NORMAL MODE APPROACH TO MODELING OF FEEDBACK STABILIZATION OF THE RESISTIVE WALL MODE

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OUTLINE

- Motivation
- Formulation for general geometry
- The open loop feedback problem
- The closed loop feedback problem
- Study of the characteristic equations
- Conclusion

I. MOTIVATION

- Stabilization of the resistive wall mode (RWM) relies on using external coils to replenish the magnetic flux diffused through the resistive shell
- In principle, a perfect feedback needs to cover the resistive shell completely with many feedback coils and sensor loops. This may not be practical for operations
- The feedback scheme in DIII–D utilizes a set of active coils, covering only one poloidal segment of the resistive shell on the outboard midplane. The sensor loops have been moved from outside the resistive wall to its inside
- In the new feedback coil design, three sets of active coils have been proposed
- Effective and practical design of the feedback system depends on the size and geometry of the coils, placement of the sensor loops and the choice of an efficient feedback algorithm
- The purpose of the present work is to provide a model for evaluating the feedback system and for the detailed comparison with experimental observation

II. FORMULATION FOR GENERAL GEOMETRY

- **A formulation is presented for the feedback stabilization of the RWM in general plasma equilibrium configurations, i.e. the present formulation is valid for helical plasma confinement systems as well as axisymmetric confinement systems. The formulation is applicable to the stabilization of general external plasma modes, i.e., this includes not only the helical external modes but also the vertical plasma displacements for the tokamak**

- **The full solution of the problem consists of three steps:**
  - — **The open loop stability problem. This is a generalization of the ideal MHD stability problem**
  - — **Formation of the excitation and sensor matrices of the external feedback coils and sensor loops**
  - — **Solution of the characteristic equations of the closed loop feedback system**

A Non-Self-Adjoint Quadratic Form for the Energy

- A general quadratic form may be obtained for the plasma energy and the vacuum energies in the presence of the external resistive wall and feedback coils

$$\delta W_p + \delta K + \delta W_v + D_w + \delta E_c = 0 \qquad (1)$$

- The above expression is in general non-self-adjoint. The non-self-adjointness results from that the energy can be injected through external coils

- The vacuum region consists of the inner vacuum (IV), between the plasma and the resistive wall; the outer vacuum (OV), between the resistive wall and the coil surface; and the external vacuum (EV), outside of the coil surfaces

The Forms of the Energy Components

- By using the magnetic potential where $\delta \boldsymbol{B} = \nabla \chi$

$$\delta W_p = \text{plasma potential energy}$$

$$\delta K = \text{kinetic energy} = \frac{1}{2} \int_p dV \left[\rho \xi^+ \bullet \frac{\partial^2 \xi}{\partial t^2} \right]$$

$$\delta W_v = \text{vacuum energy} = \delta W_{IV} + \delta W_{OV} + \delta W_{EV}$$

$$\delta W_{IV,OV,EV} = \frac{1}{2\mu_0} \int_{IV,OV,EV} dV \left(\nabla \chi \bullet \nabla \chi^+ \right)$$

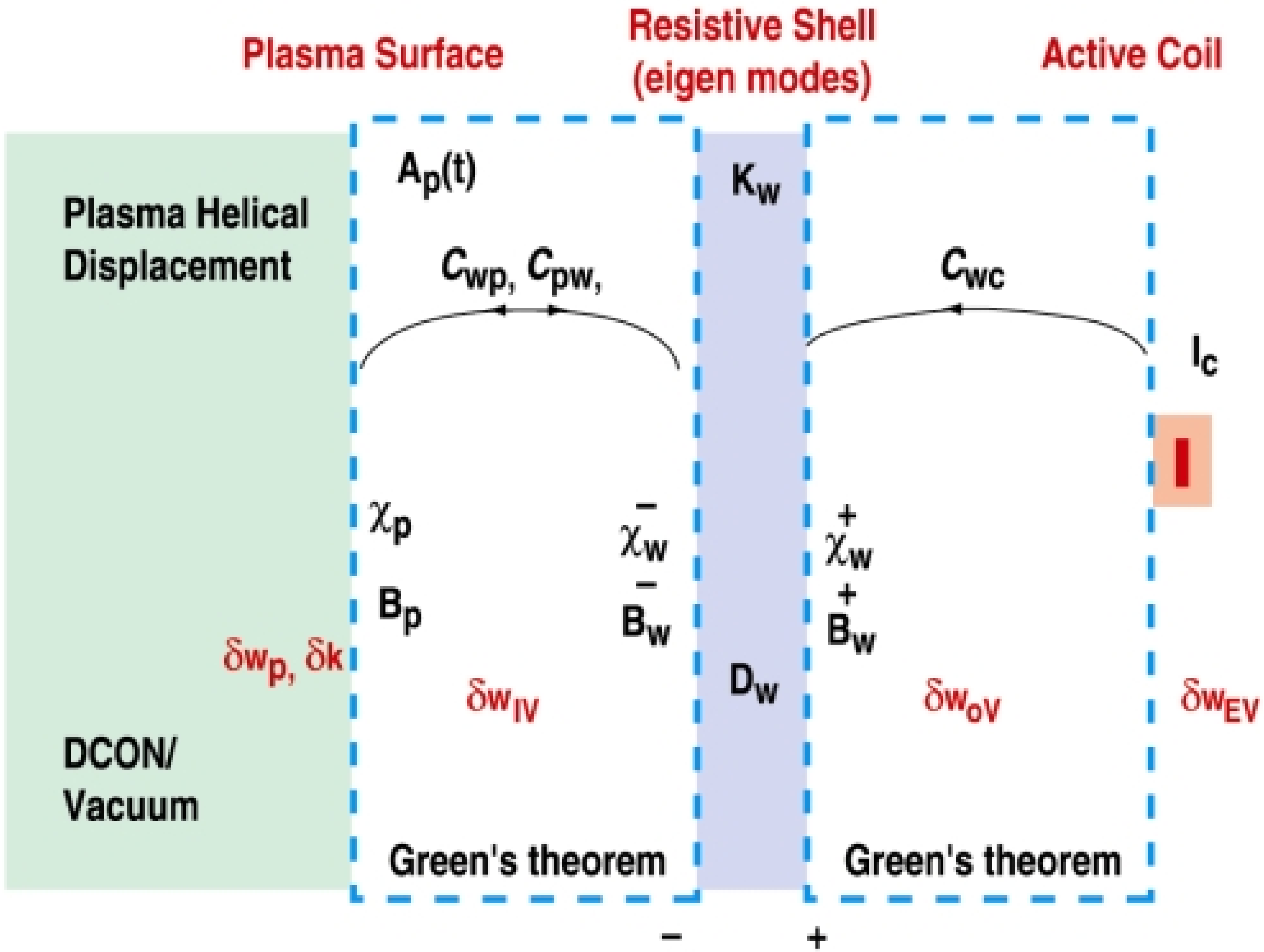
$$D_W = \text{energy dissipaed in the resistive wall}$$

$$= \frac{1}{2\mu_0} \int_W dS \left[\left(\chi^+_+ - \chi^+_- \right) \frac{\partial \chi}{\partial n} \right]$$

$$\delta E_c = \text{energy from feedback coils} = \frac{1}{2\mu_0} \int_c dS \left[\left(\chi^+_+ - \chi^+_- \right) \frac{\partial \chi}{\partial n} \right]$$

- For the resistive wall mode, the plasma kinetic energy is negligible, $\delta K = 0$
- Thin wall approximation, in which $B_n = \partial \chi / \partial n$ is continuous across the resistive wall has been used

Schematics of RWM Feedback Analysis with Toroidal Geometry



Magnetic Potential χ and Green's Function Solution

- The magnetic field on the plasma surface and resistive shell are formulated with scalar potential

$$B = \nabla\chi$$

- With Green's theorem

$$4\pi\overline{\chi}(\vec{r}) + \int_S \chi(\vec{r}\,')\vec{\nabla}' G(\vec{r},\vec{r}\,') \bullet d\vec{S} = \int_S G(\vec{r},\vec{r}\,')\vec{\nabla}'\overline{\chi}(\vec{r}\,') \bullet d\vec{S}' \quad ,$$

where $\quad d\vec{S} = \vec{\nabla} Z\, J\, d\theta d\phi$

$$2\chi(\vec{\rho}) + (1/2\pi) \int_S e^{in(\phi-\phi')}\chi(\rho')\nabla' G(\vec{r},\vec{r}\,') \bullet d\vec{S} =$$

$$(1/2\pi) \int_S e^{in(\phi-\phi')} G(\vec{r},\vec{r}\,')\, \vec{\nabla}'\chi(\vec{\rho}\,') \bullet d\vec{S}' \quad .$$

Solution for χ on the Surfaces

- The scalar potential, χ, for the magnetic field in the vacuum is solved by the methods of Chance, Phys. Plasmas 4 (1997) 2161. It calculated χ as a response to the magnetic perturbations on the surface in the vacuum, i.e., the plasma, resistive shell, and feedback coil. We thus have the relations

$$\chi_p\left(\theta_p^i\right) = \sum_{\ell_p} C_{i\ell_p}(p,p) B_{p\ell_p} + \sum_{\ell_w} C^{-}_{i\ell_w}(p,w) B^{+}_{w\ell_w} \quad .$$

$$\chi^{-}_w\left(\theta_w^i\right) = \sum_{\ell_p} C^{-}_{i\ell_p}(w,p) B_{p\ell_p} + \sum_{l_w} C^{-}_{i\ell_w}(w,w) B^{+}_{w\ell_w} \quad .$$

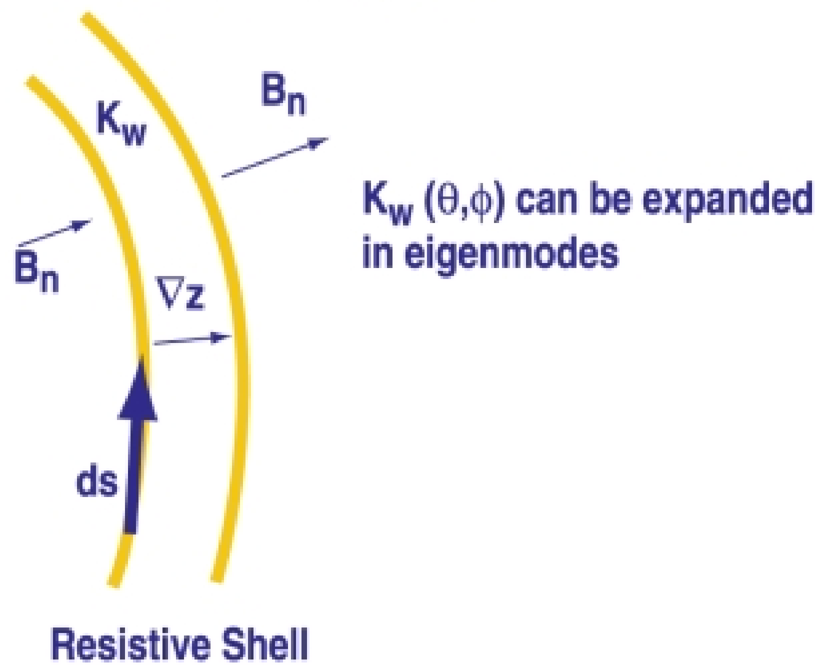
$$\chi^{+}_w\left(\theta_w^i\right) = \sum_{\ell_c} C^{+}_{i\ell_c}(w,c)\ \ell_c + \sum_{\ell_w} C^{+}_{i\ell_w}(w,w) B^{+}_{w\ell_w} \quad .$$

$C^{-}_{i\ell_p}$ (w,p) are Fourier coefficients and $B^{+}_{w\ell_w}$ coefficients of the shell eigenfunction ψ_{ℓ_w}

Resistive Wall Thin Shell Approximation

- Introducing "skin current stream fluction" K_w: $j = \nabla Z \times \nabla K_w \delta\ (z - z_w)$
- Normal magnetic field continuity $\partial\chi\ /\ \partial n\ (-) = \partial\chi\ /\ \partial n\ (+) = B_n$
- Ampere's law: $\chi(+) - \chi(-) = K_w$
- Faraday's law

$$\nabla_s \bullet [\eta \nabla_s K_w] = \partial/\partial t\ (B_n), \text{ assuming } |\nabla z| = 1$$



The Resistive Tank Eigenfunctions and D_W

- The solution can be effected through the following eigenvalue problem

$$\nabla \cdot [\eta\, |\nabla z|^2\, \nabla K_i] = -|\nabla z|^2\, \omega_i K_i \qquad (2)$$

the eigenfunctions satisfies the ortho-normality relation

$$\int_W K_i K_j\, |\nabla z|^2\, dV = \delta_{ij} \qquad (3)$$

So that if

then

$$\frac{\partial}{\partial t}\frac{\partial \chi}{\partial z} = \sum_i \frac{\partial a_i}{\partial t} K_i \sqrt{\omega_i} \qquad (4)$$

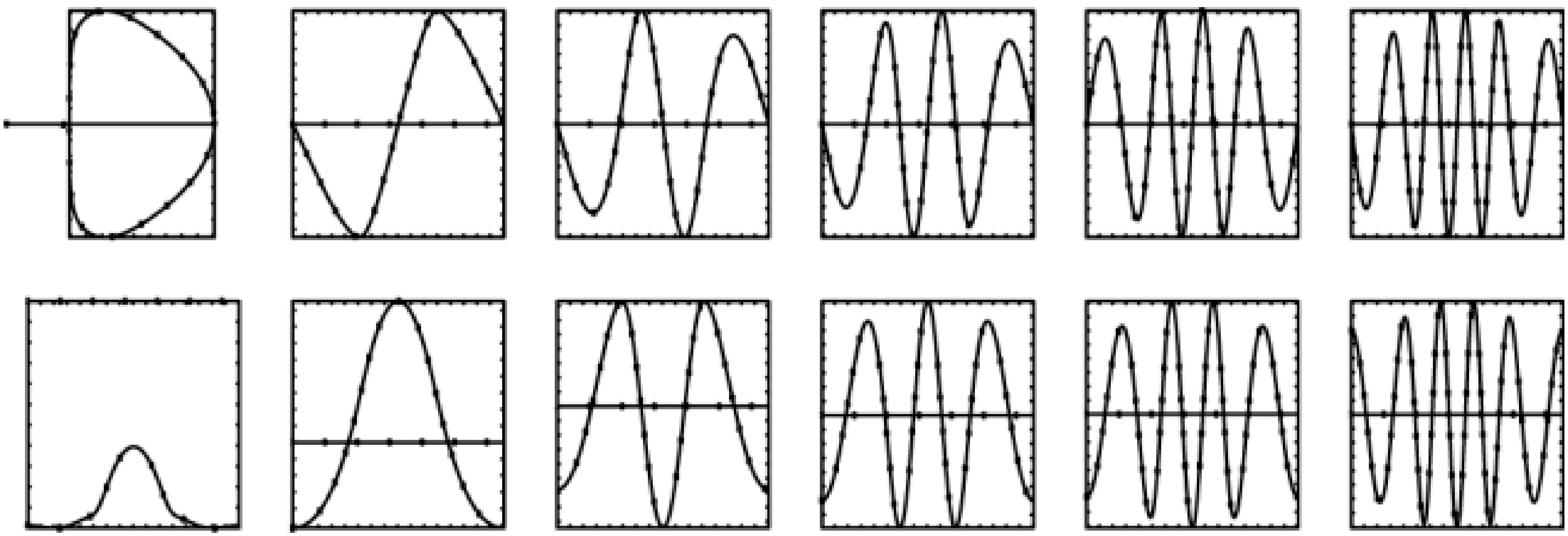
and

$$K = \sum_i \frac{1}{\sqrt{\omega_i}}\, \frac{\partial a_i}{\partial t} K_i \qquad (5)$$

$$D_W = \frac{1}{2}\sum_i \frac{\partial a_i^+}{\partial t} a_i \qquad (6)$$

Examples of The Resistive Tank Eigenfunctions

- The resistive tank eigenfunctions can be either odd or even with respect to the midplane



- m = 0 to m = 5 lowest order poloidal odd (top) and even (bottom) eigenfunctions of the resistive shell eigenmodes with the inclusion of the effect of the plasma

III. THE OPEN LOOP FEED BACK PROBLEM

- During the open loop operations, $\delta E_C = 0$. The outer vacuum region and the external vacuum regions merge into one OV region. We are left with

$$\delta W_p + \delta W_{IV} + \delta W_{OV} + D_W = 0 \quad (7)$$

- We note that in the above expression, since the magnetic potentials satisfies the Laplace equation $\nabla^2\chi = 0$, the vacuum energy may be written in terms of surface integrals

$$\delta W_{IV} = \frac{-1}{2\mu_0}\int_p dS\left(\chi + \frac{\partial\chi}{\partial n}\right) + \frac{1}{2\mu_0}\int_w dS\left(\chi + \frac{\partial\chi}{\partial n}\right) \quad (8)$$

$$\delta W_{OV} = \frac{-1}{2\mu_0}\int_w dS\left(\chi + \frac{\partial\chi}{\partial n}\right) \quad (9)$$

- In the above expressions, Eqs. (8) and (9) for the vacuum energies, the normal magnetic fields at the plasma and resistive wall surfaces are the source and the magnetic potentials are the response. These relations are solved by the VACUUM Code. The open loop expression is self-adjoint with D_W given in Eq. (6).

The Open Loop Eigenfunctions

- With $\partial/\partial t = \gamma$, it forms an eigenvalue problem. This is a generalized energy expression from that of ideal MHD. It gives us a set of orthonormal eigenfunctions with countable eigenvalues.

$$\delta W_p(i,j) + \delta W_{IV}(i,j) + \delta W_{OV}(i,j) + \frac{1}{2}\gamma_i \delta_{i,j} = 0 \qquad (10)$$

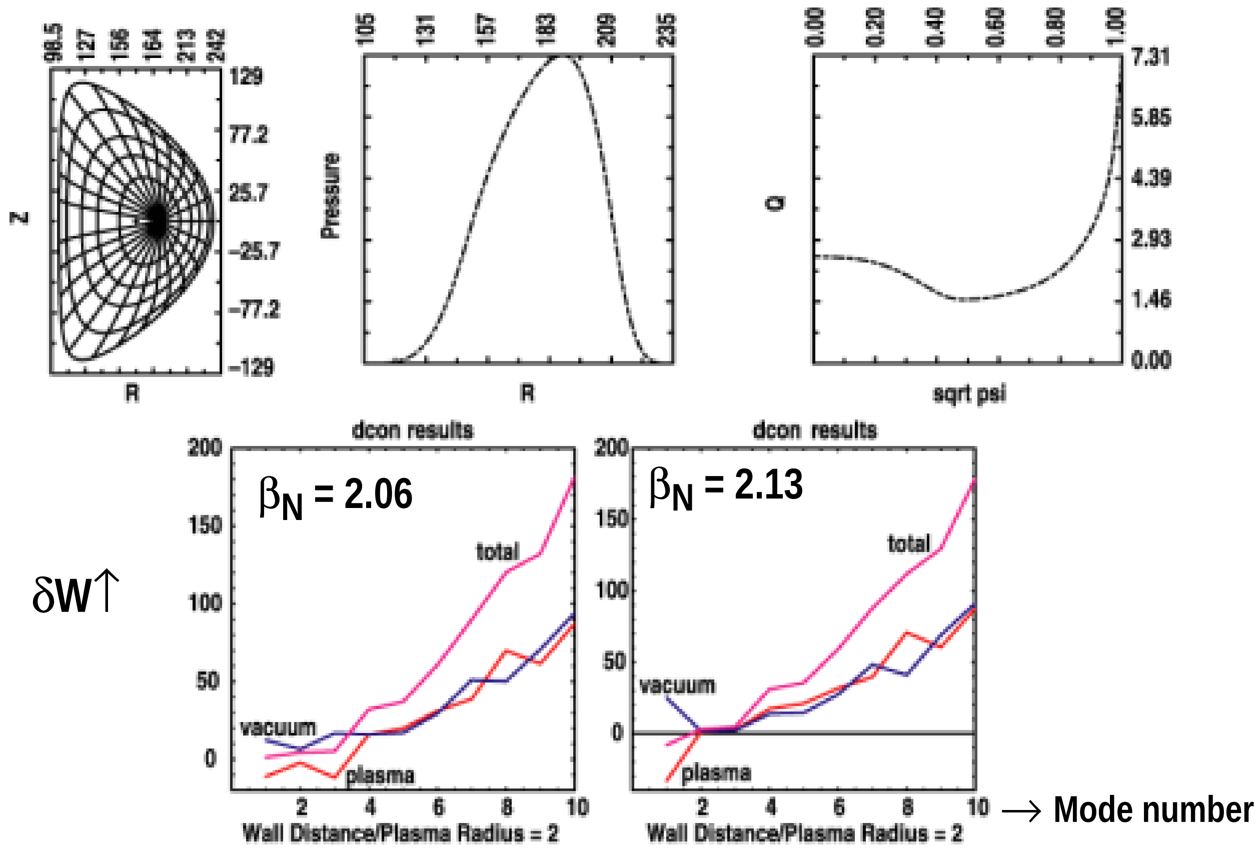
and

$$D_w(i,j) = \frac{1}{2}\gamma_i \delta_{i,j} \qquad (11)$$

- A part of this diagonalization has been done by DCON, i.e., with the $\partial_{\chi w}/\partial n = 0$. Our diagonalization utlizes the eigenfunctions and eigenvalues given by DCON together with the eigenfunctions given by the resistive tank eigenfunctions. This set is complete for arbitrary skin current distribution on the resistive wall.

Example TOQ Equilibrium and DCON Stability Results

- DIII—D equilibrium with high Elongation and Triangularity



- An equilibrium unstable with the wall at infinity and stable with a nearby conducting wall

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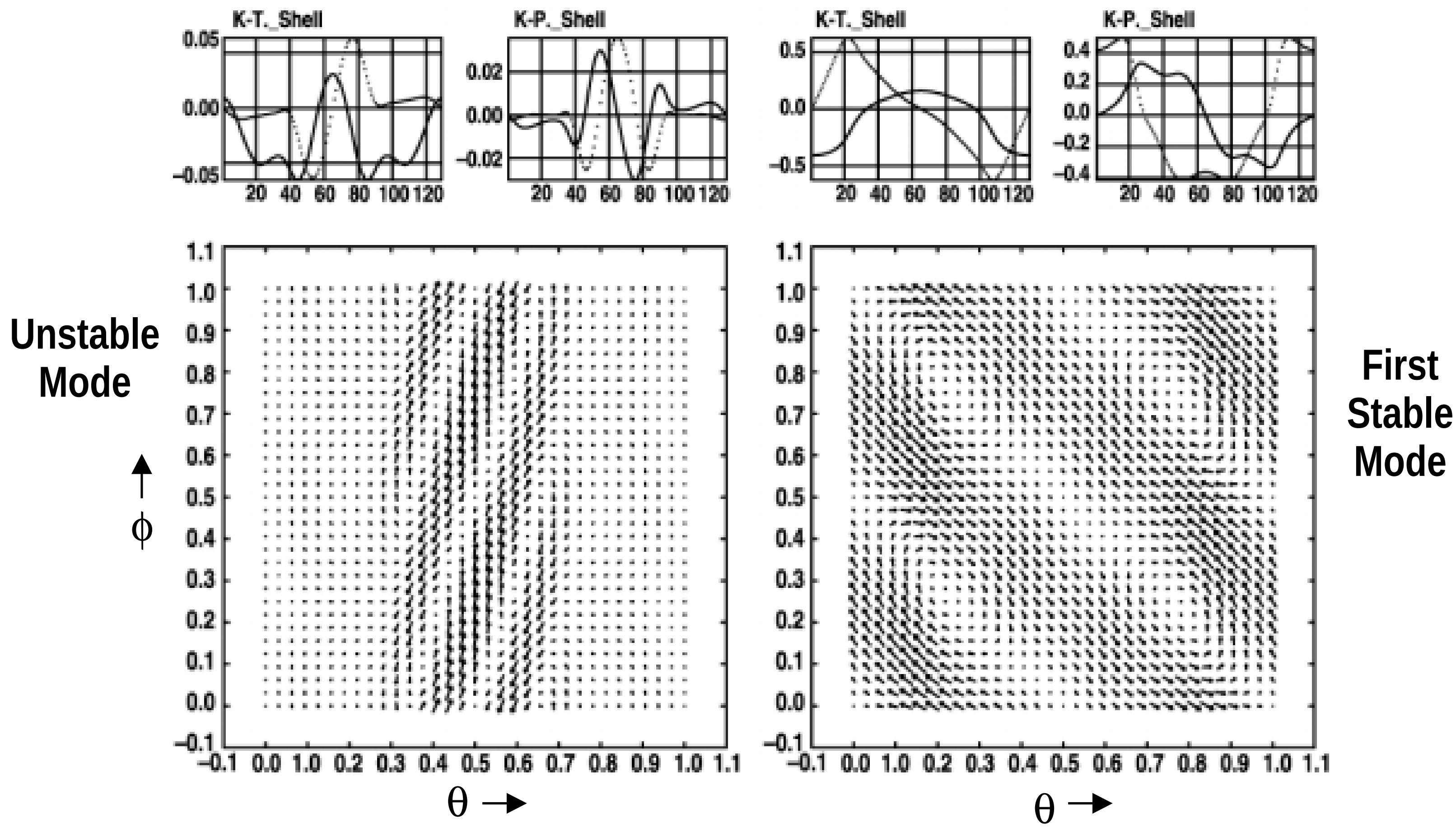
Wall Current of the Open Loop Eigenfunction

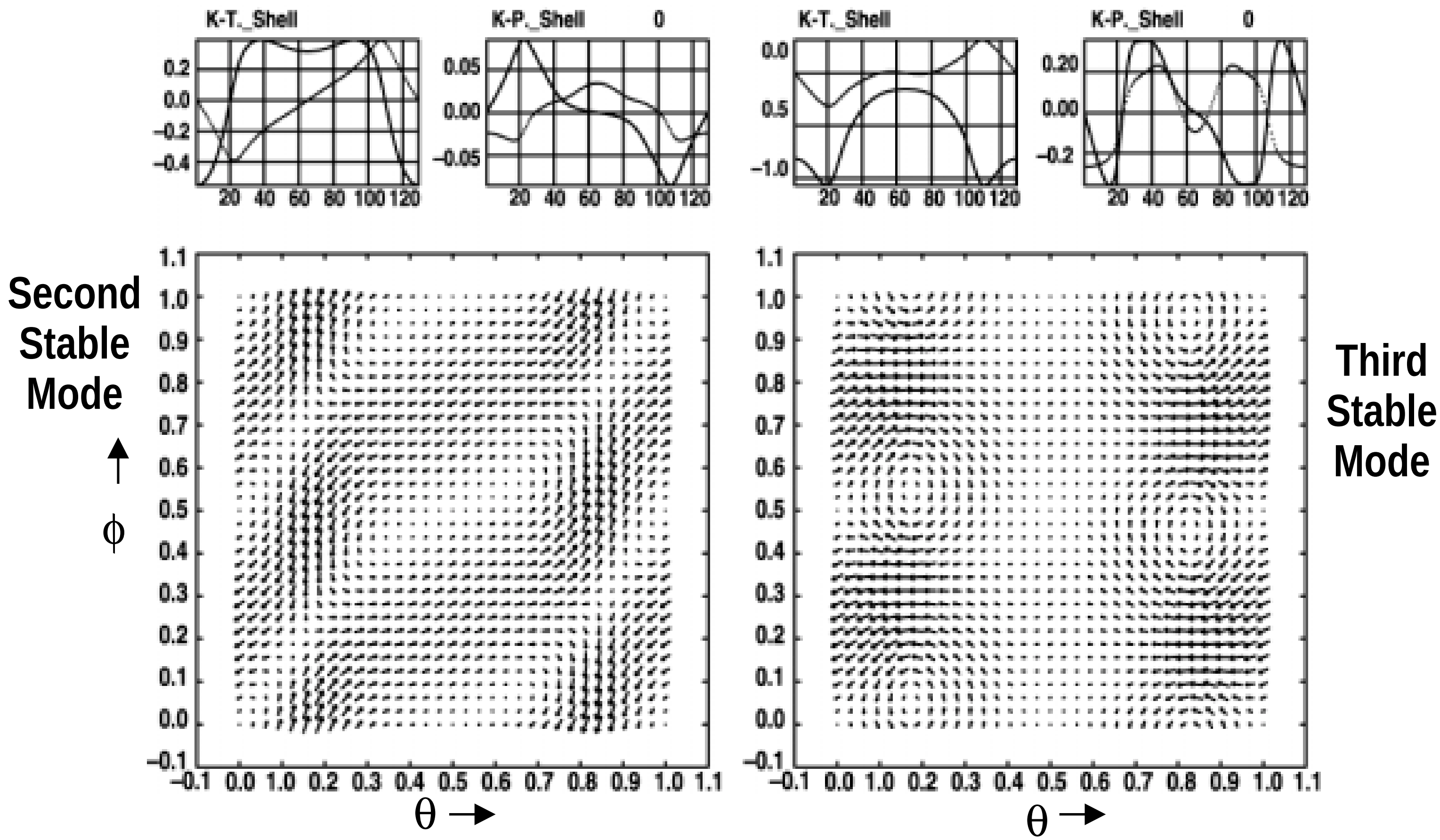
- **Lowest two eigenfunction**



- **Skin currents on the resistive wall induced by the open loop eigenfunction. On the left is the lowest mode, on the right is the second lowest mode**

Wall Current of the Open Loop Eigenfunction (II)

- Next two eigenfunction



- Skin currents on the resistive wall induced by the open loop eigenfunction. On the left is the third lowest mode, on the right is the fourth lowest mode

IV. THE CLOSED LOOP FEEDBACK PROBLEM

- During the closed loop operation, because the set of open loop eigenfunction is complete, the normal magnetic field on the resistive wall can be expanded in terms of this set of eigenfunctions. From the boundary conditions, the perturbation inside the inner surface of the resistive wall is completely determined if the value of the magnetic field is given on the resistive wall. The magnetic potential needs to deviate from a superposition of the open loop eigenfunctions only in the region outside of the outer surface of the resistive wall. Or in the OV and EV regions

$$\chi = \sum a_i(t)\chi_i + I_c\chi_{out}^c \quad (12)$$

- The χ_{out}^c not only satisfies the Laplace equation, $\nabla^2\chi_{out}^c = 0$ but also satisfies the condition that it has no perpendicular magnetic field at the resistive wall, $\partial\chi_{outW}^c / \partial n = 0$.

The Closed Loop Feedback Equations

- Substitution of Eq. (12) into the energy Eq. (1), and utilizing the properties of the open loop eigenfunctions, we obtain a set of time evolution equations for the amplitudes for the open loop eigenfunctions α_i

$$\frac{\partial \alpha_i}{\partial t} - \gamma_i \alpha_i = \sum_c I_c E_i^c \quad (13)$$

- In the above expression, *i* is the label for the open loop eigenfunction; the E_i^c gives the excitation of the open loop eigenfunctions by the external coils. They are given by the expression

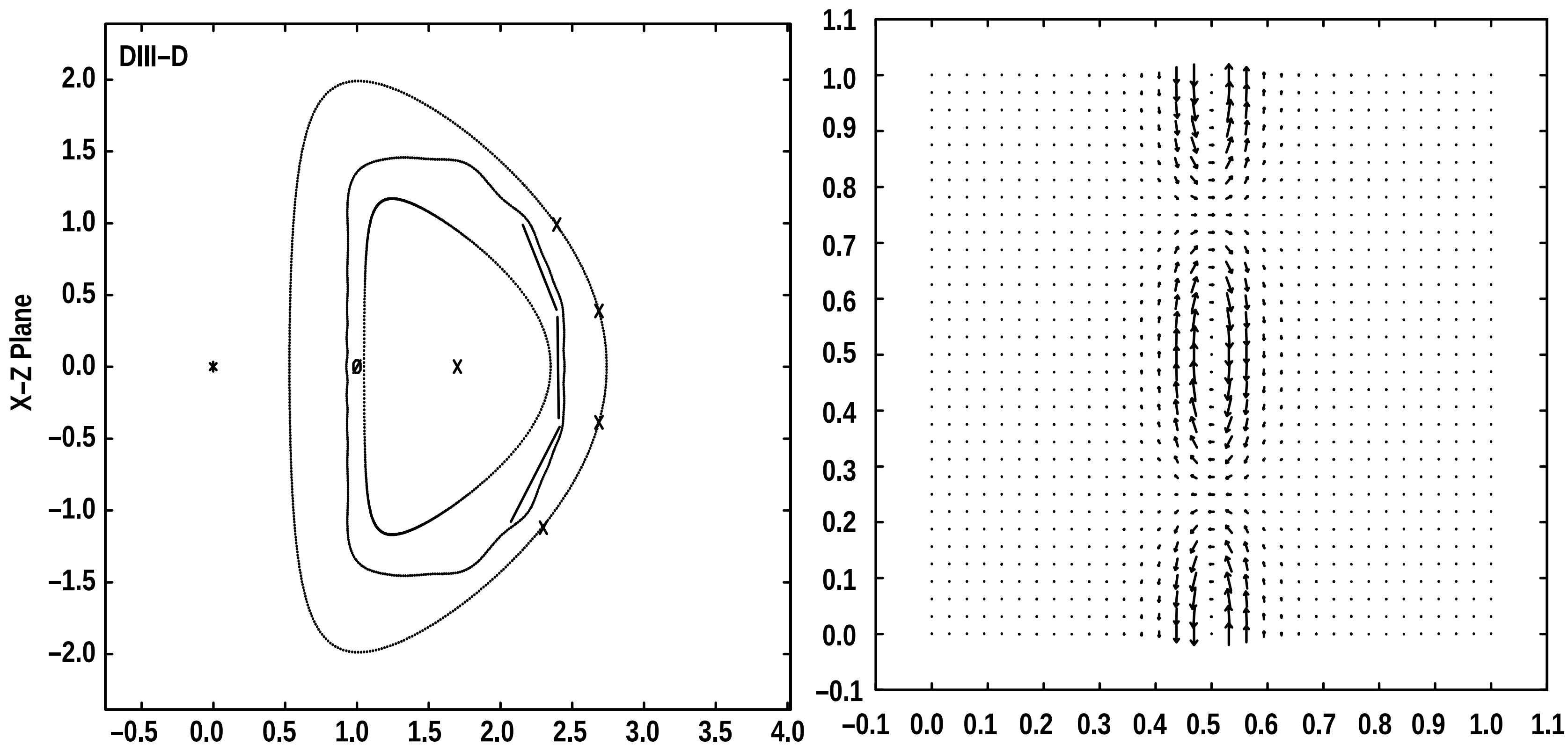
$$E_i^c = \frac{1}{\mu_0} \int_W dS \left(\chi_{out}^c \frac{\partial \chi_l}{\partial n} \right) \quad (14)$$

- The feedback system is completed by the equations that prescribes the currents in response to the amplitudes of α_i. They are written in general as

$$\frac{\partial I_c}{\partial t} + \frac{1}{\tau_c} I_c = \sum_l G_c^l F\left(\{\alpha_i\},\{I_c\}\right) \quad (15)$$

- In here, *l* is the index for the sensor loops and G_c^l is the amplifica-tion matrix and F_l is the flux detected by the sensor loop. The above set of equations is the general feedback equation for RWM

Example of Feedback Coil in DIII–D Geometry



- **Left is the coil geometry. Right is the excitation potential**

V. THE CHARACTERISTIC EQUATIONS

- The basic equations for the RWM derived in the previous viewgraph can be used for the time-dependent simulation of the feedback system. More insight may be obtained by studying the characteristics of the system. We assume that the F_ls can be approximated by linear functions. If we let $s = \partial/\partial t$ then the system of equations are given by

$$\vec{R}\vec{V} = s\vec{I}\vec{V} \tag{16}$$

- Here $\vec{R}$ is the response matrix, $\vec{V}$ is the state vector of the plasma-resistive wall system (α_i, I_c) and $\vec{I}$ is the identity matrix. The form of $\vec{R}$ is given by

$$\vec{R} = \begin{vmatrix} \vec{\Gamma} & \vec{E} \\ \vec{G}\vec{F} & \vec{L} \end{vmatrix} \tag{17}$$

- In here $\Gamma_{ij} = \gamma_i \delta_{ij}$ is a diagonal matrix, $E_{ic} = E_i^c$ is the excitation matrix and $GF_{ci} = \sum_l G_l^c \frac{\partial F_l}{\partial \alpha_i}$ is the inner product of the amplification and sensor matrices, and $L_{cc'} = -\frac{-\delta_{cc'}}{\tau_c} + \sum_l G_l^c \frac{\partial F_l}{\partial I_{c'}}$ is the modified coupling matrices of the feedback coils

V. THE CHARACTERISTIC EQUATIONS (Cont.)

- The characteristic equation is given by the determinant of

$$D(s) = \left| s\vec{I} - \vec{R} \right| \tag{18}$$

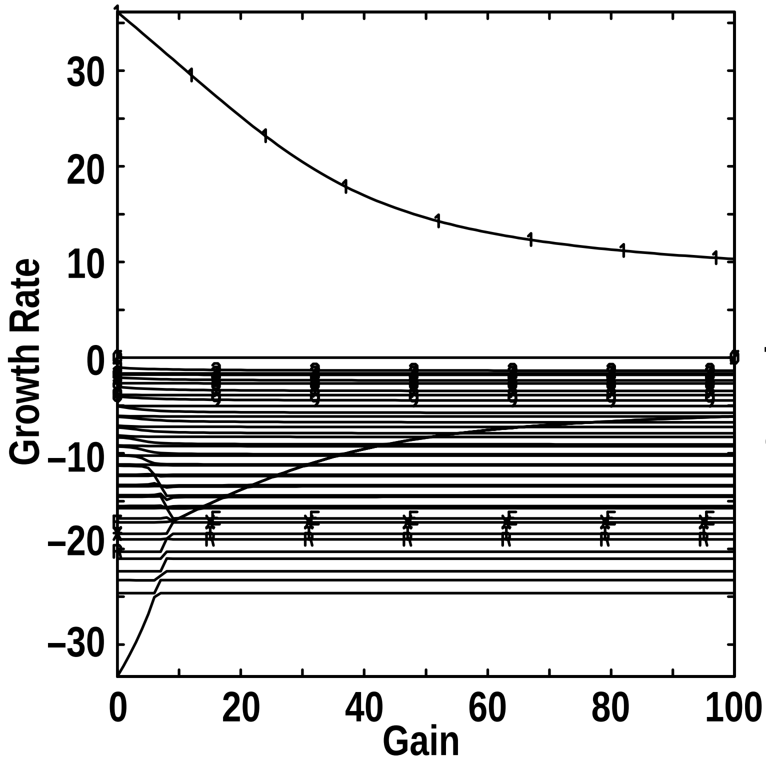
- It is interesting to note that when the sensor loops are inside of the resistive wall, then $\partial F_l / \partial I_c = 0$. Then the *L* matrix is also diagonal. The way the amplification matrix comes into the equation is substantially modified. The examples provided below will show that it is always more stable than the case when the sensor loops are outside or when $\partial F_l / \partial I_c \neq 0$.

Central Coil Alone Not Sufficient for Stabilization of β_N = 2.5

Filtering factor f: $F_{ci} \rightarrow f\, F_{ci}$ for all stable modes

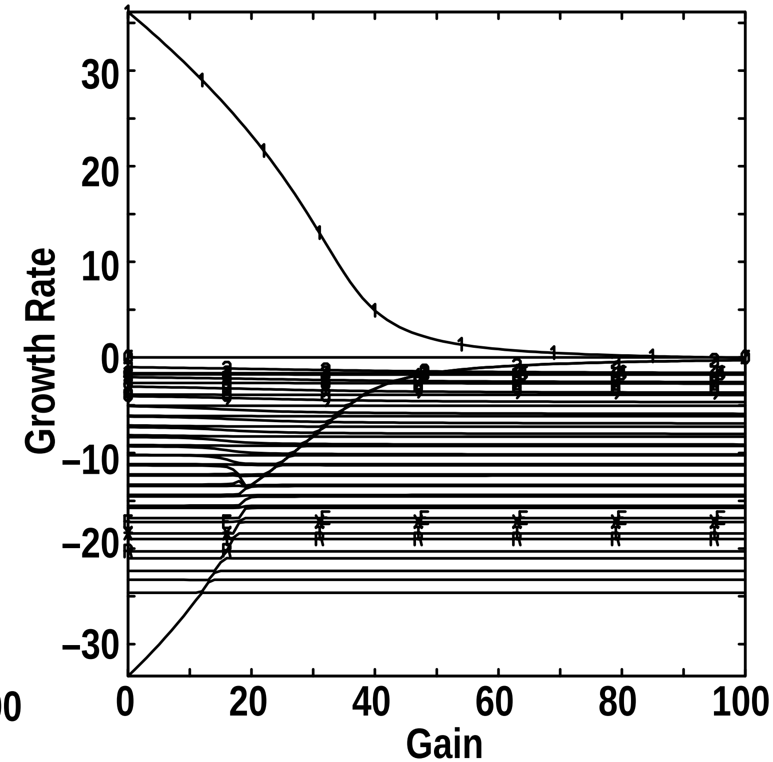
Without Filtering f = 1

Real Roots



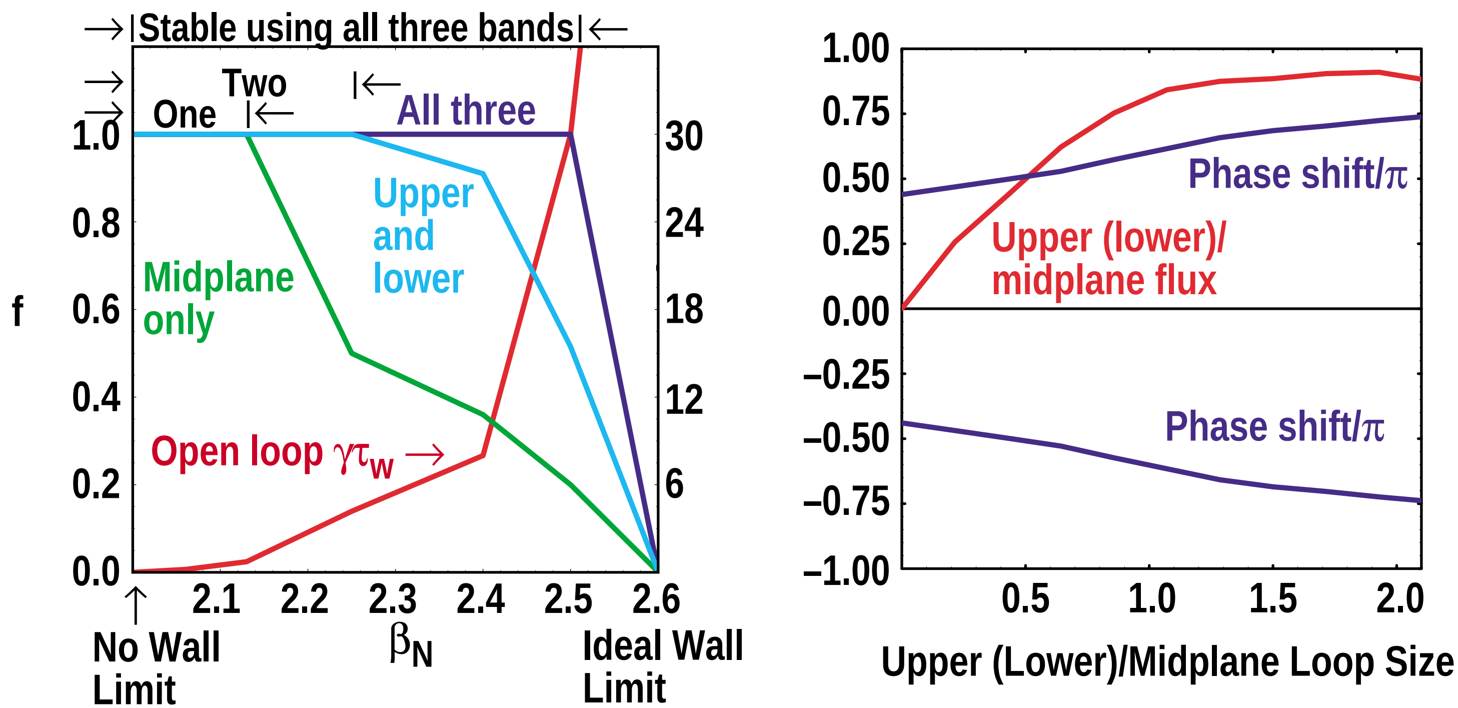
With Filtering f = 0.15

Real Roots



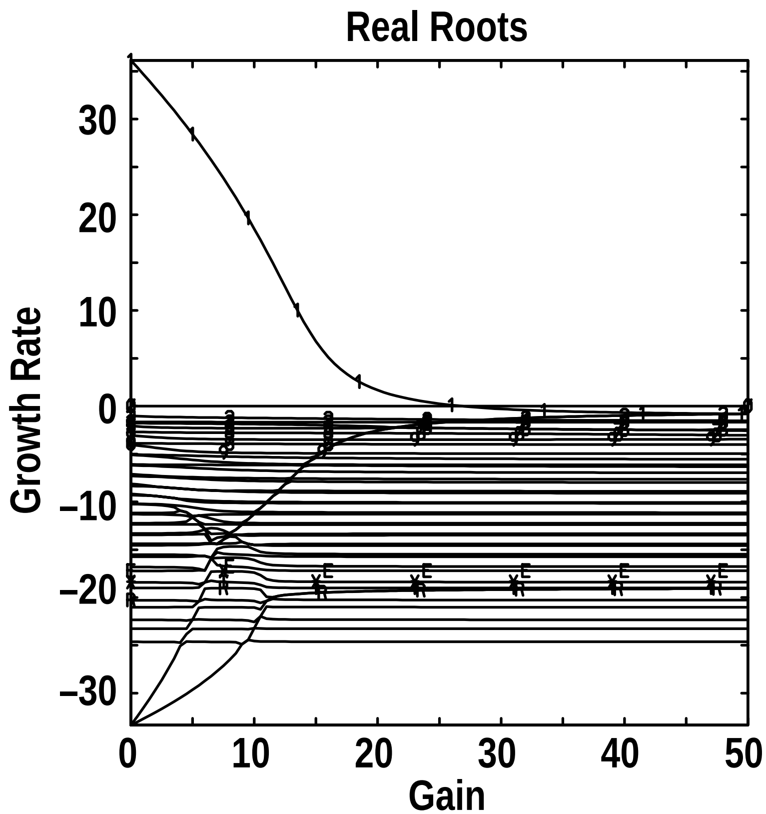
- Left is the root trajectory with one central segment of feedback coils only and with no filtering of the signal from the stable modes. Right is the root trajectory with filtering (filtering factor of 0.15)

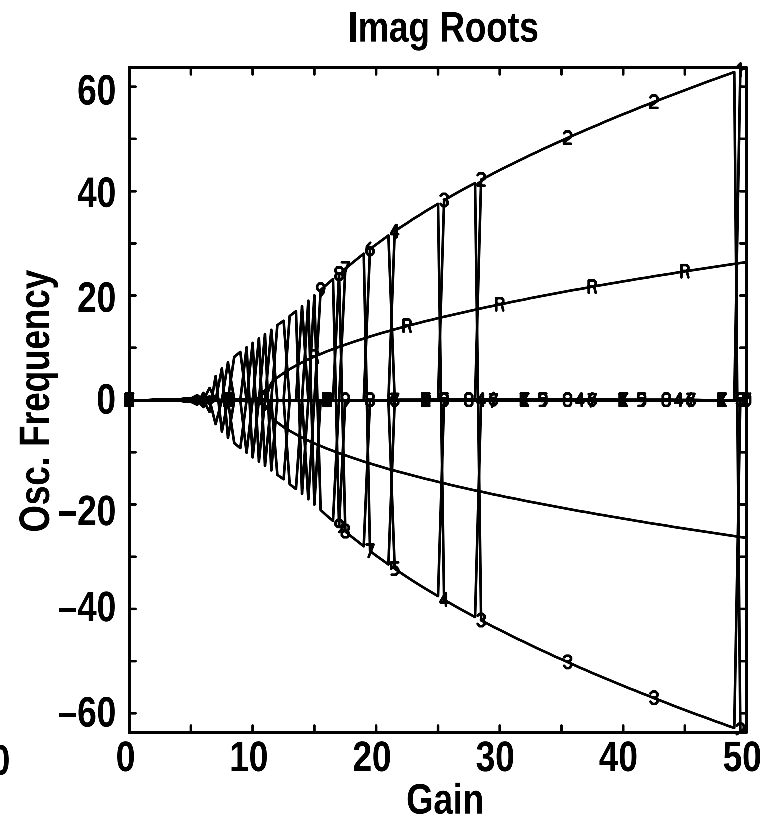
Summary of Results for the DIII–D Equilibria with Different β_N's



- Left is the growth rate and feedback requirements of the RWM as a function of β_N. Red is the growth rate. The green and blue curves are the filtering factors of the sidebands that are required to stabilize the RWM. With three segments of coils feedback stabiliza-tion is feasible for equilibria with $\gamma\tau_w$ up to 30 (with upper and lower segment size equal to that of the midplane.) Right is the flux ratio and phase angle difference due to the RWM on the upper (lower) and the midplane coil segments as a function of the size of the coils.

Three Band Feedback Sufficient for Equilibria with β_N = 2.5





- Left is the trajectory of growth rates; right is the frequencies of the roots of the characteristic equation for different gain factors by using three bands of coils and without filtering of the signal from the stable

VII. CONCLUSION

- A formulation is presented for the feedback stabilization of the RWM in general plasma.
- The full solution of the problem consists of three steps:
  - — The open loop stability problem. This is a generalization of the ideal MHD stability problem.
  - — Formation of the excitation and sensor matrices of the external feedback coils and sensor loops
  - — Solution of the characteristic equations of the closed loop feedback system
- This formulation has been numerically implemented for the toroidal geometry and can be used to study the detailed dynamical behavior of plasmas in feedback experiments.
- An increase in the number of coil segments from the central segment only to include the upper and lower bands in DIII–D geometry can have a substantial stabilization effect.