A NEW METHOD FOR SOLVING THE RESISTIVE MHD INNER LAYER

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A NEW TECHNIQUE TO SOLVE SYSTEMS OF DIFFERENTIAL EQUATIONS WITH SINGULAR SOLUTIONS CAN BE SUCCESSFULLY APPLIED TO SOLVE THE RESISTIVE INNER LAYER PROBLEM

- New numerical technique originally developed for *regular* singular points
  - Also works with *irregular* singular points arising in inner layer problem
- New method transforms singular solutions into finite solutions:
  - Extended here to systems of coupled equations for inner layer problem
  - Extracts out the dominant singular behavior and solves for remainder
  - Boundary conditions applied naturally to exclude exponential singular solutions
    - Retains exponentially decaying and Frobenius solutions and extracts interchange and tearing parity matching data Δ_+ and Δ_-
- Code can reproduce previous results (Glasser, Jardin, Tesuaro)
  - Excellent agreement over full range of parameters
  - New technique is applicable to an extended range in parameters
- Code is applied to detailed study model of Greene and Miller with nonuniform density
  - Nonuniform inertia has little effect for low growth rates
  - Significant divergence of results from uniform inertia at high growth rates:
    - Results depend on the inertia profile
    - New poles appear in interchange parity matching data for some inertia profiles
    - Stationary points appear for tearing parity growth rates where Δ is insensitive to MHD parameters over a range of growth rates

INNER LAYER EQUATIONS FOR RESISTIVE PLASMA AND FINITE β GENERALIZED TO ARBITRARY DENSITY PROFILE

- Inner layer equations for the perturbed flux Ψ, the displacement Ξ, and the perturbed current density Γ: (Glasser Jardin, Tesauro, Phys. Fluids, 27, 1225, (1984)):

Generalized to include varying mass density profile ρ

□ Inertia profile varies across inner layer

$$\frac{d^2Y}{dx^2} - U\frac{dY}{dx} - \mu Y = 0 \qquad Y = \begin{pmatrix} \Psi \\ \Xi \\ \Gamma \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & H \\ -H/(Q^2\rho) & -\rho'/\rho & 0 \\ HKQ & 0 & 0 \end{pmatrix}$$

E, F, G, H, K = Constants (from outer region)

Q = Scaled Growth Rate

$D_R = E + F + H^2$

$D_I = D_R - (1/2 - H)^2$

$$\mu = \begin{pmatrix} Q & -xQ & 0 \\ -x/(Q\rho) & x^2/(Q\rho) & -(E+F)/(Q^2\rho) \\ -x/Q & -(G-KE)Q & x^2/Q + (G+KF)Q \end{pmatrix}$$

TRANSFORMATION TO FINITE UNIT INTERVAL YIELDS EQUATION WITH ONE IRREGULAR SINGULAR POINT AT MATCHING POINT

- Variable x represents volume distance from resonant surface scaled to X_0:

$$x = (V - V_0) / X_0 \qquad X_0 = \left(\rho\, M\, \eta^2 \langle B^2\rangle^2 / \alpha^2 \Lambda^2 \langle B^2/|\nabla\psi|^2\rangle^2\right)^{1/6}$$

η = resistivity

- In the limit as η -> 0 the parameter X_0 -> 0:
  - x varies from 0 (resonant surface) to ∞ (matching point)
- Transform: $x = \dfrac{(1 - t)}{t}$
  - t varies from 0 (matching point) to 1 (resonant surface)
- Transformed coupled equations are system of ordinary homogeneous differential equations:

$$\frac{d}{dt}\left(t^2 \frac{dY}{dt}\right) + U \frac{dY}{dt} - \frac{1}{t^2}\mu\, Y = 0$$

- The point $t = 0$ is an *irregular* singular point
  - General solutions cannot be expressed solely as Frobenius series

GENERAL ATOMICS

GENERAL SOLUTION IS A LINEAR COMBINATION OF FROBENIUS AND DIVERGENT AND CONVERGENT EXPONENTIAL SOLUTIONS

- Inner layer equation is a sixth order system for the perturbed flux Ψ, the displacement Ξ and the perturbed current density Γ:
  - Six linearly independent solutions: $Y = \sum \alpha_i T_i(x) = \sum \alpha_i \mathcal{T}_i(t)$

- Asymptotic behavior as $x \to \infty$ or $t \to 0$:

$$T_{1(2)}(x) = \exp\left\{ +x^2/(2Q^{1/2}) \right\} x^{s_{+(-)}} \times S_{1(2)}(x) \quad \text{(Divergent Exponential)}$$

$$T_{3(4)}(x) = x^{p_{+(-)}} \times S_{3(4)}(x) \quad \text{(Frobenius)}$$

$$T_{5(6)}(x) = \exp\left\{ -x^2/(2Q^{1/2}) \right\} x^{s_{+(-)}} \times S_{5(6)}(x) \quad \text{(Convergent Exponential)}$$

$$s_{+(-)} = s_{+(-)}(E,F,G,H,K,Q) \qquad p_{+(-)} = -1/2 \; +(-)\, \mu \qquad (\mu = (-D_I)^{1/2})$$

- Boundary conditions:
  - Solution Parity about resonant surface $t = 1$:
    - Odd: $\Psi'(1) = \Xi(1) = \Gamma(1) = 0$ Even: $\Psi(1) = \Xi'(1) = \Gamma'(1) = 0$
  - Divergent Exponential solutions $T_{1(2)}$ eliminated: $\alpha_1 = \alpha_2 = 0$
  - Homogeneity: Choose one $\alpha_j = 1$

NEW ALGORITHM TRANSFORMS GENERAL SOLUTION INTO FINITE SOLUTION BY EXTRACTING REMAINING DOMINANT LARGE FROBENIUS SOLUTION AND SOLVING FOR REMAINDER

- Divergent exponential solution removed from general solution by boundary conditions
  - Remaining dominant solution is the large Frobenius solution T_3
  - **New algorithm developed for equations with *regular* singular points can be applied to this problem with an *irregular* singular point**
- Transformation extracts dominant Frobenius solution T_3:

$$\mathbf{Y} = \mathcal{P}\,\mathbf{Z} = \mathcal{P}\begin{pmatrix}\Psi \\ \Xi \\ \Gamma\end{pmatrix} \qquad \mathcal{P} = \begin{pmatrix} p_{11} & 0 & 0 \\ 0 & p_{22} & 0 \\ 0 & 0 & p_{33}\end{pmatrix}$$

$$\mathbf{T}_3(x) = x^{-1/2+\mu} \times \mathbf{S}_3(x)$$

$$\mathbf{S}_3(x) = \begin{pmatrix} x + \gamma_1/x + \dots \\ 1 + \gamma_2/x^2 + \dots \\ 1 + \gamma_3/x^2 + \dots \end{pmatrix}$$

$$\left.\begin{array}{l} p_{11} = x^{+1/2+\mu} \\ p_{22} = x^{-1/2+\mu} \\ p_{33} = x^{-1/2+\mu} \end{array}\right\} \quad x \to \infty \;\; (t \to 0)$$

- Final homogeneous boundary condition: Choose $\alpha_3 = 1$

NEW SYSTEM OF EQUATIONS CAN BE SOLVED BY STANDARD FINITE DIFFERENCE OR FINITE ELEMENT METHODS

- Obtain an ordinary homogeneous system of equations for Z over the unit interval:

$$\frac{d^2 Z}{d\boldsymbol{t}^2} + \mathcal{P}^{-1}\left\{ 2\frac{d\mathcal{P}}{d\boldsymbol{t}} + \frac{2}{\boldsymbol{t}}\mathcal{P} + \frac{1}{\boldsymbol{t}^2}\mathrm{U}\mathcal{P} \right\}\frac{dZ}{d\boldsymbol{t}}$$

$$+ \mathcal{P}^{-1}\left\{ \frac{d^2\mathcal{P}}{d\boldsymbol{t}^2} + \frac{2}{\boldsymbol{t}}\frac{d\mathcal{P}}{d\boldsymbol{t}} + \frac{1}{\boldsymbol{t}^2}\mathrm{U}\frac{d\mathcal{P}}{d\boldsymbol{t}} + \frac{1}{\boldsymbol{t}^4}\mu\mathcal{P} \right\} Z = 0$$

- Boundary conditions:
  - Dirichlet boundary conditions at the left edge (matching point):

$$Z(\boldsymbol{t} = 0) = 1 \quad \square \quad \tilde{\Psi}(0) = \tilde{\Xi}(0) = \tilde{\Gamma}(0) = 1$$

  - Parity conditions at the right edge (rational surface):

$$\text{Odd parity:} \quad \square \quad \tilde{\Psi}'(1) = \tilde{\Xi}(1) = \tilde{\Gamma}(1) = 0$$

$$\text{Even parity:} \quad \square \quad \tilde{\Psi}(1) = \tilde{\Xi}'(1) = \tilde{\Gamma}'(1) = 0$$

- All terms are finite but require care in numerical evaluation to avoid cancelling infinite terms

ACCURATE EXTRACTION OF LARGE AND SMALL FROBENIUS SOLUTIONS FROM SPECIFIC SOLUTION IS NONTRIVIAL

- TWIST-IR code numerically computes the complete solution by finite differences
  - Dispersion relation for growth rate Q: 2 x 2 matrix equation $D(Q) = D'$
    - Elements of D′ from the external inertia-free region:
    - D(Q) is diagonal with elements corresponding to the ratio of the leading coefficients of the large and small Frobenius solution components from the inner layer:

$\Delta_{+(-)} = \alpha_3/\alpha_4$ for even (+) and odd (-) parity solutions

  - **Frobenius expansion coefficients need to be extracted accurately to obtain the matching data D for matching to outer region solution data D′**

- Complete solution contains large and small Frobenius solutions $T_{3(4)}$ plus the two convergent exponential solutions $T_{5(6)}$:
  - Transformed linearly independent solutions to be extracted:

$$G_1(x) = \mathcal{P}\, T_3(x) = \mathcal{P}\, x^{-1/2+\mu} \times S_3(x) = \begin{pmatrix} x + \gamma_1/x + \ldots \\ 1 + \gamma_2/x^2 + \ldots \\ 1 + \gamma_3/x^2 + \ldots \end{pmatrix}$$

$$G_2(x) = \mathcal{P}\, T_4(x) = \mathcal{P}\, x^{-1/2-\mu} \times S_4(x) = x^{-2\mu} \times \begin{pmatrix} x + \delta_1/x + \ldots \\ 1 + \delta_2/x^2 + \ldots \\ 1 + \delta_3/x^2 + \ldots \end{pmatrix}$$

  - Similarly: $G_3(x) = \mathcal{P}\, T_5(x)$ and $G_4(x) = \mathcal{P}\, T_6(x)$

ALGORITHM FOR EXTRACTING MATCHING DATA REQUIRES FINDING FOUR LINEARLY INDEPENDENT SOLUTIONS

- Numerical solution solves for four linearly independent solutions U_1, U_2, U_3, U_4:
  - All with the normalization condition at the matching point: $$\mathbf{U}_k(0) = \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad k = 1, 2, 3, 4$$
  - Independent boundary conditions at the rational surface: $$\mathbf{U}_1(1) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{U}_2(1) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{U}_3(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{U}_4(1) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$
- Each is a linear combination of the four solutions G_1, G_2, G_3, and G_4:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} 1 & \omega_{12} & \omega_{13} & \omega_{14} \\ 1 & \omega_{22} & \omega_{23} & \omega_{24} \\ 1 & \omega_{32} & \omega_{33} & \omega_{34} \\ 1 & \omega_{42} & \omega_{43} & \omega_{44} \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{pmatrix}$$

- Exclude G_1 from U_2, U_3, and U_4 by subtracting U_1:

$$\tilde{U}_2 = U_2 - U_1 = U_2 - G_1 - \omega_{12} G_2 - \omega_{13} G_3 - \omega_{14} G_4$$
$$= (\omega_{22} - \omega_{12})\, G_2 + (\omega_{23} - \omega_{13})\, G_3 + (\omega_{24} - \omega_{14})\, G_4$$

$\tilde{U}_2$ is now dominated by the small Frobenius solution G_2:

- Renormalize the $\tilde{U}_k$ by $\omega_{22} - \omega_{12}$:

$$W_2 = G_2 + \varpi_{23}\, G_3 + \varpi_{24}\, G_4$$

$$\varpi_{23} = (\omega_{23} - \omega_{13})/(\omega_{22} - \omega_{12}) \qquad \varpi_{24} = (\omega_{24} - \omega_{14})/(\omega_{22} - \omega_{12})$$

NUMERICAL ALGORITHM FOR EXTRACTING MATCHING DATA IS ACCURATE FOR ALL CASES OF REAL INTEREST

- **Exclude G_2, from W_3, and W_4, by subtracting $\varpi_{k2}W_2$, taking ϖ_{k2} as the scalar product between G_2 and each W_k, (k = 3, 4), averaged over a neighborhood ε of $t = 0$ ($x \to \infty$):**

$$\varpi_{k2} = \lim_{\varepsilon \to 0} \frac{\langle W_k\, G_2\rangle_\varepsilon}{\langle W_2^2\rangle_\varepsilon} \;\square\; \lim_{\varepsilon \to 0} \frac{\langle W_k\, W_2\rangle_\varepsilon}{\langle W_2^2\rangle_\varepsilon} \qquad k = 3, 4$$

  - **In the limit as ε vanishes, this becomes asymptotically correct since the difference between G_2 and W_2 is proportional to G_3 and G_4 and vanishes faster than G_2**

- **Exclude G_2, from W_1, by subtracting $\varpi_{12}W_2$, with ϖ_{12} defined as:**

$$\varpi_{12} = \lim_{\varepsilon \to 0}\left\{\frac{\langle W_1\, G_2\rangle_\varepsilon - \langle G_1\, G_2\rangle_\varepsilon}{\langle W_2^2\rangle_\varepsilon}\right\} \;\square\; \lim_{\varepsilon \to 0}\left\{\frac{\langle W_1\, W_2\rangle_\varepsilon - \langle 1\, W_2\rangle_\varepsilon}{\langle W_2^2\rangle_\varepsilon}\right\}$$

  - **The expression on the left is also asymptotically correct since $W_1 - G_1$ has leading term proportional to G_2 and the remaining terms, proportional to G_3 and G_4 and vanish faster than G_2**
  - **The expression on the left is is accurate except when μ is an integer and the large and small Frobenius series for G_1 and G_2 are no longer linearly independent**

- **New solutions labeled $V_k = W_k - \varpi_{k2}W_2$ (k = 1,3,4) and $V_2 = W_2$**

V_k ARE SIMPLY CALCULATED FROM THE ORIGINAL SOLUTIONS AND EQUAL THE DESIRED SOLUTIONS G_k TO LEADING ORDER

- New solutions are related to the numerical solutions U_k through known coefficients :

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} W_1 - \varpi_{12} W_2 \\ W_2 \\ W_3 - \varpi_{32} W_2 \\ W_4 - \varpi_{42} W_2 \end{pmatrix} = \begin{pmatrix} U_1 - \varpi_{12}(U_2 - U_1) \\ (U_2 - U_1) / (\omega_{22} - \omega_{12}) \\ (U_3 - U_1) - \varpi_{32}(U_2 - U_1) \\ (U_4 - U_1) - \varpi_{42}(U_2 - U_1) \end{pmatrix}$$

  - The ϖ_{k2} relating the V_k and U_k are known from the inner products and $(\omega_{22} - \omega_{12})$ is known from the leading term of $U_2 = U_2 - U_1$:
  - The V_k are also clearly related to the desired solutions G_k:

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \varpi_{13} - \varpi_{12}\varpi_{23} & \varpi_{14} - \varpi_{12}\varpi_{24} \\ 0 & 1 & \varpi_{23} & \varpi_{24} \\ 0 & 0 & \varpi_{33} - \varpi_{32}\varpi_{23} & \varpi_{34} - \varpi_{32}\varpi_{23} \\ 0 & 0 & \varpi_{43} - \varpi_{42}\varpi_{23} & \varpi_{44} - \varpi_{42}\varpi_{23} \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{pmatrix} \square \left\{ \begin{array}{l} V_1 = G_1 + O(G_3, G_4) \\ V_2 = G_2 + O(G_3, G_4) \\ V_3 = O(G_3, G_4) \\ V_4 = O(G_3, G_4) \end{array} \right.$$

- The boundary conditions on the V_k are now:

$$\left\{ \begin{array}{l} V_1(0) = \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad V_k(0) = \mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad k = 2, 3, 4 \\ V_1(1) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad V_2(1) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad V_3(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad V_4(1) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{array} \right.$$

PARTICULAR SOLUTION CAN BE EXPANDED IN TERMS OF THE LINEARLY INDEPENDENT FUNCTIONS V_k

- The particular solution can now be constructed directly from the solutions V_k:

$$Z = V_1 + \Delta_2 V_2 + \Delta_3 V_3 + \Delta_4 V_4$$

  - Z automatically satisfies the boundary condition $Z(t = 0) = 1$ at the matching point since:

$$V_1(0) = 1 \qquad V_2(0) = V_3(0) = V_4(0) = 0$$

- The Δ_k are chosen to satisfy the particular parity boundary conditions at the resonant surface $t = 1$: 3 conditions and 3 unknowns

$$Z(1) = \begin{pmatrix} \tilde{\Psi}(1) \\ \tilde{\Xi}(1) \\ \tilde{\Gamma}(1) \end{pmatrix} = \begin{pmatrix} \cdot \\ 0 \\ 0 \end{pmatrix} \text{ and } Z'(1) = \begin{pmatrix} \tilde{\Psi}'(1) \\ \tilde{\Xi}'(1) \\ \tilde{\Gamma}'(1) \end{pmatrix} = \begin{pmatrix} 0 \\ \cdot \\ \cdot \end{pmatrix} \quad \text{(odd parity)}$$

or

$$Z(1) = \begin{pmatrix} \tilde{\Psi}(1) \\ \tilde{\Xi}(1) \\ \tilde{\Gamma}(1) \end{pmatrix} = \begin{pmatrix} 0 \\ \cdot \\ \cdot \end{pmatrix} \text{ and } Z'(1) = \begin{pmatrix} \tilde{\Psi}'(1) \\ \tilde{\Xi}'(1) \\ \tilde{\Gamma}'(1) \end{pmatrix} = \begin{pmatrix} \cdot \\ 0 \\ 0 \end{pmatrix} \quad \text{(even parity)}$$

ALGORITHM FINDS MATCHING DATA Δ_+ AND Δ_- DIRECTLY FROM Δ_2

- $V_1 \sim G_1$ and $V_2 \sim G_2$ up to terms of the order of the two convergent exponential solutions G_3 and G_4

□

$$\Delta_2 = \begin{cases} 1/\Delta_- & \left(\begin{matrix}\text{odd}\\ \text{parity}\end{matrix}\right) \\ 1/\Delta_+ & \left(\begin{matrix}\text{even}\\ \text{parity}\end{matrix}\right) \end{cases}$$

- Back transformation:
  and inverse variable transformation :

$Y = \mathcal{P} Z$ □ Solution eigenvector Y

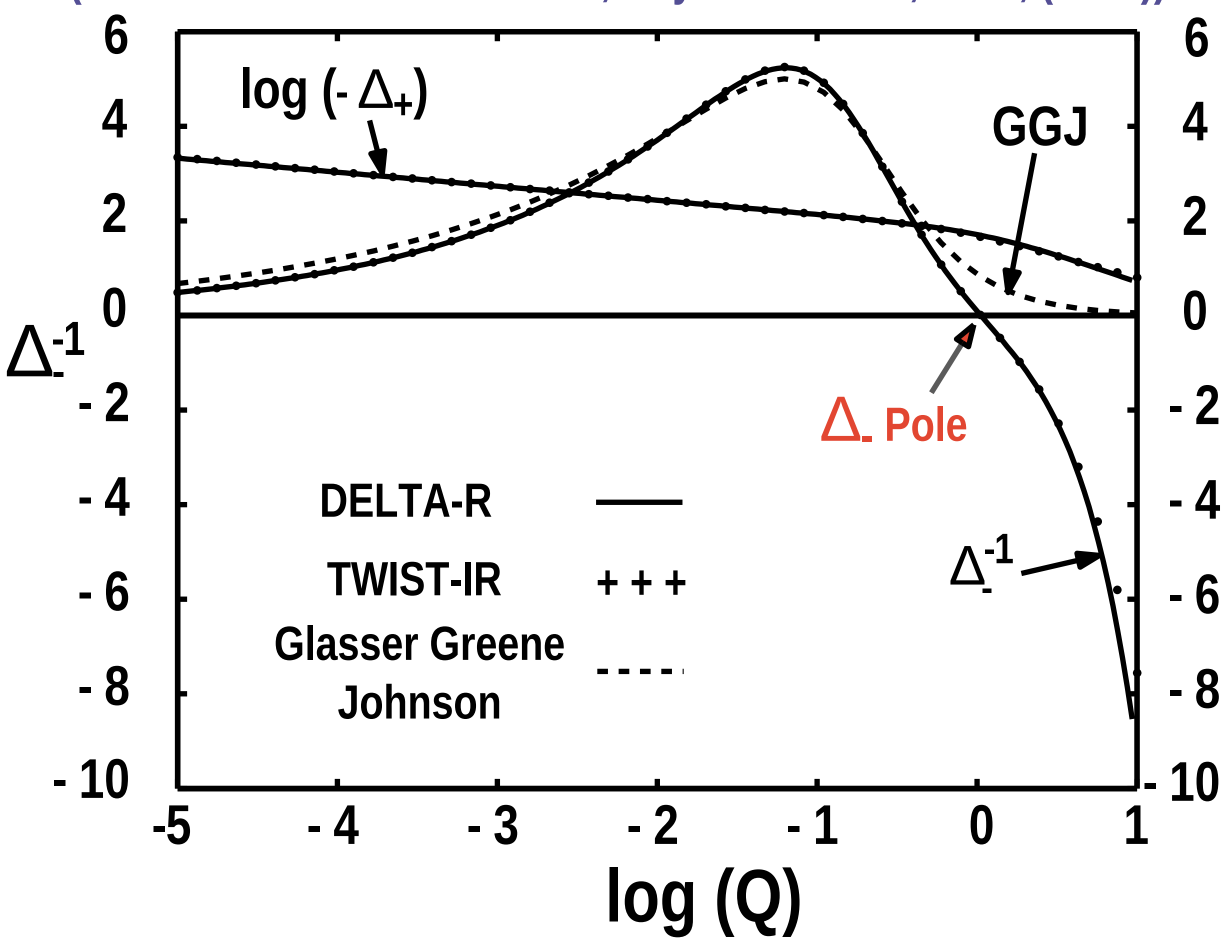
$t \to x$

- Numerical algorithm for computing both Y and the matching data Δ_+ and Δ_- works well for all parameters:
  - Even for the important pressureless case with $\mu = 1/2$
  - Even for $\mu < 1/2$ and $\mu > 1$
  - Except for the cases when μ is close to an integer and the large and small Frobenius series for G_1 and G_2, as given, are degenerate:

□ Solution for Y can still be obtained by solving for Z with the physical boundary conditions but $\Delta_{-(+)}$ are not easily extracted

BENCHMARK STUDY FOR CONSTANT DENSITY PROFILE SHOWS EXCELLENT AGREEMENT WITH PREVIOUSLY PUBLISHED RESULTS

- Comparison of TWIST-IR results with the DELTA-R code (Glasser Jardin and Tesauro, Phys. Fluids 27, 1225, (1984))



$\mu = 0.5916$
$E = -0.1$
$F = G = 0$
$K = H = 0$
$\rho = 1$

Extremely good agreement for Δ_- and Δ_+ over large range of growth rates

Divergence between DELTA-R and TWIST-IR begins only for Q > 5: DELTA-R fails for Q > 10

GENERAL ATOMICS

GREENE AND MILLER MODEL FOR NONUNIFORM DENSITY ACROSS INNER LAYER INTENDED TO RESOLVE INCONSISTENCIES IN MATCHING TO INERTIA-FREE EXTERNAL SOLUTION

- Density taken as: $\rho = \begin{cases} 1 & |x| < x_c \\ 0 & |x| > x_c \end{cases}$ (Greene and Miller Phys. Plasmas, 2, 1236, (1995))
  - Matching to outer region at $x \to \infty$ is performed where inertia is zero on both sides of the matching point to avoid inconsistency
- Density taken to vary smoothly across inner layer and vanishing as $x \to \infty$ ($t \to 0$)

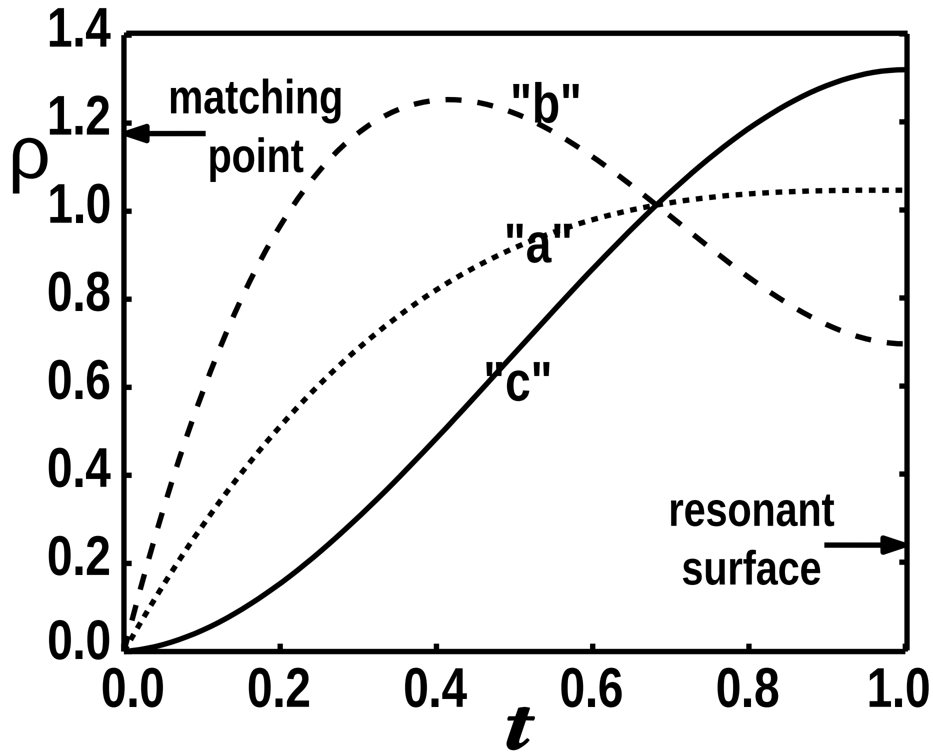
$$\rho(t) = \rho_1 + (1-t)^2 \times (k\, t^n - 2\rho_1 t - \rho 1)$$

Three density profiles taken to test effect of ρ profile across whole range of parameters

"a" $\begin{cases} \rho_1 = 1.0 \\ k = 3.0 \\ n = 1 \end{cases}$

"b" $\begin{cases} \rho_1 = 1.0 \\ k = 10.0 \\ n = 1 \end{cases}$

"c" $\begin{cases} \rho_1 = 10.0 \\ k = 1.0 \\ n = 1 \end{cases}$



DENSITY PROFILES NORMALIZED TO FIXED TOTAL MASS IN INNER LAYER TO ISOLATE PROFILE FROM TOTAL DENSITY VARIATION

- To compare different density profiles total mass must be the same to obtain same effective total inertia
  - For constant $\rho = \rho_1$, with $x = (V - V_0) / X_0$, total mass from $x = -x_1$ to $x = x_1$ is:

$$M(x_1) = 2X_0\int_0^{x_1}\rho(x)dx = M_1 = 2X_0\rho_1 x_1$$

  - Require that x_1 remains large ($x_1 >> X_0$) but finite: Otherwise $M_1 \to \infty$
- To scale to same total mass M_1: □ $\rho(x) = M_1\rho(x)/M(x_1)$
  - Then the limit $x_1 \to \infty$ can legitimately be taken

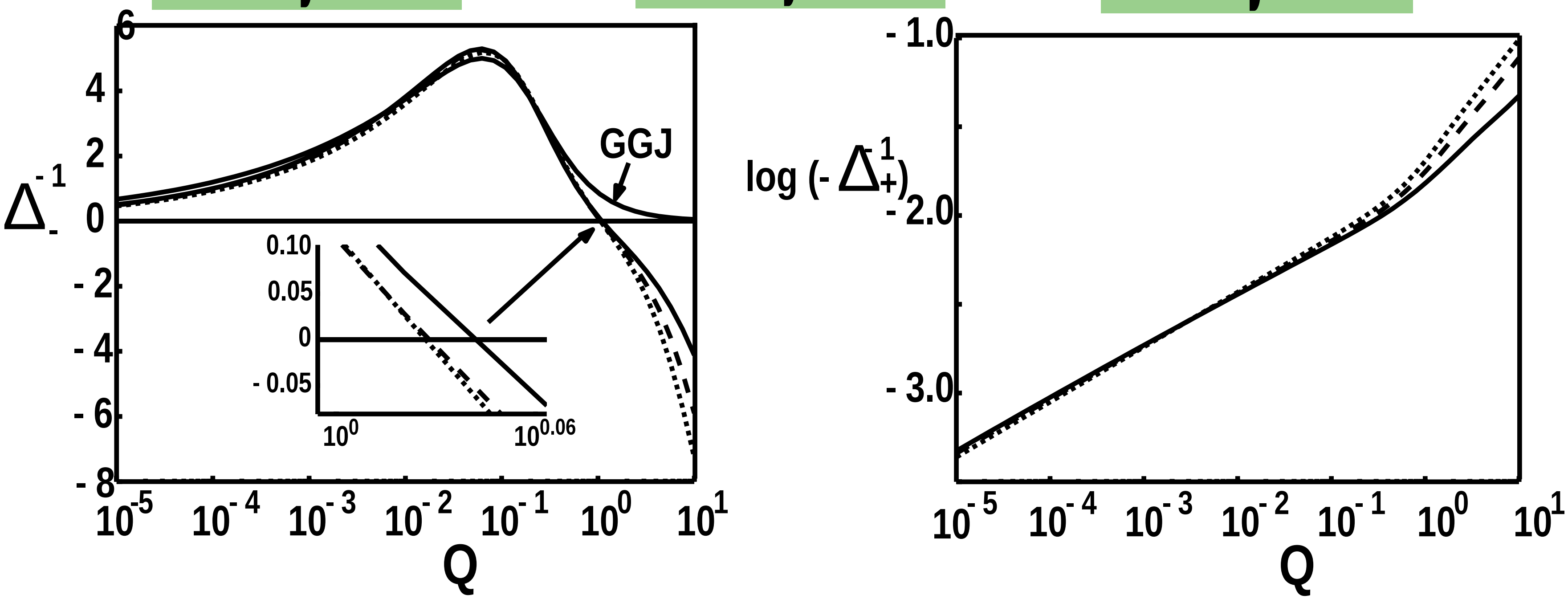
Study effect of varying M_1 (effectively ρ_1) and varying $\rho(x)$ profile independently

Both can have important qualitative as well as quantitative effects

VARYING DENSITY PROFILE HAS SMALL EFFECT ON BOTH Δ_+ AND Δ_- EXCEPT AT LARGE Q

- Three different profiles at constant M_1 normalized to Glasser Greene Johnson result:

$\rho_1 = 1$, $k = 3$, $n = 1$	$\rho_1 = 1$, $k = 10$, $n = 1$ - - - -	$\rho_1 = 10$, $k = 1$, $n = 1$ ———

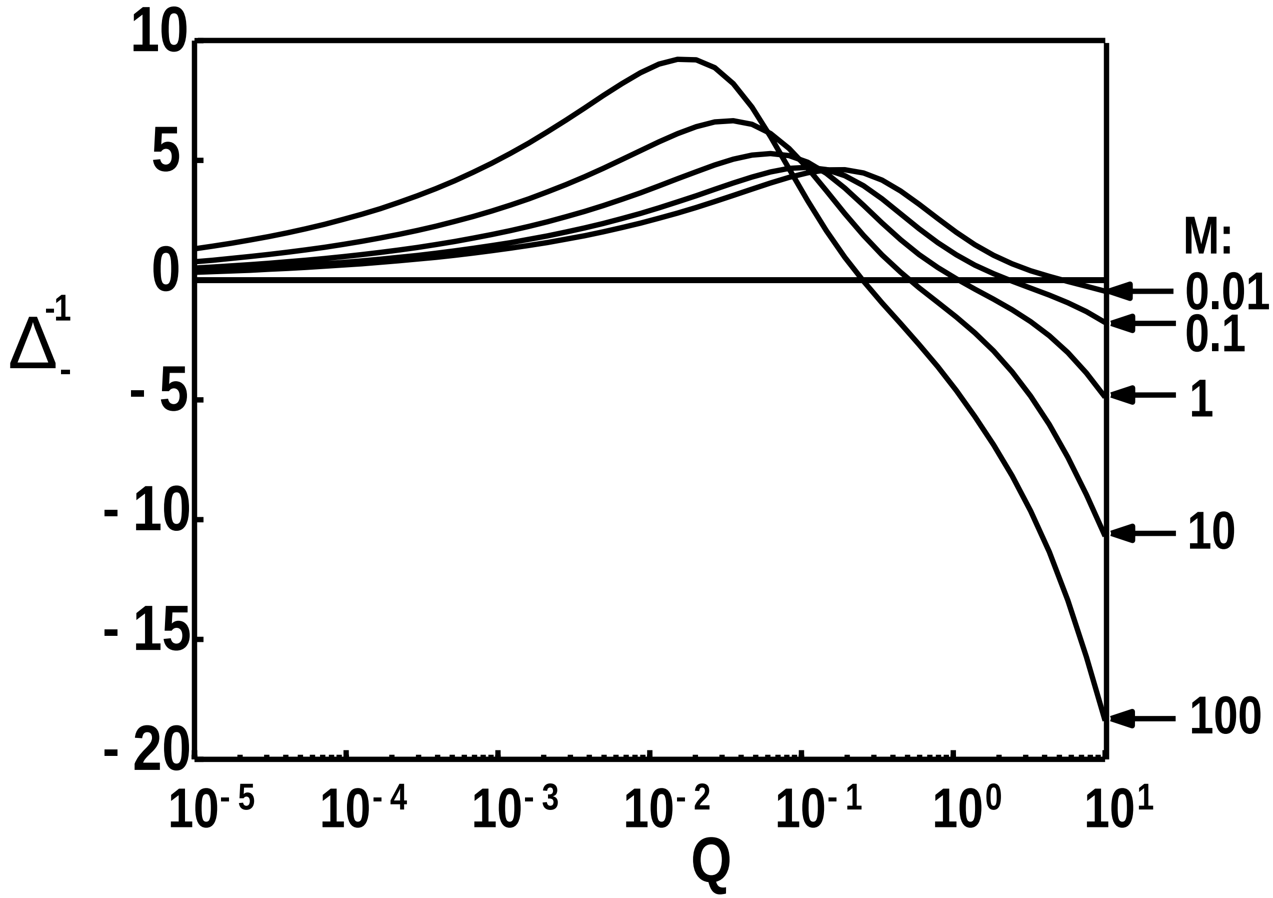


☐ Δ_- has a pole at Q ~ 1 for all three profiles in agreement with the constant density result and in contrast to the Glasser Greene Johnson formula

VARYING TOTAL INERTIA HAS A SIGNIFICANT EFFECT ON Δ_{-} AT ALL Q AND SHIFTS THE POLE POSITION

- Density profile $\rho_1 = 1$, $k = 1$, $n = 1$:

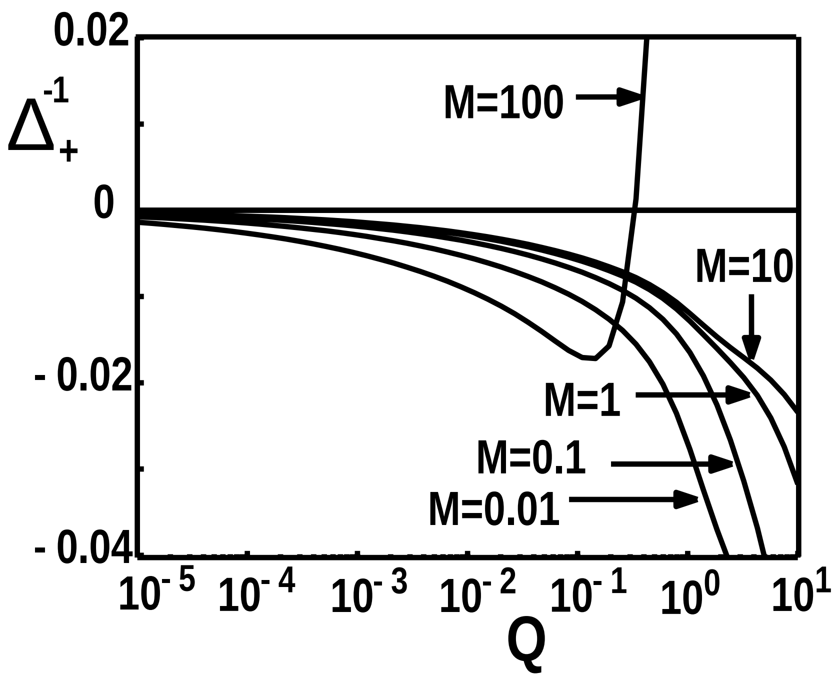
Shape of the Δ_{-}^{-1} curves remains essentially unchanged with varying total inertia but the values change considerably:

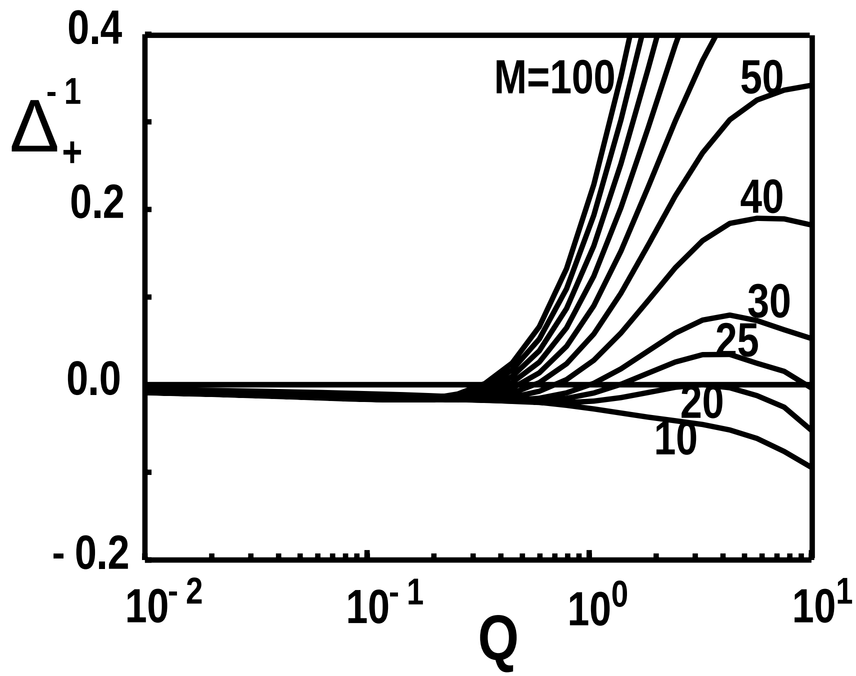


Δ_{-} pole position is roughly inversely proportional to M

NEW POLES IN Δ_+ APPEAR WITH INCREASED TOTAL INERTIA

- For density profile $\rho_1 = 1$, $k = 1$, $n = 1$, the Δ_+^{-1} curves remain monotonically decreasing for $M < 10$
  - For $M > 10$ Δ_+^{-1} begins to increase at intermediate Q
  - For $M > 20$ a new pair of poles in Δ_+ (zeros of Δ_+^{-1}) appears at high Q



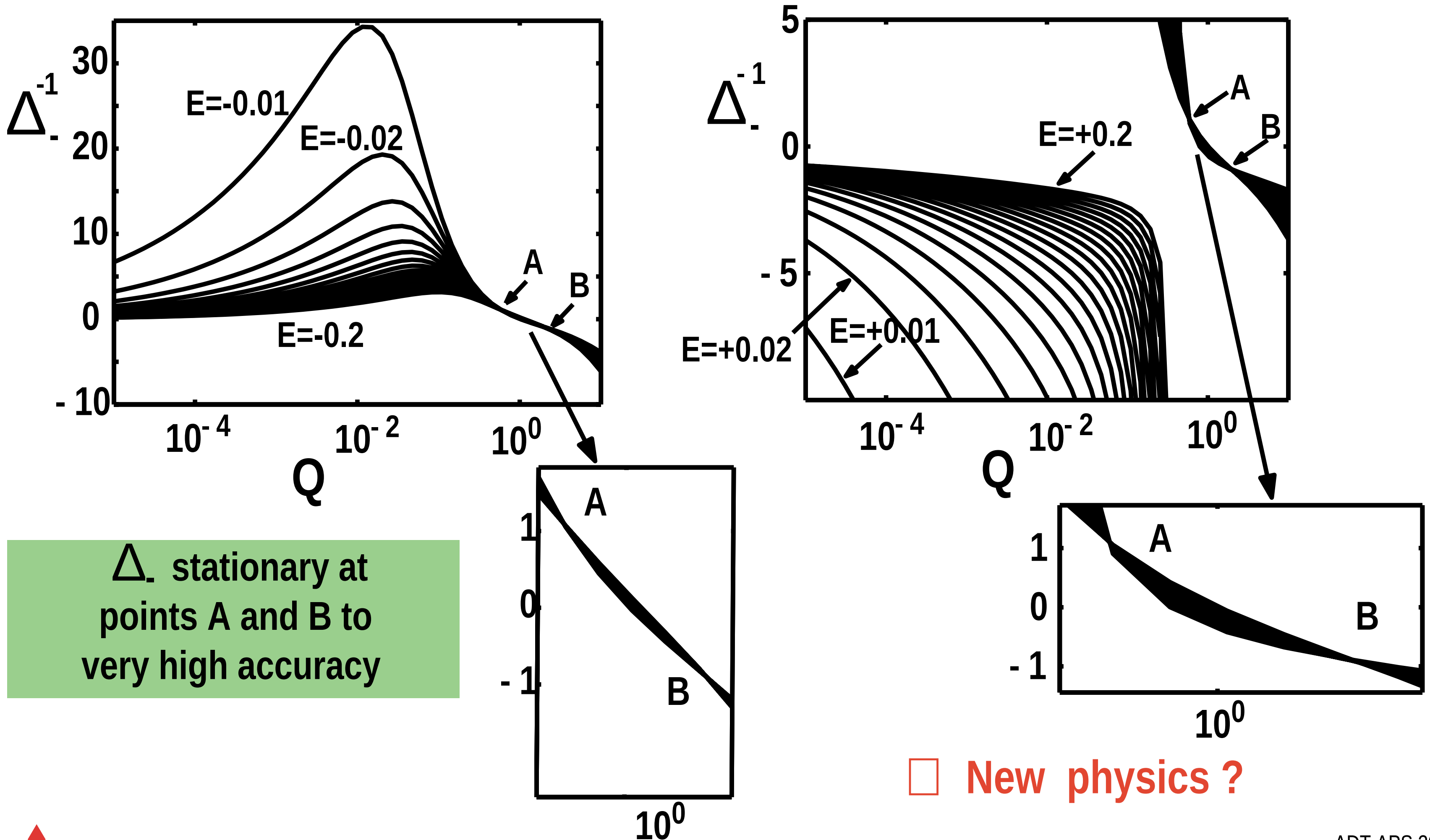


New Δ_+ poles separate in Q as M increases:
- Rightmost pole moves quickly to $Q = \infty$
- Leftmost pole moves to lower Q

- New physics enters with variable density profile:
  Is it real or not ?

STATIONARY POINTS IN $\Delta_-(Q)$ APPEAR FOR VARYING EQUILIBRIUM PARAMETER E

- Dependence of Δ_-^{-1} on Q for varying E shows a pair of stationary points at large Q where $\Delta_-^{-1}(Q)$ has no dependence on E to a high degree:
  - Density profile $\rho_1 = 1$, $k = 1$, $n = 1$
  - Δ_-^{-1} is insensitive to Q between these stationary points

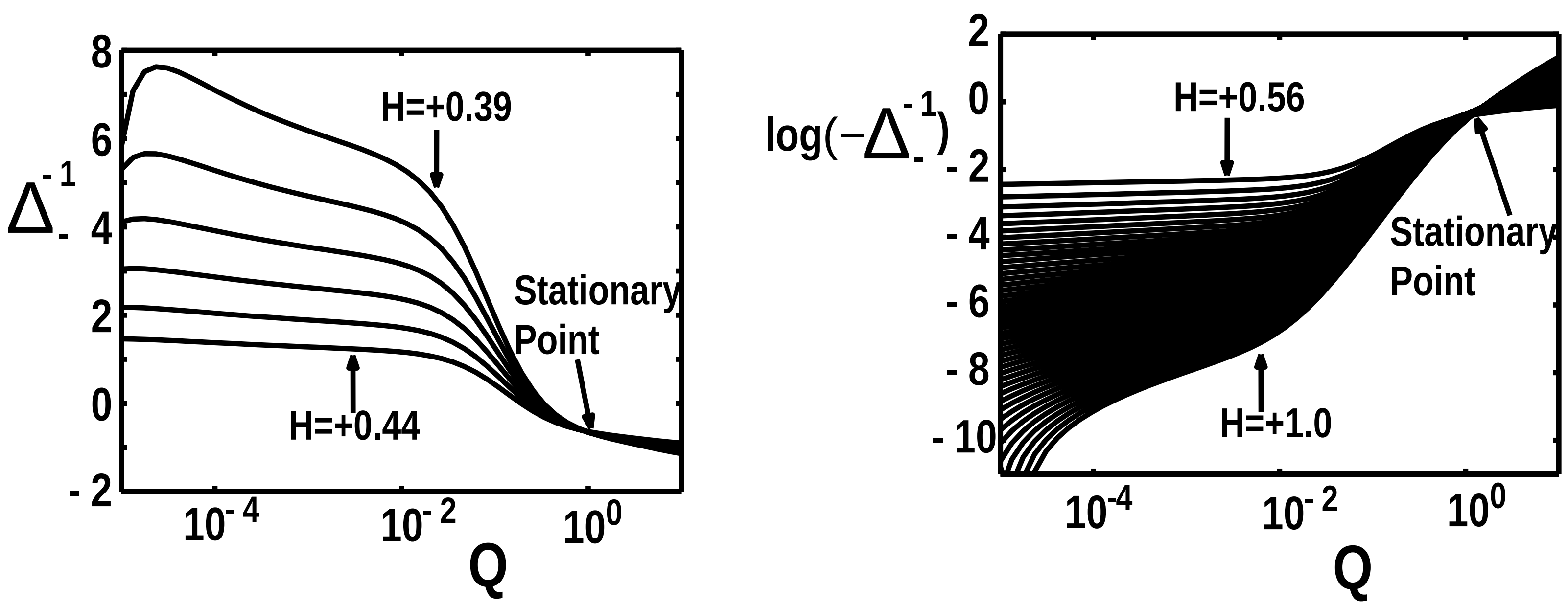


Δ_- stationary at points A and B to very high accuracy

New physics ?

STATIONARY POINTS IN $\Delta_-(Q)$ ALSO APPEAR FOR VARYING EQUILIBRIUM PARAMETER H

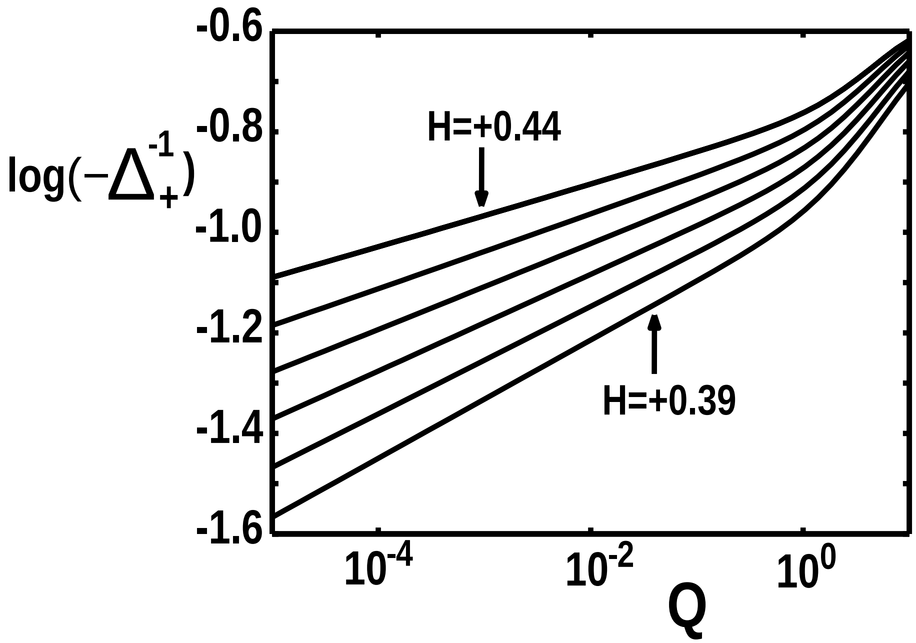
- Dependence of Δ_-^{-1} on Q for varying H shows a single stationary point at large Q where $\Delta_-^{-1}(Q)$ has no dependence on H to high accuracy:

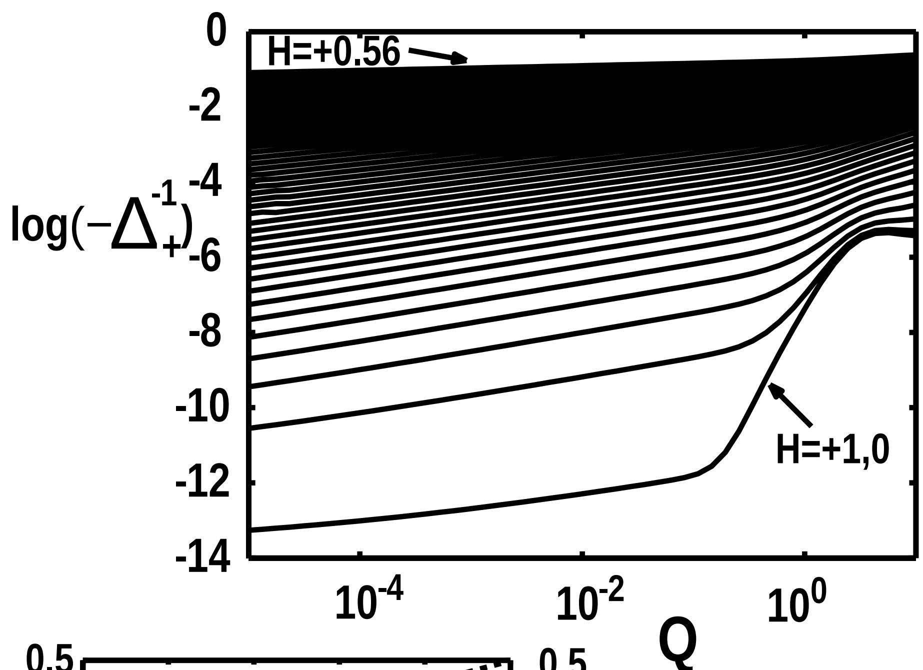


Presence of these stationary points is not understood: Solution matching data is invariant to H and E

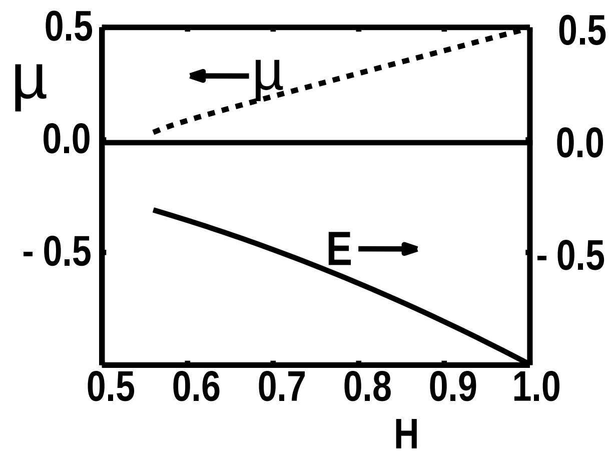
INTERCHANGE PARITY MATCHING DATA $\Delta_+(Q)$ DOES NOT EXHIBIT STATIONARY POINTS

- Dependence of Δ_+^{-1} on Q for varying H shows simple monotonic behavior with no stationary points
  - Density profile $\rho_1 = 1$, k = 1, n = 1





Both E and the Mercier parameter μ vary monotonically with increasing H



GENERAL ATOMICS

GENERAL NUMERICAL TECHNIQUE DEVELOPED TO SOLVE FOR SINGULAR SOLUTIONS ALSO APPLIES TO THE RESISTIVE MHD INNER LAYER PROBLEM WITH IRREGULAR SINGULAR POINTS

- **Technique is implemented in TWIST-IR code and is robust and accurate and reproduces previously published results:**
  - **TWIST-IR reproduces dependence of Δ_- and Δ_+ on gnormalized growth rate Q over a wide range of equilibrium parameters**
  - **TWIST-IR can be applied to Q values up to two orders of magnitude larger than was previously possible**
- **TWIST-IR code used to investigate implications of variable density model of Greene and Miller:**
  - **Dependence of matching data on profile variations and total inertia needs to be separated and treated carefully**
  - **Profile has small effect on matching data curves but varying total inertia has a large effect at large Q: □ New poles can appear in Δ_+ at large Q**
- **Stationary points are exhibited at large Q where Δ_-(Q) is invariant with variations in certain equilibrium data**
  - **Two stationary points in Δ_- with varying E, with insensitivity to Q in between**
  - **A single stationary point in Δ_- with respect to varying H**

Full significance of these features is not yet well understood