ABSTRACT

Obtaining an effective description of electron heat transport within tokamak plasmas is a long-standing problem. The separate effects of heat conduction and convection are not discernible from radial power balance analysis alone. The high power electron cyclotron heating (ECH) system on the DIII–D tokamak is a useful tool for studying heat transport because it locally increases the electron temperature. Modulating the ECH power at frequencies between 25 and 300-Hz produces a series of heat pulses in the plasma that are observed using electron cyclotron emission (ECE). Analytic solutions of the Braginskii energy conservation equation are obtained for various model assumptions, including slab and cylindrical geometries, conductive and convective transport, and damping terms. The analytic solutions are fit to the Fourier analyzed ECE data to determine the salient transport properties of the electron channel. Software tools are developed to compare multi-harmonic data to the models.

ELECTRON HEAT TRANSPORT—WHAT'S THE PROBLEM?

- Although ion heat transport is well explained by the ion temperature gradient (ITG) mode, electron heat transport remains somewhat a mystery.

- The degrees to which conduction and convection affect heat transport cannot be determined from steady-state power balance analysis, since there is only one equation, ($q = -\chi n \nabla T + UnT$), and two unknowns (χ,U).

- Studies of electron heat diffusivity that rely on sawtooth crashes exhibit modulated electron densities and enhanced transport, obscuring the desired measurement.

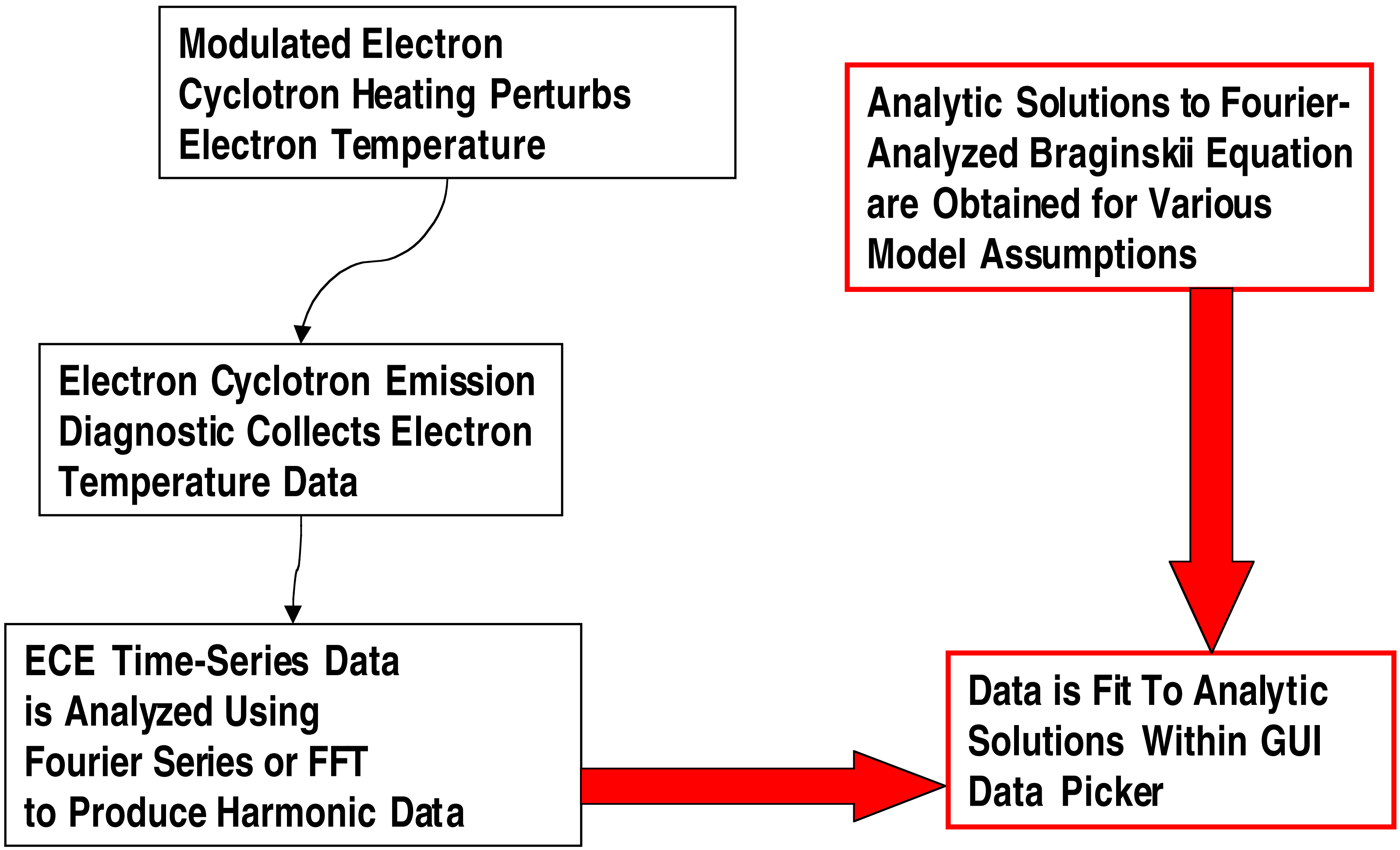
ECH + ECE — TOOLS FOR MEASURING ELECTRON HEAT TRANSPORT

- The high-power Electron Cyclotron Heating (ECH) system at DIII–D allows local perturbation of electron temperature with little direct effect on the density.

- The Electron Cyclotron Emission (ECE) diagnostic can be used to probe the electron temperature response to the local perturbations.

- By comparing analytical models to the observed responses, the roles of conduction and convection in electron heat transport can be determined.

BLUEPRINT FOR ANALYSIS



EFFECTIVE HEAT TRANSPORT EQUATIONS

Start with electron energy conservation equation:

$$\frac{\partial}{\partial t}\left(\tfrac{3}{2}nT\right) + \nabla \cdot \left\{\tfrac{5}{2}\Gamma T - n\chi(\rho,T,\nabla T)\nabla T + nU(\rho,T)T\right\} = Q$$

Electron energy — Particle Flux — Heat diffusion — Heat convection — Heat Source (or sink)

Assume periodic perturbation and expand equation to 1st order in response. This results in the effective heat transport equations for the perturbation:

$$-D\nabla^2\widetilde{T} + V\cdot\nabla\widetilde{T} + \left(\tfrac{1}{\tau} + \tfrac{3}{2}i\omega\right)\widetilde{T} = S_\omega$$

In general, D, V, S, and τ are all functions of position.
The solution of the above equation depends on our assumptions about the functional forms for these effective transport parameters.

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ANALYTIC SOLUTIONS—EXAMPLES

Slab, constant convection

$$-D\frac{d^2\tilde{T}}{dx^2}+V_0\frac{d\tilde{T}}{dx}+\left(\frac{1}{\tau}+\tfrac{3}{2}i\omega\right)\tilde{T}=S_\omega$$

Let:

$$k_\pm \equiv \frac{V_0}{2D}\pm\sqrt{\frac{V_0^{\;2}}{4D^2}+\frac{1}{D\tau}+\tfrac{3}{2}i\frac{\omega}{D}}$$

Two homogeneous solutions:

$$\tilde{T}_1(x)=e^{k_+x}$$

$$\tilde{T}_2(x)=e^{k_-x}$$

Complex Exponentials

Cylindrical, linear convection

$$-D\frac{d^2\tilde{T}}{dr^2}+\left(\frac{V_0r}{a}-\frac{D}{r}\right)\frac{d\tilde{T}}{dr}+\left(\frac{1}{\tau}+\tfrac{3}{2}i\omega+\frac{2V_0}{a}\right)\tilde{T}=S_\omega$$

Transform to:

$$z\frac{d^2\tilde{T}}{dz^2}+(1-z)\frac{d\tilde{T}}{dz}-\gamma\tilde{T}=-\frac{S_\omega a}{2V_0}$$

Where:

$$z=\frac{V_0r^2}{2aD}$$

$$\gamma=1+\frac{a}{2V_0}\left(\frac{1}{\tau}+i\tfrac{3}{2}\omega\right)$$

Two homogeneous solutions:

$$\tilde{T}_1(r)=M(\gamma;1;z)$$

$$\tilde{T}_2(r)=U(\gamma;1;z)$$

Confluent Hypergeometric Functions
(of complex first argument)

ANALYTIC SOLUTIONS FOR DIFFERENT GEOMETRY AND BOUNDARY CONDITIONS

- In general, we have two solutions with the properties:

$$\lim_{\rho \to 0} \left|\widetilde{T}_1\right| < \infty \qquad \lim_{\rho \to \infty} \left|\widetilde{T}_1\right| = \infty$$

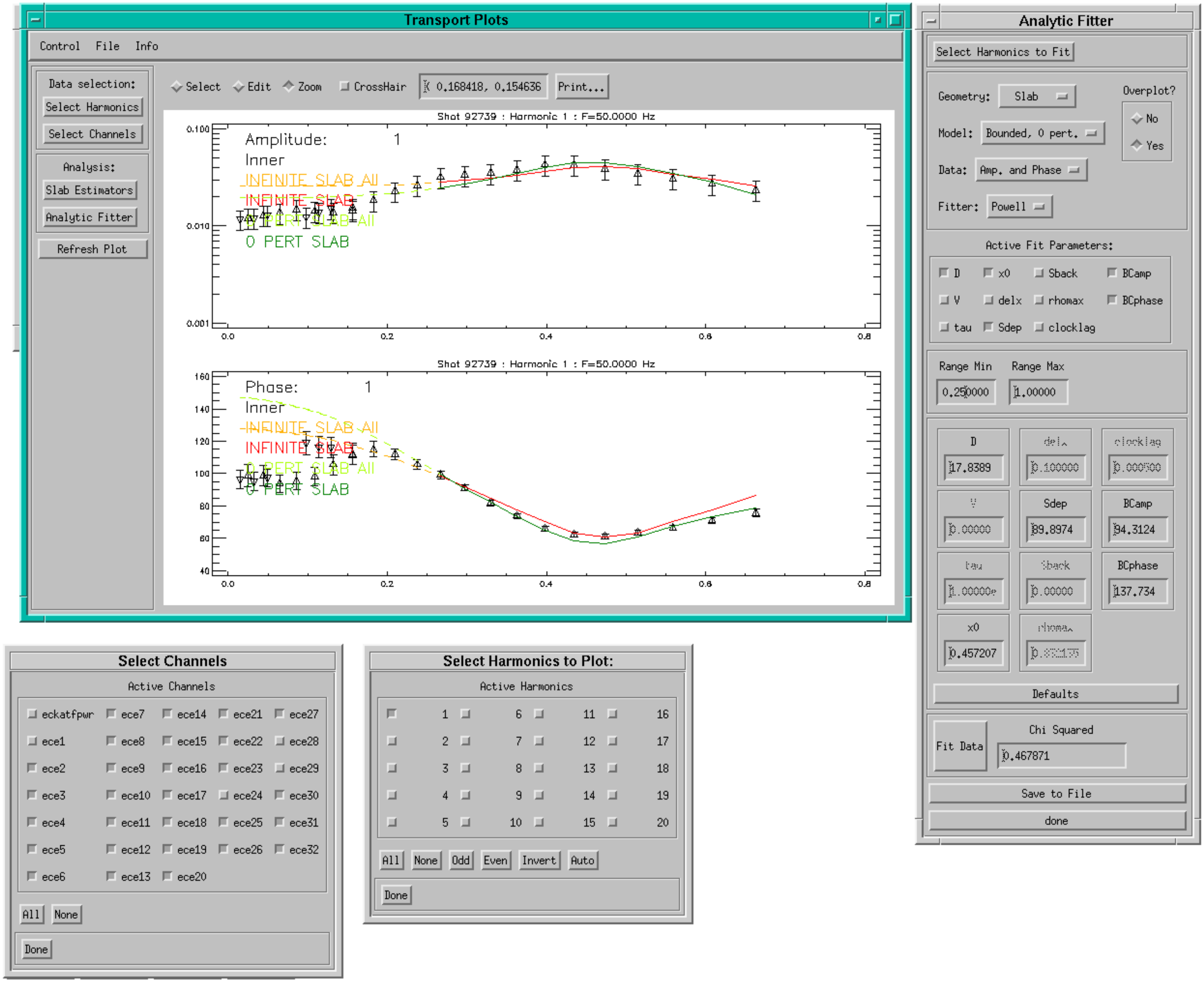
$$\lim_{\rho \to 0} \left|\widetilde{T}_2\right| = \infty \qquad \lim_{\rho \to \infty} \left|\widetilde{T}_2\right| = 0$$

- We can make full solutions by matching across source region given our choice of boundary conditions:
  - Infinite (only T_2 on outer side of source)
  - Zero perturbation at edge ($\rho = 1$)
  - Zero flux at edge
  - Given value of flux or perturbation at edge
- For now, the ECH source is modeled as a constant term between two radii, and zero elsewhere (box source).
- We then calculate the amplitude and phase of the complex response and compare with the data to determine the transport coefficients.

poetAP

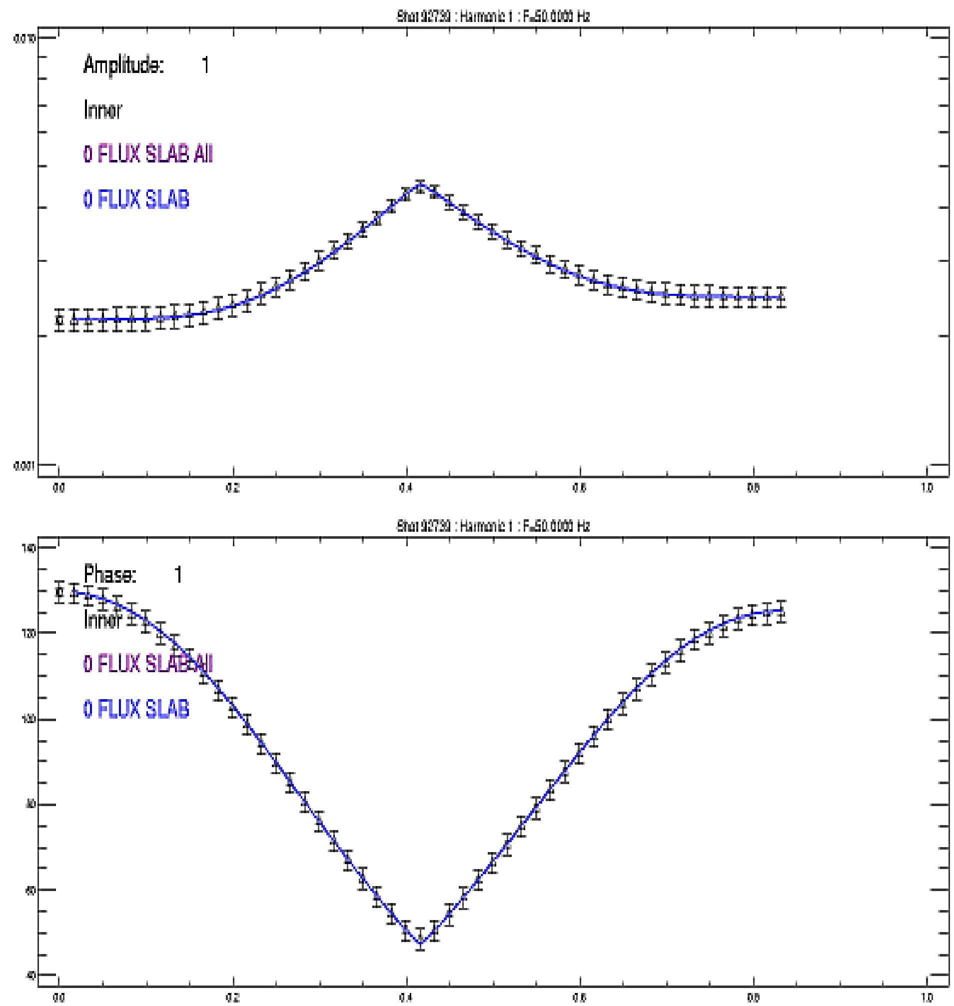
Perturbation Of Electron Temperature Analysis Package

- Multiple harmonics of a single shot can be displayed.
- Can select channels and harmonics to be displayed and fitted.
- Line fitter gives ‘slab estimators’ of the diffusion coefficient.
- Analytic fitter can fit solutions to the data, plot the results, and save relevant fit parameters to file.


Transport Plots
Control File Info
Data selection:
Select Harmonics
Select Channels
Analysis:
Slab Estimators
Analytic Fitter
Refresh Plot
Select Edit Zoom CrossHair
0.168418, 0.154636
Print...
Shot 92739 : Harmonic 1 : F=50.0000 Hz
Amplitude: 1
Inner
INFINITE SLAB All
INFINITE SLAB
0 PERT SLAB All
0 PERT SLAB
Phase: 1
Analytic Fitter
Select Harmonics to Fit
Geometry: Slab
Model: Bounded, 0 pert.
Data: Amp. and Phase
Fitter: Powell
Overplot?
No
Yes
Active Fit Parameters:
D x0 Sback BCamp
V delx rhomax BCphase
tau Sdep clocklag
Range Min Range Max
0.250000 1.00000
D 17.8389
Sdep 89.8974
BCamp 94.3124
BCphase 137.734
x0 0.457207
Defaults
Fit Data
Chi Squared
0.467871
Save to File
done
Select Channels
Active Channels
eckatfpwr ece7 ece14 ece21 ece27
ece1 ece8 ece15 ece22 ece28
ece2 ece9 ece16 ece23 ece29
ece3 ece10 ece17 ece24 ece30
ece4 ece11 ece18 ece25 ece31
ece5 ece12 ece19 ece26 ece32
ece6 ece13 ece20
All None
Done
Select Harmonics to Plot:
Active Harmonics
All None Odd Even Invert Auto
Done

ANALYTIC SOLUTIONS WERE SUCCESSFULLY BENCHMARKED TO NUMERICAL MODELS

- The fitting routines were able to reproduce J. Nelson's simulation code, under a variety of different parameters, verifying that the analytic solutions were valid.



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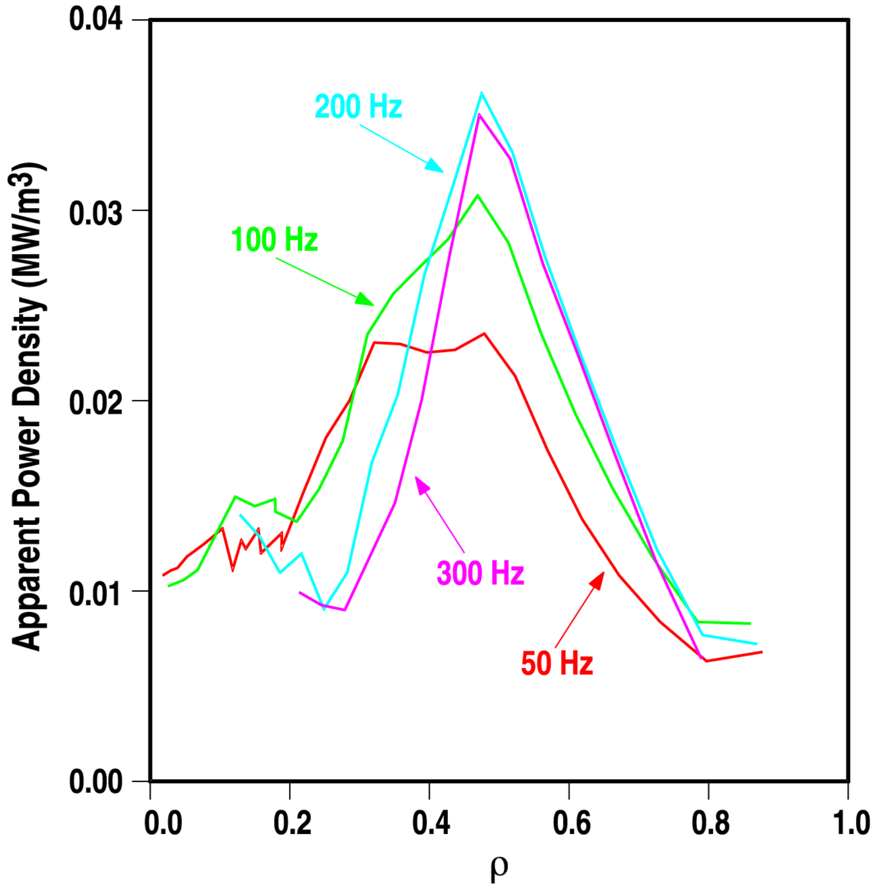
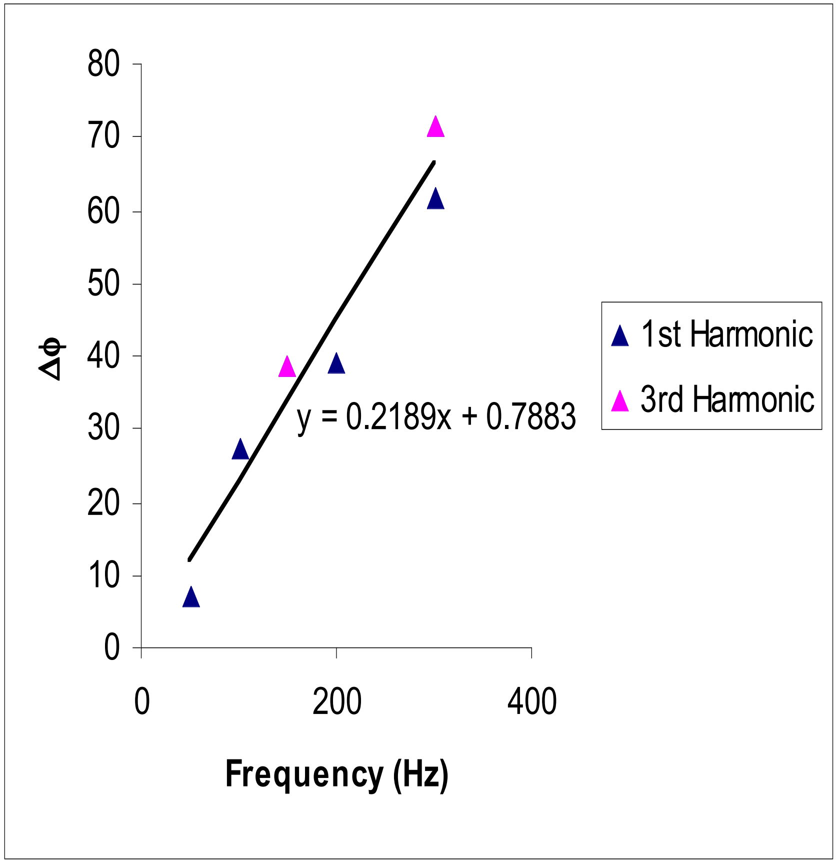
DATASET CONTAINING SCAN OF MODULATION FREQUENCY WAS ANALYZED TO DETERMINE WHETHER TRANSPORT COEFFICIENTS ARE INDEPENDENT OF FREQUENCY



- Modulation frequency scan at fixed deposition location
- Apparent power deposition narrows with increasing modulation frequency

ABERRANT PHASE FITS TRACED TO DIGITIZER ERROR

- Fitting amplitude data only gave poor fit results for the phase.
- This could be corrected by introducing a constant phase offset to the model.
- A linear relation between phase offset and frequency was found for the data.
- The offset fit the model $\Delta\phi$=360*f(Hz)*.0005
- This is interpreted as a clock skip error in the digitization (1 skip at 2kHz=.0005 s)
- Model was expanded to allow arbitrary digitizer error.



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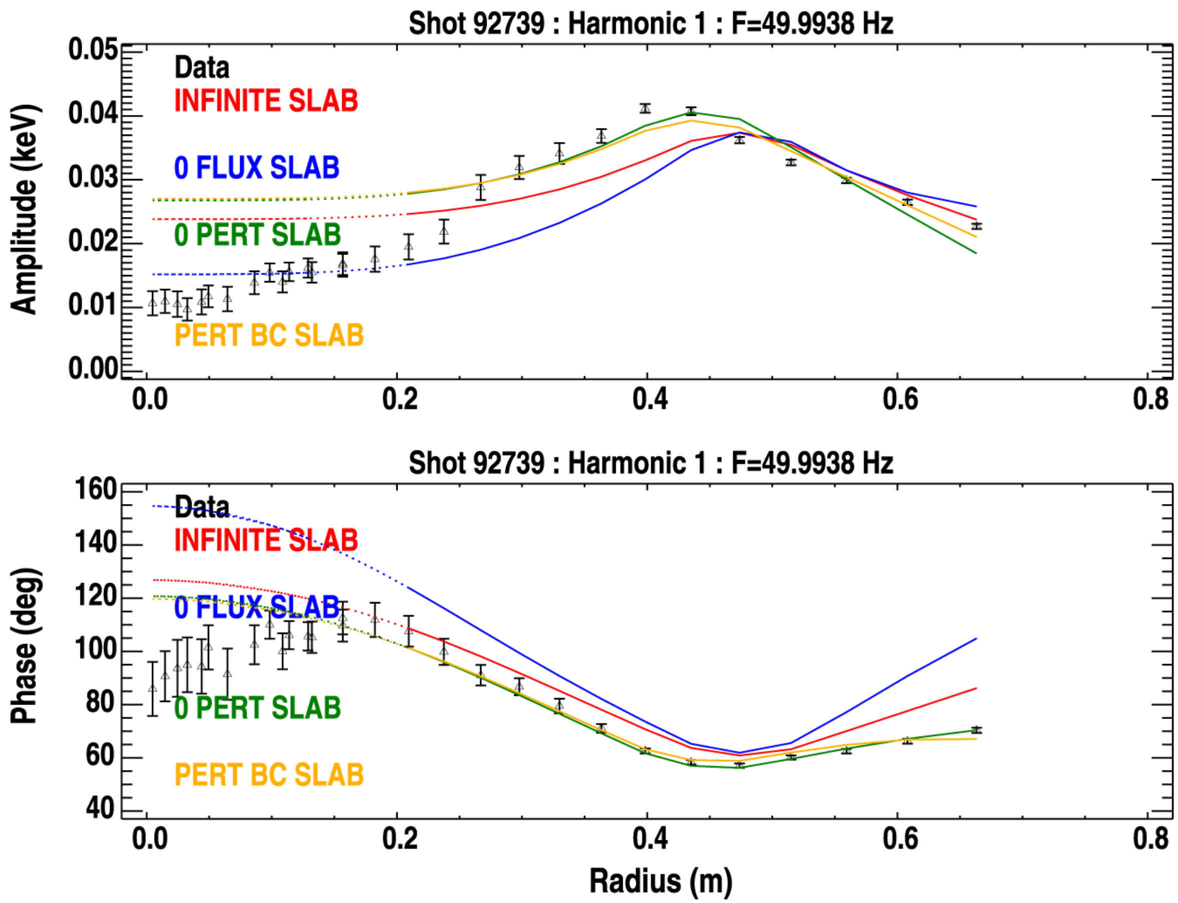
CHOICE OF BOUNDARY CONDITIONS AFFECTS EDGE FITS



- D = 18–32 m^2/s, V = 0, $\tau = \infty$, Geometry = slab
- No perturbation boundary condition works well, fixed edge perturbation ($\tilde{T}$ = 0.018 keV) works ever better

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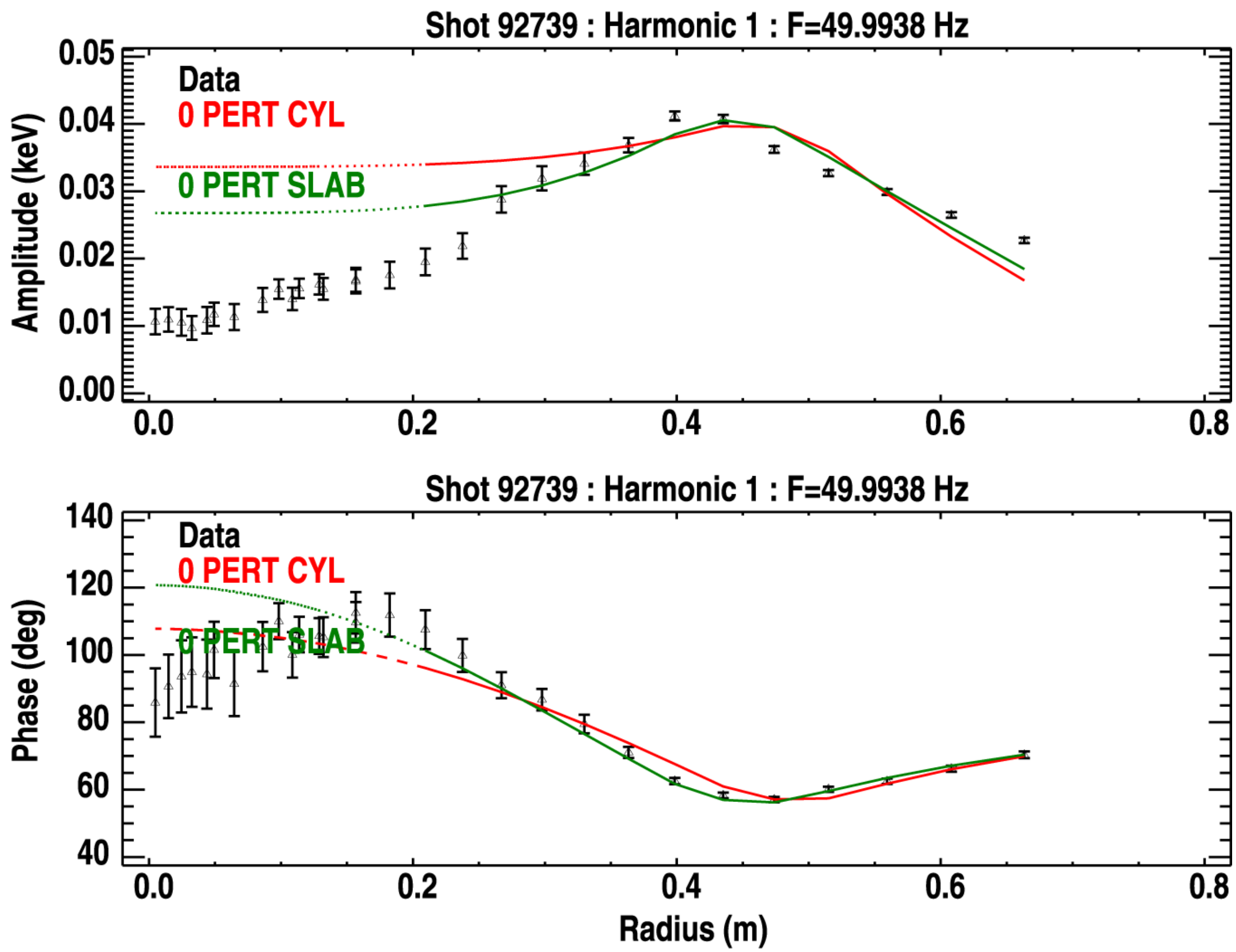
CHOICE OF GEOMETRY AFFECTS FITS NEAR AXIS



- D = 25-30 m^2/s, V = 0, $\tau = \infty$, B.C. = no perturbation

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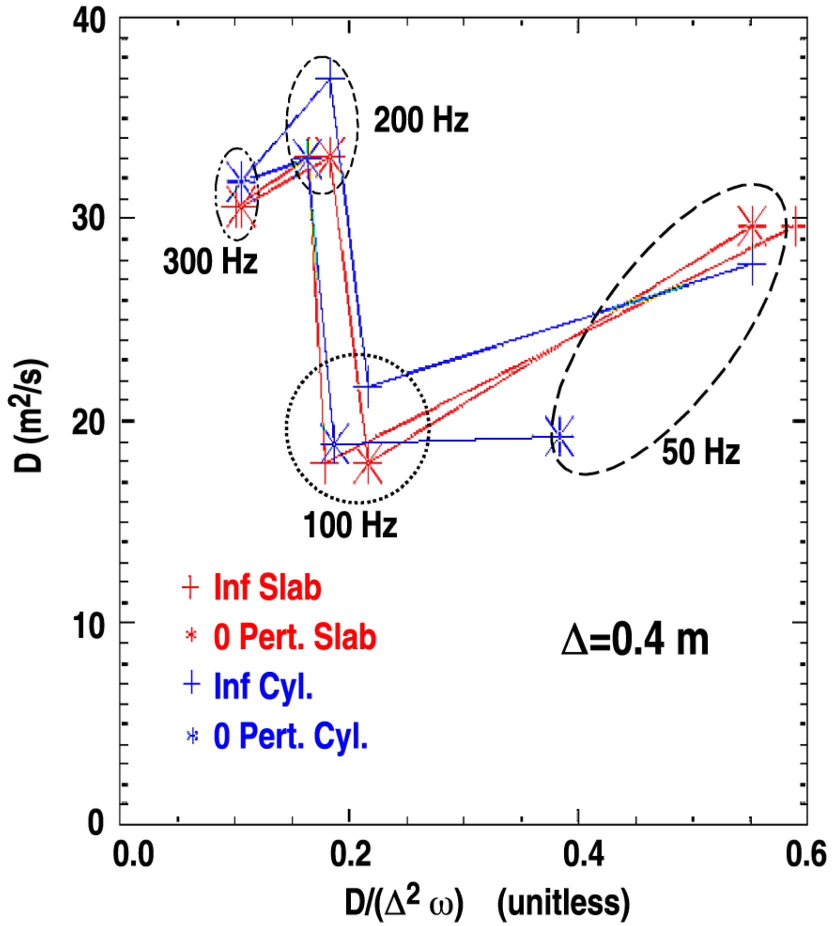
FITTED DIFFUSION COEFFICIENT IS INSENSITIVE TO MODEL FOR HIGH FREQUENCIES

- The range over which the boundary affects the determination of D should decrease with modulation frequency.
- We say that the boundary strongly affects a region Δ when $D/(\Delta^2\omega)>1$.
- Spread in D based on different model fits decreases with modulation frequency (Δ fixed).



FUTURE WORK

- Extend analytic solutions:
  - Different forms of D,V, τ
  - Real Geometry
  - Different Boundary Conditions
- Add the ability to load multiple shots and fit multiple harmonics simultaneously.
- Incorporate Justin Nelson's numerical simulation code into fitter.
- Try to gain better physical understanding of fit parameters.

CONCLUSIONS

- **Amplitudes and phases of heat pulses from modulated ECH can be qualitatively described by simple analytic models**
  - Analytic solutions to energy balance equation derived for various assumptions about geometry and boundary conditions
  - Fitter constructed to determine transport parameters from data

- **A few preliminary results have been obtained:**
  - Some data has a phase offset due to digitization error – easily corrected.
  - Adding a convective term does not significantly improve the fits, however, FFT error bars are large, and fits for linear convection in cylindrical coordinates were not obtainable.