Current Holes

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Presented at the American Physical Society
Division of Plasma Physics Meeting
Orlando, Flordia

November 11-15 , 2002

GENERAL ATOMICS

Introduction

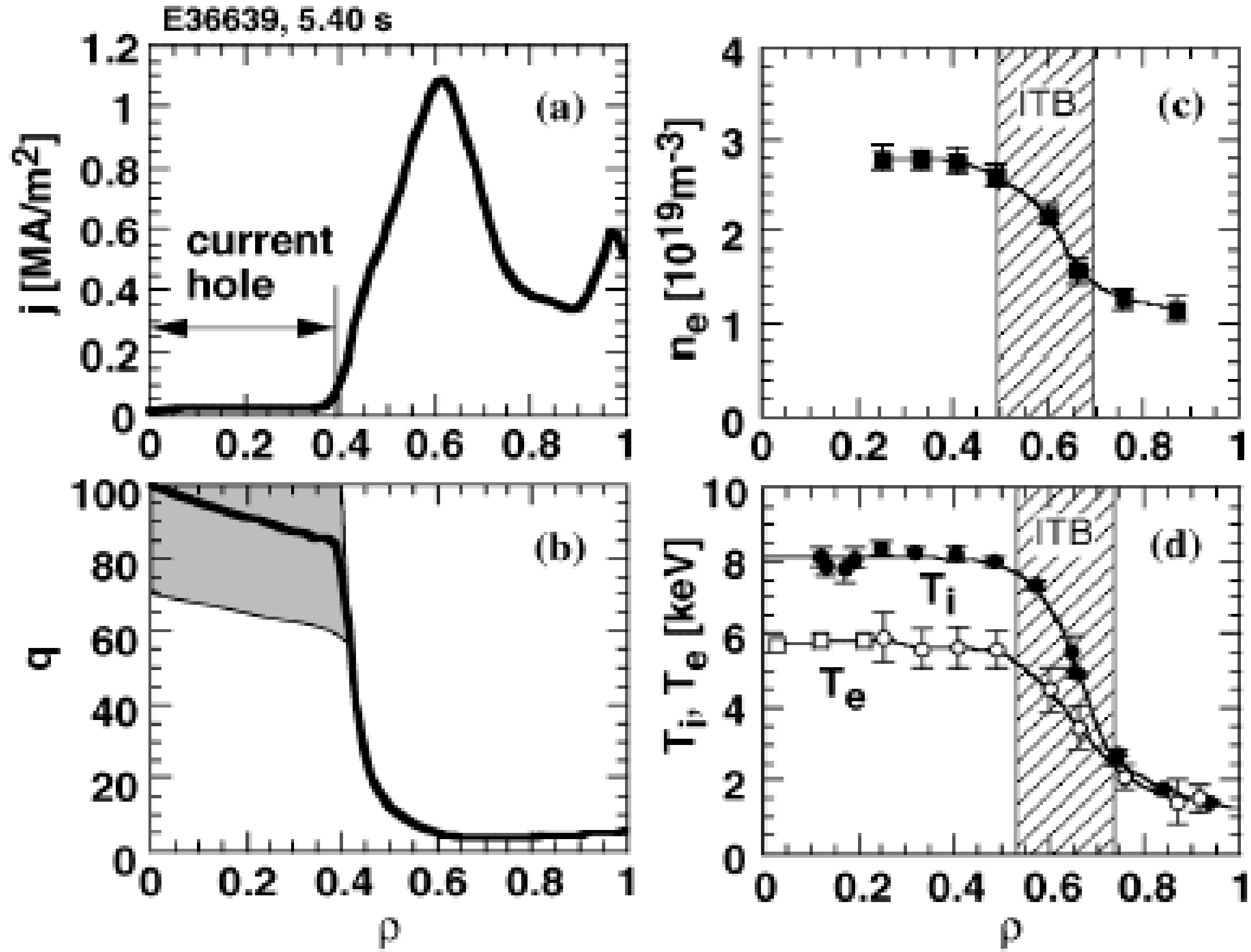
- DIII-D, JET, and JT-60U found a “current hole” in the central core.

— Formation: Off axis non-inductive current (bootstrap, rf) generates a negative electric field which diffuses inwards causing transient negative on-axis current. This excites the n = 0, m = 1 resistive kink mode which “flattens” the central current to zero. (Huysmans, PRL, 87 (2001)

— Properties after formation.

- ❑ Robust discharge. Large negative magnetic shear region outside hole stabilizes ballooning modes and ITG modes).
- ❑ Good confinement. Clearly identifiable ITBs structure observed in the outer region.
- ❑ Slow hole decay, ~ 5 s in JT-60U, yet longer than the current resistive time scale.

- Current holes may improve tokamak performance by (1) Eliminating the need for a loop voltage and non-inductive current drive (2) Enlarging the region of negative shear.



JT-60U Current Hole

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T. Fujita, T. Oikawa, et al, Phys Rev Lett 87 (2001) 245001

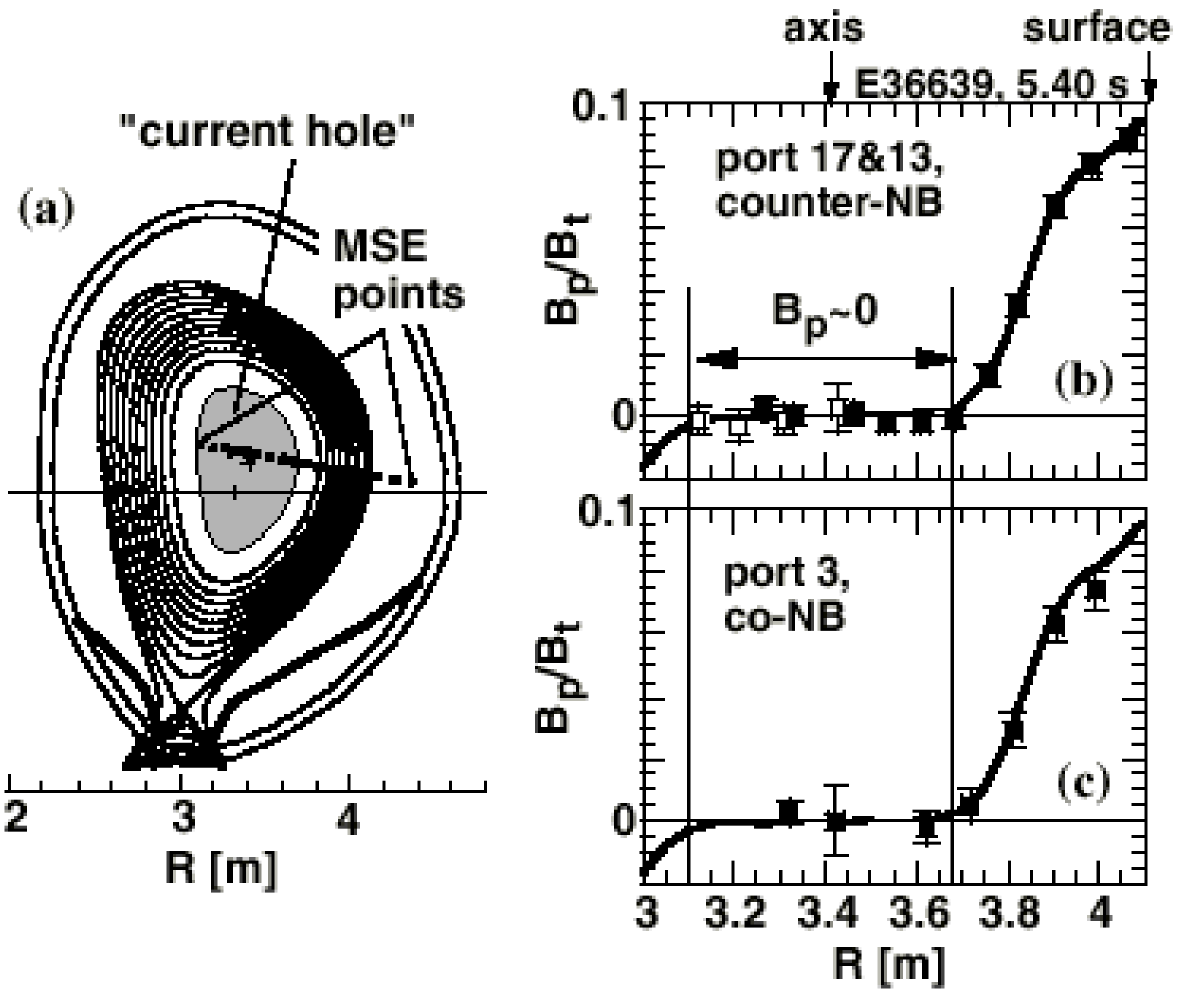


FIG. 1. (a) Poloidal cross section of plasma and MSE points. The region of the "current hole" is shown by the shaded area. (b), (c) B_p/B_t as a function of major radius. Rectangles denote measured data, and solid lines denote calculated values from equilibrium reconstruction. (b) Channels viewing a counter-NB.

What Sustains Current Hole?

- EXPLANATION: Torkil Jensen (GA-A 23898) proposed the idea that the current hole might be sustained by particle flow across flux surfaces, i.e., outward convection of magnetic flux counters inward diffusion, and prevents annihilation of flux at 0-point.

— Once a finite-sized hole is created it then become possible to “fit” a distributed particle source inside the hole (ionized beam particles or pellets). The effect of the outflow is analogous to the solar wind’s effect on the Earth’s magnetic field.

—Needs sufficiently large particle fueling rate inside the hole, and the stronger the source the larger the hole.

How to Solve for Current Hole Equilibrium

- **Magnetic fields and Currents**

$$\vec{B}_p = \nabla\Psi \times \hat{\phi}/R \qquad B_\phi = F(\Psi)/R \qquad \vec{J}_p = \frac{1}{\mu_0}\nabla F \times \hat{\phi}/R \qquad J_\phi\hat{\phi} = \frac{1}{\mu_0}\nabla \times \vec{B}_p$$

- **Grad-Shafranov equation in cylindrical coordinates (R, ϕ, Z) with axi-symmetry**

$$\Delta^*\Psi \equiv R^2\nabla \cdot \left(\frac{\nabla\Psi}{R^2}\right) = -\mu_0 R J_\phi = -\mu_0 R^2 p' - FF'$$

- <u>Main Idea</u>. Find an equilibrium solution with a current hole that is also consistent with a steady-state Ohms law. This entails relating surface functions p′ and FF′ to the supplied particle source S and the neoclassical Bootstrap current λ.

Current Closure Condition and Steady State

- Outside the current hole: div $(\vec{J}_{||} + \vec{J}_{\perp}) = 0$, where $\vec{J}_{\perp} = (B \times \nabla p)/B^2$

→ $\vec{B} \cdot \nabla \left(\frac{J_{||}}{B} - \frac{Fp'}{B^2} \right) = 0$, so $\frac{J_{||}}{B} - \frac{Fp'}{B^2} = C(\Psi)$. Taking flux surface average <...> resolves $C(\Psi)$.

- Current closure shows that the parallel current has two distinct parts

$$J_{||} = \underbrace{\frac{B\langle J_{||}B\rangle}{\langle B^2\rangle}}_{\text{force-free}} + \underbrace{\frac{Fp'}{B}\left[1 - \frac{B^2}{\langle B^2\rangle}\right]}_{\text{Pfirsch-Schluter}}$$

- With no loop voltage and auxiliary current drive, the only remaining force-free parallel current in steady-state is the neoclassical bootstrap current: In banana regime this is

$\langle J_{bs}B\rangle = g(\Psi)F(\Psi)p'$ Limits: $g \to 1$ if trapped fraction $= 1$, $g \to \sqrt{\varepsilon}$ if $\varepsilon << 1$.

Y.R. Lin Liu, et al Phys Plasmas (1999) proposed a general form for $g(\Psi)$.

- The nature of current closure imposes a constraint not connected to the || Ohms law

$p' = \frac{-F'\langle B^2\rangle}{\mu_0 F[1 - g(\Psi)]}$ For normal pressure profiles $F' < 0$, plasma is diamagnetic!

Ideas on Constructing a Phenomenological Ohms Law

- **The Key Dilemma** The toroidal resistivity in tokamaks is Spitzer-like with neoclassical corrections, yet particle transport is anomalously larger than neoclassical predictions.

Experimental Magnetic Reynolds number is much greater than neoclassical expectations

$$R_m^{exp} > 10 \; , \; R_m^{neo} \sim (a/R)^{1/2}\beta_p \sim 1 \quad Rm = \frac{\text{current resistive time}}{\text{particle confinement time}}$$

- **Possible Reconciliation**. Turbulent fluctuations are long-wavelength modes, $k_{||}$, k_ϕ << $k_\perp$. Thus the fluctuation-induced electron friction force is anomalously large in the $\perp$ direction, but remains classical in the $||$, ϕ direction

- **Approach** Poloidal current has two parts $\perp$ diamagnetic part and a $||$ parallel streaming part. Presumably turbulent fluctuations affect only frictional forces having to do with the $\perp$ part and leave the $||$ part unaffected. So we assign an anomalous resistivity η_A to the $\perp$ part and a Spitzer $||$ resistivity η to the parallel part.

Toroidal Current and Particle Continuity

- Toroidal current density from new Ohms's law

$$\eta R J_\phi = -\vec{v} \cdot \nabla\Psi + \eta F(\Psi)\langle J_{bs} B\rangle / \langle B^2 \rangle$$

Ironically, the toroidal friction is classical yet opposes an anomalous cross-field flow.

- Flux surface average the continuity equation

$$\rho V' \langle \vec{v} \cdot \nabla\Psi \rangle = -S(\Psi) \quad \text{differential volume } V' = 2\pi \oint \frac{R d\ell}{|\nabla\Psi|}$$

$S(\Psi)$ = rate of particles supplied inside the volume enclosed by flux surface Ψ

Then $S(\Psi_{max})$ = particle source inside the current hole.

Since the theory and computation of neutral beam/pellet deposition is well at hand, it would seem natural to specify the real source $S(\Psi)$ instead of the commonly used surface functions. This leads to more exotic equilibrium solutions, e.g., the current hole.

Relate Particle Source to Pressure gradient

- Flux-surface average toroidal current from Ohm's law, then combine with continuity

$$\rightarrow \quad S(\Psi) = 2\pi n\eta \oint \frac{R^2 J_\phi d\ell}{|\nabla\Psi|} - n\eta V' F(\Psi)\langle J_{bs} B\rangle / \langle B^2\rangle$$

- Next step is to replace J_ϕ in the above equation with the right-hand side of the Grad-Shafranov equilibrium equation. Then use the explicit form of the bootstrap current. We then obtain an equation connecting p', and FF' to S

$$p'\left[\langle R^2\rangle - g(\Psi)F^2/\langle B^2\rangle\right] + FF'/\mu_0 = \frac{S(\Psi)}{n(\Psi)\eta(\Psi)V'(\Psi)}$$

- Finally we use current closure to eliminate F' in the above equation to get

$$p'(\Psi) = \frac{S(\Psi)\langle B^2\rangle}{n\eta V'[\langle R^2\rangle\langle B^2\rangle - F^2]}$$

By specifying the particle source function $S(\Psi)$, the pressure gradient adopts an "organic" quality: p' automatically goes to zero near a developing current hole where $V' \rightarrow \infty$.

New Form of GS Equation in terms of Particle Source

$$\Delta^* \Psi = \frac{-\mu_0 S(\Psi) R^2}{n\eta V' \langle R^2 \rangle} H(\Psi) \quad \text{where } H(\Psi) = \frac{\langle B_p^2 \rangle \langle R^2 \rangle / F^2 + \langle R^{-2} \rangle \langle R^2 \rangle - (1-g) \langle R^2 \rangle / R^2}{\langle B_p^2 \rangle \langle R^2 \rangle / F^2 + \langle R^{-2} \rangle \langle R^2 \rangle - 1}$$

This form corrects T. Jensen's expression (GA-A23898) in which $H(\Psi) = 1$ as a result of artificially forcing $F' = 0$. Note: the $H(\Psi)$ function is non-singular and well-behaved, since the denominator is positive definite by a Schwartz inequality: $\langle R^{-2} \rangle \langle R^2 \rangle - 1 > 0$.

- Nondimensional form: $r = R/R_0$, $z = Z/R_0$, $\psi = \Psi/\Psi_{max}$, $\tilde{V}' = \oint \frac{r ds}{|\nabla \psi|}$, $ds = d\ell / R_0$,

  Normalized source profile: $G(\psi) = S(\psi)/S(1)$, and $R_0 \equiv$ inboard major radius

$$\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} + \frac{r^2 G(\psi) H(\psi) Rm}{\langle r^2 \rangle \tilde{V}'} = 0, \quad \text{BCs: } \psi = 0 \text{ on bounding surface,}$$

$\psi = 1$ is maximum internal value

Critical parameter is hole Magnetic Reynolds Number $Rm \equiv \frac{\mu_0 S(1)}{2\pi n(1) \eta(1) R_0} \sim \mu_0 v_r r_{hole} / \eta$.

More on H(ψ) function

- Additional definitions and normalizations used in H(ψ):

$B_{\phi 0} \equiv$ Vacuum toroidal magnetic field at inboard radius R_0 , $F_0 \equiv R_0 B_{\phi 0}$, $f(\psi) \equiv F(\psi)/F_0$.

To determine Ψ_{max}, choose a nominal poloidal magnetid field B_{p0}, and inverse aspect ratio ε_0 . Then $\Psi_{max} \equiv R_0^2 B_{p0}\varepsilon_0$. Everything is known in H except f(ψ), which we determine later.

$$H(\psi) = \frac{\dfrac{B_{p0}^2}{B_{\phi 0}^2}\dfrac{\varepsilon_0^2}{\tilde{V}'}\dfrac{\langle r^2\rangle}{f^2}\oint |\nabla\psi|\, ds/r + \langle r^2\rangle\langle r^{-2}\rangle - (1-g)\langle r^2\rangle / r^2}{\dfrac{B_{p0}^2}{B_{\phi 0}^2}\dfrac{\varepsilon_0^2}{\tilde{V}'}\dfrac{\langle r^2\rangle}{f^2}\oint |\nabla\psi|\, ds/r + \langle r^2\rangle\langle r^{-2}\rangle - 1}$$

- Estimation of magnitude: $H(\psi) \sim \dfrac{\varepsilon_0^2 + g(\psi)}{\varepsilon_0^2 + 2\varepsilon_0^2 + 21\varepsilon_0^4/8\ldots} \sim \dfrac{1}{3\varepsilon_0^{3/2}}$

- In JT-60U H(ψ) ~ 3. T. Jensen (GA-A23898) computed equilibria with H(ψ) $\equiv$ 1. Interestingly, in limit of zero bootstrap current, and infinite aspect ratio, H(ψ) $\rightarrow$ 1.

Numerical Scheme

- Choose Rm, and specify functional form of source function $G(\psi)$ the Bootstrap coefficient $g(\psi)$. Choose edge q_0, and ε_0. Then $B_{p0}^2 / B_{\phi 0}^2 = \varepsilon_0^2 / q_0^2$.
- Choose the vacuum field $f(\psi) = 1$ as a reasonable initial guess for $f(\psi)$.
- Put 2-D GS equation into finite difference form, and iterate solution using T. Jensen's Jacobi algorithm. After each iteration, renormalize ψ such that its maximum is $\psi = 1$.
- Use this 2-D solution to solve a "cheap" 1-D flux-surface-averaged GS equation to obtain the next iteration on $f(\psi)$. Note: $\tilde{V}'$ is of order ε_0^2 in the following 1-D form,

$$\frac{df}{d\psi} = \frac{f(1-g)\dfrac{d}{d\psi}\oint |\nabla\psi|\, ds/r}{\oint |\nabla\psi|\, ds/r + gf^2\langle r^{-2}\rangle \dfrac{\tilde{V}'}{\varepsilon_0^2}\dfrac{q_0^2}{\varepsilon_0^2}} \sim \frac{(1-g)}{gf\langle r^{-2}\rangle}\frac{\varepsilon_0^2}{q_0^2}\frac{\varepsilon_0^2}{\tilde{V}'}\frac{d}{d\psi}\oint |\nabla\psi|\, ds/r, \quad \text{BC:} \quad f = 1 \text{ at } \psi = 0$$

Here we used the relation $\langle \nabla \cdot \vec{A} \rangle = \dfrac{1}{V'}\dfrac{d}{d\Psi}\left(V'\langle \vec{A} \cdot \nabla\Psi\rangle\right)$. Physically $\oint |\nabla\psi|\, ds/r \sim I_\phi(\psi)$.

- Alternate back and forth between 2-D and 1-D equations until convergence obtains.

Special Case: T. Jensen's "Super Hole" Solution

- Suppose bootstrap current is zero ($g = 0$). Then solution of the 1-D equation for f gives $f(\psi) = \text{const}\ I_\phi(\psi)$. So without a bootstrap current the toroidal magnetic field is also zero in the current hole — a "Super Hole". Next, we show how this case connects to T. Jensen's "pressure driven tokamak" (Phys. Plasmas, 1996).

- In large aspect ratio circular cross section limit $I_\phi \propto \rho B_\theta \propto \rho d\psi / d\rho$ (ρ = minor radius).

  → $q = \text{const}\ \rho^2$, and the magnetic shear = 2, regardless of the $f(\psi)$ profile.

Choosing Jensen's power-law form: $f(\psi) = (1 - \psi)^\nu$, we find a trivial equation for ψ

→ $\rho d\psi / d\rho = \text{const}\ (1 - \psi)^\nu$

BCs: $\psi = 1$ at hole radius $\rho = \rho_h$, and $\psi = 0$ at plasma boundary $\rho = a$.

→ $\psi(\rho) = 1 - \left[\dfrac{\ln(\rho/\rho_h)}{\ln(a/\rho_h)}\right]^{1/(1-\nu)}$ confirms Jensen's solution using a different approach!

Note: Index ν must lie in the interval $0.5 < \nu < 1$, to avoid singular currents at hole radius.

Calculations for JT60U

- Plasma parameters in current hole region (Fujita, Oikawa, et al PRL, 2001)

  $T_e = 5500$ eV $\quad n = 2.9 \times 10^{19}$ m^{-3} $\quad Z_{eff} = 3 \quad R_0 = 2.5$ m

  $$\sigma_\perp = \frac{9700\ T_e^{3/2}}{Z_{eff} \ln \Lambda} = 8.3 \times 10^7 \text{ A/V-m} \quad , \quad \sigma_{||} = 1.7 \times 10^8 \text{ A/V-m}$$

- Beam ionization rate in current hole (Fujita, private communication to Jensen 2002)

  $S(1)_{ion} = 2.5 \times 10^{20}$ ions/s

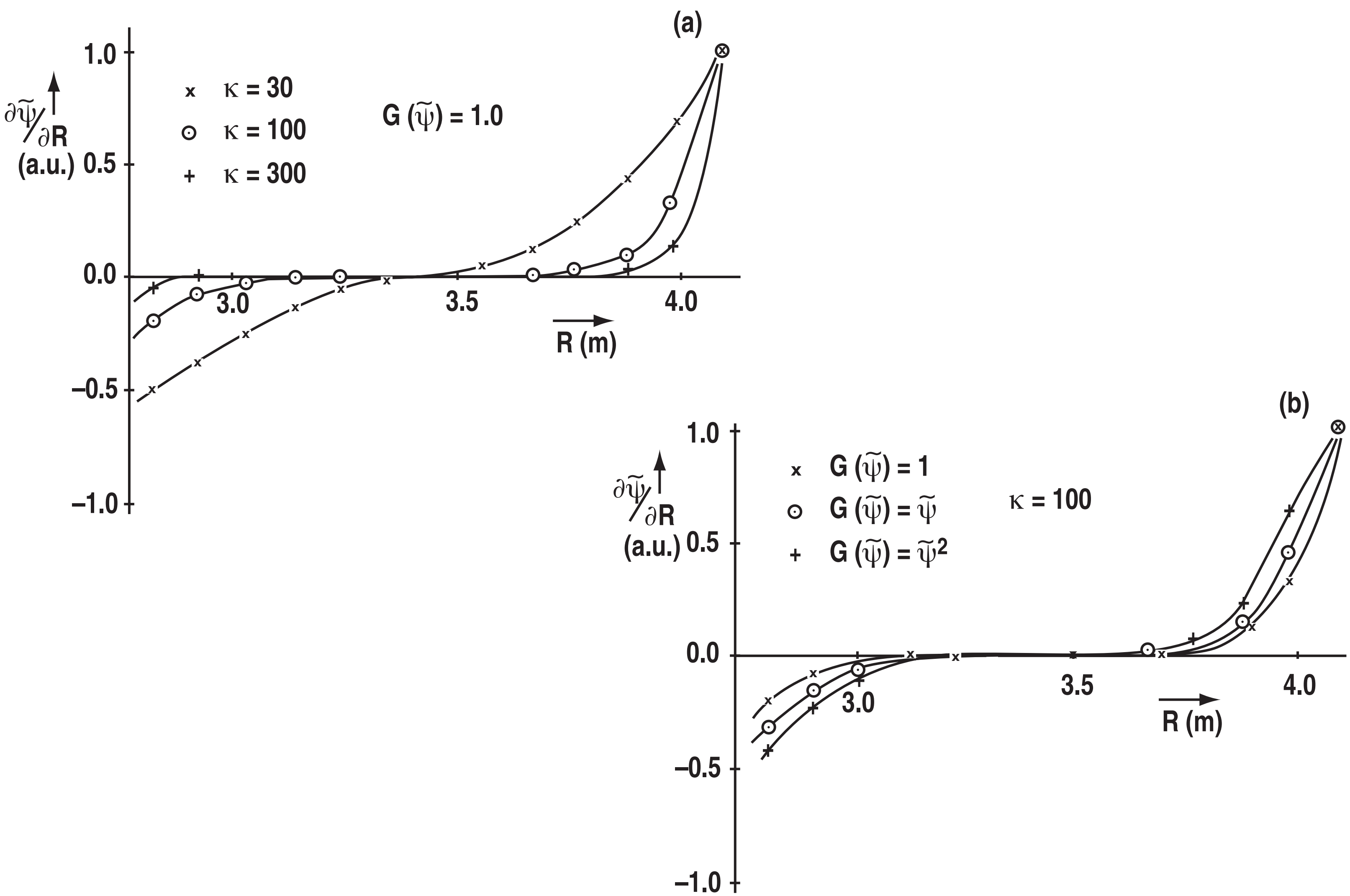
- Magnetic Reynolds number in hole assuming $S(1) = S(1)_{ion}$. This is an upper limit!

$$Rm = \frac{\mu_0 S(1) \sigma_\perp}{2\pi n R_0} = 54 \quad Rm^{eff} \sim Rm \times H = 54 \times 3 = 160.$$

→ Caveat: Because large orbit beam particles may thermalize outside the hole the fueling source could be considerably less than the ionization source.

Crude estimate in JT-60: $S(1) \sim 0.35\ S(1)_{ion}$, so possibly $Rm \sim 19$ and $Rm^{eff} \sim 57$.

CURRENT HOLE SIZE VERSUS Rm



From T.H. Jensen, GA-A23898

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GENERAL ATOMICS

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Conclusion

- The basic idea is that a finite sized hole, once formed by other processes, allows one to "fit" a distributed mass source inside the hole volume. Outward flow prevents poloidal flux from collapsing towards the magnetic axis and sustains the hole with no or little loop voltage required.

- We developed and enlarged the model of T. Jensen (GA-A23898). The improved model is more internally consistent since it include the proper current closure constraint which was missing in the Jensen model i.e., ($F' \neq 0$). We included the bootstrap current effect necessary for a steady-state current hole equilibrium without auxiliary current drives.

- We combined the equilibrium and transport equations (Ohms law and continuity) to obtain a new form of the Grad-Shfranov equation specifying only the particle source distribution inside a flux surface $S(\Psi)$, (plus a temperature profile). It has 2 main input parameters, Rm, and q_0 at edge. Like in the Jensen model we expect to find that

— Hole size increase with magnetic Reynolds number Rm.

— Hole sizes may be in better agreement with the JT-60U experiments using this model.