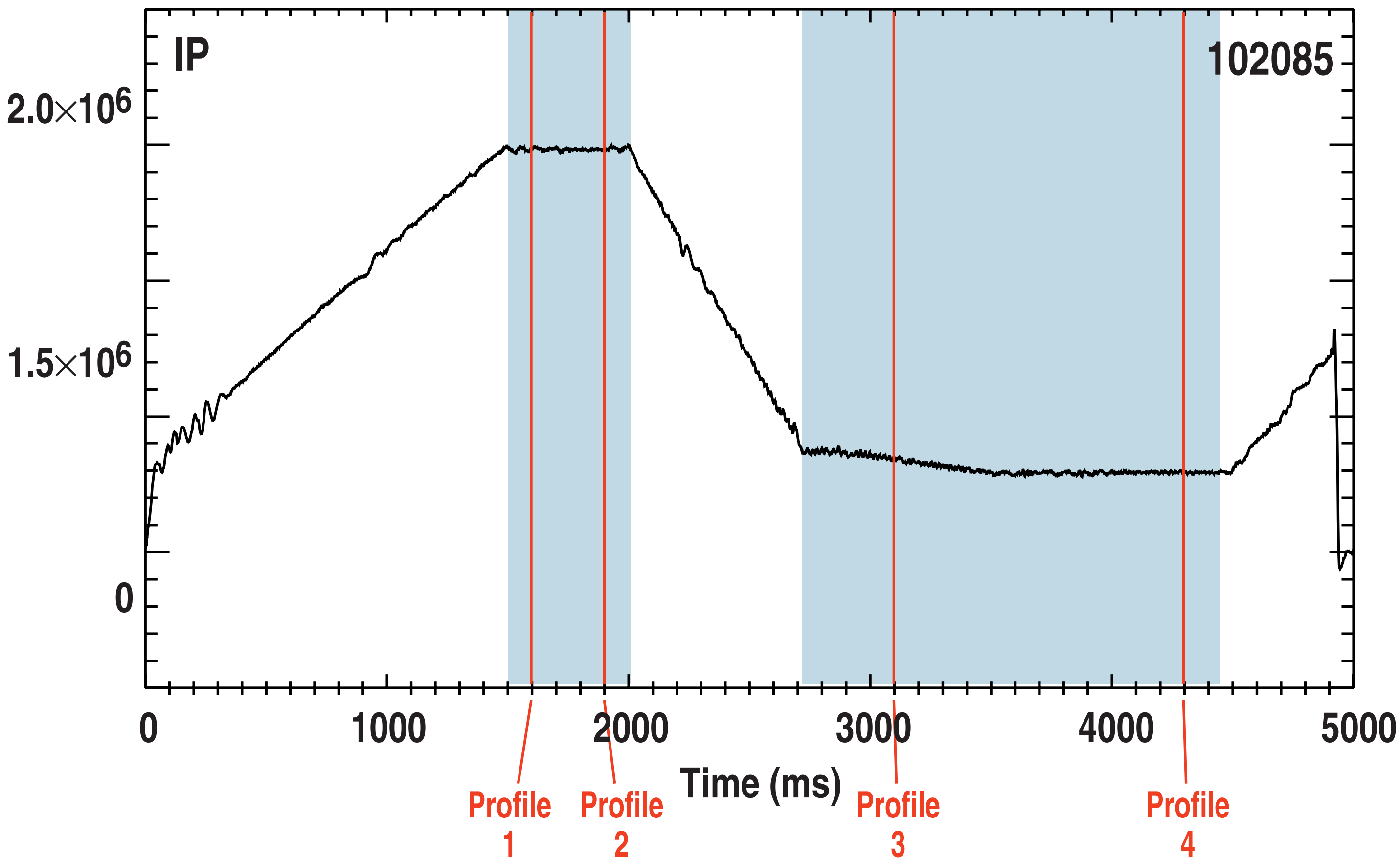
Need a Model to Predict Experimentally Observed Density Profiles

DENSITY PROFILE SHAPES IN DIII–D L–MODE PLASMAS DEPEND ON THE q PROFILE

- Change q by current ramps

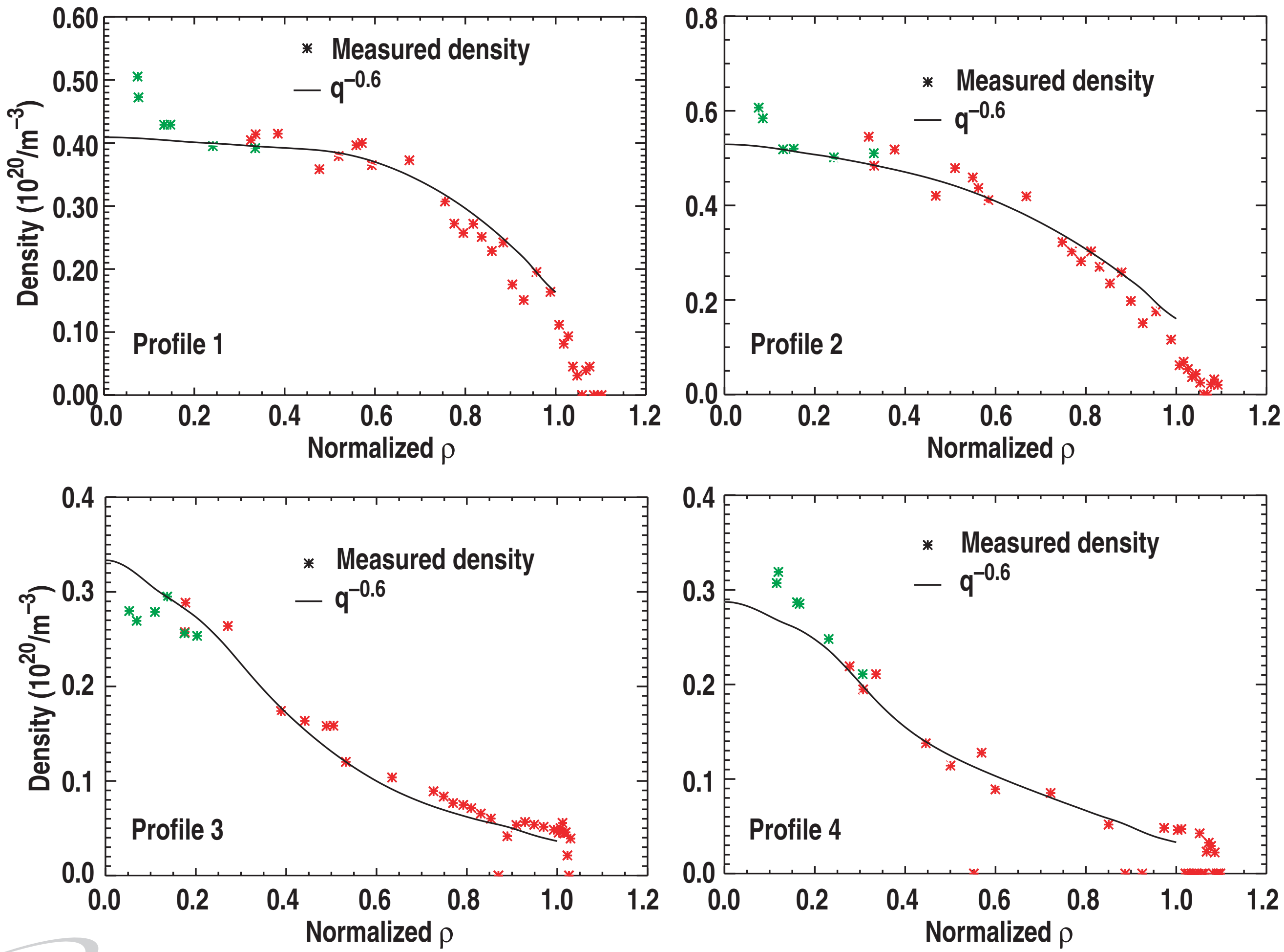


DIII–D NATIONAL FUSION FACILITY SAN DIEGO

084-02/jy

DENSITY PROFILES IN DIII–D L–MODE PLASMAS HAVE A SHAPE LIKE $(q)^{-0.6}$

- q is the usual safety factor



DIII–D
NATIONAL FUSION FACILITY
SAN DIEGO

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This experimentally observed result can be explained if,

$$\Gamma = -Dn\left[\frac{1}{n}\frac{\partial n}{\partial \rho} + \xi \frac{1}{q}\frac{\partial q}{\partial \rho}\right],$$

Then with large D and small Γ,

$$n_e \propto (q)^{-\xi}$$

This result can be obtained from the Drift Kinetic Equation using a certain ordering and certain set of assumptions

(It is well known that this relation for the particle transport can be obtained by expressing the Kinetic Equation in terms of the actions and action angles)

Detailed Theory For Model

The Drift Kinetic Equation is given by, (We consider only the ions)

$$\frac{\partial \bar{f}}{\partial t} + \left(\vec{\mathbf{v}}_{\parallel} + \vec{\mathbf{v}}_{d}\right)\cdot \nabla \bar{f} + e\left(\vec{\mathbf{v}}_{\parallel} + \vec{\mathbf{v}}_{d}\right)\cdot \vec{\mathbf{E}}\,\frac{\partial \bar{f}}{\partial K} = C\left(\bar{f}\right).$$ with,

$$\bar{f} = \left\langle \bar{f}\right\rangle_{ens} + \tilde{f}$$

$$\vec{\mathbf{v}}_{d} = \left\langle \vec{\mathbf{v}}_{d}\right\rangle_{ens} + \tilde{\mathbf{v}}_{d} \equiv \vec{\mathbf{v}}_{D} + \tilde{\mathbf{v}}_{E} \equiv \vec{\mathbf{v}}_{d0} + \vec{\mathbf{v}}_{E0} + \tilde{\mathbf{v}}_{E}$$

$$\vec{\mathbf{v}}_{D} = \frac{\hat{\mathbf{b}}}{\Omega} \times \left[\left(\mathrm{v}_{\parallel}^{2} + \mathrm{v}^{2}\right)\frac{\nabla B}{2B} + \frac{e\nabla\Phi_{0}}{m}\right] \qquad \tilde{\mathbf{v}}_{E} = -\frac{c\nabla\tilde{\Phi}\times\hat{\mathbf{b}}}{B}$$

Then we make the following ordering assumptions;

$$\frac{\tilde{f}}{\bar{f}} \sim \frac{f_1}{f_M} \sim \frac{e\tilde{\Phi}}{T} \sim \frac{\tilde{n}}{n} \sim \frac{a}{qR} \sim \varepsilon << 1\,,\quad \varepsilon \sim \rho_L / a\,,\ \text{and}\ k_{\parallel}/k_{\perp} \sim \varepsilon,\ \nu \sim \varepsilon^{2}$$

$$\bar{f} = \left\langle \bar{f}_0\right\rangle_{ens} + \left\langle \bar{f}_1\right\rangle_{ens} + \tilde{f} + O(\varepsilon) \equiv f_M + f_1 + \tilde{f} + O(\varepsilon);\qquad C(f) \sim \varepsilon \ \text{and}\ \langle E_{\parallel}\rangle \sim \varepsilon$$

The Drift Kinetic Equation for the Ions then becomes,

$$\vec{\mathbf{v}}_{\parallel}\cdot\nabla f_1+\vec{\mathbf{v}}_{d0}\cdot\nabla f_M \approx -\left[\left\langle\tilde{\mathbf{v}}_E\cdot\nabla\tilde{f}\right\rangle_{ens}+e\mathrm{v}_{\parallel}\left\langle\tilde{E}_{\parallel}\frac{\partial\tilde{f}}{\partial K}\right\rangle_{ens}+e\left\langle\vec{\mathbf{v}}_{d0}\cdot\tilde{\mathbf{E}}\frac{\partial\tilde{f}}{\partial K}\right\rangle_{ens}\right]$$

and,

$$\frac{\partial\tilde{f}}{\partial \mathrm{t}}+\left(\vec{\mathbf{v}}_{\parallel}+\vec{\mathbf{v}}_d\right)\cdot\nabla\tilde{f}+\tilde{\mathbf{v}}_E\cdot\nabla f_M-e\left(\vec{\mathbf{v}}_{\parallel}+\vec{\mathbf{v}}_d\right)\cdot\nabla\tilde{\phi}\frac{\partial f_M}{\partial K}=0$$

The radial component of the particle flux is given by,

$$\Gamma=\int d^3\mathrm{v}\left\langle\left\langle\vec{\mathbf{v}}_d\cdot\nabla\psi\, f\right\rangle_{ens}\right\rangle_{fs}/A,\ \text{or}$$

$$\Gamma=\Gamma_a+\Gamma_E=\int d^3\mathrm{v}\left\langle\vec{\mathbf{v}}_{d0}\cdot\nabla\psi\, f_1\right\rangle_{fs}/A+\int d^3v\left\langle\left\langle\tilde{\mathbf{v}}_E\cdot\nabla\psi\,\tilde{f}\right\rangle_{ens}\right\rangle_{fs}/A\,.$$

Note: $\Gamma_E=\left\langle\tilde{n}\tilde{\mathbf{v}}_E\right\rangle_{ens,fs}\cdot\hat{\mathbf{r}}$

Need to obtain more explicit expression for Γ_a

Take $v_\parallel/B$ moment of equation for f_1 and orbit average

$$\left\langle \frac{\mathrm{v}_\parallel}{B}\vec{\mathbf{v}}_\parallel \cdot \nabla f_1 \right\rangle_{\mathrm{o}} + \left\langle \frac{\mathrm{v}_\parallel}{B}\vec{\mathbf{v}}_{d0} \cdot \nabla f_M \right\rangle_{\mathrm{o}} =$$

$$\approx \left\langle \frac{\mathrm{v}_\parallel}{B}\left[\left\langle \tilde{\vec{\mathbf{v}}}_E \cdot \nabla \tilde{f}\right\rangle_{ens} + e\mathrm{v}_\parallel \left\langle \tilde{E}_\parallel \frac{\partial \tilde{f}}{\partial K}\right\rangle_{ens} + e\left\langle \vec{\mathbf{v}}_{d0} \cdot \tilde{\mathbf{E}} \frac{\partial \tilde{f}}{\partial K}\right\rangle_{ens} \right] \right\rangle_{\mathrm{o}}$$

Use annihilation operator $\left\langle \vec{\mathbf{v}}_\parallel \cdot \nabla F\right\rangle_{\mathrm{o}} = 0$, **and**

$$\left\langle \frac{\mathrm{v}_\parallel}{B}\vec{\mathbf{v}}_\parallel \cdot \nabla f_1 \right\rangle_{\mathrm{o}} = -\frac{\Omega}{IB}\left\langle \vec{\mathbf{v}}_{d0} \cdot \nabla \psi \, f_1 \right\rangle_{\mathrm{o}},$$ (using $\vec{\mathbf{v}}_d \cdot \nabla\psi = \mathrm{v}_\parallel I \vec{\mathbf{B}} \cdot \nabla\left(\mathrm{v}_\parallel / B\right)$)

and

$$\left\langle \int d^3\mathrm{v} F\right\rangle_{fs} = \int d^3\mathrm{v} \frac{\mathrm{v}_\parallel}{B}\left\langle \frac{B}{\mathrm{v}_\parallel}\right\rangle_b \langle F\rangle_{\mathrm{o}},$$

to obtain,

$$\Gamma_a = \frac{IB}{\Omega}\left\langle \int d^3\mathrm{v} \frac{v_\parallel}{B}\left[\left\langle \tilde{\vec{\mathbf{v}}}_{\mathrm{E}} \cdot \nabla \tilde{f}\right\rangle_{ens} + e\mathrm{v}_\parallel \left\langle \tilde{\boldsymbol{E}}_\parallel \frac{\partial \tilde{f}}{\partial K}\right\rangle_{ens} + e\left\langle \vec{\mathbf{v}}_{d0} \cdot \tilde{\mathbf{E}} \frac{\partial \tilde{f}}{\partial K}\right\rangle_{ens} \right]\right\rangle_{fs} \Bigg/ A$$

small (the $e\mathrm{v}_\parallel \langle \tilde{E}_\parallel \, \partial\tilde{f}/\partial K\rangle_{ens}$ term) *small* (the $e\langle \vec{\mathbf{v}}_{d0}\cdot\tilde{\mathbf{E}} \, \partial\tilde{f}/\partial K\rangle_{ens}$ term)

Particle Flux Has Two Terms

$$\Gamma = \Gamma_a + \Gamma_E = \left(\frac{IB}{2\Omega} \int d^3\mathrm{v} \left\langle \frac{\mathrm{v}_\parallel}{B} \left\{ \left\langle \tilde{\mathbf{v}}_E \cdot \nabla \tilde{f} \right\rangle_{ens} \right\} \right\rangle_{fs} - \int d^3\mathrm{v} \left\langle \left\langle \tilde{\mathbf{v}}_E \cdot \nabla\psi\, \tilde{f} \right\rangle_{ens} \right\rangle_{fs} \right) \Big/ A$$

Need expression for $\tilde{f}$ from,

$$\frac{\partial \tilde{f}}{\partial t} + \left(\vec{\mathbf{v}}_\parallel + \vec{\mathbf{v}}_d \right) \cdot \nabla \tilde{f} + \tilde{\mathbf{v}}_E \cdot \nabla f_M - e\left(\vec{\mathbf{v}}_\parallel + \vec{\mathbf{v}}_d \right) \cdot \nabla \tilde{\phi} \frac{\partial f_M}{\partial K} = 0$$

Fourier transform in time and perpendicular space

$$\mathrm{v}_\parallel \frac{\partial \tilde{f}_{\omega,k}}{\partial \ell} - i\left(\omega - \omega_d \right) \tilde{f}_{\omega,k} = i\omega_{*d} \frac{e\tilde{\phi}_{\omega,k}}{T} f_M - \mathrm{v}_\parallel \frac{\partial}{\partial \ell} \left(\frac{e\tilde{\phi}_{\omega,k}}{T} \right) f_M$$

with

$$\omega_d = \vec{k} \cdot \vec{\mathbf{v}}_d \;;\quad \omega_{*d} = \omega_* - \omega_d \;;\quad \omega_* = k_\perp \frac{cT}{eB} \frac{1}{L_n} \left[1 + \left(\frac{K}{T} - \frac{3}{2} \right) \eta_i \right]$$

Replace $\partial/\partial\ell$ with $\square 1/\square qR$, (neglect $k_{\|}v_{\|}$ with respect to ω_d)

$$-i\left(\omega_k - \omega_d\right)\tilde{f}_{\square,k} \square \frac{v_{\|}}{\square qR}\tilde{f}_{\square,k} = i\omega_{*d}\frac{e\tilde{\phi}_k}{T}f_M + \frac{v_{\|}}{\square qR}\left(\frac{e\tilde{\phi}_{\square,k}}{T}\right)f_M,$$

then

$$\tilde{f}_{\square,k} = -\frac{e\tilde{\phi}_{\square,k}}{T}f_M + \frac{\left(\omega_k - \omega_d\right)^2 - \omega_{*d}\left(\omega_k - \omega_d\right)}{\left(\omega_k - \omega_d\right)^2 + \left(v_{\|}/\square qR\right)^2}\frac{e\tilde{\phi}_{\square,k}}{T}f_M$$

$$\square\frac{i\left(\omega_k - \omega_*\right)v_{\|}/\square qR}{\left(\omega_k - \omega_d\right)^2 + \left(v_{\|}/\square qR\right)^2}\frac{e\tilde{\phi}_{\square,k}}{T}f_M .$$

Then when $\omega_k \sim \omega_d$, the non-adiabatic part is,

$$\tilde{f}_{\square,k} \approx -i\left(\omega_k - \omega_*\right)\frac{\square qR}{v_{\|}}\frac{e\tilde{\phi}_{\square,k}}{T}f_M .$$

The inverse transform of the non-adiabatic part can be written as,

$$\tilde{f}_{non-adiabatic} \sim i\,\square\square_{*}\,\frac{\square qR}{\mathrm{v}_{\parallel}}\,\frac{e\tilde{\square}(t)}{T}\,f_M.$$

Where *i* is a notational reminder that $\tilde{f}_{non-adiabatic}$ is approximately 90° out of phase with respect to $\tilde{\square}$.

Then (after some manipulation)

$$\left\langle \tilde{\mathbf{v}}_{\mathrm{E}} \cdot \square\tilde{f} \right\rangle_{ens} \square \frac{cRB_{\square}B_{\square}}{B^2}\, e\left\langle \tilde{E}_{\square}\, i\square\square_{*}\tilde{\square} \right\rangle_{ens} \frac{1}{T} \left[\frac{\square qR}{\mathrm{v}_{\parallel}} \frac{\partial f_M}{\partial \square} + f_M \frac{\partial}{\partial \square} \frac{\square qR}{\mathrm{v}_{\parallel}} \right]$$

Remember,

$$\square = \square_a + \square_E = \left[\frac{IB}{2\square} \square d^3\mathrm{v} \left\langle \frac{v_{\parallel}}{B} \left\{ \left\langle \tilde{\mathbf{v}}_{\mathrm{E}} \cdot \square\tilde{f} \right\rangle_{ens} \right\} \right\rangle_{fs} \square \square d^3\mathrm{v} \left\langle \left\langle \tilde{\mathbf{v}}_{\mathrm{E}} \cdot \square\square\,\tilde{f} \right\rangle_{ens} \right\rangle_{fs} \right] \Big/ A$$

Particle Flux is then given by,

$$\square A = \square \frac{IB}{2\square} \square d^3\mathrm{v} \left\langle \frac{\mathrm{v}_{\|}}{B} \left[\frac{cRB_{\square}B_{\square}}{B^2} e\left\langle \tilde{E}_{\square} i \square \square_{*} \tilde{\square} \right\rangle_{ens} \frac{1}{T} \left[\frac{\square qR}{\mathrm{v}_{\|}} \frac{\partial f_M}{\partial \square} + f_M \frac{\partial}{\partial \square} \frac{\square qR}{\mathrm{v}_{\|}} \right] \right] \right\rangle_{fs}$$

$$+ Ic \square d^3\mathrm{v} \left\langle \frac{B_{\square}}{B^2} \frac{1}{T} \left[\frac{\square qR}{\mathrm{v}_{\|}} \right] e\left\langle \tilde{E}_{\square} i \square \square_{*} \tilde{\square} \right\rangle_{ens} f_M \right\rangle_{fs}$$

If we assume that $\left\langle \tilde{E}_{\square} i \square \square_{*} \tilde{\square} \right\rangle_{ens}$ **is independent of velocity,**
then the last term velocity integrates to zero and,

$$\square = \square \frac{IB}{2\square} \left\langle \left[\frac{cRB_{\square}B_{\square}}{B^2} e\left\langle \tilde{E}_{\square} i \square \square_{*} \tilde{\square} \right\rangle_{ens} \frac{1}{T} \left[\frac{\square qR}{B} \frac{\partial n}{\partial \square} + n \frac{\partial}{\partial \square} \frac{\square qR}{B} \right] \right] \right\rangle_{fs} / A,$$

We can then approximate,

$$\square \square \square \frac{D}{q} \left[q \frac{\partial n}{\partial \square} + \square n \frac{\partial q}{\partial \square} \right[$$

Outline Of Particle Transport Model

In the drift approximation, the radial component of the particle flux is given by,

$$\Gamma = \int d^3\mathrm{v} \left\langle \left\langle \vec{\mathbf{v}}_d \cdot \nabla\psi\, f \right\rangle_{ens} \right\rangle_{fs} / A, \quad \textbf{or}$$

$$\Gamma = \Gamma_a + \Gamma_E = \underbrace{\int d^3\mathrm{v} \left\langle \vec{\mathbf{v}}_{d0} \cdot \nabla\psi\, f_1 \right\rangle_{fs} / A}_{\Gamma_a} + \underbrace{\int d^3\mathrm{v} \left\langle \left\langle \tilde{\mathbf{v}}_E \cdot \nabla\psi\, \tilde{f} \right\rangle_{ens} \right\rangle_{fs} / A}_{\Gamma_E} .$$

Note: $\Gamma_E = \left\langle \tilde{n} \tilde{\mathbf{v}}_E \right\rangle_{ens,fs} \cdot \hat{\mathbf{r}}$ **,** $\tilde{\mathbf{v}}_E$ **is the fluctuating ExB velocity**

Derive Expression for □$_a$

We will show that

$$\square_a \equiv \square d^3\mathrm{v}\left\langle\left\langle \vec{\mathbf{v}}_{d0}\cdot\hat{\mathbf{r}}\, f_1\right\rangle_{ens}\right\rangle_{fs} \square \frac{IB}{2\square}\square d^3\mathrm{v}\left\langle \frac{\mathrm{v}_{\parallel}}{B}\left\{\left\langle \tilde{\mathbf{v}}_{\mathrm{E}}\cdot\square\tilde{f}\right\rangle_{ens}\right\}\right\rangle_{fs}$$

$$I(\square) \equiv BR \;\; ; \;\; \square \equiv Ion\,Cyclotron\,Frequency$$

□$_a$ is a neoclassical type of turbulent flux which is due to the drifts of the particles which are perturbed by the turbulence.

f_1 is the quasi stationary first order distribution function which is generated by the fluctuations. f_1 can be formally solved for using the Drift Kinetic Equation (see detailed theory or outline below)

The Drift Kinetic Equation for the Ions has two parts,

$$\vec{\mathbf{v}}_{\parallel}\cdot\nabla f_1+\vec{\mathbf{v}}_{d0}\cdot\nabla f_M \approx -\left[\left\langle\tilde{\mathbf{v}}_E\cdot\nabla\tilde{f}\right\rangle_{ens}+e\mathrm{v}_{\parallel}\left\langle\tilde{E}_{\parallel}\frac{\partial\tilde{f}}{\partial K}\right\rangle_{ens}+e\left\langle\vec{\mathbf{v}}_{d0}\cdot\tilde{\mathbf{E}}\frac{\partial\tilde{f}}{\partial K}\right\rangle_{ens}\right]$$

and,

$$\frac{\partial\tilde{f}}{\partial t}+\left(\vec{\mathbf{v}}_{\parallel}+\vec{\mathbf{v}}_d\right)\cdot\nabla\tilde{f}+\tilde{\mathbf{v}}_E\cdot\nabla f_M - e\left(\vec{\mathbf{v}}_{\parallel}+\vec{\mathbf{v}}_d\right)\cdot\nabla\tilde{\phi}\frac{\partial f_M}{\partial K}=0$$

Use annihilation operator $\left\langle\vec{\mathbf{v}}_{\parallel}\cdot\nabla F\right\rangle_o=0$, **and**

$$\frac{\Omega}{IB}\left\langle\vec{\mathbf{v}}_{d0}\cdot\nabla\psi\, f_1\right\rangle_o=-\left\langle\frac{\mathrm{v}_{\parallel}}{B}\vec{\mathbf{v}}_{\parallel}\cdot\nabla f_1\right\rangle_o,\quad \left(\text{using } \Omega\vec{\mathbf{v}}_d\cdot\nabla\psi=-\mathrm{v}_{\parallel}I\vec{\mathbf{B}}\cdot\nabla\left(\mathrm{v}_{\parallel}/B\right)\right)$$

This then yields,

$$\Gamma_a=\int d^3\mathrm{v}\left\langle\left\langle\vec{\mathbf{v}}_{d0}\cdot\hat{\mathbf{r}}\, f_1\right\rangle_{ens}\right\rangle_{fs}\approx\frac{IB}{2\Omega}\int d^3\mathrm{v}\left\langle\frac{\mathrm{v}_{\parallel}}{B}\left\{\left\langle\tilde{\mathbf{v}}_{\mathrm{E}}\cdot\nabla\tilde{f}\right\rangle_{ens}\right\}\right\rangle_{fs}$$

$I(\psi)\equiv BR\ ;\ \Omega\equiv Ion\,Cyclotron\,Frequency$

The perturbed part of the Drift Kinetic Equation

$$\frac{\partial \tilde{f}}{\partial t} + \left(\vec{\mathbf{v}}_{\parallel} + \vec{\mathbf{v}}_{d}\right)\cdot\nabla\tilde{f} + \tilde{\mathbf{v}}_{E}\cdot\nabla f_{M} - e\left(\vec{\mathbf{v}}_{\parallel} + \vec{\mathbf{v}}_{d}\right)\cdot\nabla\tilde{\phi}\,\frac{\partial f_{M}}{\partial K} = 0$$

also provides an approximate value for

$$\left\langle \tilde{\mathbf{v}}_{E}\cdot\nabla\tilde{f}\right\rangle_{ens} \approx \frac{cRB_{\theta}B_{\phi}}{B^{2}}\, e\left\langle \tilde{E}_{\theta}\, i\omega\,\square_{*}\tilde{\phi}\right\rangle_{ens} \frac{1}{T}\left[\frac{\square qR}{\mathrm{v}_{\parallel}}\frac{\partial f_{M}}{\partial \psi} + f_{M}\frac{\partial}{\partial \psi}\frac{\square qR}{\mathrm{v}_{\parallel}}\right]$$

The q dependence comes from that fact that $k_{\parallel} \sim 1/qR$

The velocity integral then yields

$$\Gamma_{a} \approx -\frac{IB}{2\pi}\left\langle \left[\frac{cRB_{\theta}B_{\phi}}{B^{2}}\, e\left\langle \tilde{E}_{\theta}\, i\omega\,\square_{*}\tilde{\phi}\right\rangle_{ens} \frac{1}{T}\right]\left[\frac{\square qR}{B}\frac{\partial n}{\partial \psi} + n\frac{\partial}{\partial \psi}\frac{\square qR}{B}\right]\right\rangle_{fs}$$

We can then approximate,

$$\Gamma_{a} \approx -\frac{D}{q}\left[q\frac{\partial n}{\partial \psi} + \square n\frac{\partial q}{\partial \psi}\right]$$

Particle Transport Model

$$\square = \square_a + \square_E = \square D \left(\square \frac{\partial n}{\partial \square} + \square \frac{1}{q}\frac{\partial q}{\partial \square} n \square \right) + \left\langle \tilde{n}\tilde{\mathbf{v}}_E \right\rangle \cdot \hat{\mathbf{r}}$$

$\square_{\mathbf{a}}$ $\square_E$

D, □, $\tilde{n}$, and $\tilde{v}_E$ can probably only be obtained by a nonlinear turbulence simulation. Or $\square_{\mathbf{a}}$ can be obtained directly from a turbulence simulation code which can calculate

$$\square_a \ \square \ \frac{IB}{2\square} \square d^3\mathrm{v} \left\langle \frac{v_{\parallel}}{B} \left\{ \left\langle \tilde{\mathbf{v}}_{\mathrm{E}} \cdot \square \tilde{f} \right\rangle_{ens} \right\} \right\rangle_{fs}$$

CONCLUSIONS

There are two contributions to the particle flux

$$\Gamma = \Gamma_a + \Gamma_E = \int d^3\mathrm{v} \left\langle \vec{\mathbf{v}}_{d0} \cdot \hat{\mathbf{r}}\, f_1 \right\rangle_{fs} + \int d^3\mathrm{v} \left\langle \left\langle \tilde{\mathbf{v}}_E \cdot \hat{\mathbf{r}}\, \tilde{f} \right\rangle_{ens} \right\rangle_{fs}$$

With certain approximations, Γ_a yields pinch velocities dependent on q

$$\Gamma_a \approx -\frac{D}{q}\left(q\frac{\partial n}{\partial \psi} + \alpha h \frac{\partial q}{\partial \psi} \right),$$

i.e. Vpinch ~ -∇ ln q

Other approximations yield pinch terms of the types

Vpinch ~ 1/2 ∇ ln T or **Vpinch ~ ∇ ln Tq**

Empirical Studies of Particle Transport Model

- **Use total particle flux from TRANSP or ONETWO run**
- **Use Particle Transport Model to match the DIII-D density profiles**
- **Gain insight into the relative importance of $\square_a$ and $\square_E$**
- **Determine where the pinch terms are important**

$$\square = \square_a + \square_E = \square D_a \left(\frac{\partial n}{\partial \square} + \square \frac{1}{q} \frac{\partial q}{\partial \square} n \right) + \left\langle \tilde{n} \tilde{\mathbf{v}}_E \right\rangle \cdot \hat{\mathbf{r}}$$

$\square_{\mathbf{a}}$ $\square_E$

Assume $\square_E \equiv \square D_E \frac{\partial n}{\partial \square}, \quad D_a = \frac{2}{3} \square_i, \quad and \quad D_E = \square D_a$

then adjust $\square$, and $\square$ to match density profile

Need to solve

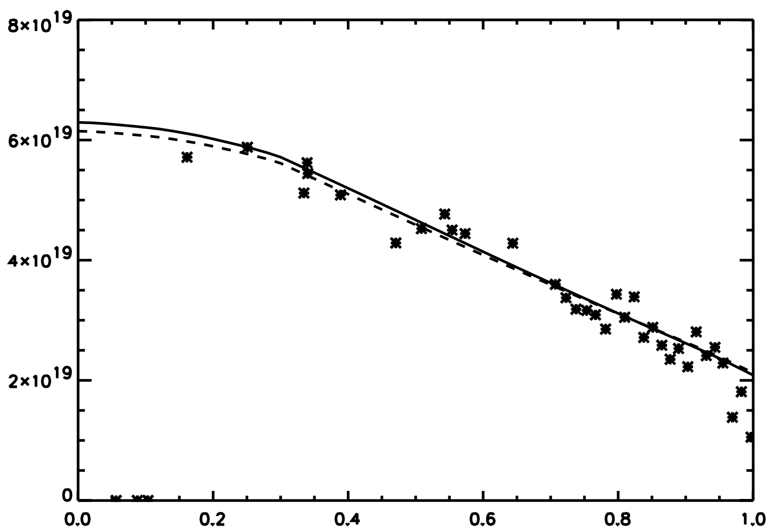
$$\square = \square D_a \left(\frac{\partial n}{\partial \square} + \square \frac{1}{q} \frac{\partial q}{\partial \square} n \right) \square D_E \frac{\partial n}{\partial \square},$$

for $\mathbf{n}(\square)$ with $\square$ and $\square_{\mathbf{i}}$ from ONETWO or TRANSP

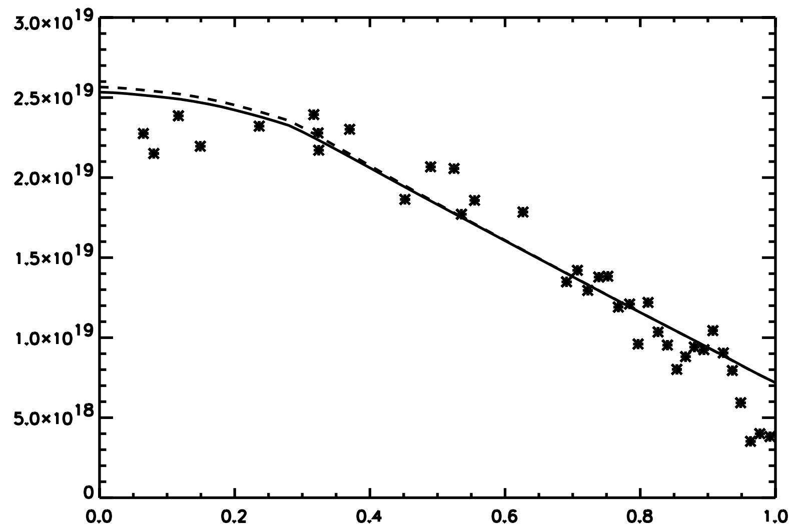
L-mode Plasmas

- **Most L mode density profiles can be matched with**
  □ = 1.0 and 0.25 < □ < 0.35 or □ = 0.0 and 0.7 < □ < 0.75

High density:
□ = 1.0 and □ = 0.35
□ = 0.7 and □ = 0.0



Low density:
□ = 1.00 and □ = 0.25
□ = 0.75 and □ = 0.0

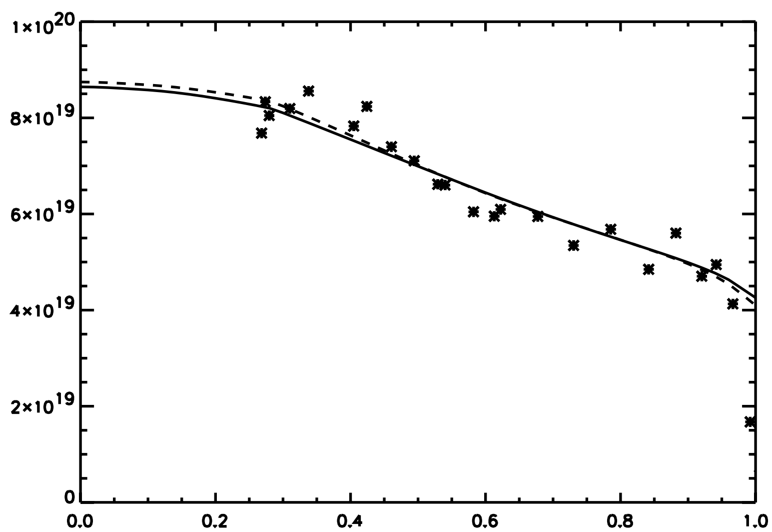


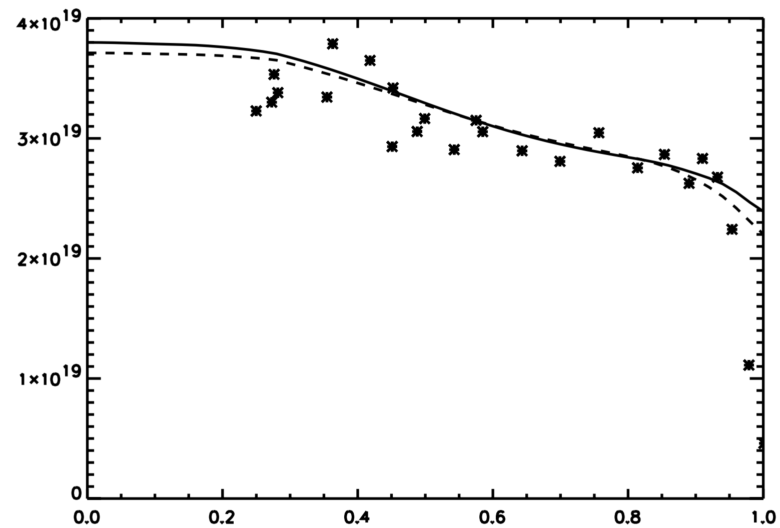
Sawtoothing ELMy H-mode Plasmas

- **Many sawtoothing ELMy H-mode density profiles are matched with □ = 1.0 and 2.0 < □ < 3.5 or □ = 0.0 and 0.15 < □ < 0.3**

High density:
□ = 1.0 and □ = 2.0
□ = 0.3 and □ = 0.0

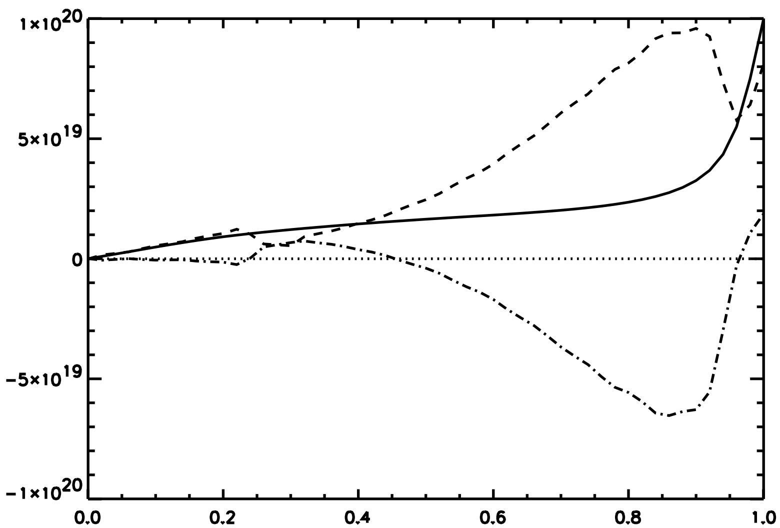
Low density:
□ = 1.00 and □ = 3.5
□ = 0.15 and □ = 0.0



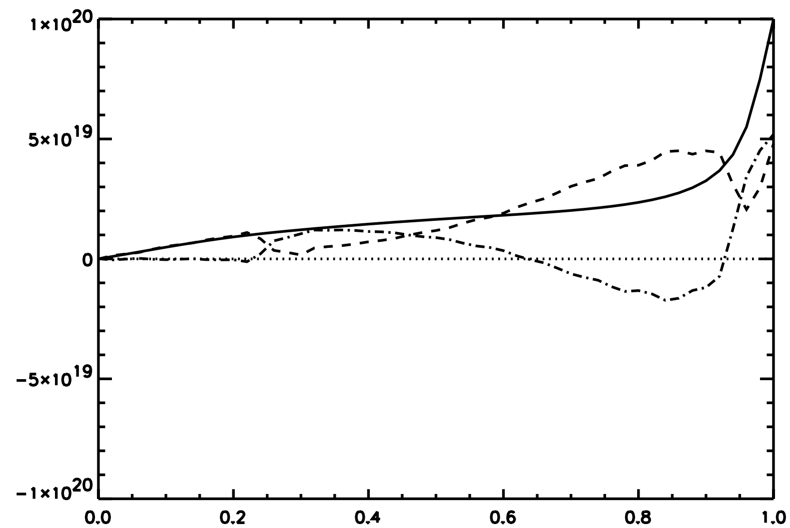


Relative Contribution of □$_a$ and □$_E$

High Density L-mode
□= 1.0 and □= 0.35

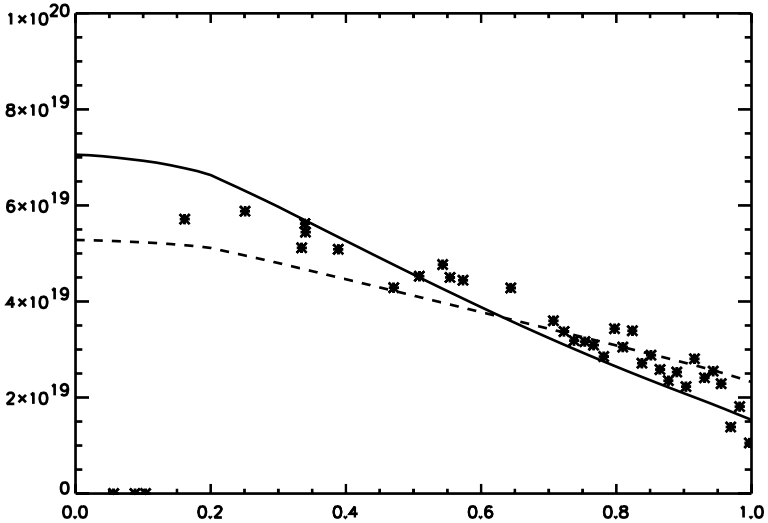


High Density L-mode
□= 0.80 and □= 0.1



Example of Fits which are NOT so Good

High density L-mode:
□= 0.7 and □= 0.35
□= 1.0 and □= 0.0



Conclusions from Empirical Studies of Particle Transport Model

- **This Particle Transport Model Has The Potential For Matching DIII–D Density Profiles**

- **Turbulence Simulation Codes Should Be Used to Calculate $\square_a$ and $\square_E$ to Verify if This Model is Correct**

APPEAL FOR HELP

Will Someone with a Turbulence Simulation Code Please Calculate

$$\square_a \; \square \; \frac{IB}{2\square} \square d^3 \mathrm{v} \left\langle \frac{\mathrm{v}_{\parallel}}{B} \left\{ \left\langle \tilde{\mathbf{v}}_{\mathrm{E}} \cdot \square \tilde{f} \right\rangle_{ens} \right\} \right\rangle_{fs}$$

and Compare it with

$$\square_E = \square d^3 \mathrm{v} \left\langle \left\langle \tilde{\mathbf{v}}_E \cdot \hat{\mathbf{r}} \, \tilde{f} \right\rangle_{ens} \right\rangle_{fs}$$

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