A Mechanism for Tearing Mode Onset Near Ideal Stability Boundaries

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GENERAL ATOMICS

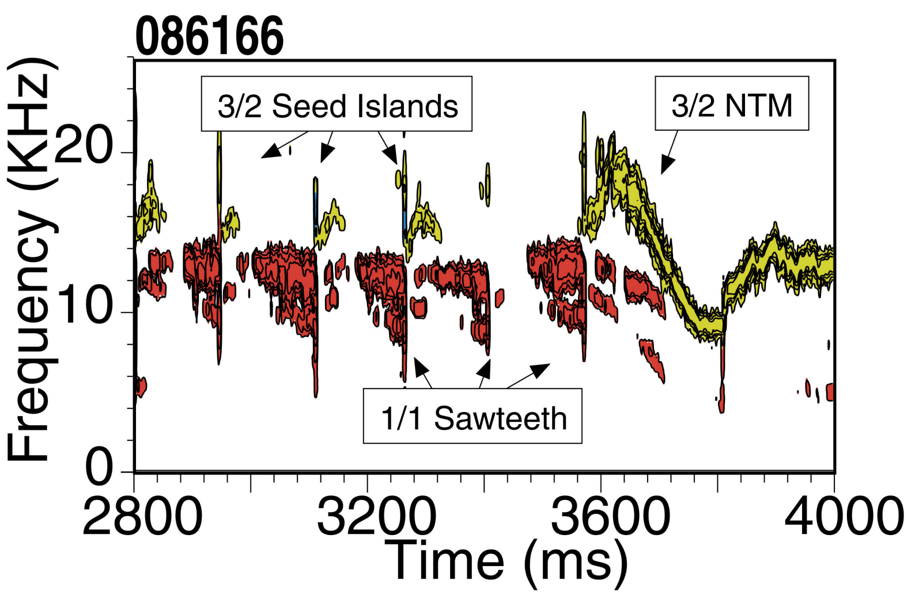
DIII-D NATIONAL FUSION FACILITY SAN DIEGO

New Onset Mechanism Addresses Two Common NTM Onset Puzzles

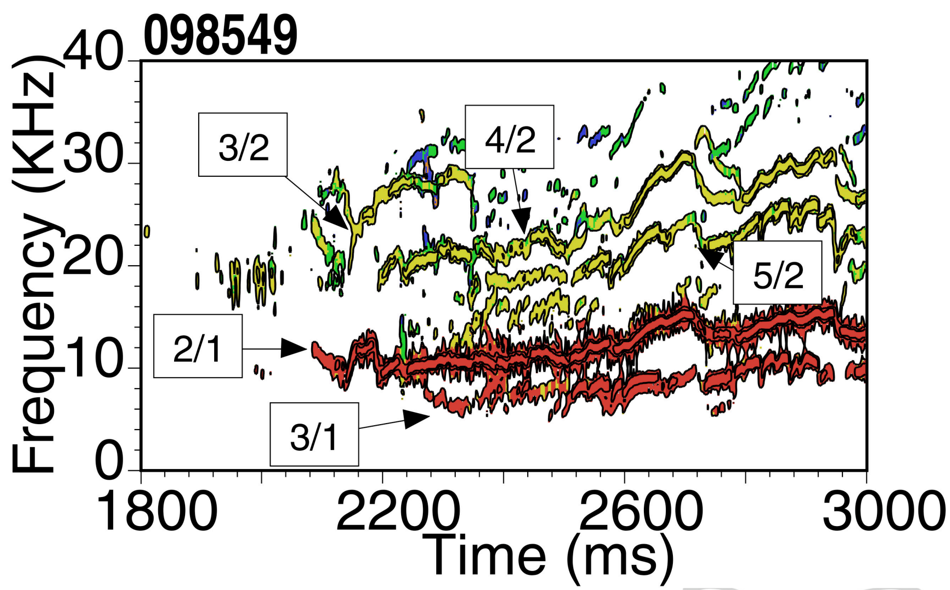
NTMs are linearly stable but nonlinearly unstable, ie. Metastable

- Need "seed" or triggering event

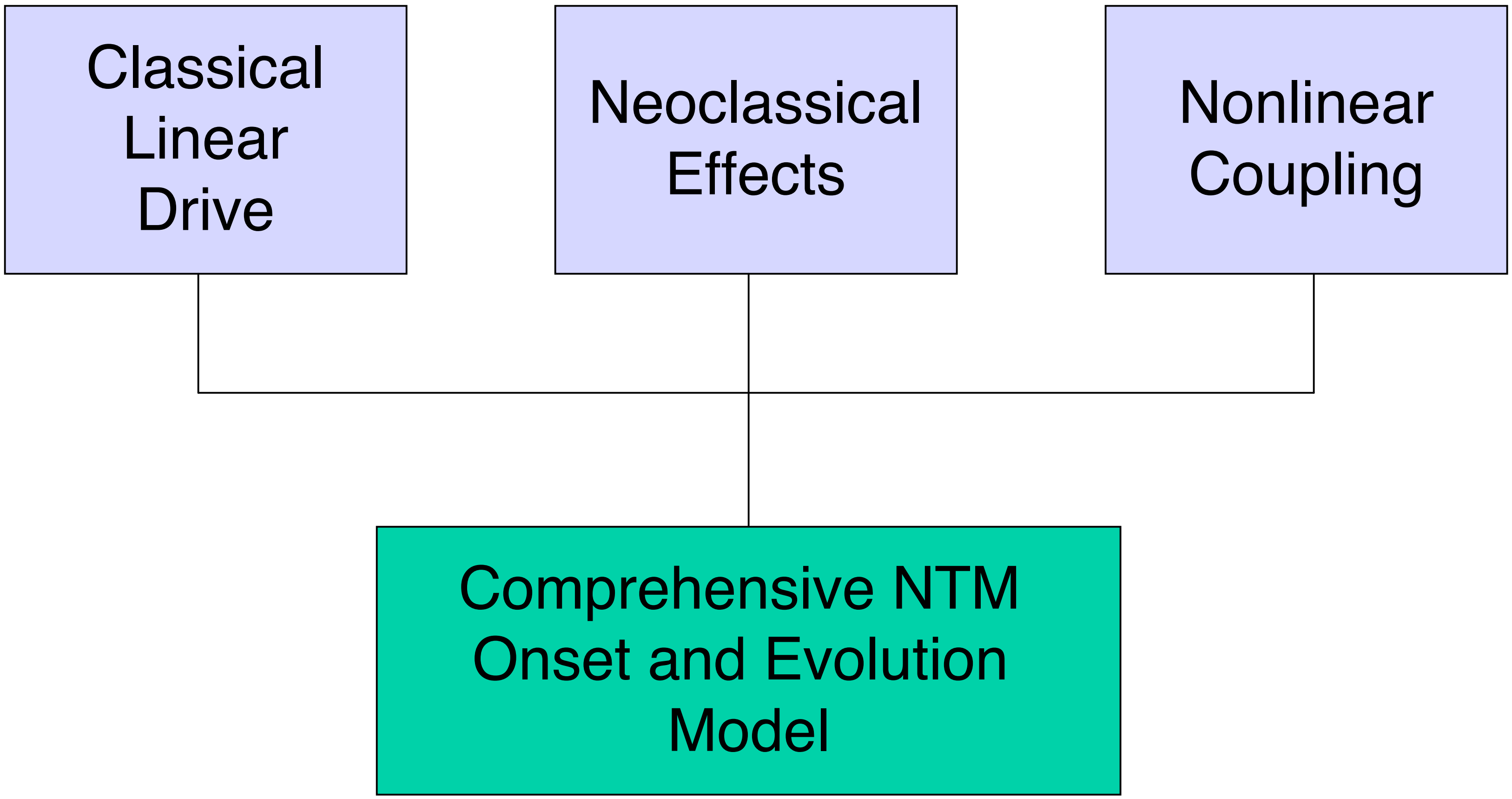
Why Does The n^{th} Sawtooth Seed an NTM?



What Causes Seedless NTMs?



A Comprehensive Model of NTM Onset Emerges From a Combination of Theories



All three components are necessary to explain the onset and evolution

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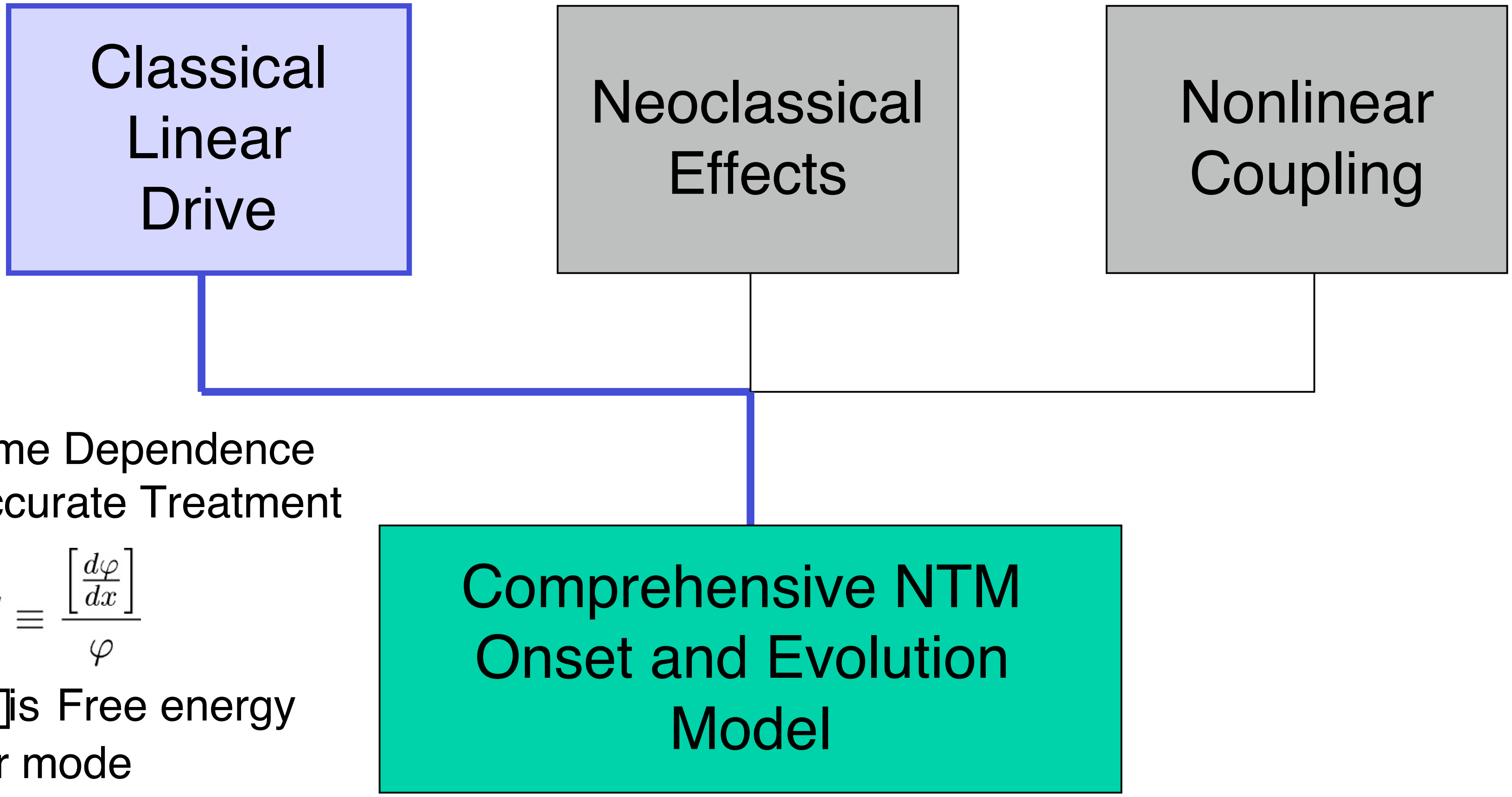


Time Dependence
Accurate Treatment

$$\triangle' \equiv \frac{\left[\frac{d\varphi}{dx}\right]}{\varphi}$$

□□is Free energy for mode

All three components are necessary to explain the onset and evolution

Outline

Thesis: □□(t) depends strongly on the equilibrium parameters near ideal limit _ key physics for NTM onset in some cases.

Island Evolution Equation : essential physics of nonlinear evolution

Analytic model : the basic physics of the time dependence of □□

Sawtooth Seeding of 3/2 NTM : DIII-D discharge which encompasses **nonlinear**, **neoclassical** and **classical** effects is comprehensively analyzed showing the critical time dependence in □□.

Pole Experiment : theoretical predictions of the effects of time dependence in □□are compared to results from a DIII-D experiment to isolate these effects.

Island Evolution Equation Captures Essential Physics Necessary to Describe Onset and Early Evolution

Coupling not included

□□often assumed constant negative

With accurate time dependence and weak coupling, onset and early evolution can be predicted

$$\frac{dw}{dt} = \frac{\eta^*}{k_0}\left[\Delta^* + \frac{k_1}{w}\left(D_{nc} + \frac{D_R}{\alpha_s - H}\right) + \frac{D_{pol}}{w^3}\right]$$

where

$$\Delta^* = \Delta'\left(\frac{w}{2}\right)^{-2\alpha_l}(-4D_I)^{1/2}$$

$$\frac{\eta^*}{k_0} \sim \frac{r_s^2}{\tau_r}$$

$$k_1 = \frac{w^2}{w^2 + w_d^2}$$

$$D_{nc} = k_2\sqrt{\epsilon}\beta_\theta \frac{L_q}{L_p}$$

$$D_{pol} = D_{nc}\frac{\rho_{i\theta}}{a^2}\frac{L_q}{L_p}g$$

Classical drive

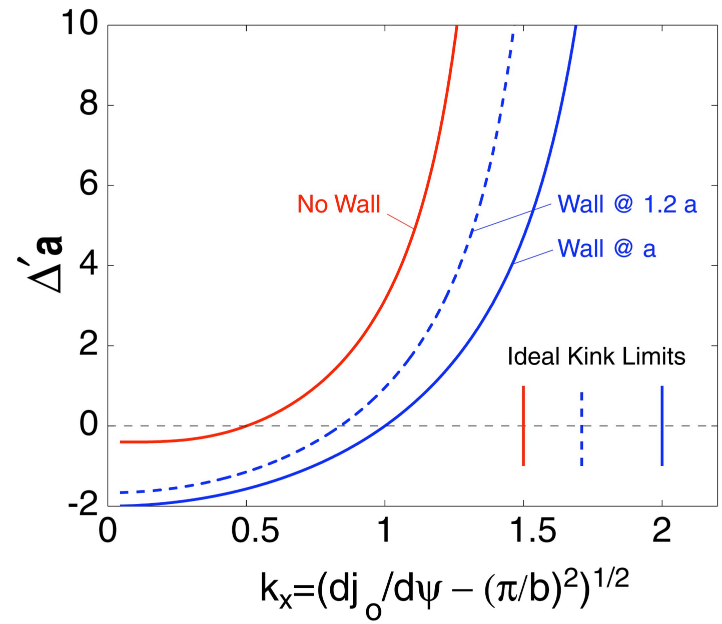
Neoclassical drive

Polarization drive

Analytic Models of Linear Tearing Stability Show Strong Variation of □□Near Ideal Instability

Small change in equilibrium and/or conducting wall dramatically affects □□near ideal limit due to

POLE DISCONTINUITY at ideal limit.



The Pole Form is Ubiquitous
Among Derivations, Numeric and Analytic

j_o□constant, slab geometry:

With a conducting wall at a:

$$\triangle' a = -2k_x a \cot(k_x a)$$
$$k_x = \sqrt{j_o' - k_y^2}$$

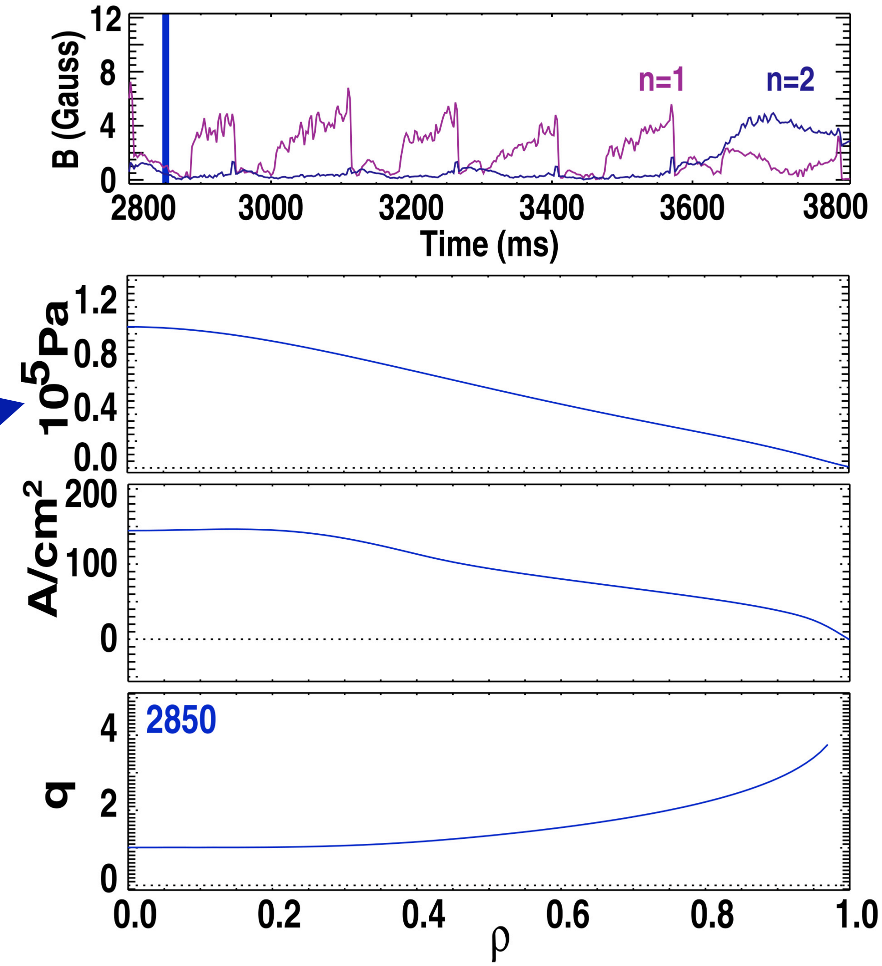
With a vacuum region from a to b:

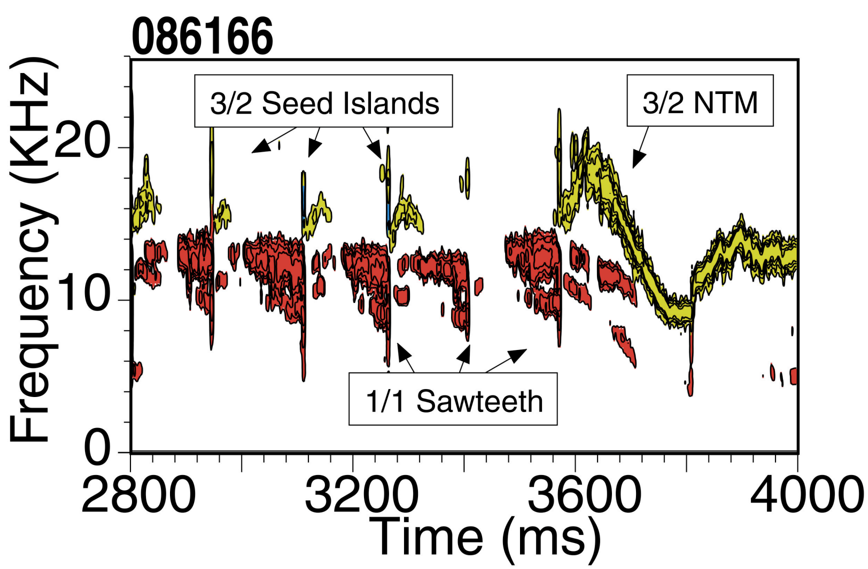
$$\triangle' a = 2k_x a \frac{E\sin(k_x a) + \cos(k_x a)}{E\cos(k_x a) - \sin(k_x a)}$$
$$E = \frac{e^{k_x a} - e^{k_x(2b-a)}}{e^{k_x a} + e^{k_x(2b-a)}}$$
$$k_x = \sqrt{j_o' - k_y^2}$$

Slight Increase in Core Pressure Destabilizes a Seed Island from a Sawtooth and Causes the Onset of a 3/2 NTM

Equilibrium reconstructions between sawteeth

Most pronounced profile variation is core pressure just before onset.





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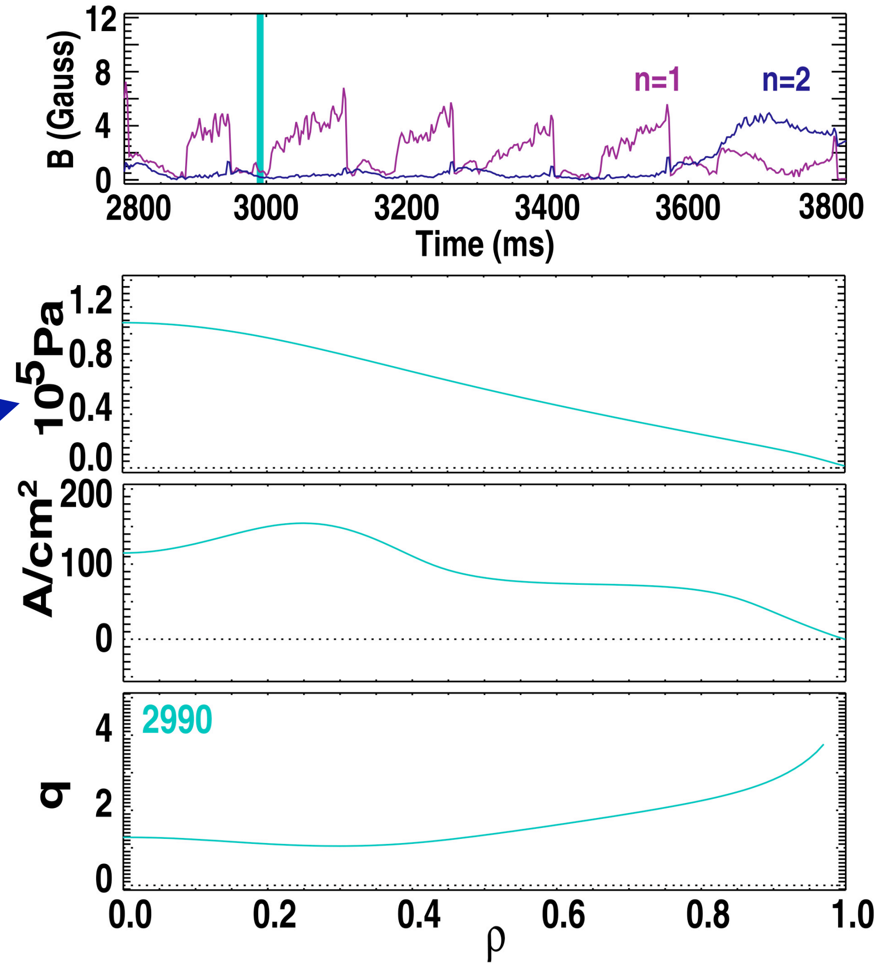

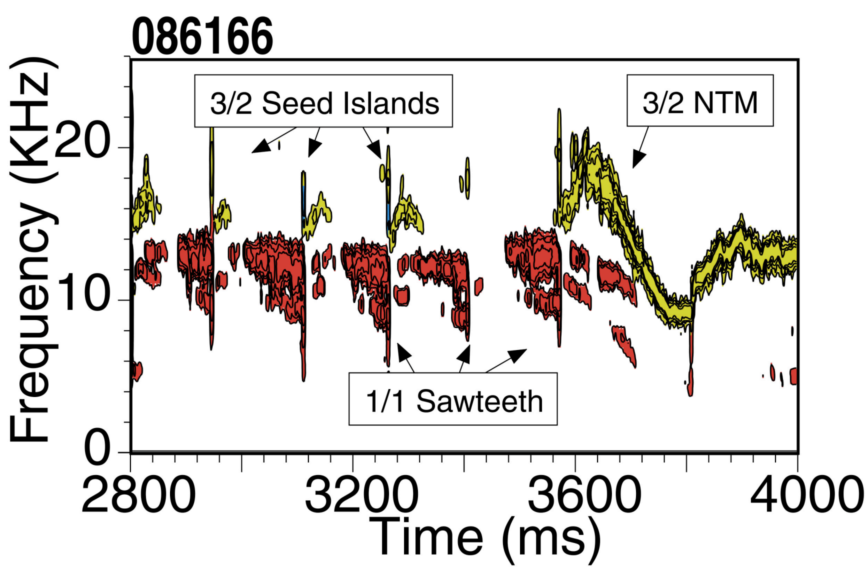

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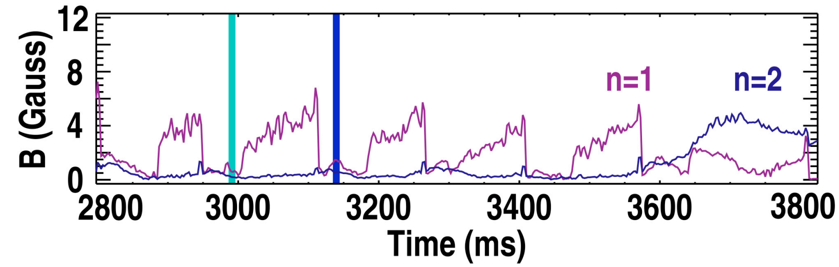

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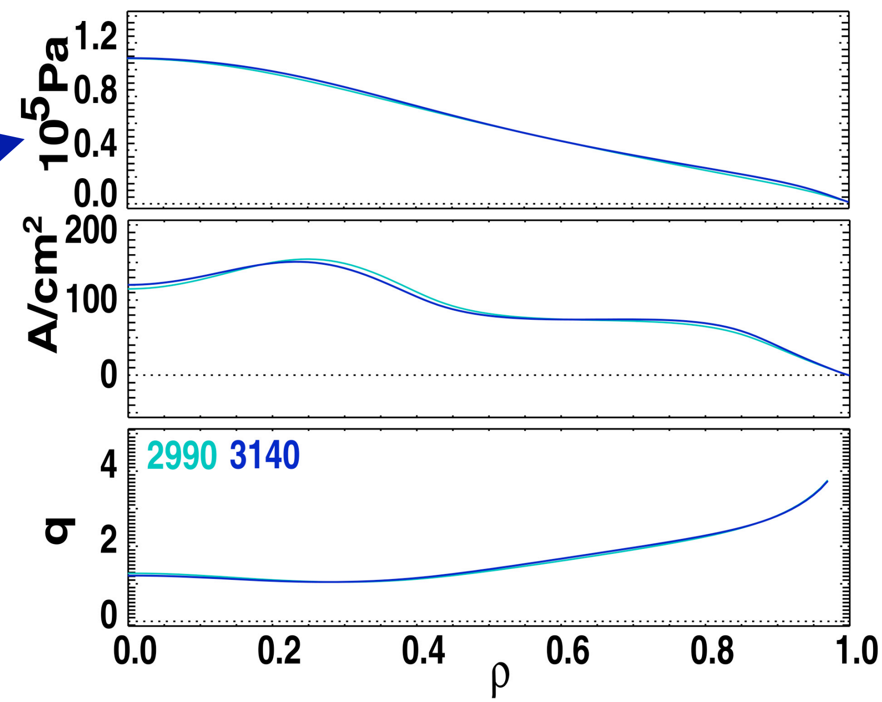

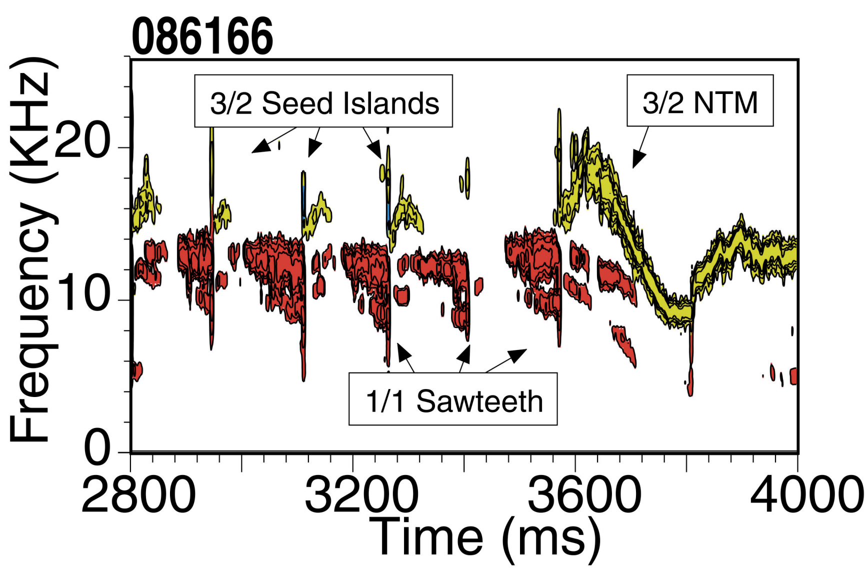

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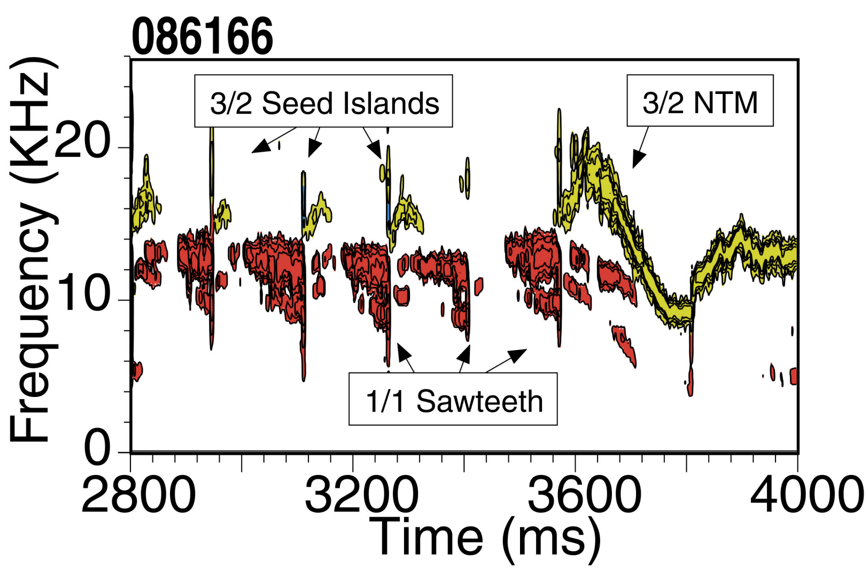

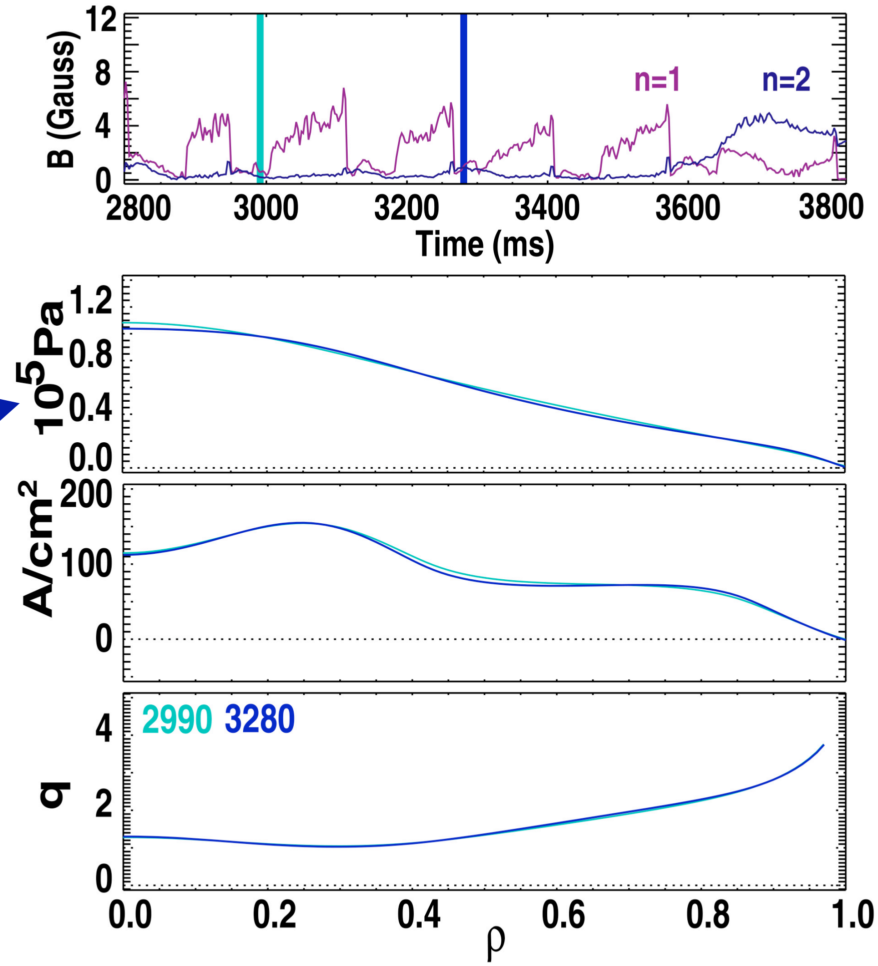

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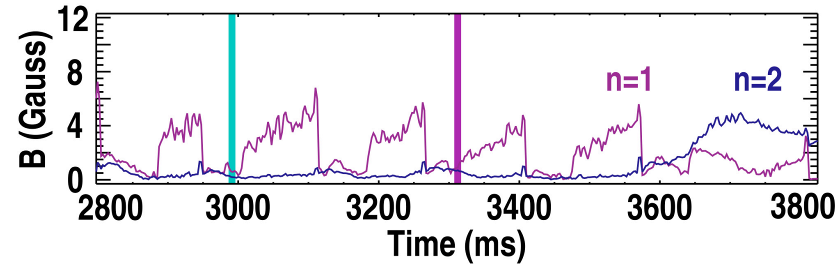

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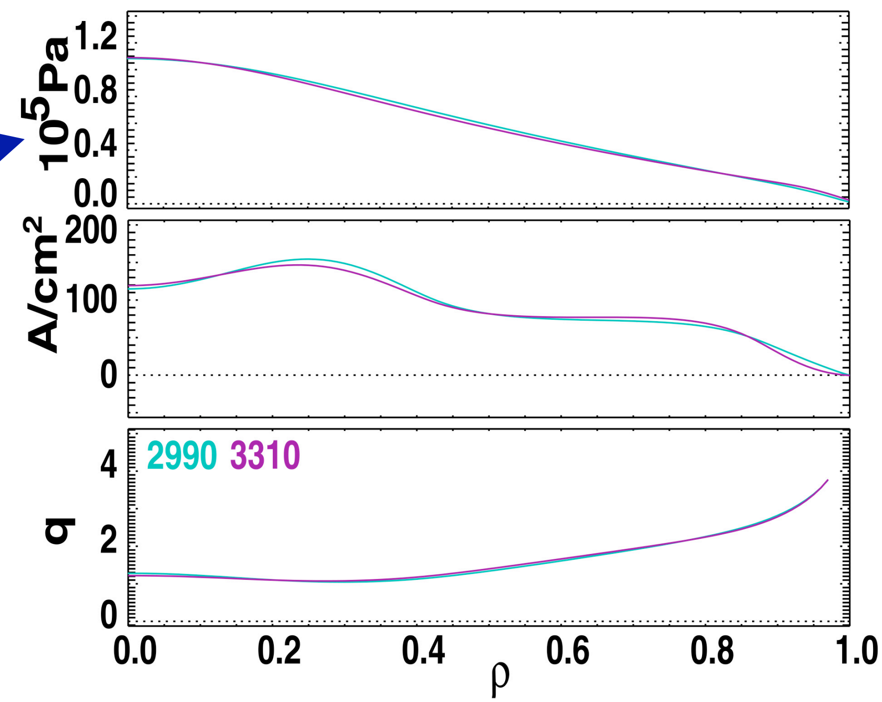

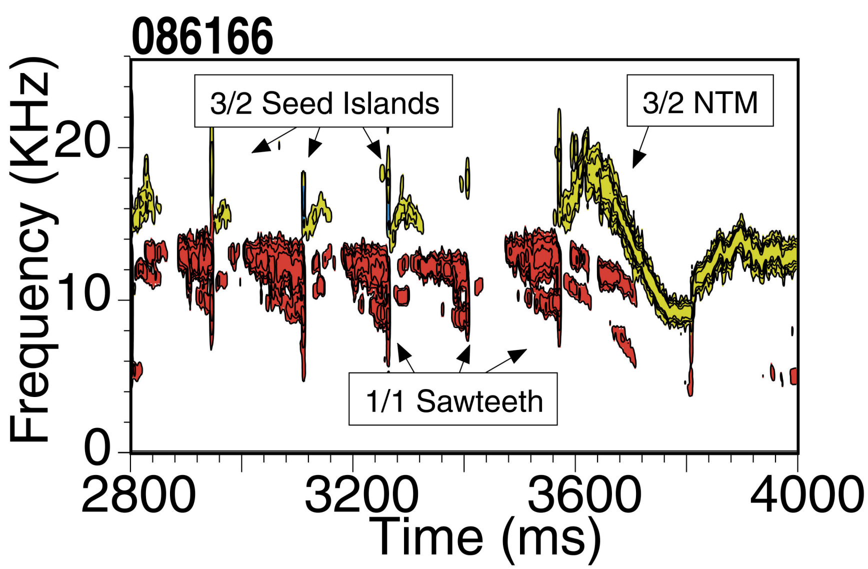

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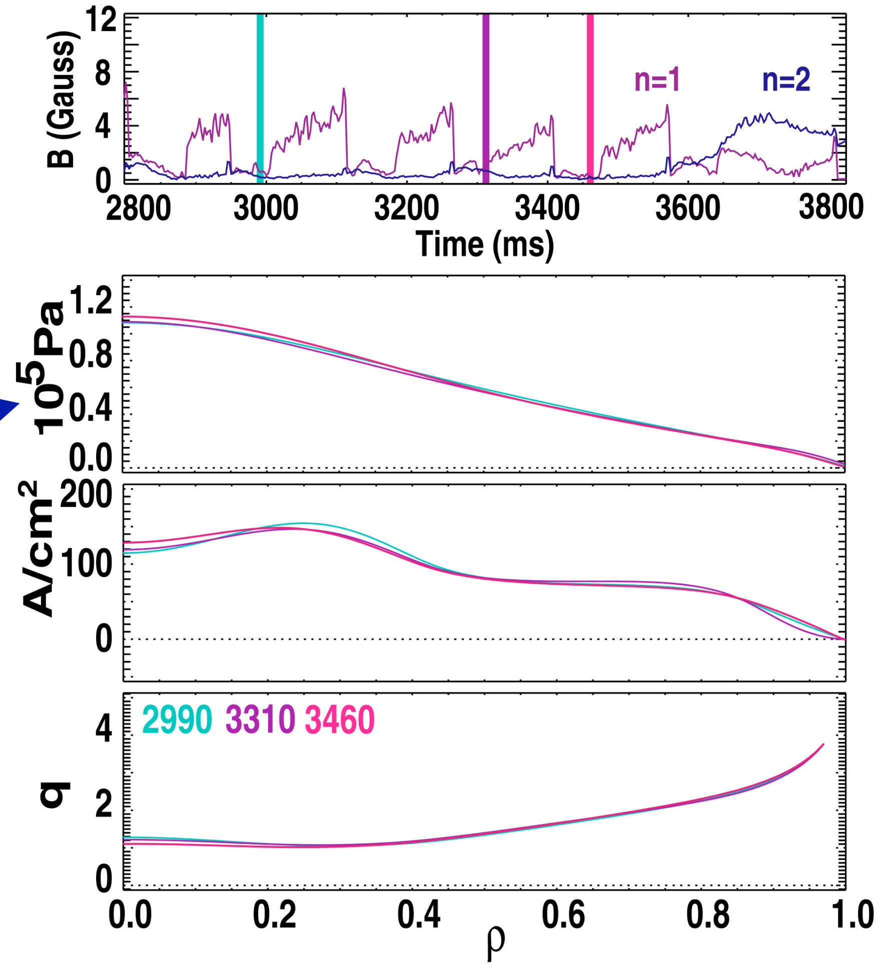

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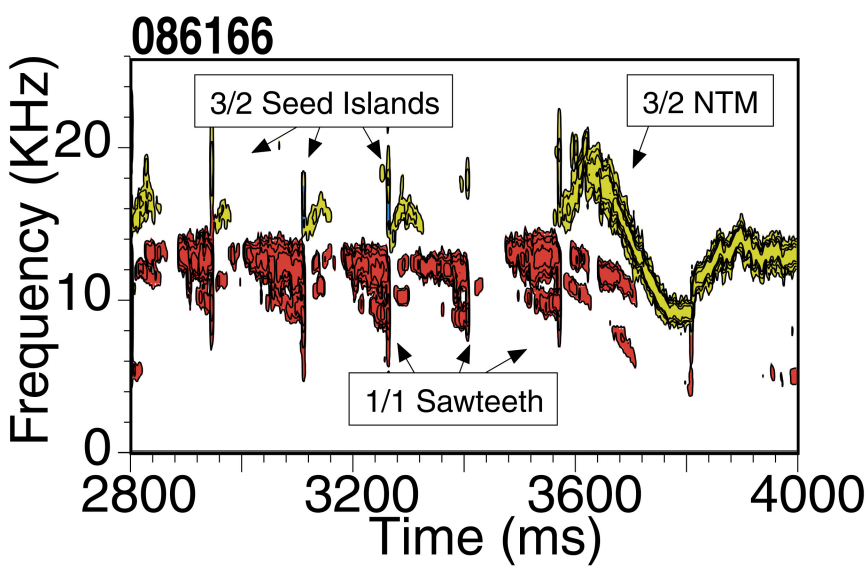

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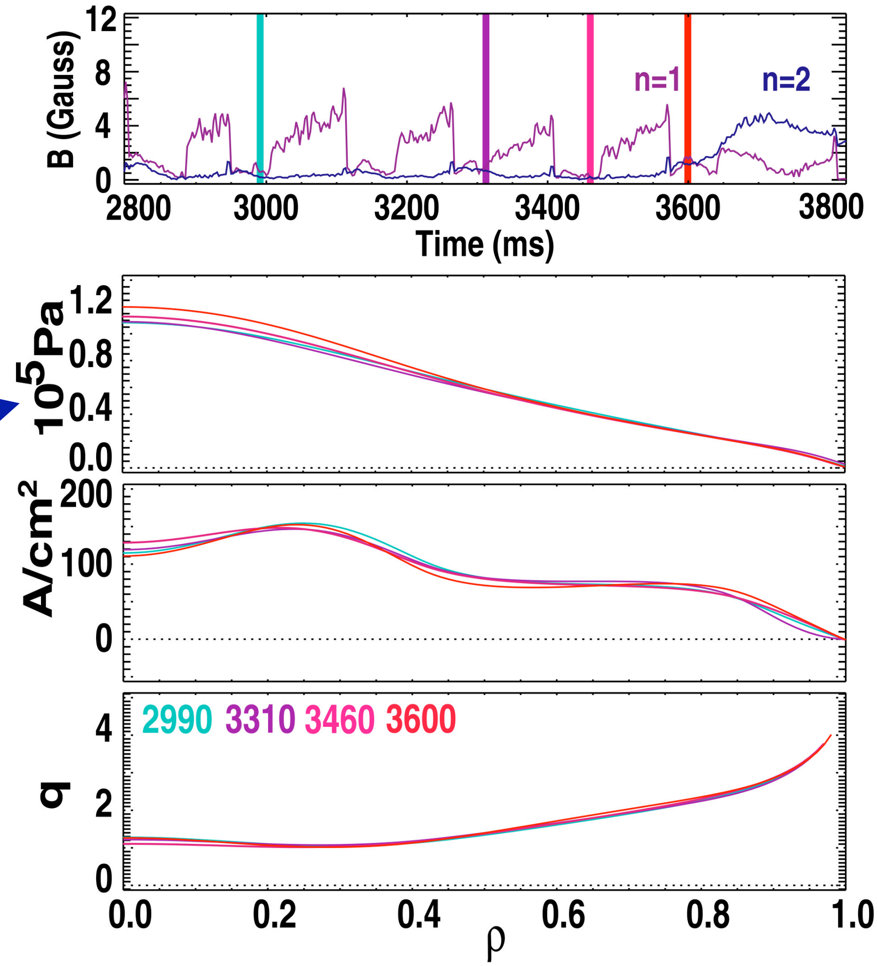

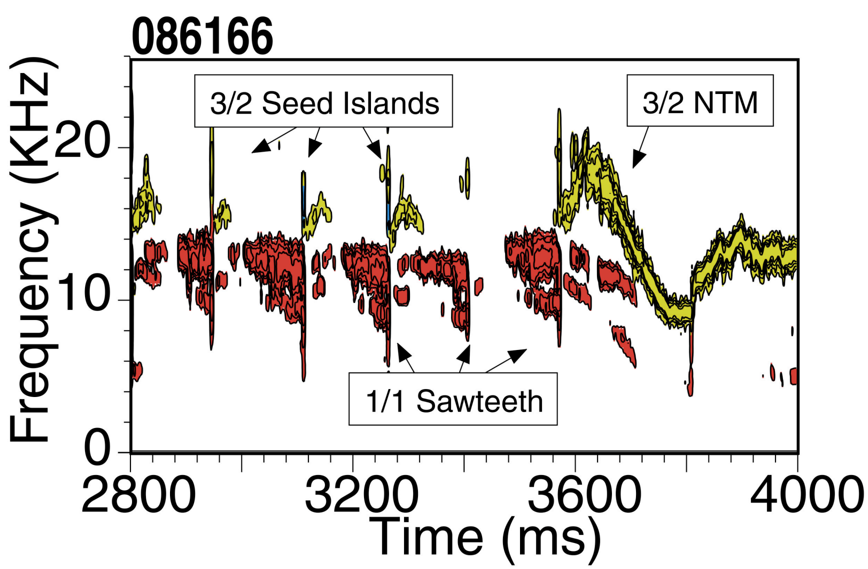

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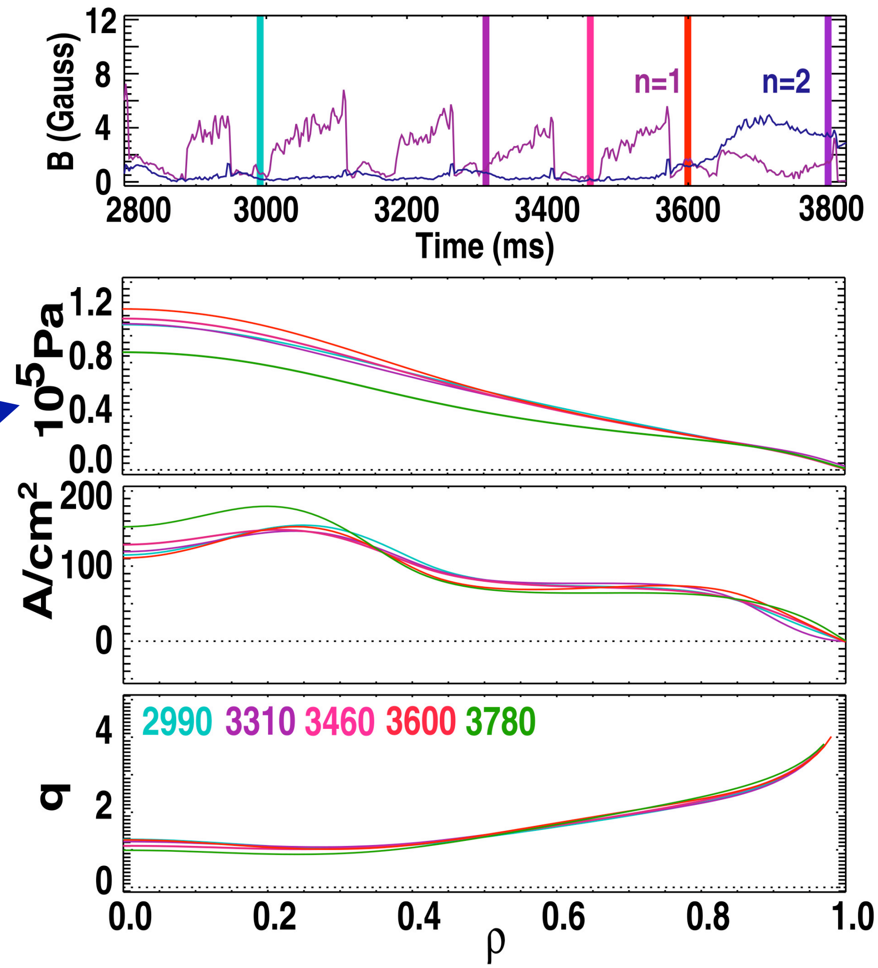

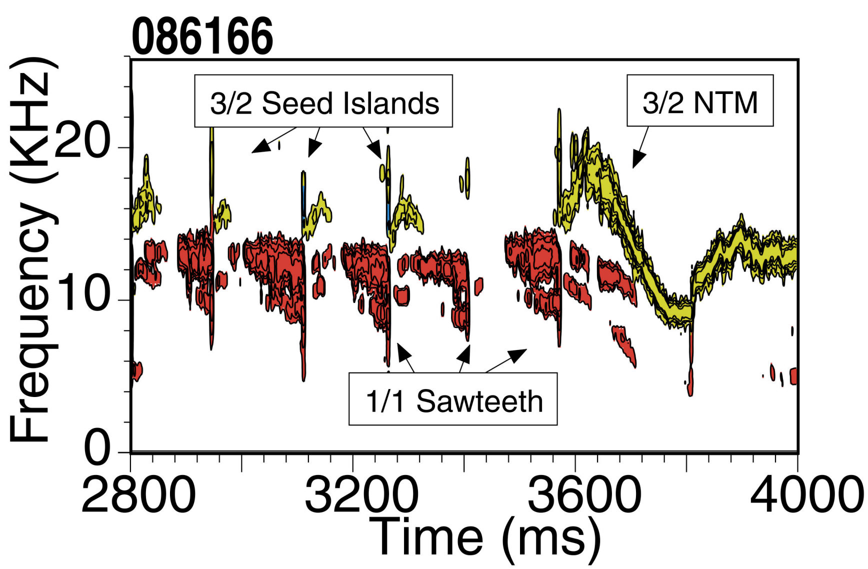

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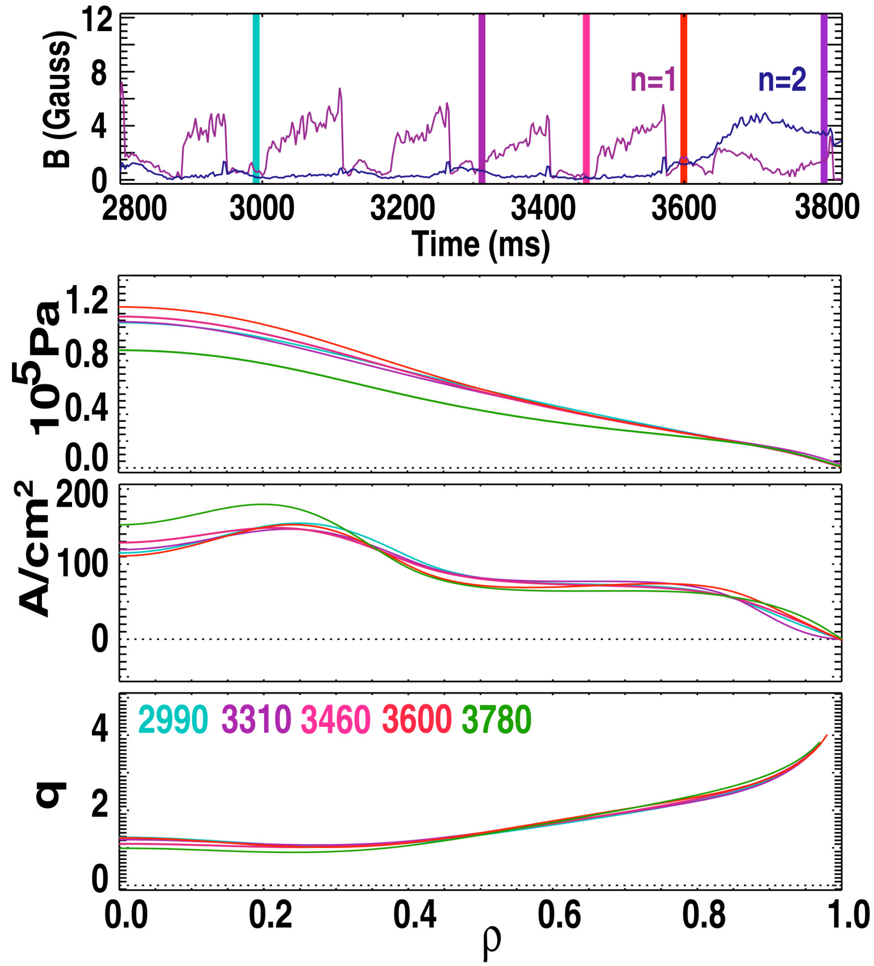
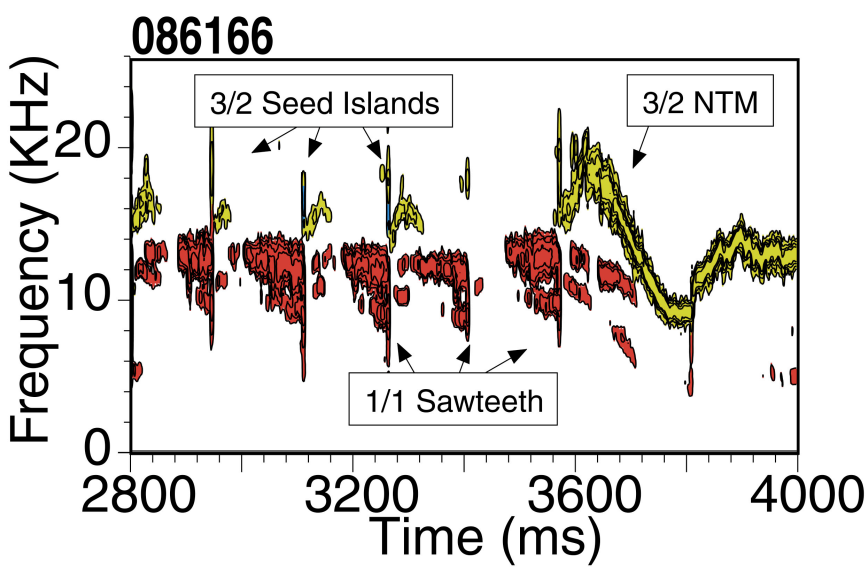
Slight Increase in Core Pressure Destabilizes a Seed Island from a Sawtooth and Causes the Onset of a 3/2 NTM

Point is to answer: why did the last sawtooth set off the 3/2 NTM while the several preceding similar sawteeth did not.





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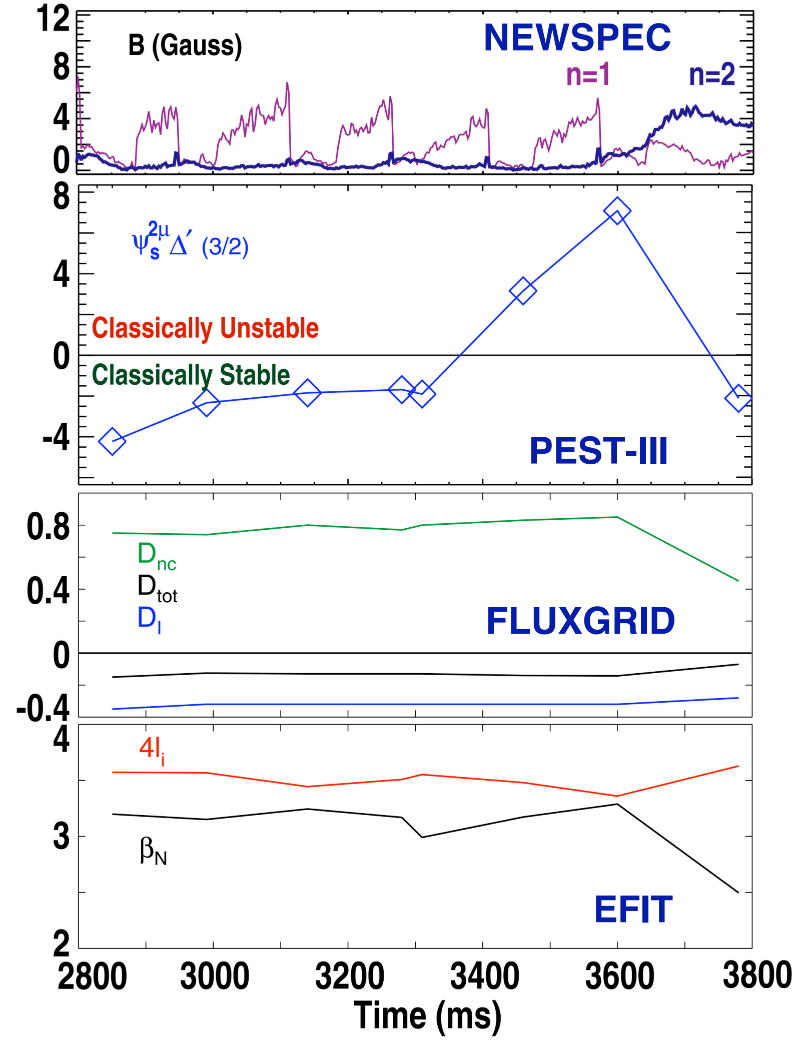
Neoclassical Terms Remain Constant Between Sawtooth Periods, While Increases Sharply

Destabilization of seed island and transition to NTM state is driven primarily by change in

$_N$ approaches no-wall limit at onset of 3/2 NTM _ pole is underlying mechanism for destabilization



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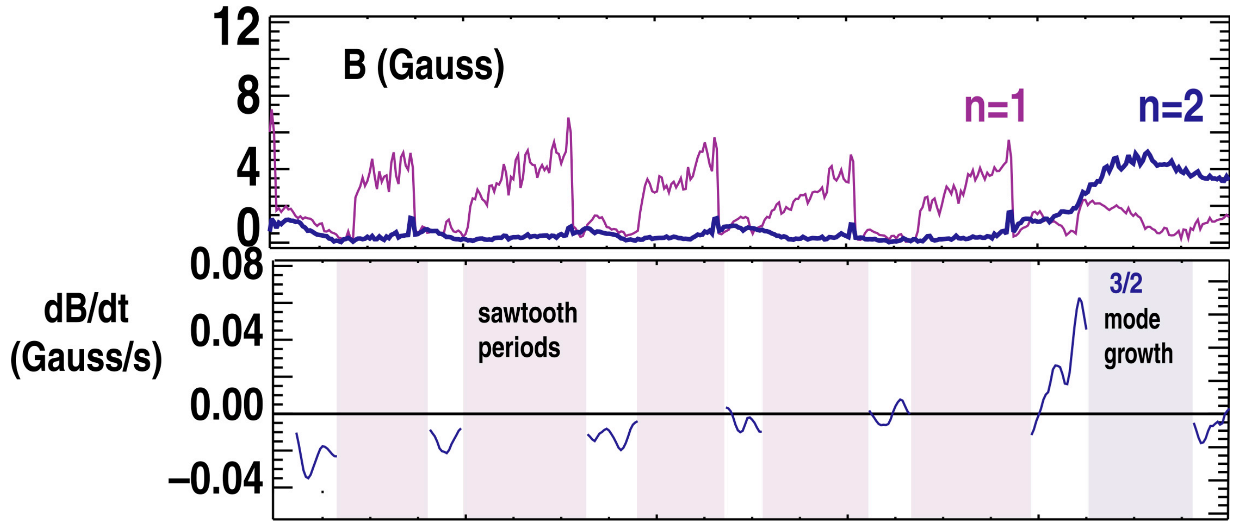
To Test This Proposition Island Evolution is Reconstructed From Experiment and Compared to Theory

$$w = \left(\frac{16 r R_o |\tilde{B}_r|}{m s B_{\phi o}}\right)^{1/2}$$

$$|\tilde{B}_r| = \frac{1}{2}\left(\frac{b}{r}\right)^{m+1} |\tilde{B}_\theta|$$



w from Magnetic Probe Signals used to compare dw/dt from theory and experiment

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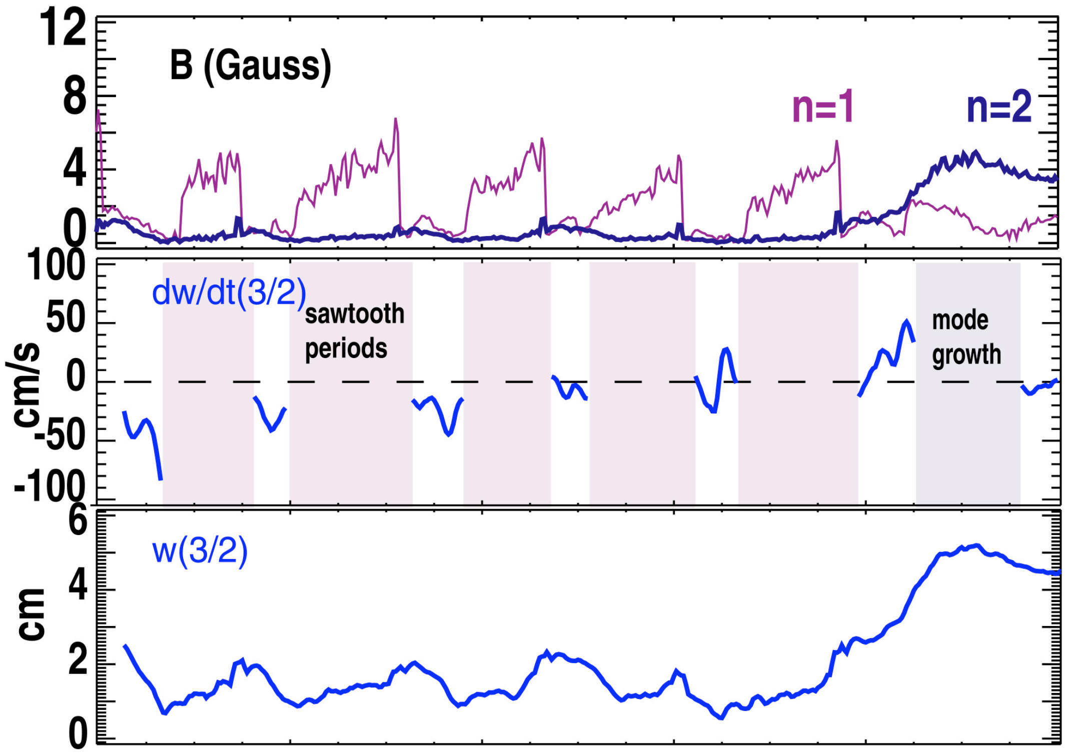
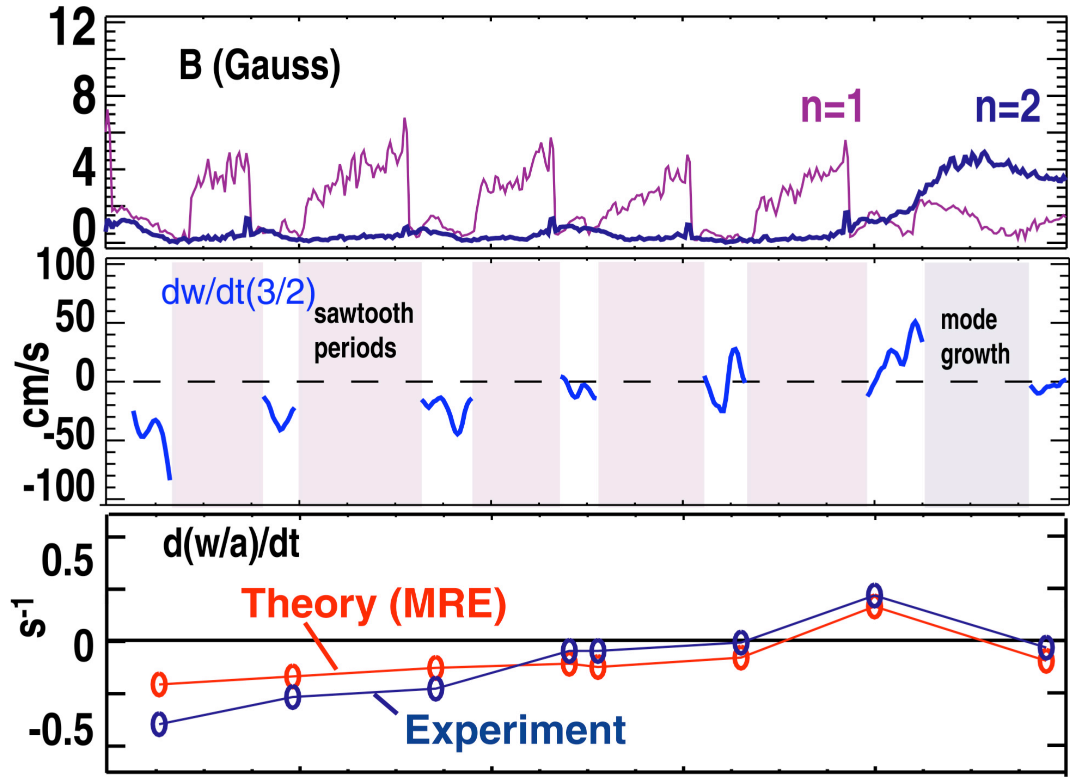


dw/dt from Island Evolution Equation Agrees with dw/dt from Experimental Data

Island Evolution Equation gives right answer even without nonlinear coupling an(with axisymmetric □□.

D_{pol} determined from □^{2} minimization agrees in sign with analytic value, but is smaller.

D_{pol}= -2.3e-5



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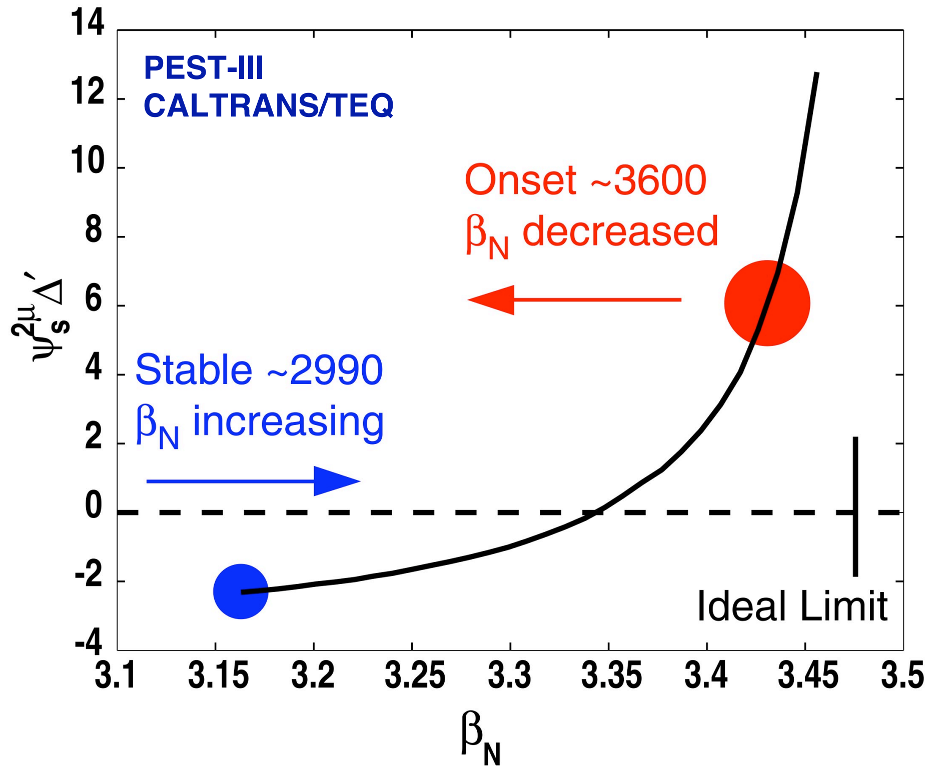
Increase in Core Pressure Causes □□to Sharply Increase Due to Approach of Ideal Limit

Pressure profile at 2990 was gradually modified to approximate profile at 3600, showing transition.

Approach to n=2 ideal limit increases 3/2 □□.

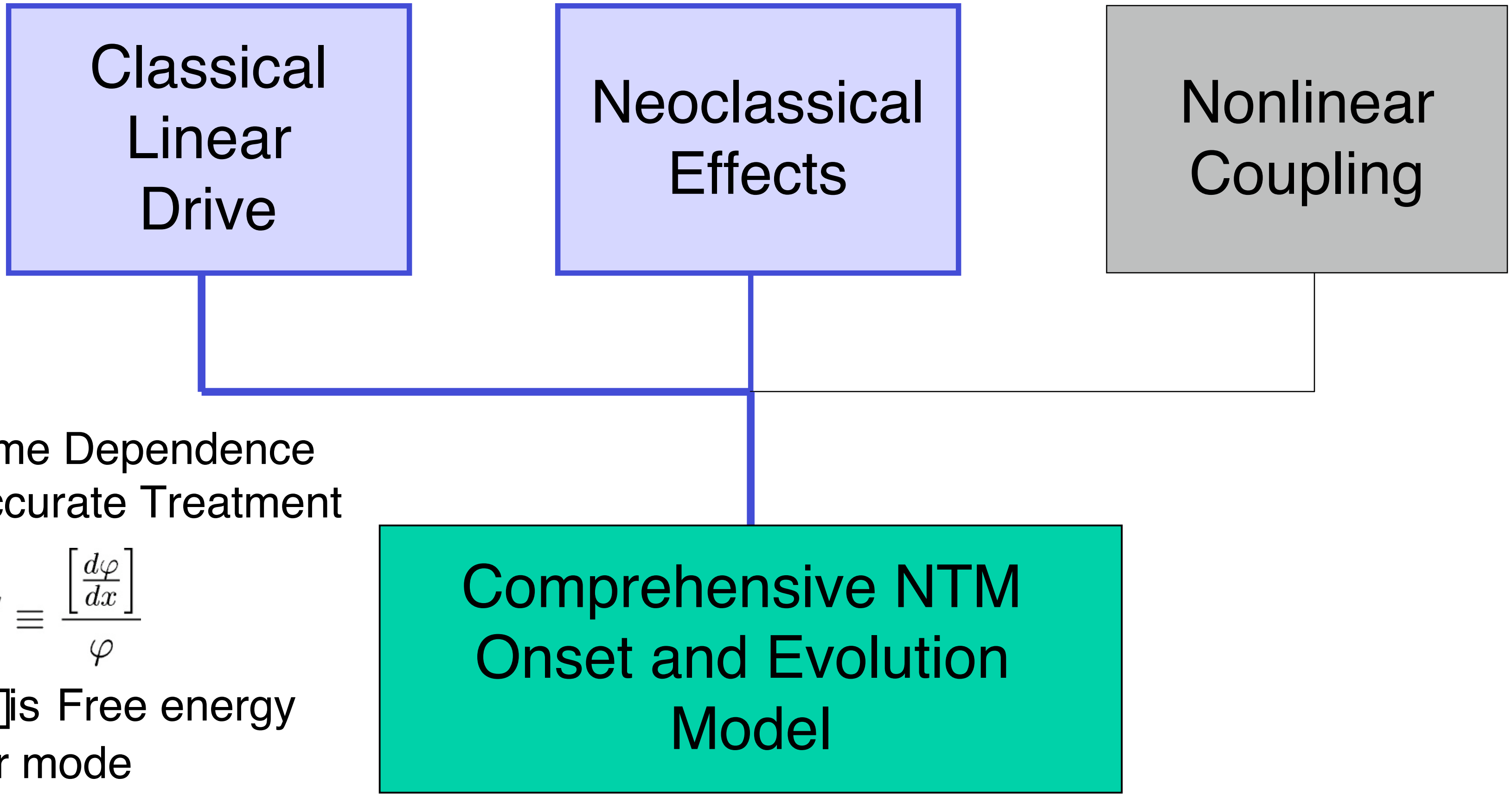
Pole causes rapid change in □□(□) as □ changes slightly



Nonlinear coupling between n=1 and n=2 modes not addressed by this model

How will this affect the stability and evolution?

A Comprehensive Model of NTM Onset Emerges From a Combination of Theories



Time Dependence
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$$\triangle' \equiv \frac{\left[\frac{d\varphi}{dx}\right]}{\varphi}$$

□□is Free energy for mode

All three components are necessary to explain the onset and evolution

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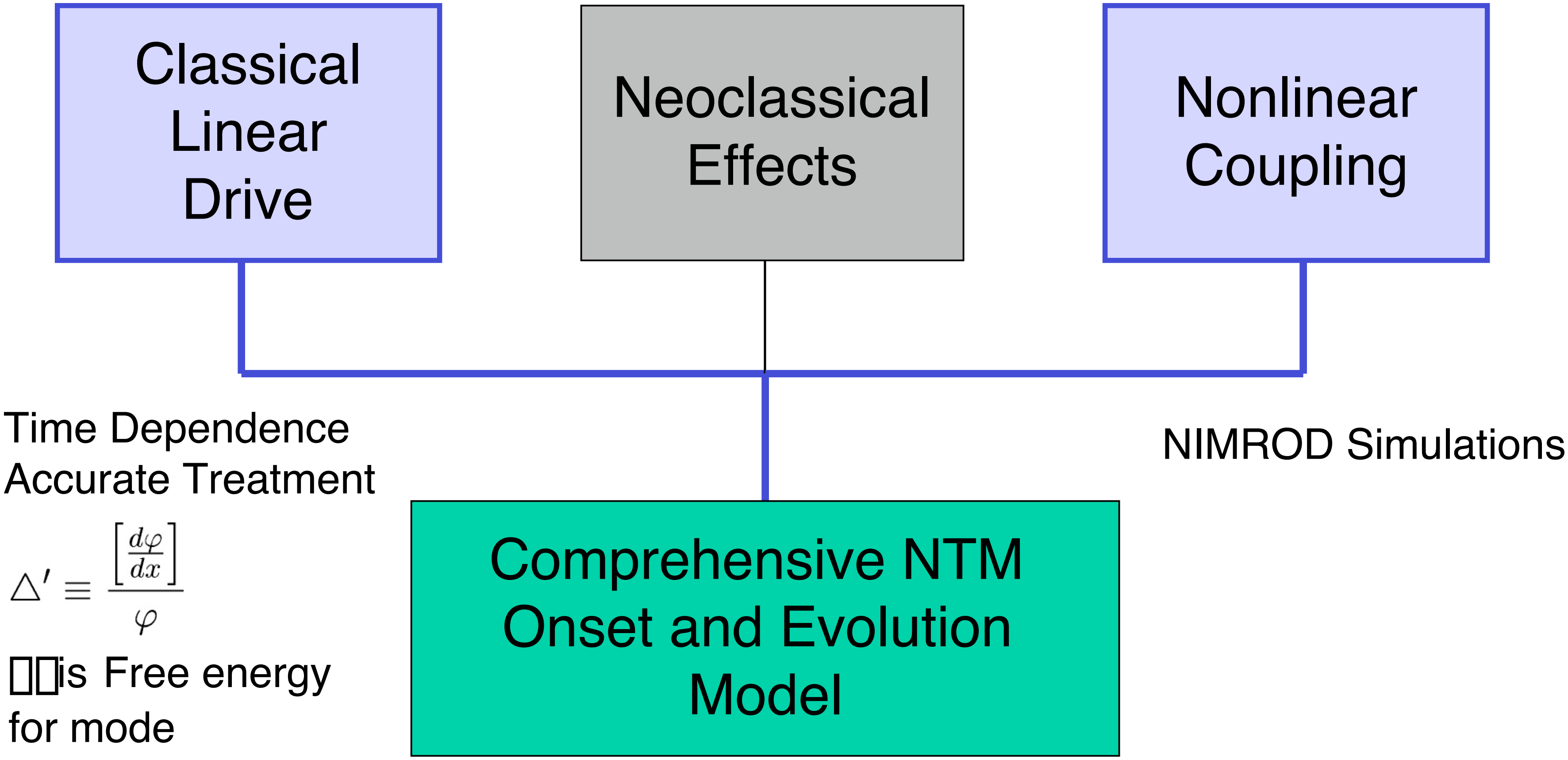

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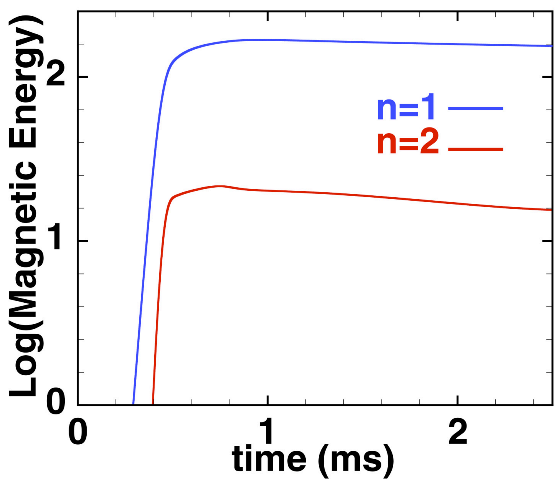
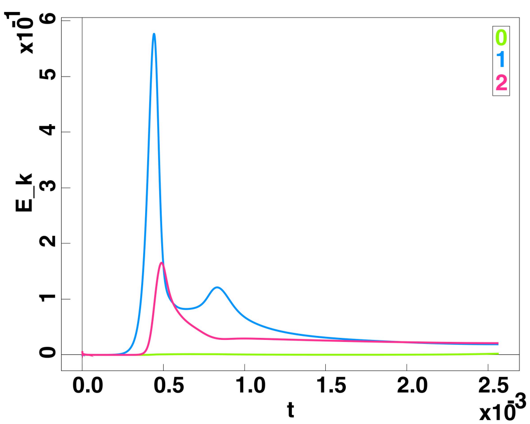
NIMROD Simulations of two discharge times show unstable n=1 mode and driven n=2 mode, in agreement with experiment

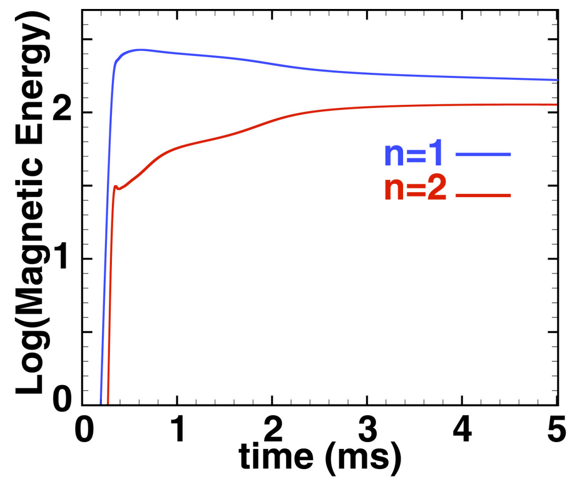
t_{exp}=2990



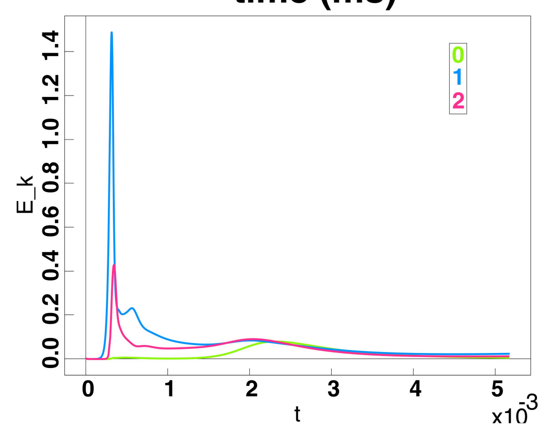
Stable
n=2 decays
after n=1
drive begins
to reduce



t_{exp}=3600

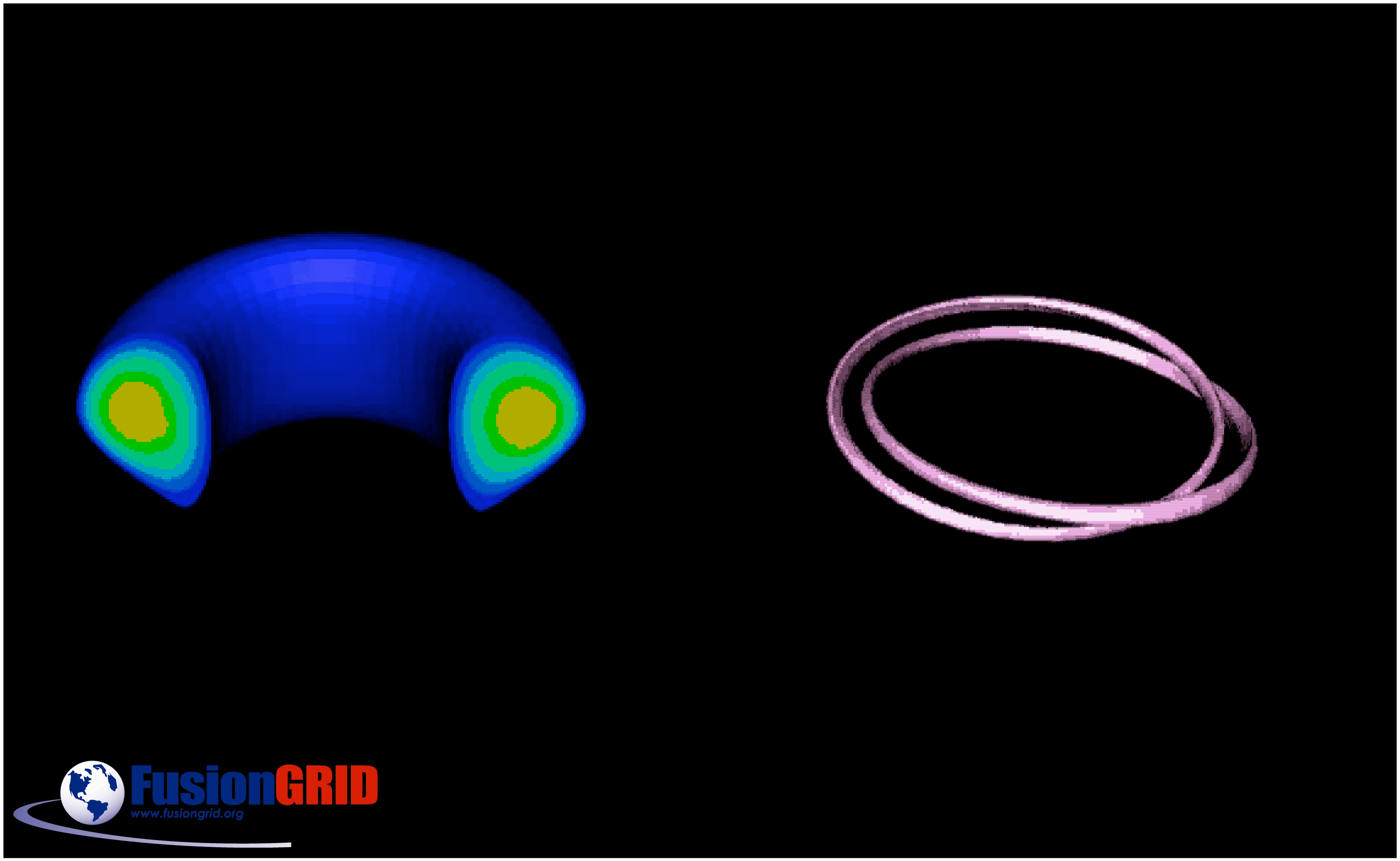


Unstable
n=2 grows
after n=1
drive begins
to reduce



$S=\tau_R/\tau_A=2.3e6$
approaching
realistic conditions

Studying the structure and nonlinear evolution of these modes with NIMROD can lead to new intuition and new physics



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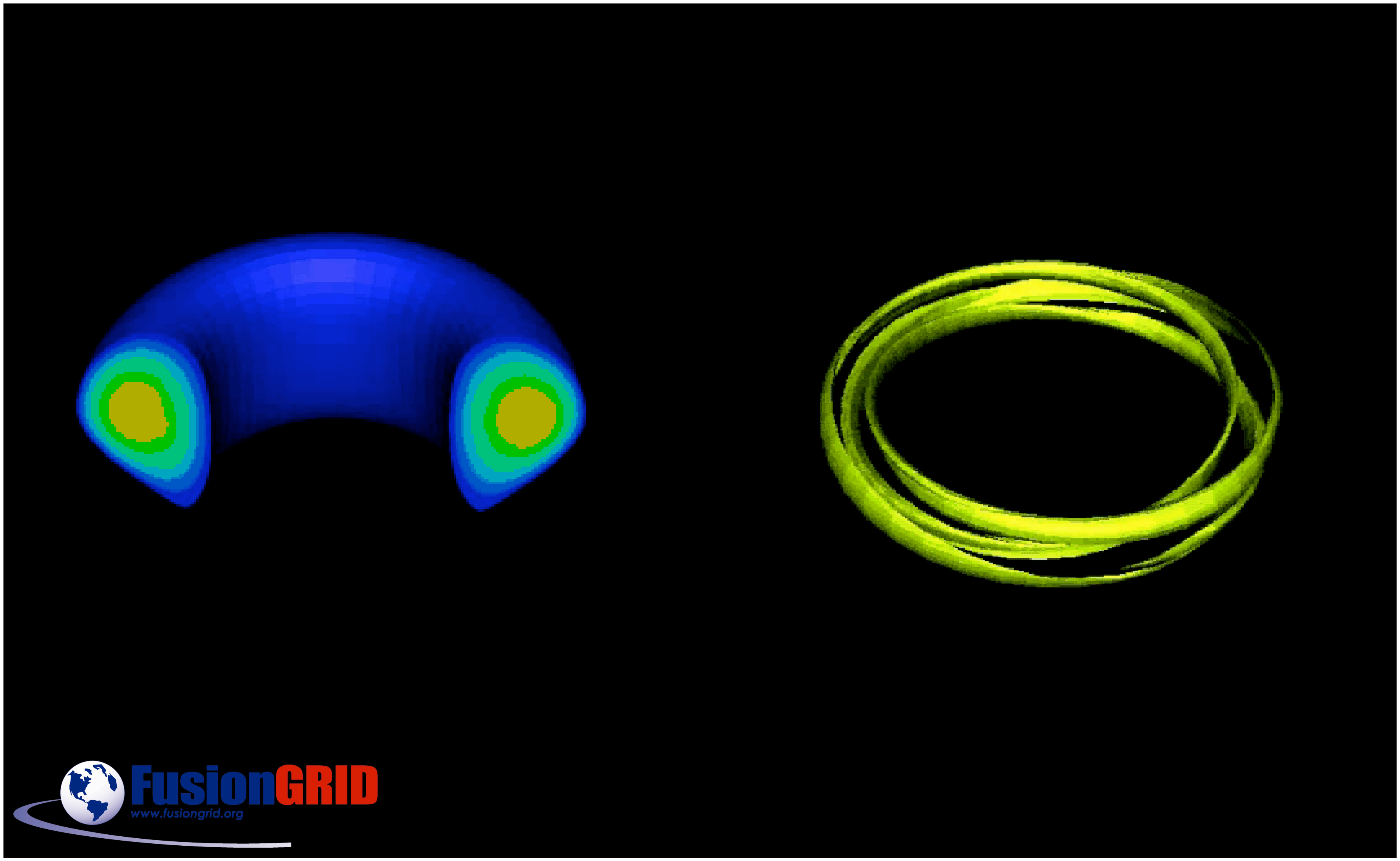

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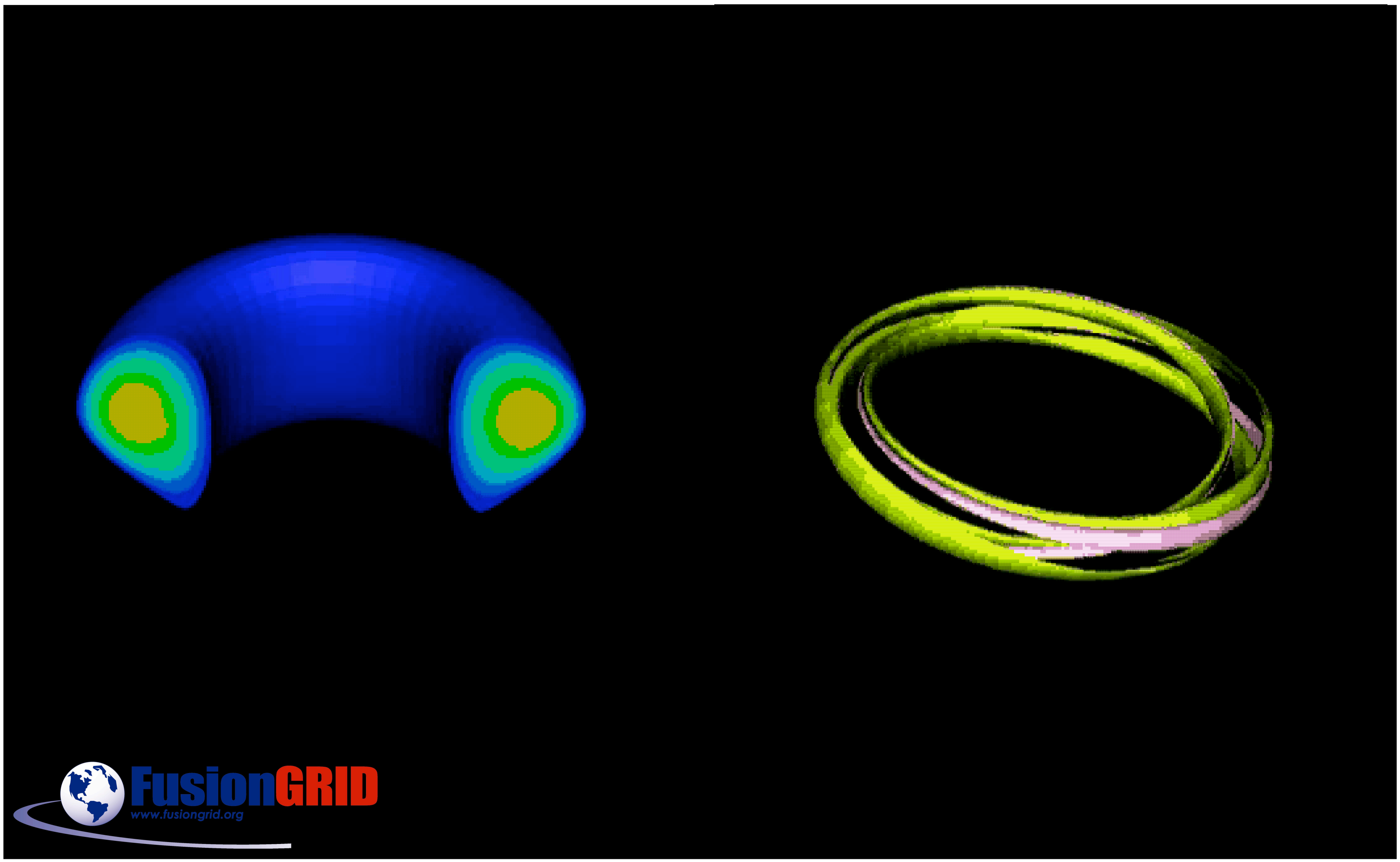

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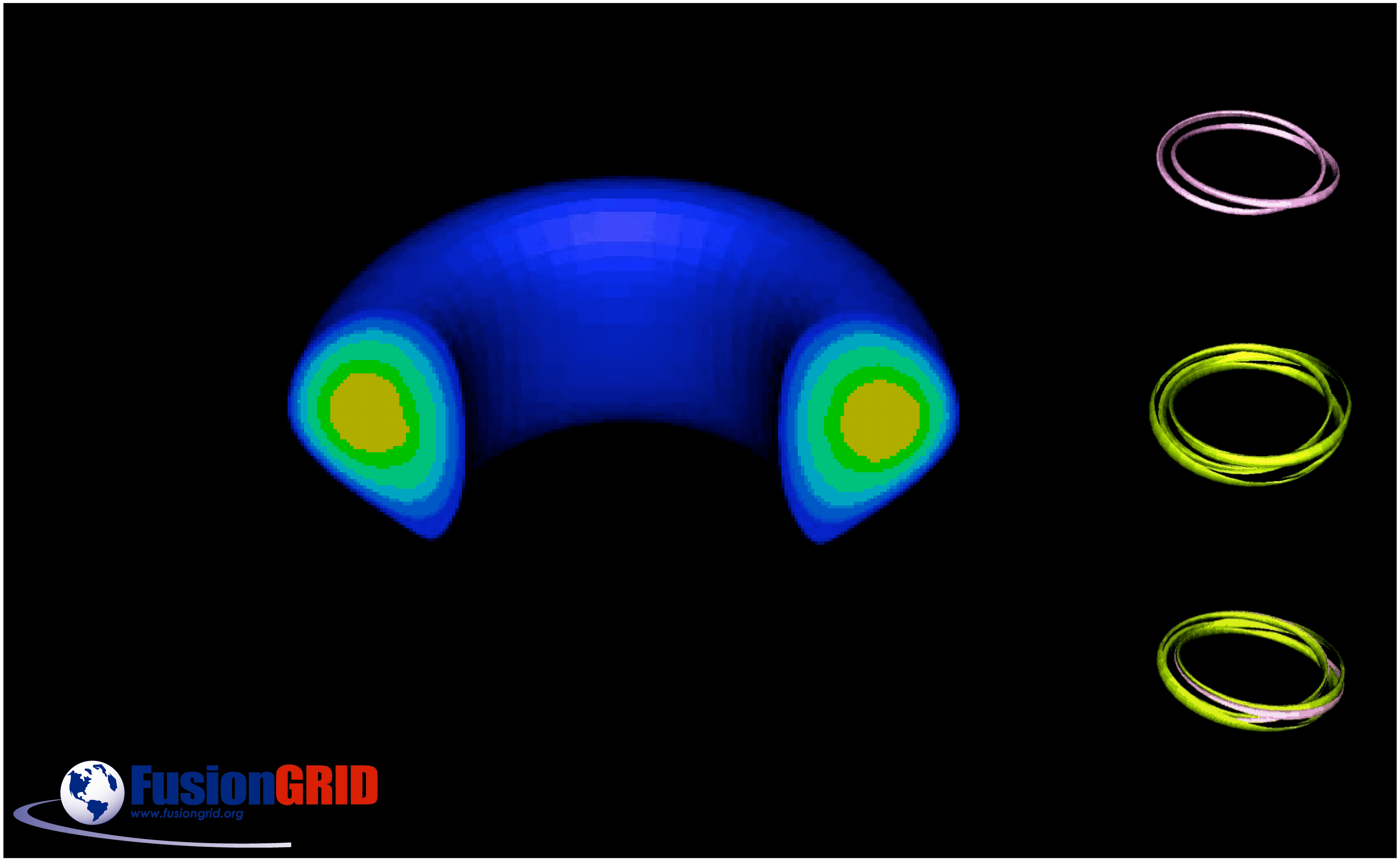


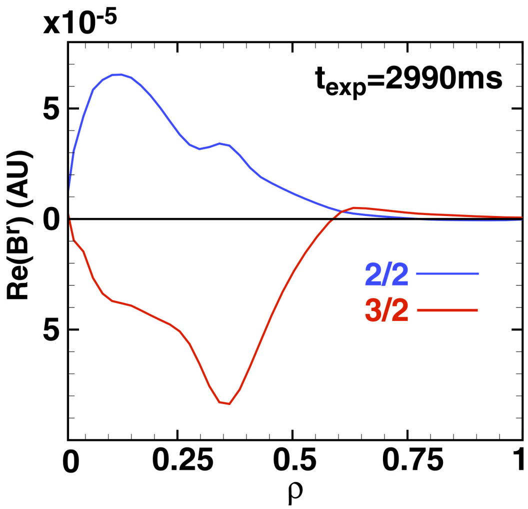
At later times, after saturation of n=1, the m=3 component of n=2 is dominant

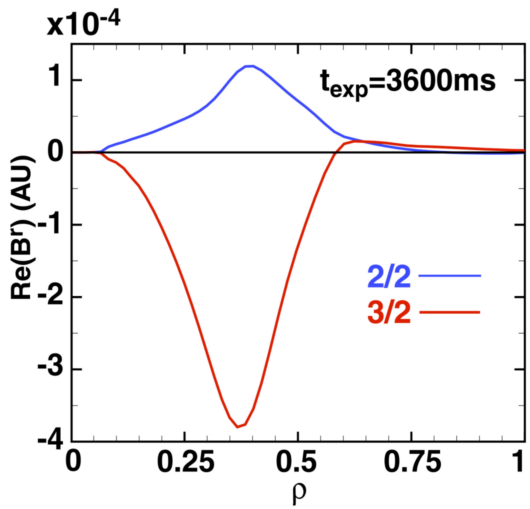
3/2 decays in simulation of 2990ms

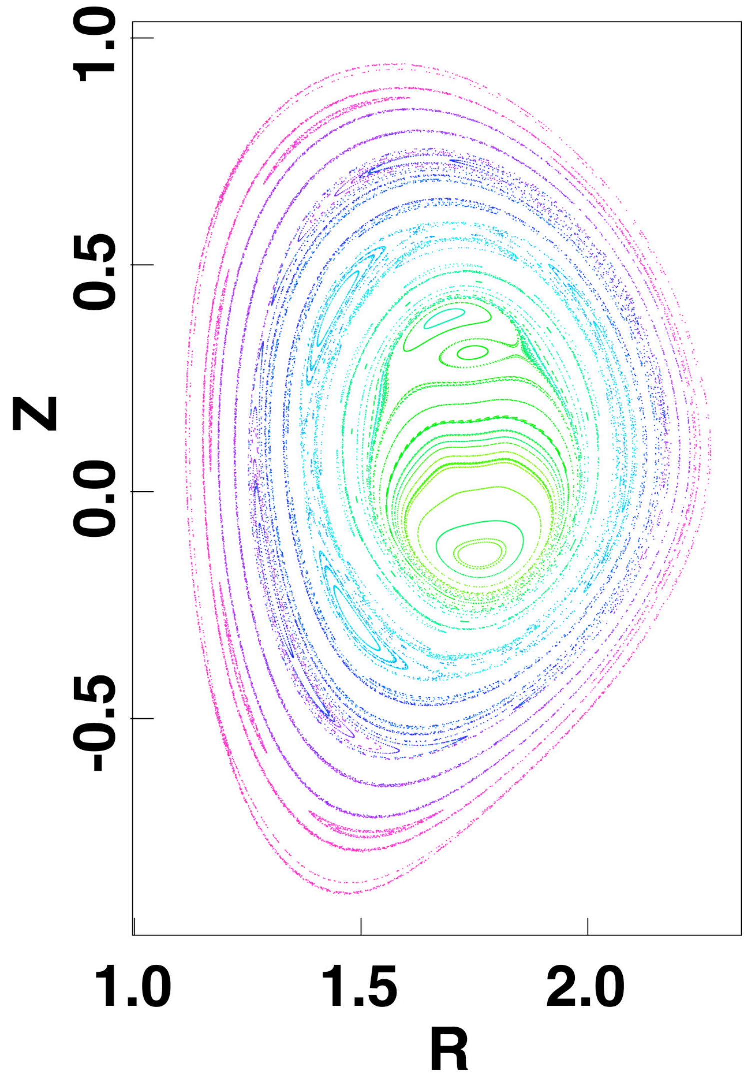
3/2 is larger and grows to saturation in simulation of 3600ms

Eigenfunctions show m=3 to be dominant n=2 component.

m=1 is dominant n=1.





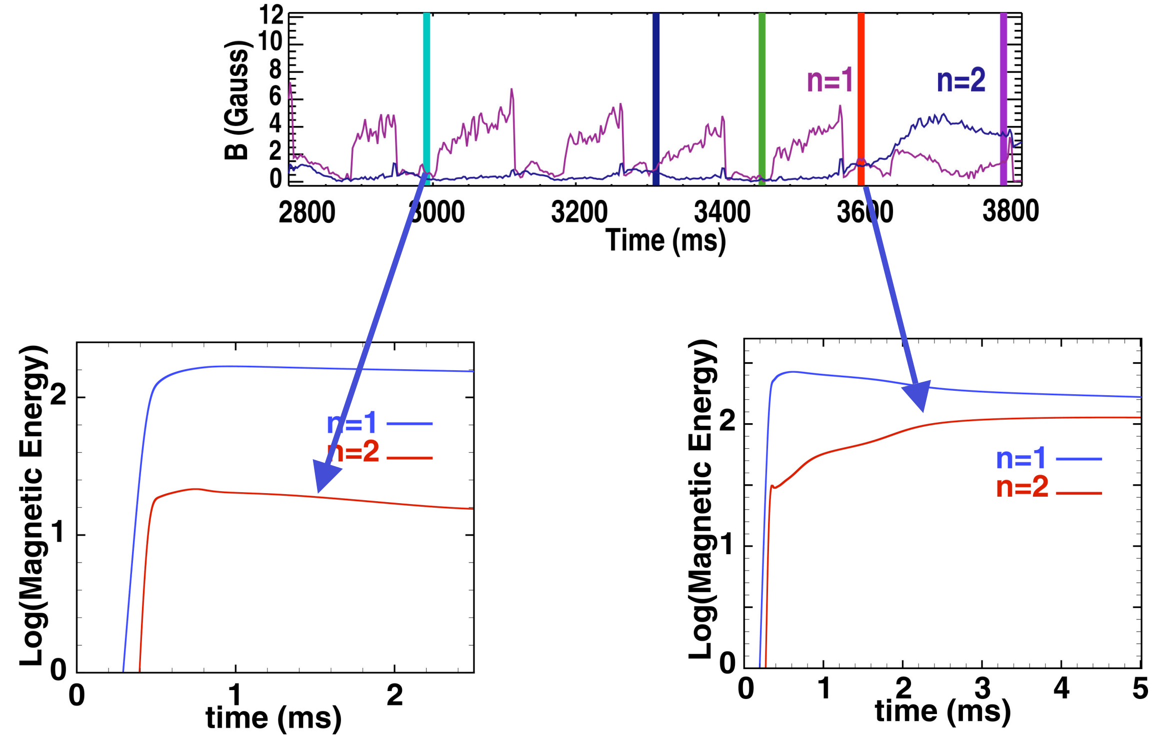


Dynamics of NIMROD Simulations Indicate That Linear □□ Drive Affects Islands Of Finite Width

m/n=3/2 is unstable for 3600ms, when n=1 mode decays.

Nonlinear coupling effects do not prevent later seed island growth.

Equilibrium at 3600ms is closer to ideal limit and has larger linear drive.



The pole in □□effects the stability of NTM seed islands

DIII-D Experiment was Designed and Performed to Determine Effect of Poles in □□on Tearing Stability

Experiment designed to isolate □□pole mechanism

- avoid other modes
- vary d$□_N$/dt on approach to onset of 2/1

Prediction was made that for spontaneous NTMs, evolution should depend on rate of approach to ideal boundary.

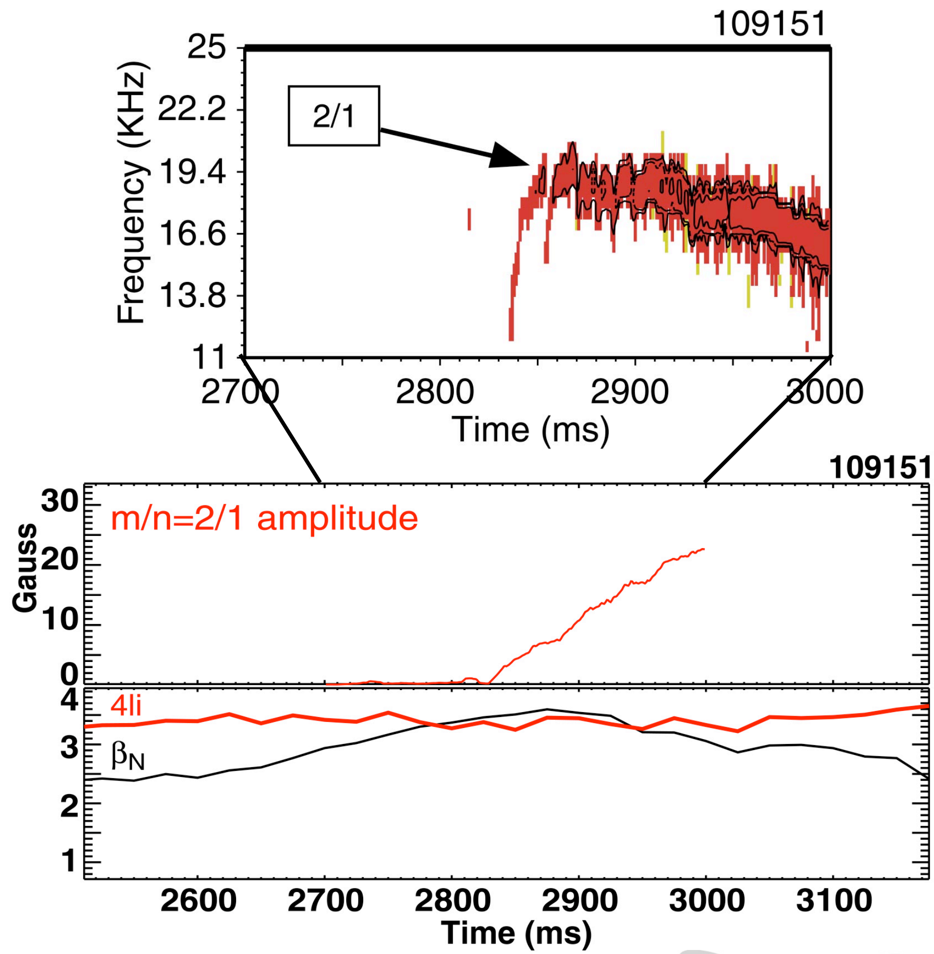
- Measure $□_N$ at point where w~w_d as a function of d$□_N$/dt

Forced reconnection gives random w at random t, and should not show a correlated function like □□(□)

Spontaneous/Seedless NTMs were Generated to Isolate Effect of □□Pole Onset Mechanism

Neutral beam injection applied at increasing rates up to □ limit for 2/1 NTM onset.

Experimental data indicates spontaneous onset in many cases.



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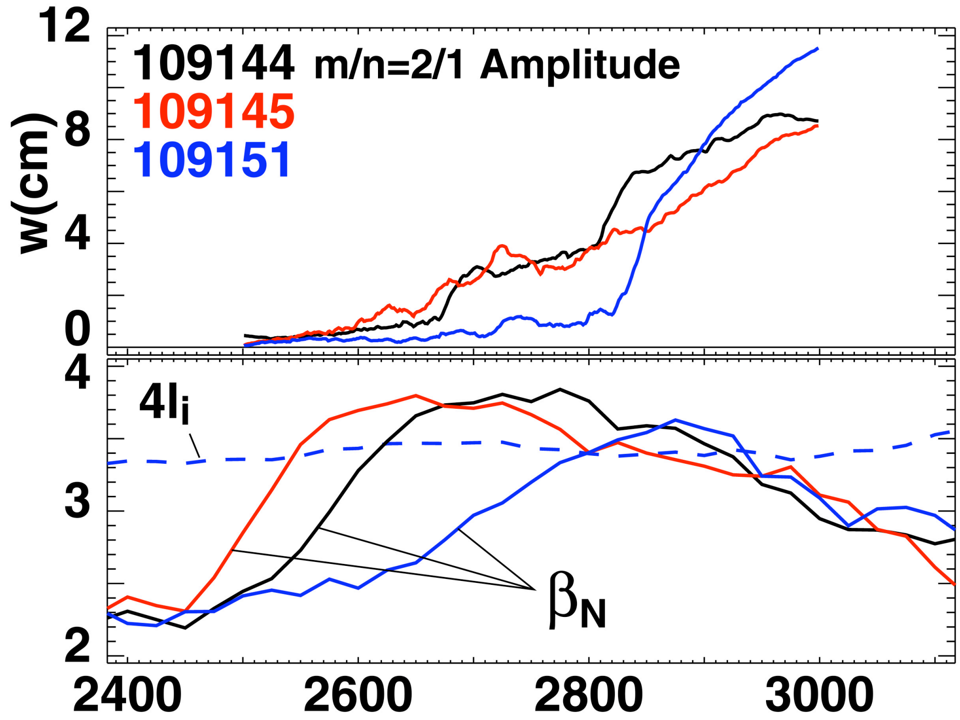

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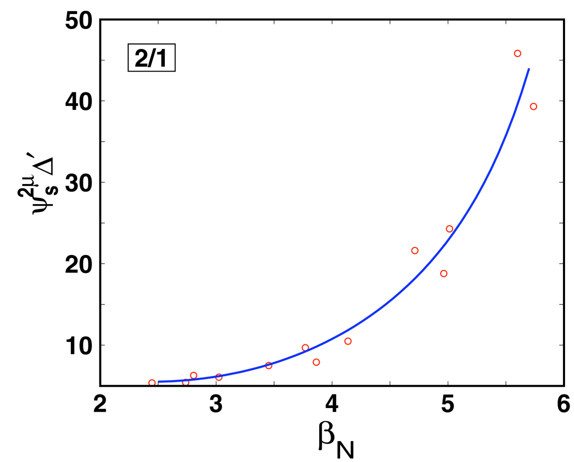
Experimental Data Confirms New Theoretical Prediction from □□Pole Model

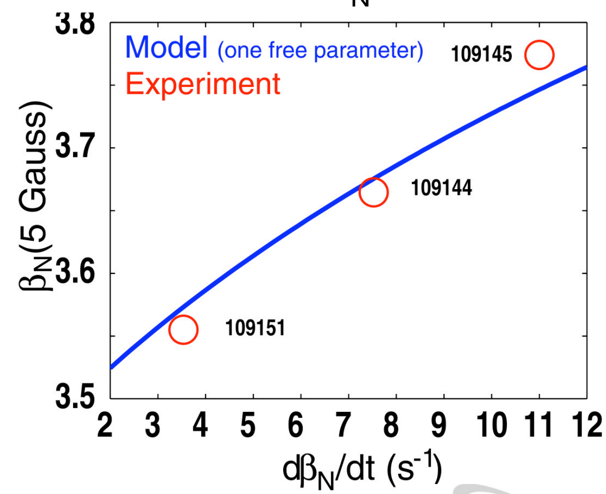
Model uses □$_N$(t) from experiment and □□(□$_N$) in island evolution equation.

Pressure profiles modified in model equilibria and □□at 2/1 surface calculated. Result is a function □□(□$_N$).

D_{pol} is single constant free parameter fit to find □$_N$ vs. d□$_N$/dt at mode onset.

Results support hypothesis that □□is increasing rapidly in time, consistent with theoretical pole model.



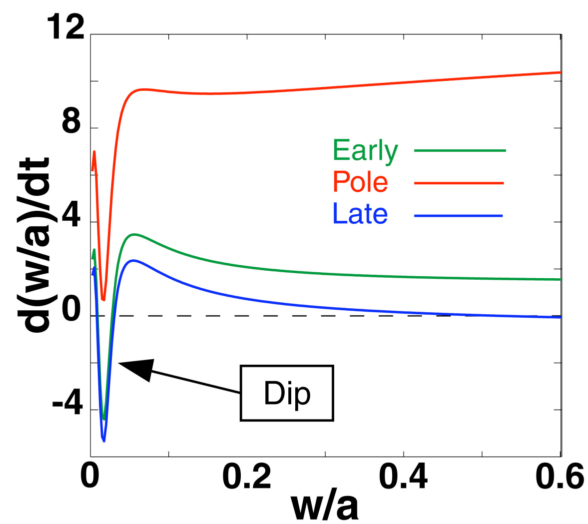


Modeling of β_N Pole Experiment Reproduces dw/dt Lull Shortly After Onset

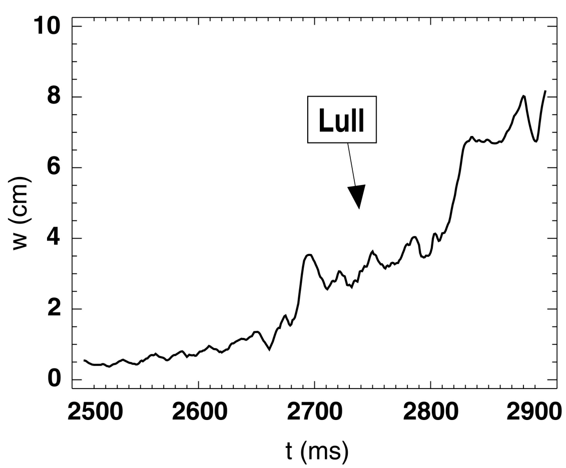
D_{pol} term negligible at small island width. A model for D_{pol} is assumed which decays at $w<w_b$.

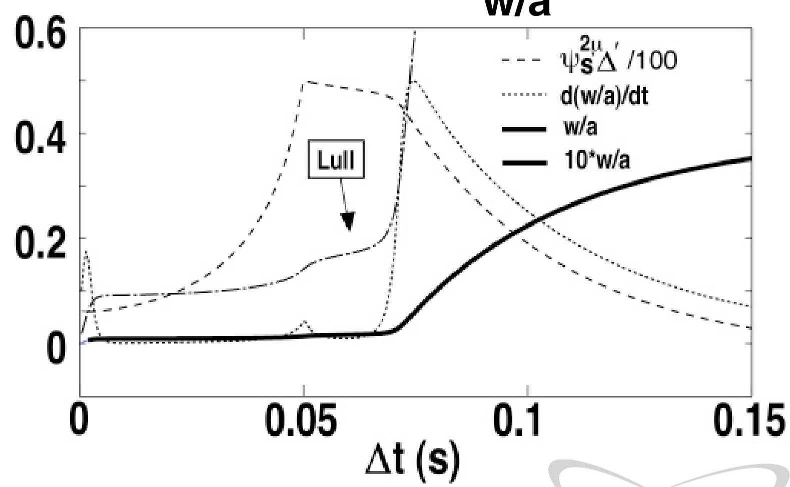
$$\frac{dw}{dt} = \frac{\eta^*}{k_0}\left(\Delta^*\left[\beta_N(t)\right] + \frac{wD_{tot}}{w_d^2 + w^2} + \frac{wD_{pol}}{w_b^4 + w^4}\right)$$

Resulting phase space plots have a Dip in growth rates.



$\beta_N(t)$ driving function reproduces Lull in growth





GENERAL ATOMICS

DIII-D NATIONAL FUSION FACILITY SAN DIEGO

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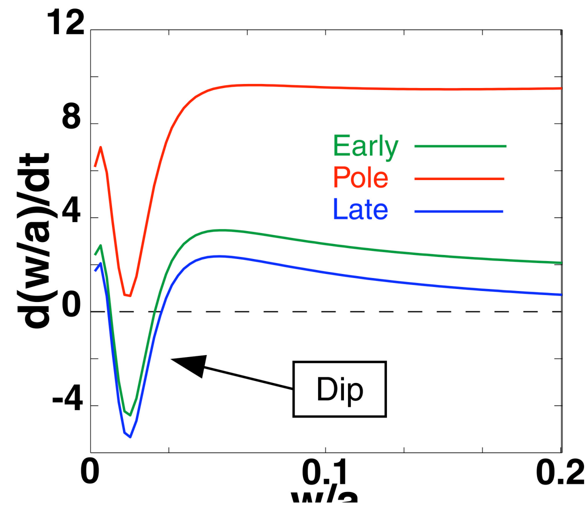
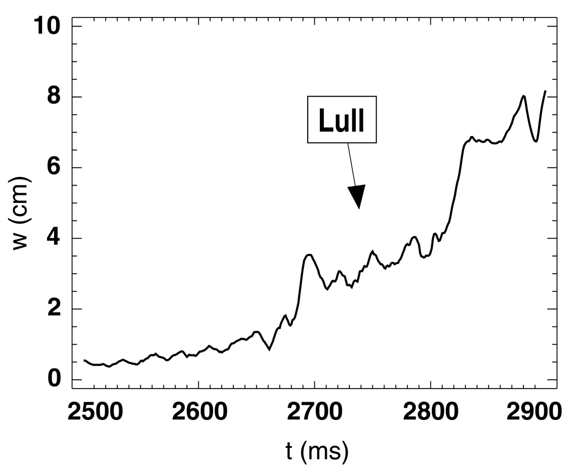
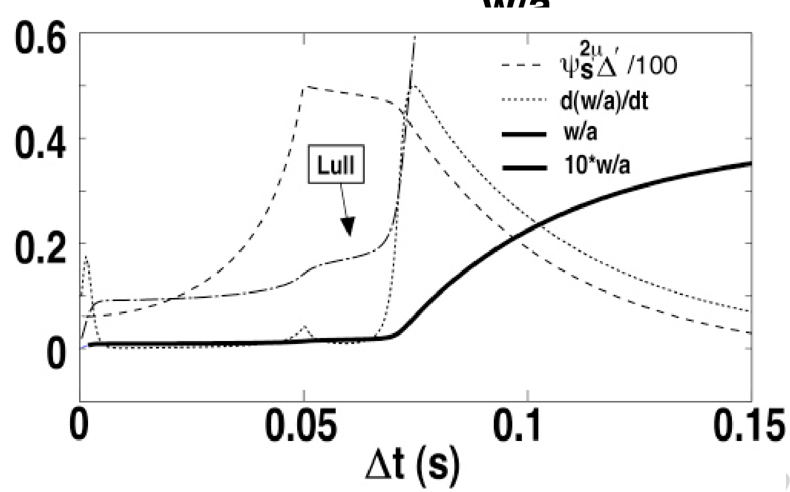


□N(t) driving function reproduces Lull in growth





Summary of Results

- Increasing □ sharply increases □□near ideal limit, due to approaching a pole, destabilizing NTMs
  - In a sawtooth seeding a NTM case, eventual onset was caused by increased □□due to approach of a pole
  - Nonlinear NIMROD simulations confirm that axisymmetric □□ is meaningful for nonlinearly coupled finite islands

- DIII-D experiment confirmed effect of poles in □□on NTM stability with varying rates of d□/dt causing spontaneous NTMs
  - Data from early evolution are in agreement with prediction, both as function of d$□_N$/dt and detailed time evolution.