ABSTRACT

Heat transport by electrons in tokamak plasmas is currently a subject of intense study. Transport analysis using power balance analysis does not allow one to separate heat convection and conduction, but this can be accomplished using the dynamic response of the electon temperature from periodic heat pulses. The plasma response is analyzed using the linearized Braginskii equation. On the DIII–D tokamak, localized electron cyclotron heating (ECH) is used to generate a train of heat pulses which is monitored using electron cyclotron emission. Two different methods have been implemented for the analysis of periodic temperatue perturbations: the fast Fourier transform (FFT) and a Fourier series using the modulation frequency of the ECH and its harmonics. The Fourier series technique holds promise for smaller uncertainty estimates. The expected dependance of the temperature perturbations in space and time has also been studied using numerical simulations of the heat pulses in realistic toroidal geometry. The results of the simulations will be compared to experimental measurements to determine the salient features of the model.

INTRODUCTION

- The contributions of conduction and convection to energy transport can be determined using the dynamic response of the electron temperature ($\tilde{T}$) from periodic heat pulses

- We desired to take a fresh look at heat pulse propagation in tokamak plasmas:
  - How is the electron temperature response best analyzed?
  - How do assumptions built into the transport model (e.g., geometry, boundary conditions) affect the predicted heat pulse propagation?
  - How much complexity is required in the transport model (e.g., convection, damping) to fit the measurements in a statistically meaningful way?

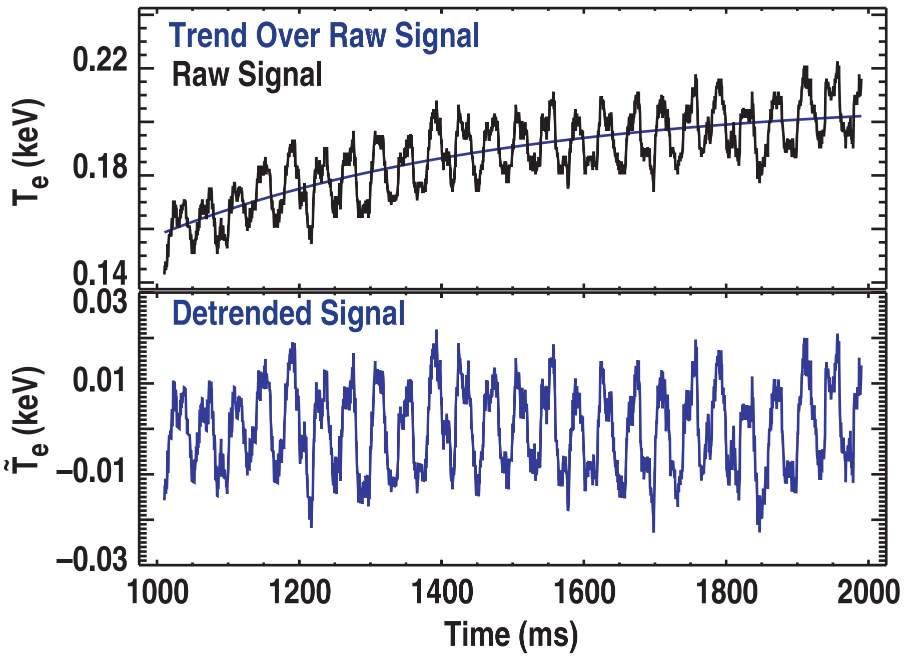
- For these experiments, electron cyclotron heating (ECH) provides a localized heat source for the electrons, while electron cyclotron emission (ECE) measures the electron temperature response at fixed locations

OUTLINE/SUMMARY OF WORK PERFORMED

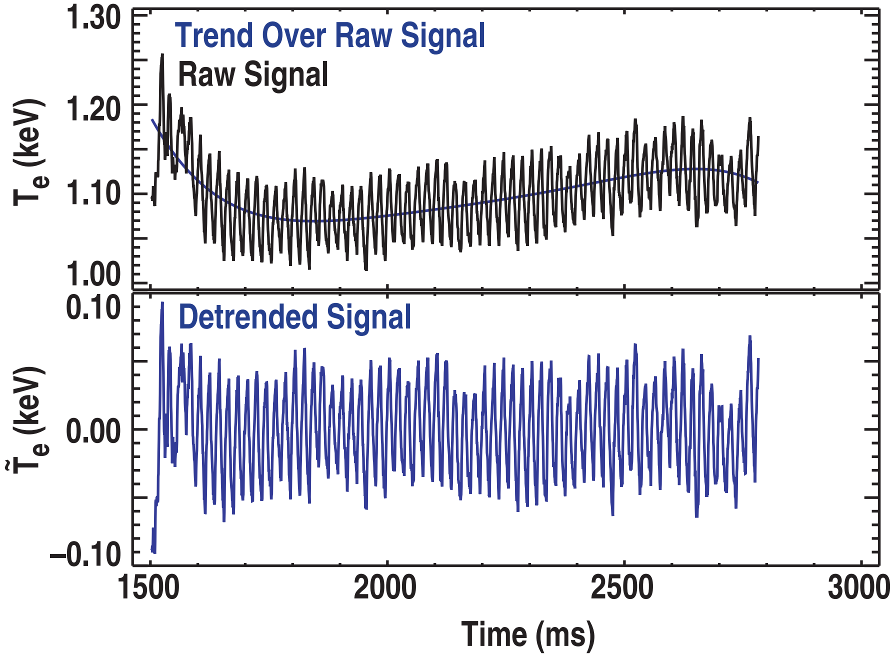
- Explored different methods for detrending the slow time dependence of the data
- Compared two methods for transforming heat pulse propagation data from time domain to frequency domain: Fast Fourier Transforms (FFT) and Fourier Series Fitting
- Built simulator that numerically solves complex coupled differential equations derived from linearized Braginskii equation
- Explored the effect of plasma geometry and boundary conditions on the heat pulse propagation using simulator
- Investigated the dependence of heat pulse propagation on transport model, i.e., conduction, convection, damping, etc.

DIFFERENT DETREND METHODS CAN BE USED TO REMOVE SLOW TIME DEPENDENCE FROM SIGNALS

- Exponential $\propto (1-e^{-t/\tau})$



- Polynomial (degree 6)



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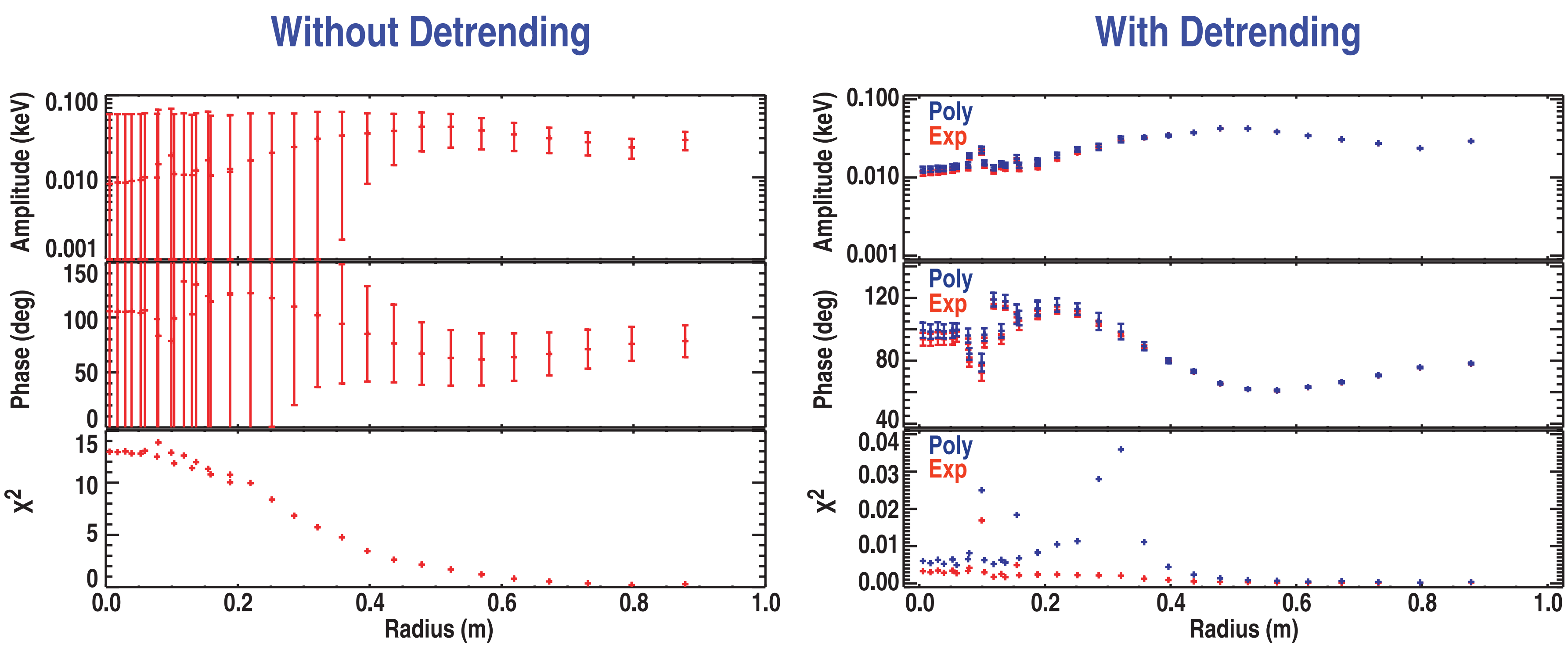
ERROR BARS ARE GREATLY REDUCED BY DETRENDING (CAN BE REDUCED FURTHER BY SELECTING APPROPRIATE METHOD)



- Fourier series method used

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FAST FOURIER TRANSFORMS (FFTs) ARE COMMONLY USED TO CONVERT FROM TIME DOMAIN TO FREQUENCY DOMAIN

- We wish to determine the amplitudes and relative phases of the measured heat pulses as they propagate radially through the plasma

- The discrete Fourier transform of an N-element function f(t) is defined as

$$F(u) = \frac{1}{N} \sum_{t=0}^{N-1} f(t) \exp\left[-j2\pi ut/N\right]$$

- Fourier components are computed up to the Nyquist frequency ($1/2\Delta t$)

- Phase of the electron temperature perturbation ($\tilde{T}$) is measured relative to the phase of the heat source

AS AN ALTERNATIVE TO FFTs, DATA CAN BE FIT BY FOURIER SERIES AT HARMONICS OF THE SOURCE FREQUENCY

- S = signal $\hat{S}$ = fit function

- $$\hat{S} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \cos\omega t & \sin\omega t & \cos 2\omega t & \sin 2\omega t & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{pmatrix} = MC$$

  where nω < Nyquist frequency

- Fitting Fourier series to data gives:

  $$S = \hat{S} = MC \quad \Rightarrow \quad M^T S = M^T MC \quad \Rightarrow \quad \left(M^T M\right)^{-1} M^T S = C$$

- Fitting uncertainties are well defined:

  $$\sigma_S^2 = \frac{\left(S-\hat{S}\right)^T \left(S-\hat{S}\right)}{(no.\ of\ points)}$$

- The phases (ϕ) and amplitudes (A) are found by:

  $$\phi = -\mathrm{atan}\left(\frac{c_{i+1}}{c_i}\right) \qquad A = \sqrt{c_i^2 + c_{i+1}^2}$$

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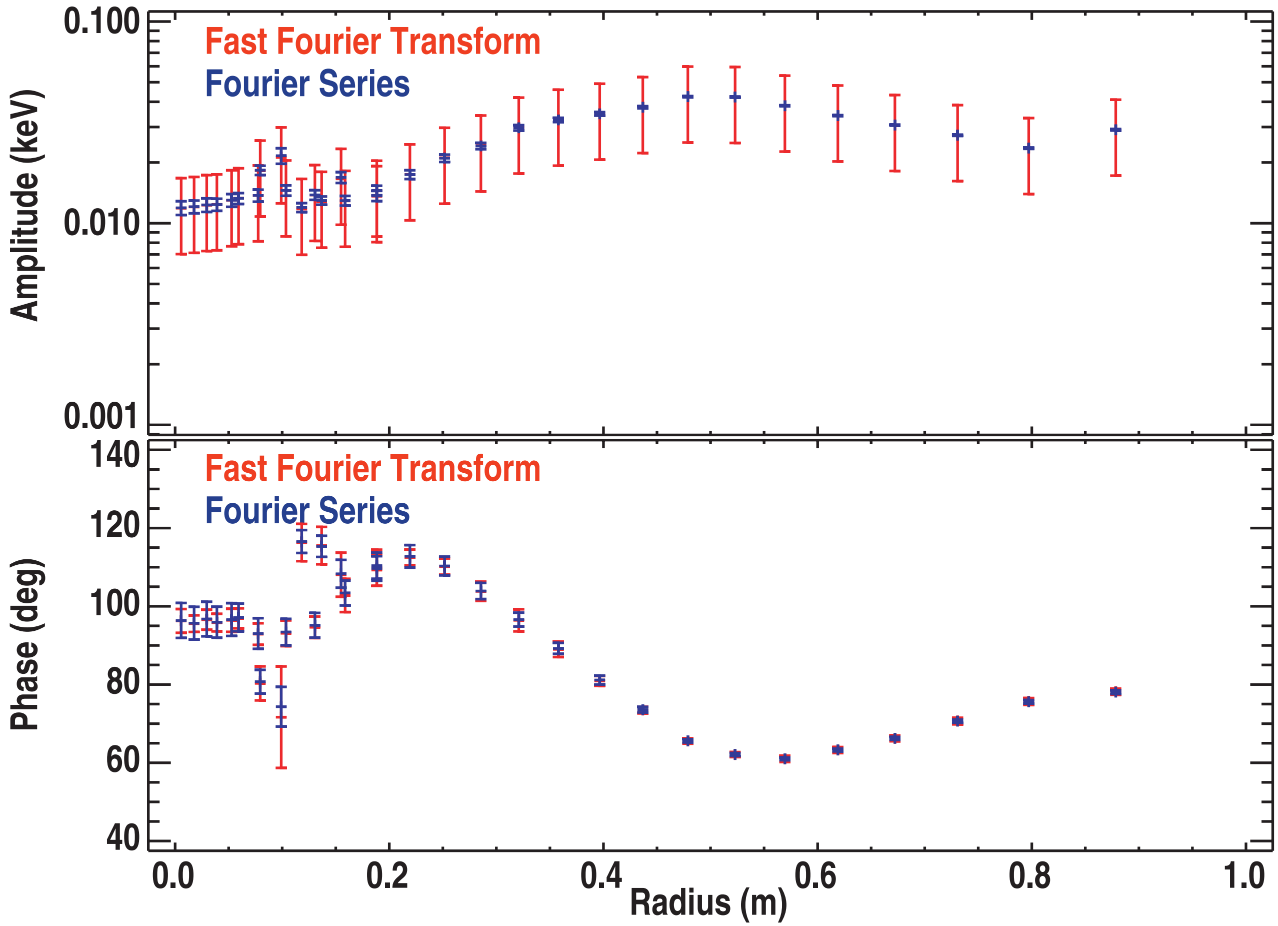
AMPLITUDES AND PHASES AGREE BETWEEN FOURIER SERIES AND FAST FOURIER TRANSFORM, BUT FOURIER SERIES HAS SMALLER UNCERTAINTIES IN THE AMPLITUDES



- However, Fouries series analysis is extremely sensitive to the base frequency (similar to the behavior of lock-in amplifiers)

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SIMULATOR WAS BUILT TO NUMERICALLY SOLVE LINEARIZED BRAGINSKII ENERGY EQUATION FOR HEAT PULSE PROPAGATION

- **Linearized Braginskii equation:**

$$-\chi \frac{d^2\tilde{T}}{dr^2} + \mathrm{U}\frac{d\tilde{T}}{dr} + \left(\frac{1+\frac{3}{2}\,\mathrm{i}\omega\tau}{\tau}\right)\tilde{T} = Sources$$

- **Slab:** $\chi = D$ conduction

$U = V$ convection

$$\frac{1}{\tau} = \frac{1}{\tau_o} + \frac{dV}{dX} \quad \text{damping}$$

- **Cylinder:** $\chi = D$

$$U = V - \frac{D}{r} - \frac{dD}{dr}$$

$$\frac{1}{\tau} = \frac{1}{\tau_o} + \frac{V}{r} + \frac{dV}{dr}$$

- **Torus:** $\chi = D$

$$U = V - \frac{D}{r} - D\frac{d(\ln H)}{dr} - \frac{dD}{dr}$$

$$\frac{1}{\tau} = \frac{1}{\tau_o} + \frac{V}{r} + V\frac{d(\ln H)}{dr} + \frac{dV}{dr}$$

$$where\ H = \frac{V'}{4\pi^2 R_o \rho}$$

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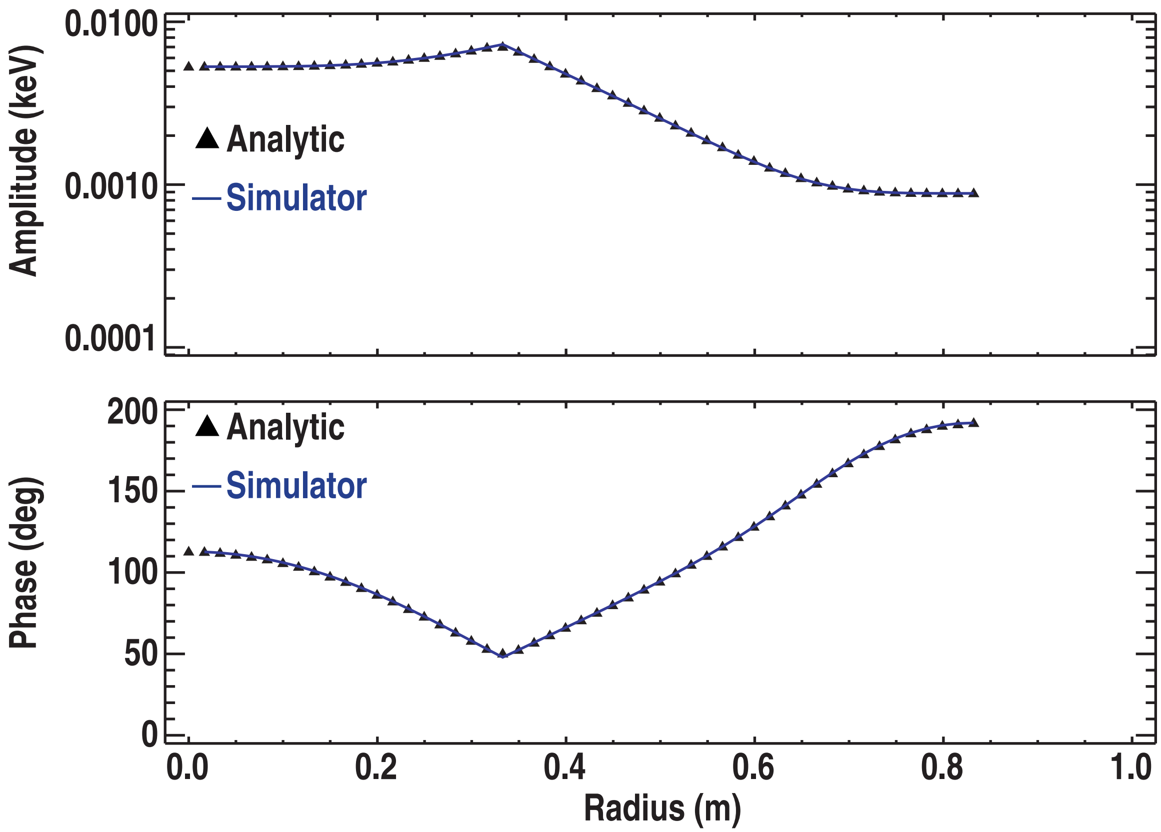
SIMULATOR WAS SUCCESSFULLY BENCHMARKED AGAINST ANALYTICAL SOLUTIONS (SEE ADJOINING POSTER BY PINO)



$D = 10\ m^2/s$

$V = 0$

$\tau = 0.01\ s$

$\omega/2\pi = 50\ Hz$

Geometry: cylindrical

Boundary condition:

$$\left.\frac{d\tilde{T}}{dx}\right|_{\rho=1} = 0 \text{ (no flux)}$$

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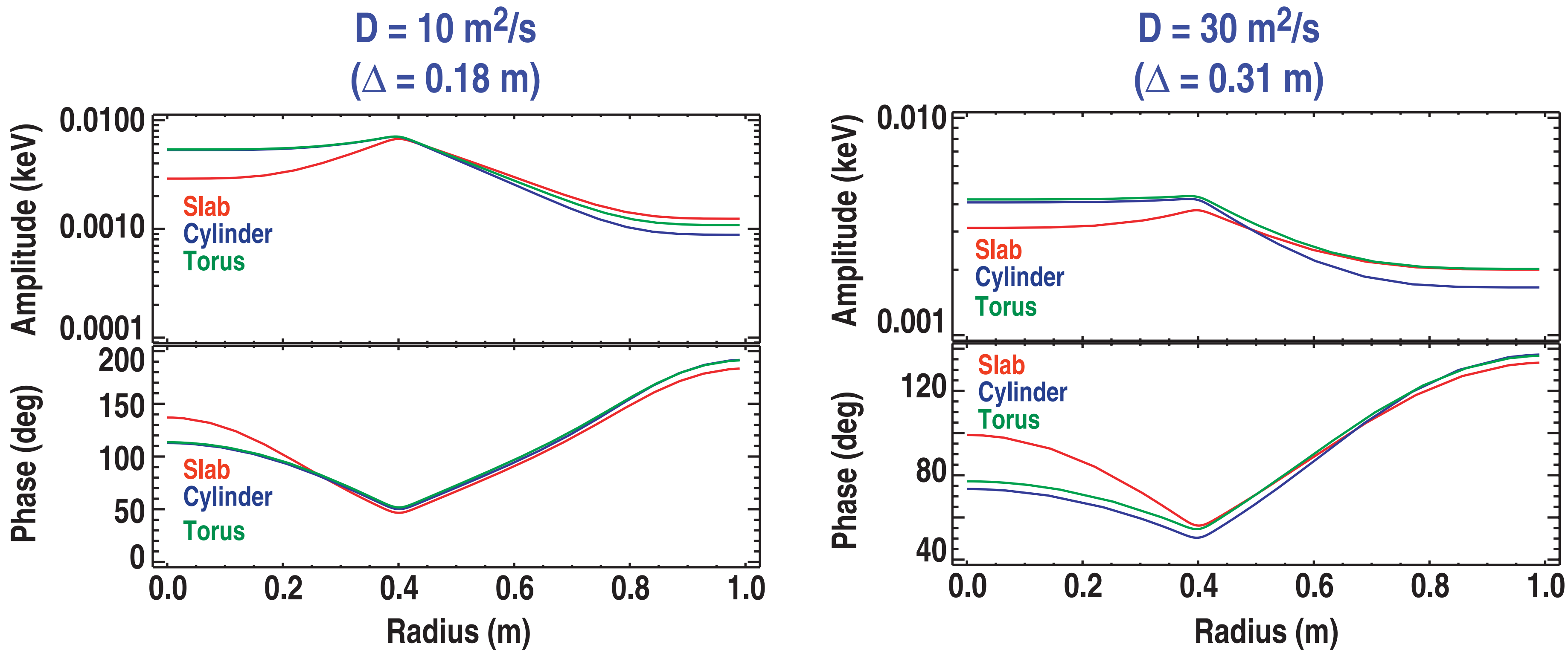
DIFFERENCES IN HEAT PULSE PROPAGATION BETWEEN CYLINDRICAL AND TOROIDAL GEOMETRY INCREASE AS TRANSPORT INCREASES



- $V = 0$, $\tau = 0.01$ s, $\omega/2\pi = 50$ Hz, boundary condition = no flux
- $\Delta = \sqrt{D/\omega}$ is characteristic scale length

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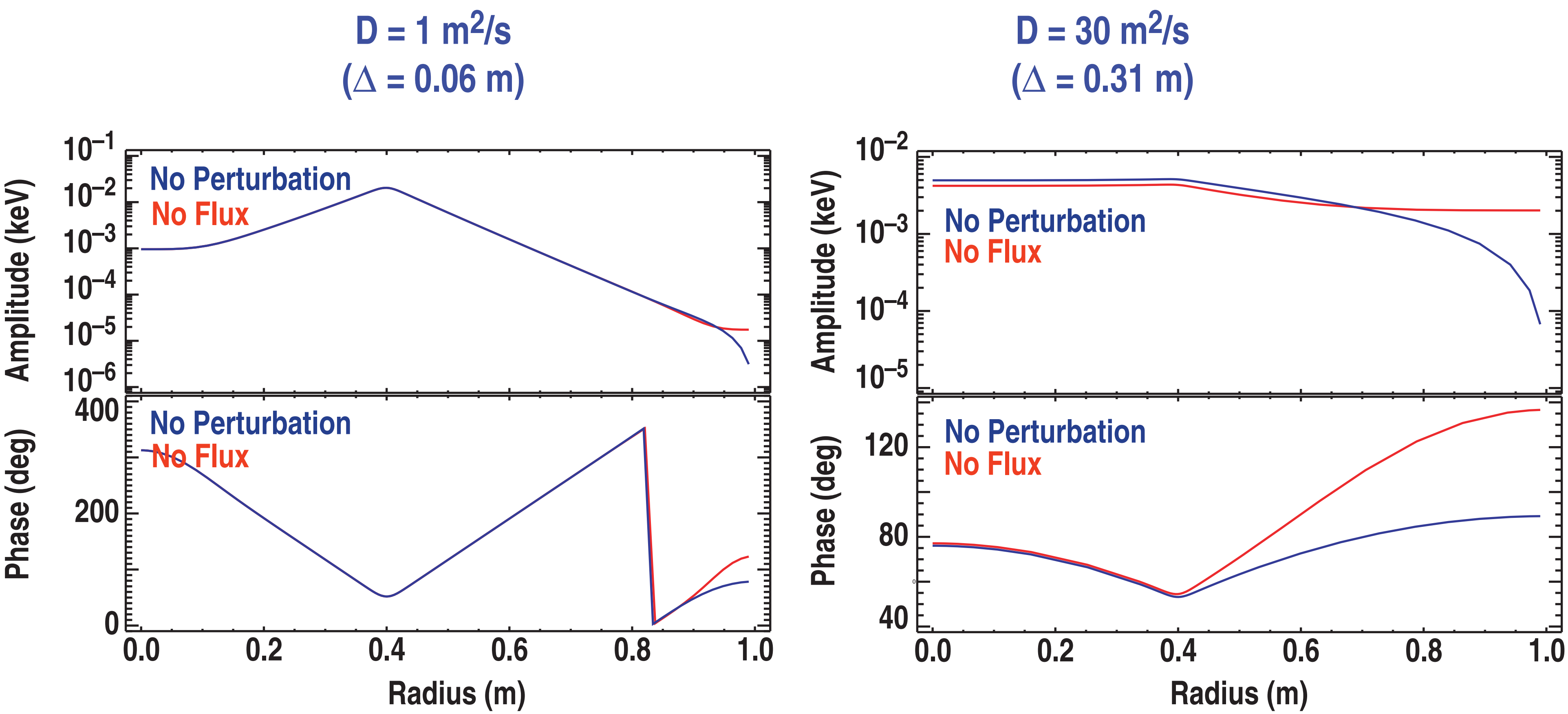
SENSITIVITY OF HEAT PULSE PROPAGATION TO BOUNDARY CONDITIONS INCREASES AS TRANSPORT INCREASES



- $V = 0$, $\tau = 0.01$ s, $\omega/2\pi = 50$ Hz, geometry = torus

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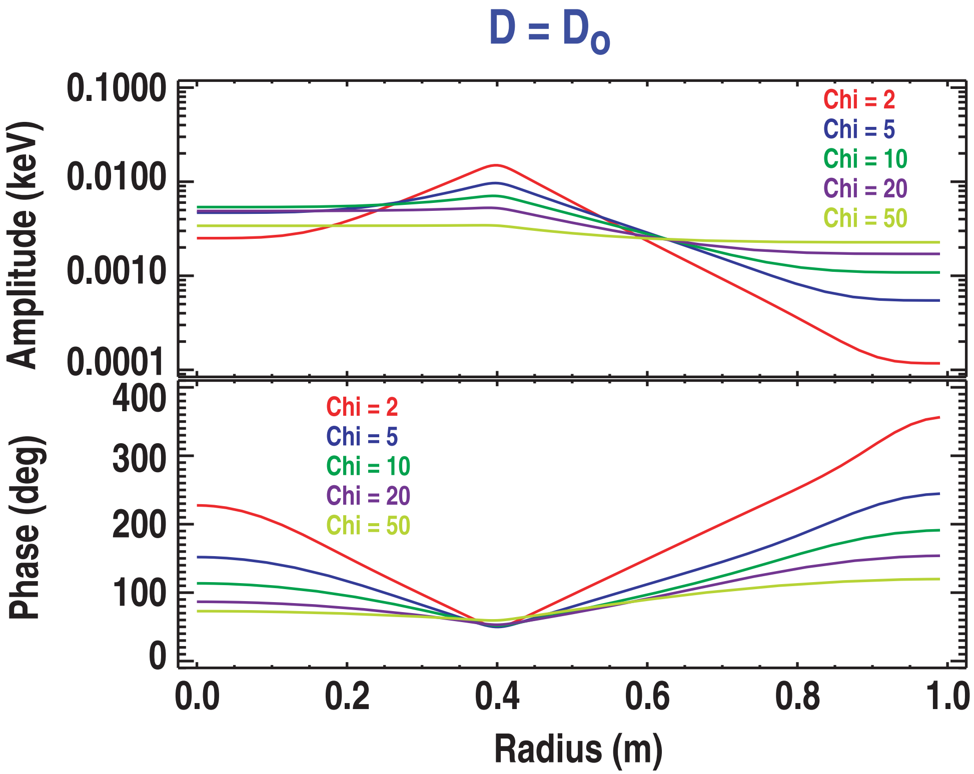
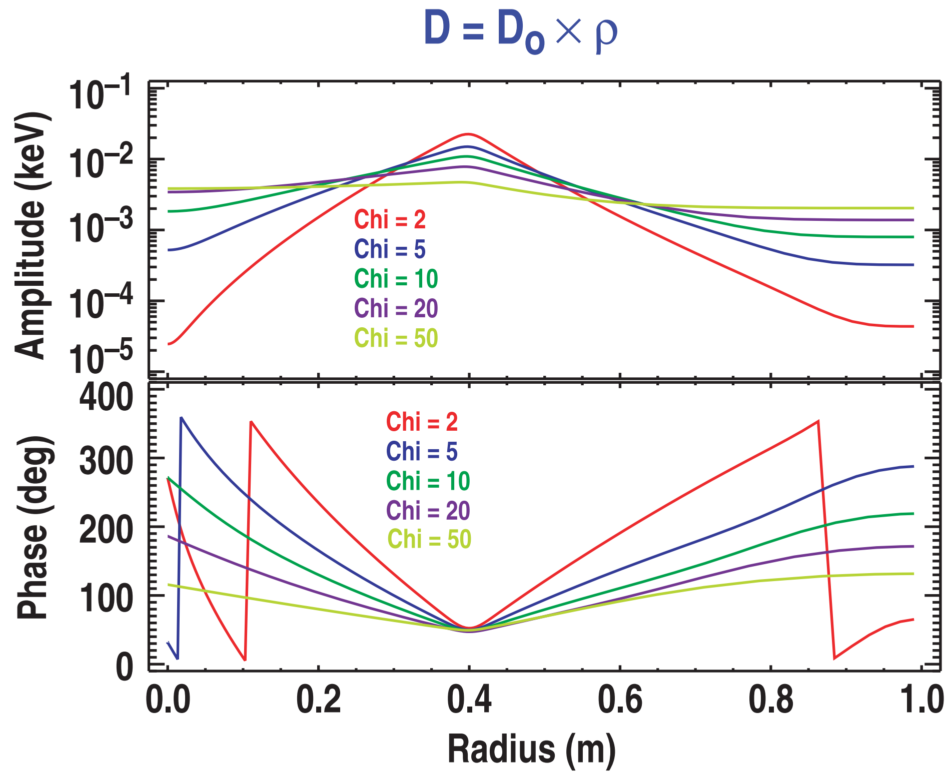
HIGH LEVELS OF TRANSPORT FLATTENS OUT THE AMPLITUDE AND PHASE DEPENDENCE WITH RADIUS





- $V = 0$, $\tau = 0.01$ s, $\omega/2\pi = 50$ Hz, geometry = torus, B.C. = no flux

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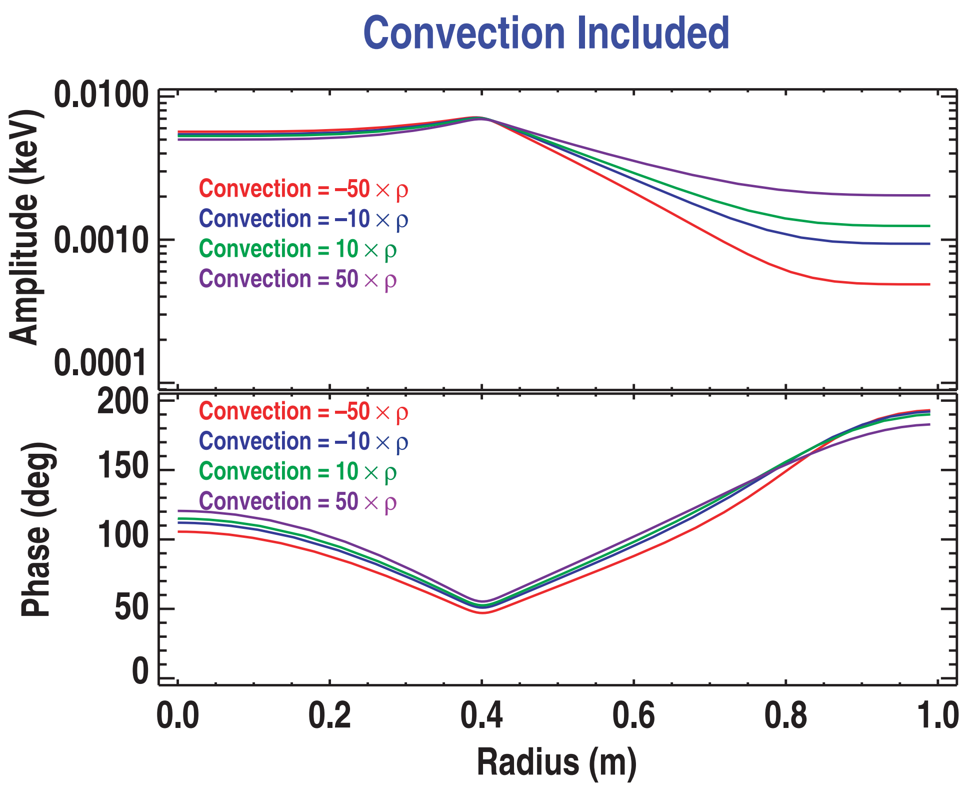
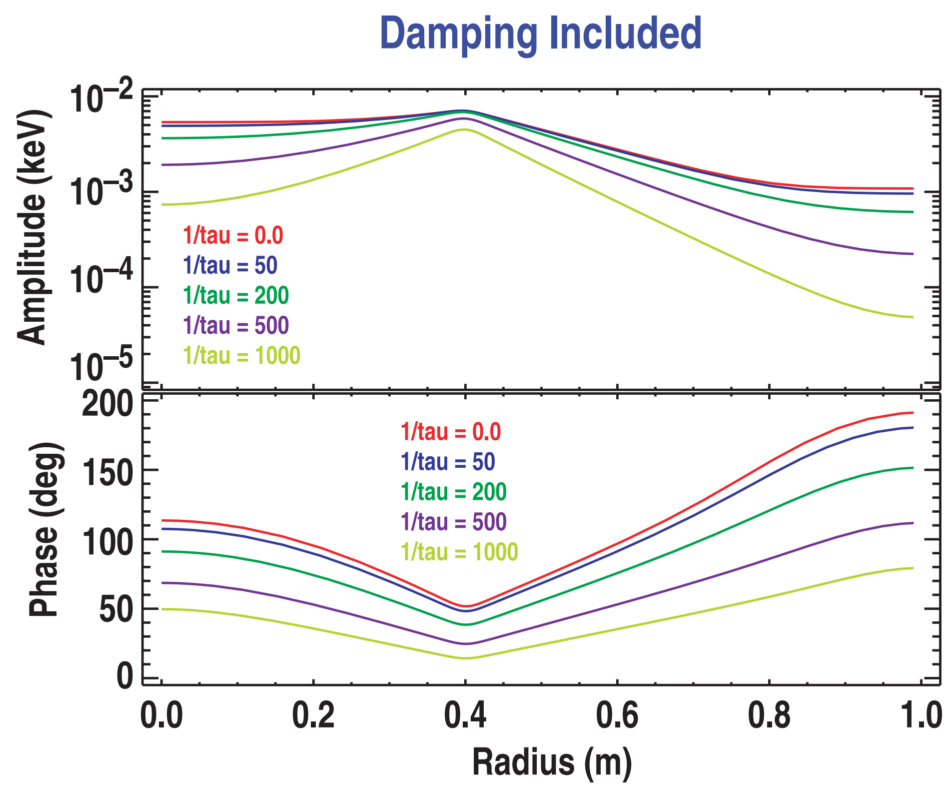
ADDING CONVECTION AND/OR DAMPING TO THE MODEL SIGNIFICANTLY CHANGES THE HEAT PULSE PROPAGATION





- $D = 10\ m^2/s$, $\omega/2\pi = 50$ Hz, geometry = torus, B.C. = no flux

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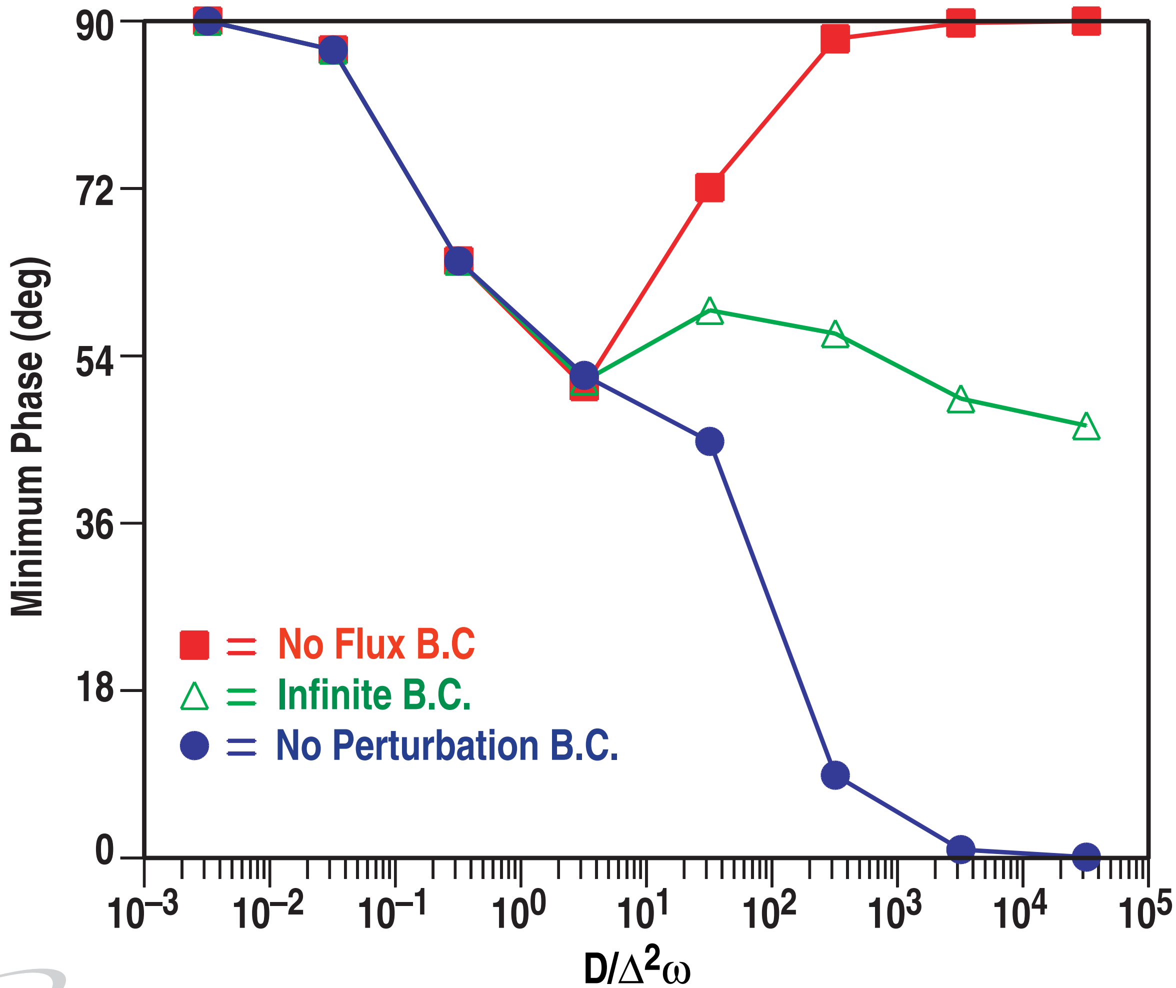
ONLY NO PERTURBATION ($\tilde{T}$ = 0) BOUNDARY CONDITION GIVES CORRECT PHASE LAG FOR HIGH TRANSPORT LEVELS

- Dimensionless parameter $D/\Delta^2\omega$ governs minimum phase



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CONCLUSIONS

- Detrending slow time evolution of signals reduces uncertainties in Fourier amplitudes and phases
  - Polynomial (degrees 3-6) fits are widely applicable
- Converting data from time domain to frequency domain using Fourier Series fits gives smaller error bars than FFT's, but the former is sensitive to choice of base frequency
- Adding convection and/or damping to transport model changes heat pulse propagation in unique ways
  - We should be able to determine whether convection/damping is important from experimental data
- Differences in heat pulse propagation between cylindrical and toroidal geometry, and between different choices of boundary conditions, are more apparent at high levels of transport
- No flux ($d\tilde{T}/dx = 0$) and infinite boundary conditions give unphysical minimum phase lags at high $D/\Delta^2\omega$, but no perturbation ($\tilde{T} = 0$) boundary condition does not suffer from this problem