$





# The Effects Of Elongation and Triangularity On Turbulent Transport Have Been Investigated Using The Miller Equilibrium Model

- Nine parameters are required to describe the local equilibrium using Miller geometry<sup>1</sup>:  $\kappa$  (elongation),  $\delta$  (triangularity),  $q$ ,  $\hat{s}$  (magnetic shear),  $\alpha$  (normalized pressure gradient),  $A=R_0/r$ ,  $\partial_r R_0$ , along with gradient factors of  $\kappa$  and  $\delta$  ( $s_\kappa$  and  $s_\delta$ )
- For D-shaped plasmas, the shape of a flux surface is specified in terms of the major radius  $R$  and height  $Z$  as a function of the poloidal angle  $\theta$ :

$$R = R_0 + r \cos[\theta + (\sin^{-1}\delta)\sin\theta]$$

$$Z = \kappa r \sin\theta$$

- Systematic nonlinear scans in  $\kappa$  and  $\delta$  were performed for the STD case with  $\partial_r R_0=0$ ,  $\alpha=0$ ,  $\beta=0$ 
  - For  $\kappa$  scans, we also varied  $s_\kappa = (r/\kappa)\partial_r \kappa \approx (\kappa-1)/\kappa$
  - For  $\delta$  scans, we also varied  $s_\delta = (r/(1-\delta^2)^{0.5})\partial_r \delta \approx \delta/(1-\delta^2)^{0.5}$

<sup>1</sup> R. L. Miller, et al, *Phys. Plasmas* 5, 973 (1998).

# Adiabatic Electron Simulations Show The Ion Energy Transport Decreasing Linearly With Increasing Elongation

- $\kappa$  varied from 1 to 2 for STD case using Miller geometry

- kappa gradient factor  $s_\kappa$  varied along with  $\kappa$
- GYRO results for  $\chi_{\text{natural}}$  where

$$\chi_{\text{natural}} = \langle |\nabla r|^2 \rangle \chi_{\text{ITER}}$$

For concentric ellipses where “r” is the midplane minor radius:

$$\langle |\nabla r|^2 \rangle = (1 + \kappa^2) / (2\kappa^2)$$

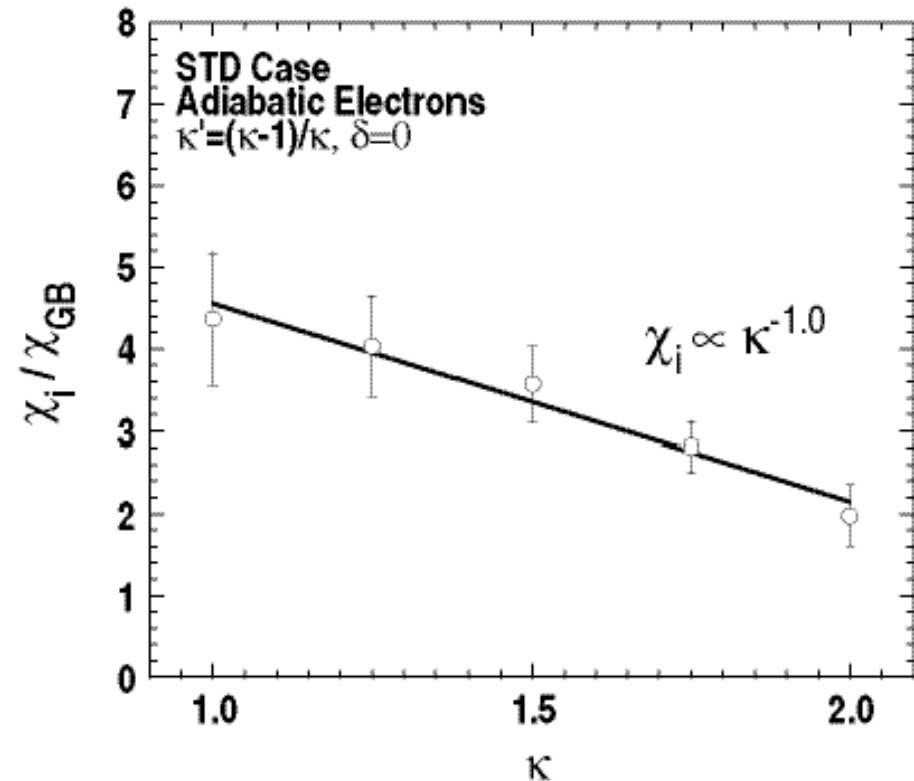
$$B_{\text{unit}} = (\rho/r) (d\rho/dr) B_0 \approx \kappa^2 B_0, \text{ so}$$

$$\chi_{\text{ITER}} \approx 2 / (1 + \kappa^2) \chi_{\text{natural}}$$

- Previous NL gyrofluid simulations by Waltz<sup>1</sup> showed no  $\kappa$  dependence for  $\chi_{\text{natural}}$  at  $q=2$  (and  $q=3$ )

<sup>1</sup> R. E. Waltz, and R.L. Miller, *Phys. Plasmas* 6, 4265 (1999).

## Adiabatic Electrons STD case w/ $q=2$ , $\delta=0$



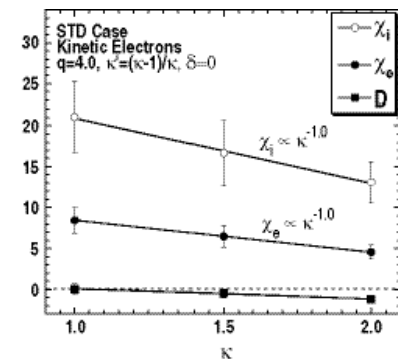
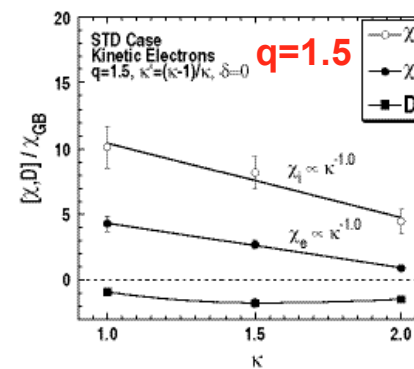
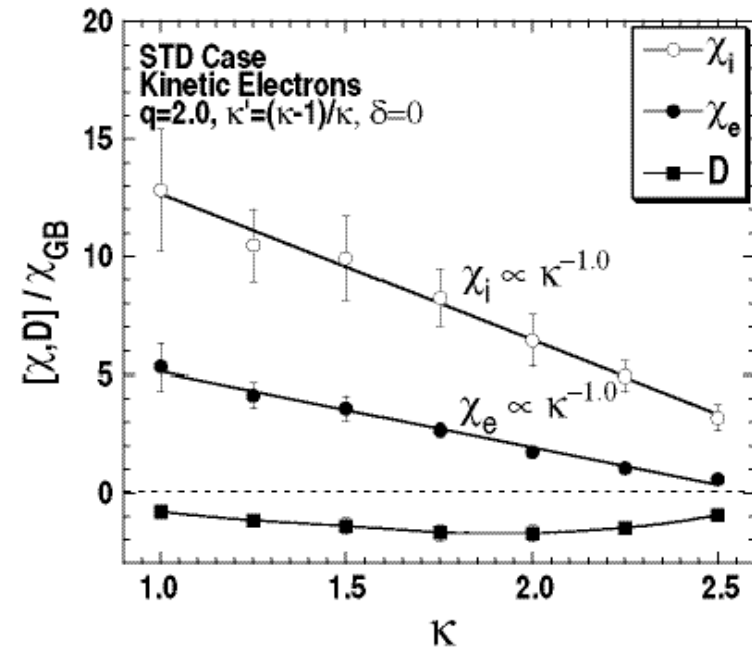
$$\chi_{\text{natural}} = \chi / \chi_{GB} \text{ where } \chi_{GB} = c_s \rho_s^2 / a$$

# Kinetic Electron Simulations Also Shows Energy Transport Decreases Linearly With Increasing Elongation

- **Linear decrease in both  $\chi_e$  and  $\chi_i$  valid for  $\kappa$  scans at different safety factors**

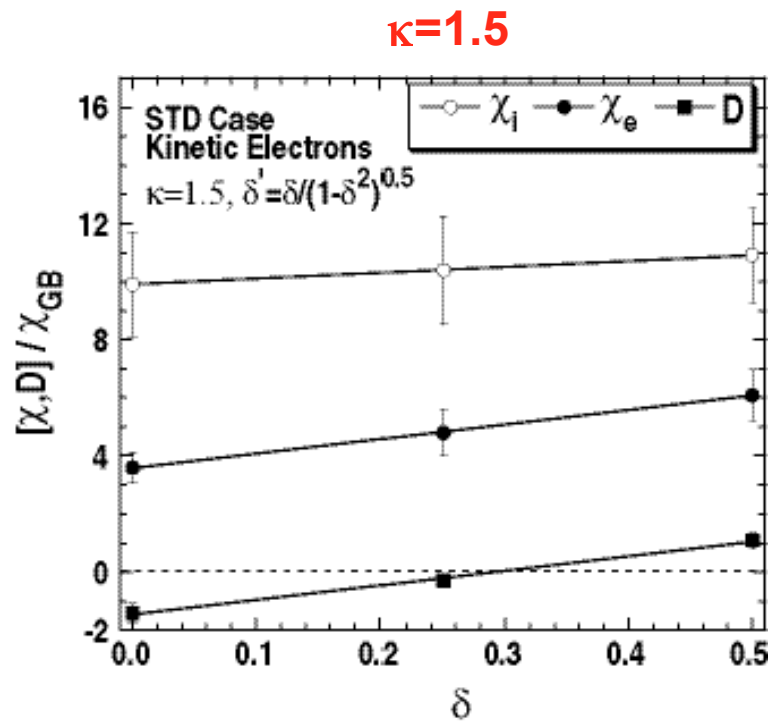
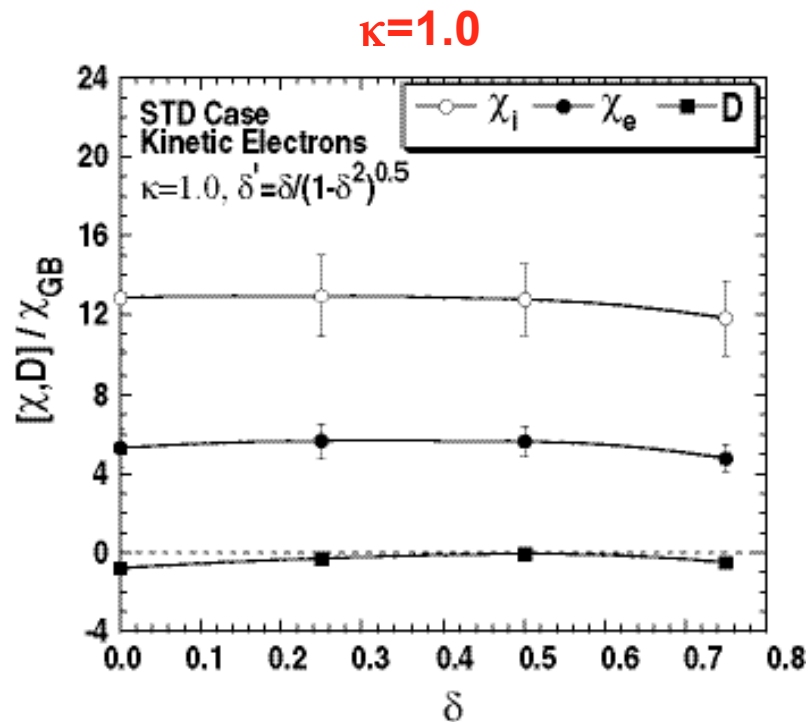
- $\kappa$  varied from 1.0 to 2.0 for different  $q$  values and for different shear values using STD parameters with  $\hat{s}=1.0$
- Miller geometry with  $\delta=0$ ; kappa gradient factor  $s_\kappa=(\kappa-1)/\kappa$  varied along with  $\kappa$
- Offset linear gives best fit to NL results. Growth rate at  $k_\theta \rho_s=0.3$  shows same dependence
- Particle transport shows little or no  $\kappa$  dependence

Caveat:  $D$  is small and negative  
 $\rightarrow$  follows  $\kappa^{-1}$  dependence if  $D$  is positive (see shear scan)



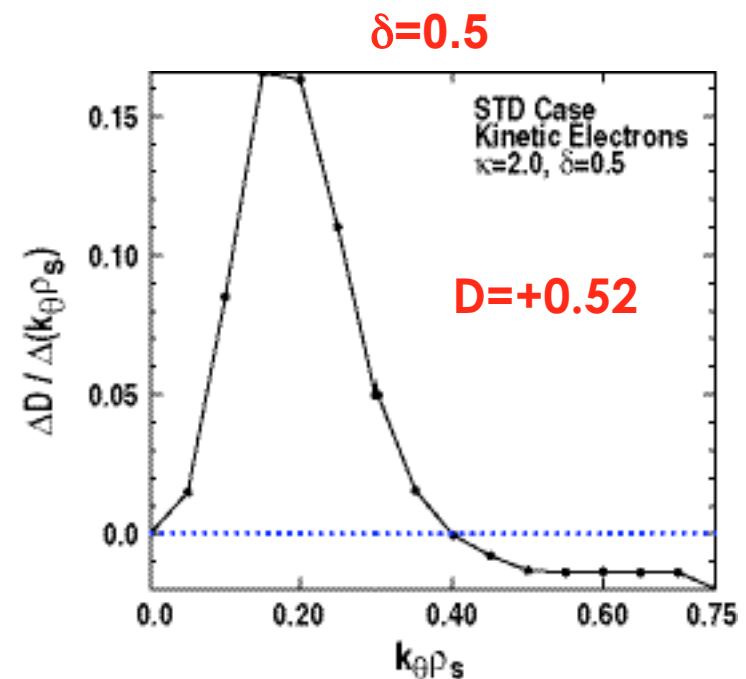
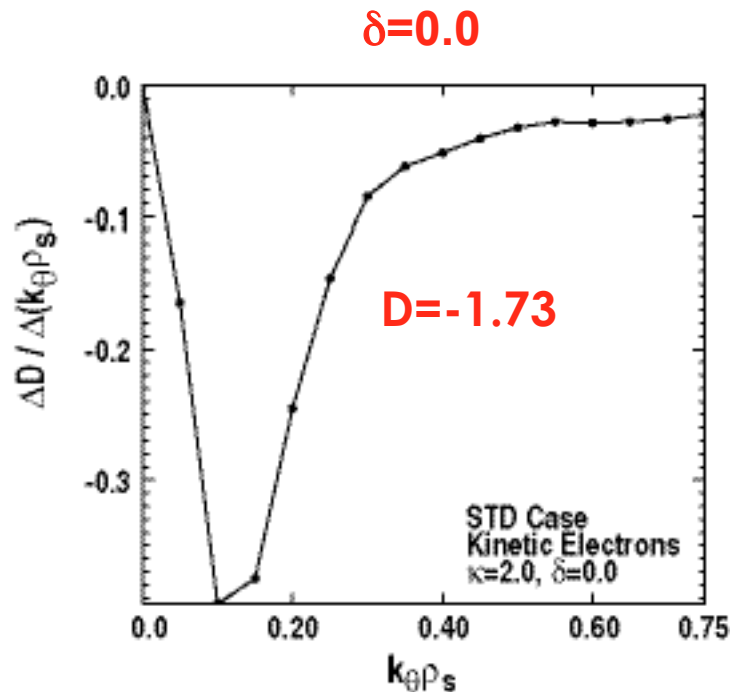
# Dependence of Transport on Triangularity Weak For Circular Plasmas, Stronger For Elongated Plasmas

- $\delta$  varied for  $\kappa=1.0, 1.5,$  and  $2.0$  using STD parameters
  - Miller geometry, delta gradient factor  $s_\delta$  varied along with  $\delta$
- Stronger  $\delta$  dependence for  $\kappa=1.5, 2.0$  cases compared to  $\kappa=1.0$  case
  - $\delta$  destabilizing, especially for  $\chi_e$  and  $D$ , going from  $\delta=0.0$  to  $\delta=0.5$



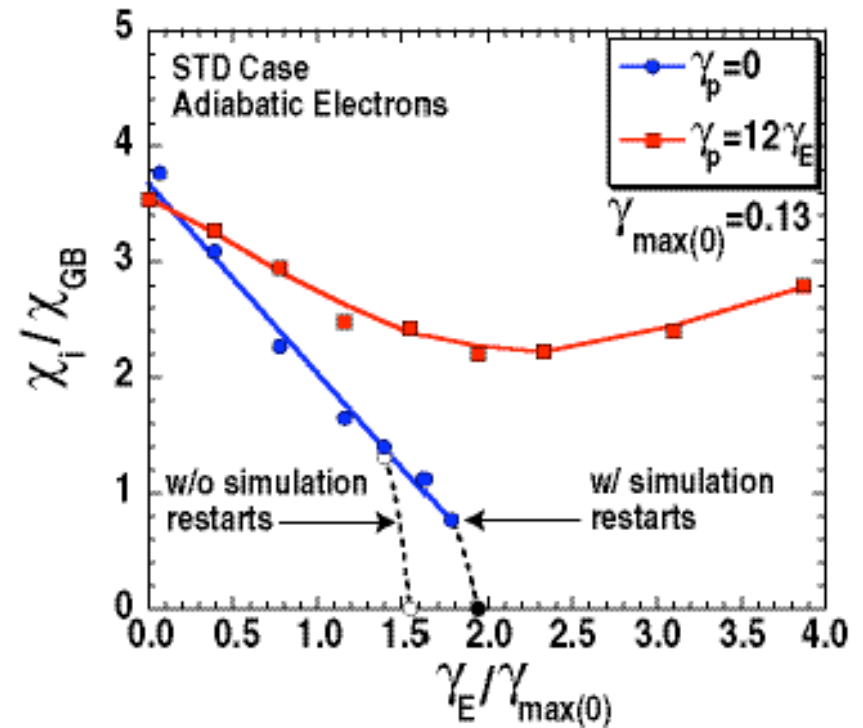
# Triangularity Strongly Impacts Particle Transport Spectrum For Elongated Plasmas

- $\delta$  varied from 0.0 to 0.5 for STD case w/  $\kappa=2.0$
- Particle transport changes from  $D/D_{GB}=-1.73$  to  $D/D_{GB}=+0.51$ 
  - Transport from low k modes changes sign
  - Less of an effect at  $\kappa=1.0$  ( $D/D_{GB}=-0.8 \rightarrow D/D_{GB}=-0.1$  when  $\delta=0.0 \rightarrow 0.5$ )



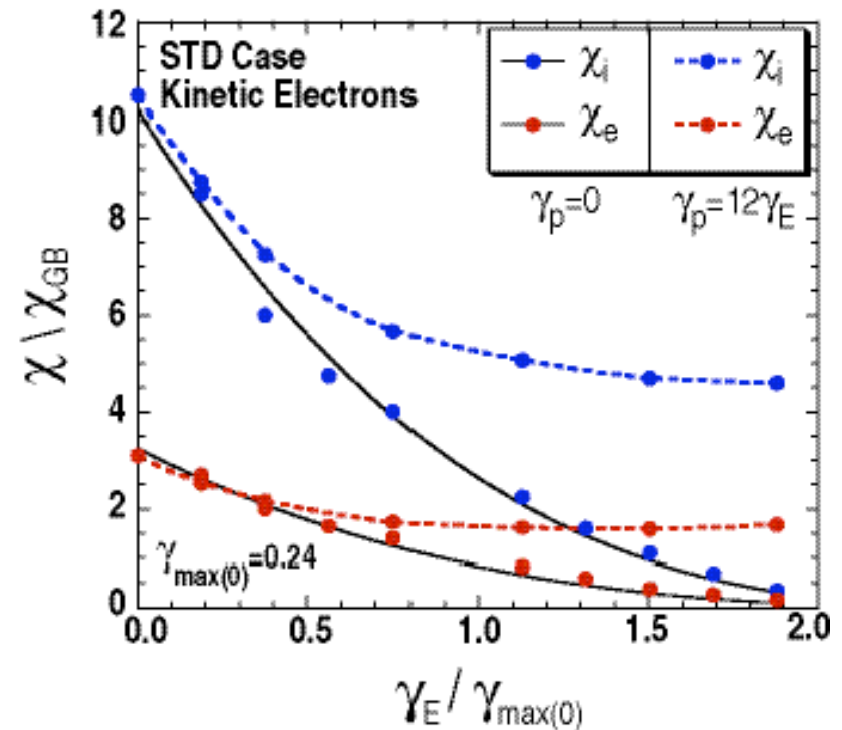
# Nonlinear Gyrokinetic Simulations With Adiabatic Electrons Show Higher ExB Shear Quench Point Compared to Gyrofluid Simulations

- ExB shear quench rule originally developed from gyrofluid ITG simulations which showed a quench point at  $\gamma_E = \gamma_{\max}$
- Using GYRO with no parallel velocity shear ( $\gamma_p = 0$ ), two quench points can be found depending on how the ExB shear is applied in the simulations
  - ITG transport quenches at  $\gamma_E = 1.6\gamma_{\max}$  when  $\gamma_E$  applied at onset of simulation
  - Quench point higher near  $\gamma_E = 2\gamma_{\max}$  when  $\gamma_E$  applied in a restart from simulation without  $\gamma_E$  included
- Purely toroidal rotation case with  $\gamma_p = (Rq/r)\gamma_E = 12\gamma_E$  shows that transport not quenched by any level of  $\gamma_E$



# ExB Shear Quench Rule Remains Valid With Addition Of Kinetic Electrons In Nonlinear Gyrokinetic Simulations

- **ExB shear quench point near  $\gamma_E = 2\gamma_{\max}$  for ions and electrons**
  - Same quench point found for the adiabatic electron case
  - Also valid for TEM cases (e.g. STD case w/  $a/L_n=3$ ,  $a/L_T=1$ ) and for negative shear (e.g. STD case w/  $s=-0.5$ )
- **Gradual reduction to zero transport near quench point**
  - Sharp drop in  $\chi$  near quench point seen in adiabatic runs
  - NOT seen with kinetic electrons



- **Transport not quenched when parallel velocity shear included (assuming purely toroidal rotation with  $\gamma_p = (Rq/r) \gamma_E = 12\gamma_E$ )**

**16 modes**  
 $k_\theta \rho_s \leq 0.75$   
 $\gamma_{\max 0} = 0.27$

# Summary

- **A large database of over 320 nonlinear GYRO gyrokinetic simulations has been created comprising of various parameter scans around several reference cases**
  - Scans in  $R/a$ ,  $r/a$ ,  $q$ ,  $s$ ,  $\alpha$ ,  $a/L_n$ ,  $a/L_T$ ,  $\nu$ ,  $\beta$ ,  $T_i/T_e$ ,  $\kappa$ ,  $\delta$ , dilution, and ExB shear with and without kinetic electrons (most runs w/ kinetic electrons)
- **Aspect ratio scan shows strong reduction in transport below  $A=2$ , weak reduction going from  $A=3$  to  $A=5$** 
  - Strong particle pinch near  $A=2$
  - At high  $A$ , low- $k$  modes drive a pinch while higher- $k$  modes drive an outward flow
- **Safety factor scans show that the ion and energy transport exhibits linear  $q$  dependence for both ITG and TEM dominated cases**
  - Spectral downshift with increasing  $q$  which contributes to most of  $q^{-1}$  dependence of transport
  - $D$  also exhibits linear  $q$  dependence if  $D$  is positive. If  $D$  is negative (pinch), then overall  $q$  dependence weak due to change in sign of passing electron contribution as  $q$  is varied
  - GAM amplitude increases with increasing  $q$ ; amplitude and frequency consistent with analytic calculations by Hinton, et al.
  - $n=0$  amplitude greater than neighboring  $n>0$  amplitudes at low  $q$ , comparable amplitudes at higher  $q$  values



## Summary (cont.)

- **Magnetic shear scans show that particle diffusivity can change sign as shear is varied**
  - At null flow point, some modes drive a pinch while other modes drive an outward flow
  - Ratio of ion to electron transport changes dramatically with shear ( $\chi_i/\chi_e$  largest for large positive shear, smallest for negative shear)
- **Adiabatic and kinetic electron simulations robustly show that  $\chi_{\text{natural}}$  exhibits an inverse linear dependence on elongation**
  - Unlike previous gyrofluid ITG simulations by Waltz, we find  $\kappa^{-1}$  regardless of  $q$  and  $s$  values
  - Particle transport also exhibits  $\kappa^{-1}$  dependence if  $D$  is positive
- **Dependence on triangularity strongest for highly elongated plasmas, weak effect for circular shaped plasmas**
  - Triangularity destabilizing for both energy and particle transport
  - Triangularity has significant impact on particle transport spectrum at high elongation; transport from low- $k$  modes can change sign as  $\delta$  is varied
- **ExB shear quench rule remains valid in the presence of kinetic electrons with a quench point at  $\gamma_E = 2\gamma_{\text{max}}$  for ions and electrons**
  - Quench rule valid for both ITG and TEM cases
  - Transport may not be quenched at higher  $q$  values if parallel velocity shear included in simulations