Continuum Gyrokinetic code simulations in a noncyclic finite-width radial annulus

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Introduction and Motivation

- Gyrokinetic code GYRO to contains all physics of low frequency (<< ion cyclotron) plasma turbulence assuming only that the ion gyroradius is less than magnetic field gradient length
  - Nonlinear
  - Electromagnetic and finite β
  - Real tokamak geometry
- Continuum (fluid-like) methods in 5-dimensional space (r, θ, n, ε, λ)
  - Possible advantage over particle codes: implicit advance of electron parallel motion
- 3-modes of operation:
  - flux-tube or high-n ballooning mode representation BMR Δn=5-10 ρ^* -> 0
    to be bench-marked with Dorland 's new gyrokinetic flux tude code GS2
  - wedge or full radius Δn=5-10 ρ^* small but finite
  - full torus Δn=1 Global MHD modes
- Why full radius? Shear in the ExB velocity known to have a powerful stabilizing effect. But shear in the diamagnetic velocity can be just as large and cannot be treated at $\rho^* = 0$. Also to quantify *avalanches* and *action at a distance* effects

Some fundamentals about flux tube codes

- "Flux tube" codes follow a field line to make (Δr, $r\Delta\theta$, $R\Delta\phi$) covers of a radial (r,θ) annulus and have cyclic radial boundary conditions
- Δr X $r\Delta\theta$ is typically $100\rho_s$ X $100\rho_s$ but ρ_s is vanishingly small.

Since $k_y\rho_s = (nq/r)\rho_s$ is finite (say .05 to 0.9), the toroidal mode numbers n are arbitrarily large.

- Although, $1/L_T = -d \ln T/ dr$ is finite, the variation of $T = \Delta r/L_T$ or any other plasma variable is insignificant, i.e. there are no profile (shear) variation effects.

...only magnetic shear $\hat{s}=d \ln q(r)/d \ln r$

- Because of there are no variation of coefficients and we have cyclic BC, most flux tube code have a radial Fourier transform space: $r_m= r_0 +x_m$ grid is replaced with a $k_x = -i\, d/dr$ grid.

For finite s^, this is called the Ballooning Mode Representation

- Because we want to study general profile variation, not just ExB velocity shear,

but diamagnetic velocity shear at finite ρ_s/r, in a finite Δr annulus with noncyclic radial BC, we are developing a code GYRO with a radial grid.

__The 1D linear modes (for a given n,k_x) became 2D modes (for a given n) : much more difficult

- In our first stage (where we are now), the profile variation is turned off and we have cyclic BC to benchmark with the Ballooning Mode code GS2 with a k_x grid

__This talk is a progress report on the first stage

The next stage: non-cyclic BC finite radial annulus at finite ρ_s

- The next stage will operate with non-cyclic BC (e.g. 0 gradient BC) over a finite annulus, say r=0.4 to 0.6 with general profile variation. We don't want 0 BC which allows no in-out flow.

- Since in-flow will not equal out-flow, and the driving temperature gradients will begin to relax, we need a scheme to add a running "incremental" sources to the n = 0 gyrokinetic equation:

$$d[F(r)+\tilde{f}_0(r,\theta)/dt + r^{-1}d/dr[r\sum_n \tilde{V}_{ExB\_x-n}(r,\theta)\tilde{f}_n(r,\theta)] = r^{-1}d/dr[r\left\langle\left\langle\sum_n \tilde{V}_{ExB\_x-n}(r,\theta)\tilde{f}_n(r,\theta)\right\rangle\right\rangle +$$

$F(r)$ is the input fixed background Maxwellian, $\tilde{f}_0(r,\theta)$ is the n=0 (zonal flow) perturbation,

$\sum_n \tilde{V}_{ExB\_x-n}(r,\theta)\tilde{f}_n(r,\theta)$ in the nonlinear term is the x-directed flux, and

$r^{-1}d/dr[r\left\langle\left\langle\sum_n \tilde{V}_{ExB\_x-n}(r,\theta)\tilde{f}_n(r,\theta)\right\rangle\right\rangle$ is the incremental source .

$\langle\langle\ \rangle\rangle$ is local time-local space maxwellized average $\Rightarrow$ defined scale separation

- We hope to approach "full radius" say r=0.3 to 0.7 then r=0.2 to 0.8 as resolution allows: 200-300 r-grid replacing the current 100 r-grid.

Motivation

- To compare a simulation χ's directly with experiment, in addition to the ITG drive, we need trapped electrons, finite-beta (passing electrons), equilibrium scale ExB shear stabilization . . . . and finite ρ_s profile shear effects.

- To treat the finite ρ_s profile shear effects, we need a finite radial annulus without cyclic BC, but still maintaining the input profiles.

- The plan for direct annulus comparison of experiments with a few simulations having truly comprehensive physics (including finite-β and real geometry), is a different strategy from previous work.

  Namely, make many runs with limited physics (e.g. ITG only) to normalize transport models with more comprehensive physics, e.g. GLF23

Gyrokinetic Equations

- **Poisson's Equation:** $-\lambda_{D0}^2 \nabla^2 \hat{\phi} = \sum_s \iint z\left[-(z\hat{n}/\hat{T})\hat{\phi} + \langle g \rangle\right]$, $\langle g \rangle$ **is non-adia. gyro-ave. DF**

- **Ampere's Law:** $-\rho_{s0}^2 \nabla^2 \hat{A}_{\|} = (\beta_{e0}/2)\sum_s \iint z\hat{v}_{\|}\langle \hat{g} \rangle$,

where the phase space integral $\iint \Rightarrow \Sigma_\sigma \pi^{-3/2} \int_0^\infty d\hat{\varepsilon}\hat{\varepsilon}^{1/2} \exp(-\hat{\varepsilon})(\pi/2)\int_0^{1/B} d\lambda B/(1-\lambda B)$.

- **Nonlinear Gyrokinetic Equation [2,3] :**

$$\partial_{\hat{t}}\hat{g} + \hat{v}_{\|}(\hat{b}\cdot\hat{\nabla}\theta)\partial_\theta \hat{g} = z\hat{n}/\hat{T}\partial_{\hat{t}}\langle \hat{U} \rangle$$
$$-i\omega_E(-(z\hat{n}/\hat{T})\langle \hat{U} \rangle + \langle g \rangle) + i\varpi_D \hat{g} - i\hat{n}\varpi_* \langle \hat{U} \rangle + \left\{\langle \hat{U} \rangle, \hat{g}\right\} - \left\{\hat{g}, \langle \hat{U} \rangle\right\} + C\hat{g},$$

where "hat" quantities are normalized, and $U = \hat{\phi} - \hat{v}_{\|}\hat{A}_{\|}$ **is the effective potential.**

- **The low-n MHD rule of neglecting** $A_\perp$ **while forcing the curvature drift to equal the grad-B drift is very good even for high-n.**

- The curvature drift operator which acts only on $\hat{g}$ is

$$\hat{\varpi}_D = -iz\hat{T}(2/R_0)\rho_{s0}(B_0/B)\{C(\theta,\hat{\varepsilon},\lambda)\ [inq/\hat{r}]\ + S(\theta,\varepsilon,\lambda)\ [i(nq/\hat{r})\eta_k + (|\nabla r|/\eta_q)D_{\hat{r}}]\}\ .$$

where C is the cosine-like normal curvature, S the sine-like geodesic curvature [1]

- The diamagnetic term is $\varpi_* = -i(B_0/B_{unit})\rho_{s0}[inq/\hat{r}][1/\hat{L}_n + 1/\hat{L}_T(\hat{\varepsilon}-3/2)]$.

- The E×B equilibrium rotation frequency is $\hat{\omega}_E = -i(B_0/B_{unit})\partial_{\hat{r}}\hat{\Phi}_0\rho_{s0}\,[inq/\hat{r}]$.

- The nonlinear term $\{X,Y\} \equiv \sum_{n',n''=n-n'} (B_0/B_{unit})\rho_{s0}\left[in''q/\hat{r}\right]X_{n''}\left[\partial_r + i(n'q/\hat{r})\eta_k\,\eta_q/|\nabla r|\right]Y_{n'}$

_parallel nonlinearity O(ρ*) not shown

- We define $[inq/\hat{r}] \equiv inq/\hat{r} + (B_t/B)\,(\hat{R}q/\hat{r})\,(\hat{b}\cdot\hat{\nabla}\theta)\partial_\theta$. $B_{unit} = B_0\rho d\rho/rdr$

The nq/r terms are *fast* and the ∂_θ terms are *slow*. where again we use D_r when ∂_r acts on g.

- C is the pitch angle scattering operator.

Field Line Following Coordinate System most efficient

- (r, θ, α)   r = midplane minor radius flux surface label
  θ = poloidal angle labeling Miller's local MHD equilibrium[1]
  which generalizes infinite aspect ratio circular s-α model with finite aspect ratio, Shafranov shift, ellipticity, and triangularity

  $\alpha = \zeta - \int_0^\theta \hat{q} d\theta$ field aligned angle in place of toroidal angle ζ

  $$\hat{q} = \hat{b}\cdot\nabla\zeta / \hat{b}\cdot\nabla\theta, \;\; \hat{b}\cdot\nabla\alpha = 0, \; \hat{b}\cdot\nabla r = 0, \; q = \int_0^{2\pi} \hat{q}\, d\theta / 2\pi$$

- Fourier decomposition of perturbations: $\phi = \Sigma_n \phi_n(r,\theta)\exp(-in\alpha)$ requires $\phi_n(r,\pi) = \phi_n(r,-\pi)\exp(-in2\pi q)$ where *phase factor* $\exp(-in2\pi q)$ is 1 at singular surfaces

- parallel derivative $\nabla_{||} = (\hat{b}\cdot\nabla\theta)\partial_\theta \Rightarrow (1/Rq)\partial_\theta$

- perpendicular derivatives on *fast* part : $\exp(-in\alpha)$

  $\nabla^f_{\perp y} = -in\nabla\alpha\cdot\hat{b}\times\hat{x} = ik_\theta\eta_q(\theta)$, where $k_\theta \equiv nq/r$ with $\eta_q(\theta) = (rB/RB_\theta)/q \Rightarrow 1$

  $\nabla^f_{\perp x} = -in\nabla\alpha\cdot\hat{x} = ik_\theta\eta_q(\theta)\eta_k(\theta)$ $\eta_k(\theta) \Rightarrow \hat{s}\theta - \alpha\sin(\theta)$

- additional *slow* derivatives on ϕn(r,θ) : $\nabla^S_{\perp y} = (B_t/B_p)(\hat{b}\cdot\nabla\theta)\partial_\theta$

  radial derivative is a mixture of *fast* and *slow*: $\nabla^{fs}_{\perp x} = |\nabla r|\partial_r$ $|\nabla r| \Rightarrow 1$

  Ballooning Mode Representation(BMR) retains only the fast not slow O(ρ*) derivatives

Normalizing Units, Parameters, and Gyro-Average

- Te(0) and ne(0) for temperature and density
  a = r of last closed flux surface for length
  $c_{s0} = [T_e(0)/M_i]^{1/2}$ for velocity
  a/c_{s0} for time
  $|e|\phi/T_e(0) = \hat{\phi}$ and $(c_{s0}/c)|e|A/T_{e0} = \hat{A}$ for potentials

- $g = \hat{g}(r,\theta,\hat{\varepsilon},\lambda,\sigma)n_e(0)F_M$ for non-adiabatic distribution function

  $\hat{\varepsilon} = \varepsilon/T, \lambda = \mu/\varepsilon, \sigma = \mathrm{sgn}(v_{||})$ and since (ε,μ) are the constants of motion

  $\partial_r \hat{g} = n_e(0)F_M D_r \hat{g}$ where $D_r\hat{g} = \partial_r \hat{g} + \partial_r\lambda\partial_\lambda + (\hat{\varepsilon}/L_T)\partial_{\hat{\varepsilon}}\hat{g} - [(\hat{\varepsilon}-3/2)/L_T]\hat{g}$

  Note all terms beyond $\partial_r \hat{g}$ are small O(ρ^*) dropped in BMR limit.

- Parameters: the central ρ^* [$\rho_{s0} = c_{s0}/(eB_0/M_i c)$], Debye length λ_{D0}, and electron beta β_{e0}.

- The gyro average $\langle\phi\rangle = \Sigma_n \exp(-in\alpha)\langle\phi\rangle_n$ expand the arguments of $\langle\phi\rangle_n$ to first order $\rho_\perp/r$, so that $\langle\phi\rangle_n = \oint d\alpha_g/2\pi \exp(-in\bar{\rho}_\perp\cdot\nabla\alpha)\phi_n(r+\bar{\rho}_\perp\cdot\nabla r, \theta+\bar{\rho}_\perp\cdot\nabla\theta)$.

  As a first approximation the *slow* $\bar{\rho}_\perp\cdot\nabla\theta$ can be neglected.

Numerical Methods

- Electron parallel motion term is much faster than drift frequencies we are trying to follow, hence need an implicit method which solves for the fields U simultaneously with the advance of g
- We use a split time step method: $g_1 = g_0 + dt * RHS_1$, $g_2 = g_1 + RHS_2$, ...

_ ε, λ, θ-parallel process

- Nonlinear step: 4-th order Runge-Kutta with sub stepping at constant U.

  Conservative form with centered multi-point derivatives crucial for $\sum_{n,m} |g_{n,m}|^2$ conservation.

_ n, ε, r-parallel process

- Pitch-angle collision step: n, ε, r-parallel

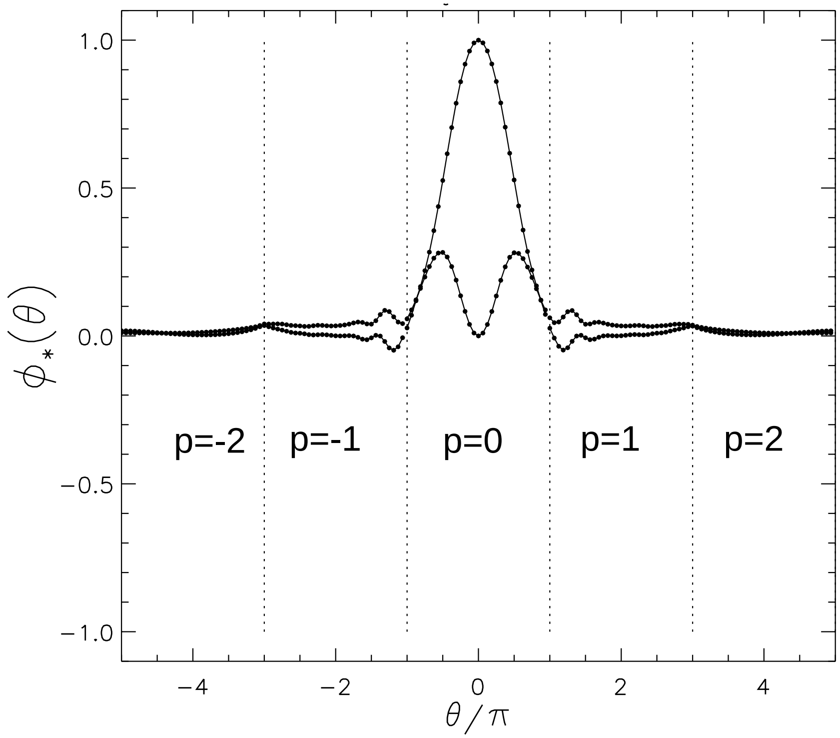
_ n, ε, λ -parallel process

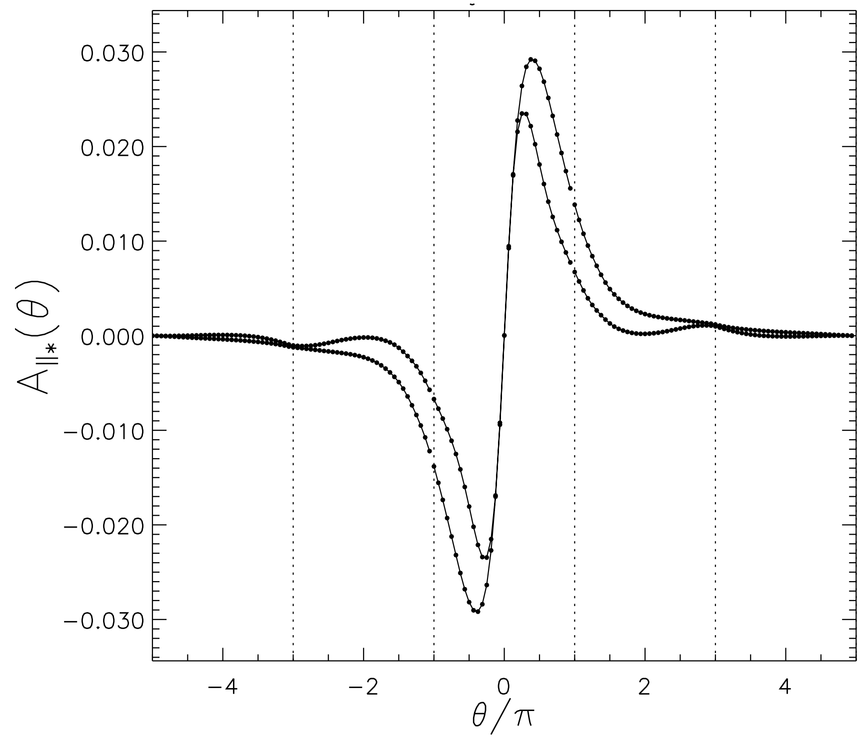
- r-step: g-advance for geodesic curvature drift motion (d/dr part) with high order derivative.
- θ-step: implicit g - advance of parallel, linear ExB, normal curvature drift motion (kx =-i d/dr=0 part), with Poisson-Ampere field simultaneous U-solve

- Most difficult numerical problem has been to stability advance the neutrally stable n=0, low-kx component over very long run times. Required the θ-step in special conservative form griding and r-step & θ-step repeated as a corrector on a predictor
- Used high-k_x damper for electron runs. This avoids resolving singular surface electron layer
- "banding" or nearest neighbor in gyro-averaging and 5-pt radial derivatives to avoid N_r^2 scaling.

Ballooning Mode (k_x-space) Analysis

finite beta $(\gamma,\omega) = (0.167,-0.361)_{GYRO}$ vs $[0.175,-0.357]_{GKS}$



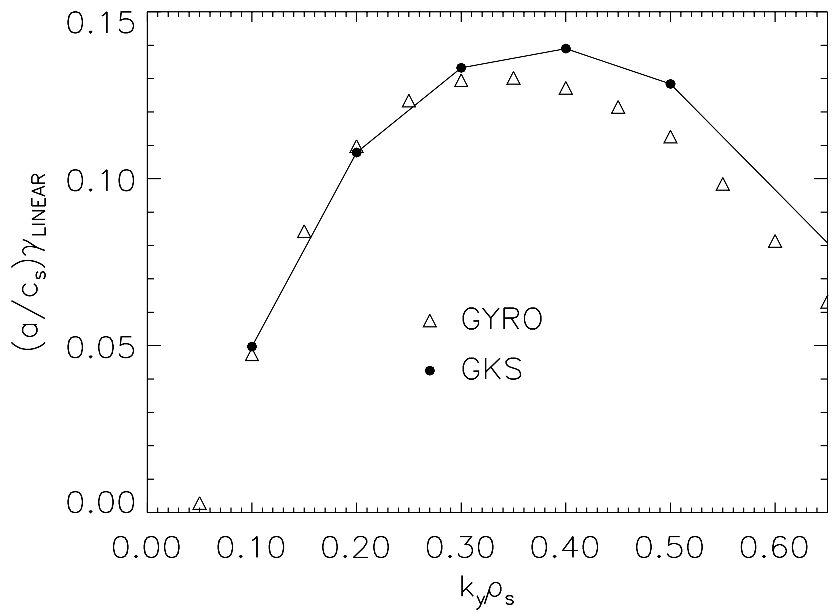


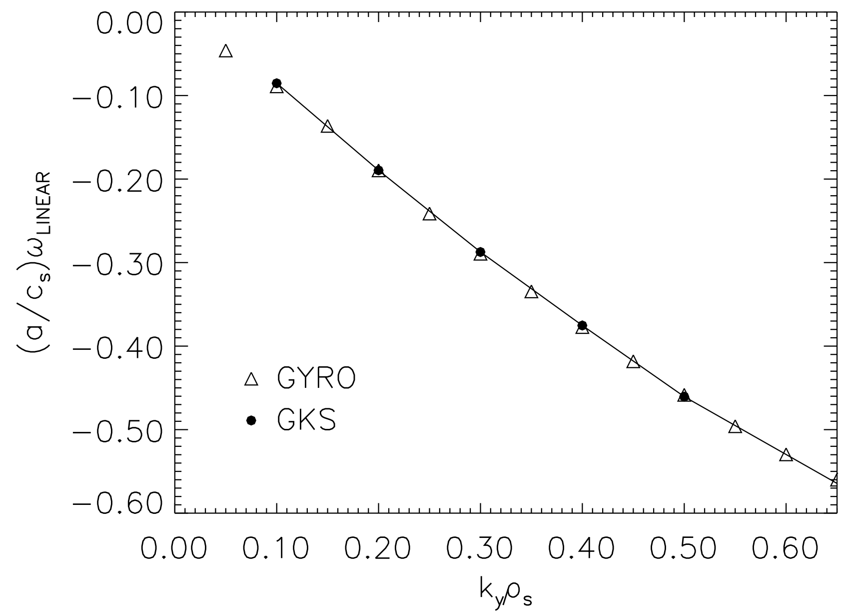
$k_y\rho_s = 0.3$, $a/L_T=3$, $a/L_n=1$, $R/a=3$, $r/a=0.5$, $s=1$, $q=2$, $T_e=T_i$, $\beta_e= 0.004$ ($\beta_{e\_crit} = 0.055$)

$n_e=5 =(3,2)$, $n_\lambda=26$, $n_\theta=32/2\pi$, $n_r=5$ ($x=m\,\Delta r$, $k_x = p\,\Delta k_x$) $\phi(x_m,\theta)= \sum_{p=-2}^{p=2} \phi_p(\theta)\exp(i\pi mp/2)$

Linear Growth Rates and Frequencies: GYRO vs GKS

ITG adiabatic-electron standard case



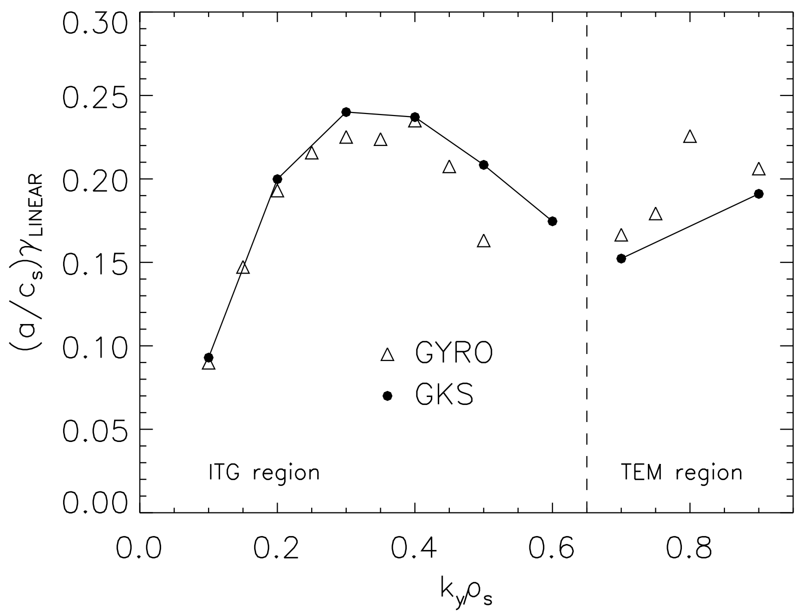


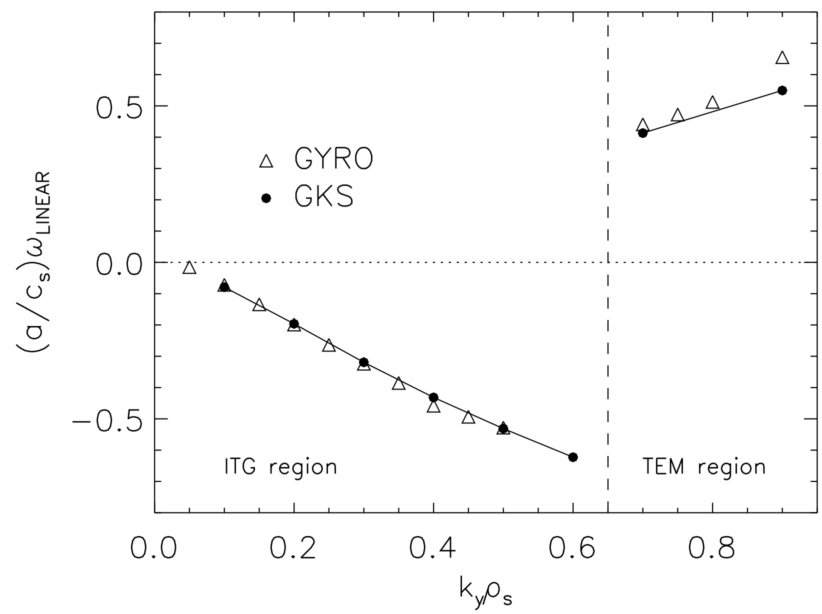
$a/L_T=3$, $a/L_n=1$, $R/a=3$, $r/a=0.5$, $s=1$, $q=2$, $T_e=T_i$,

$n_e=5=(3,2)$, $n_\lambda=26$, $n_\theta=32/2\pi$, $n_r=96$, $L_x= 80\ \rho_s$, $n_n= 15$

Linear Growth Rates and Frequencies: GYRO vs GKS

electrostatic nonadiabatic electron standard case





a/L_T=3, a/L_n=1, R/a=3, r/a=0.5, s=1, q=2, T_e=T_i,

n_e=5 =(3,2), n_λ=26, n_θ=32/2π, n_r=96, L_x= 80 ρ_s , n_n= 15

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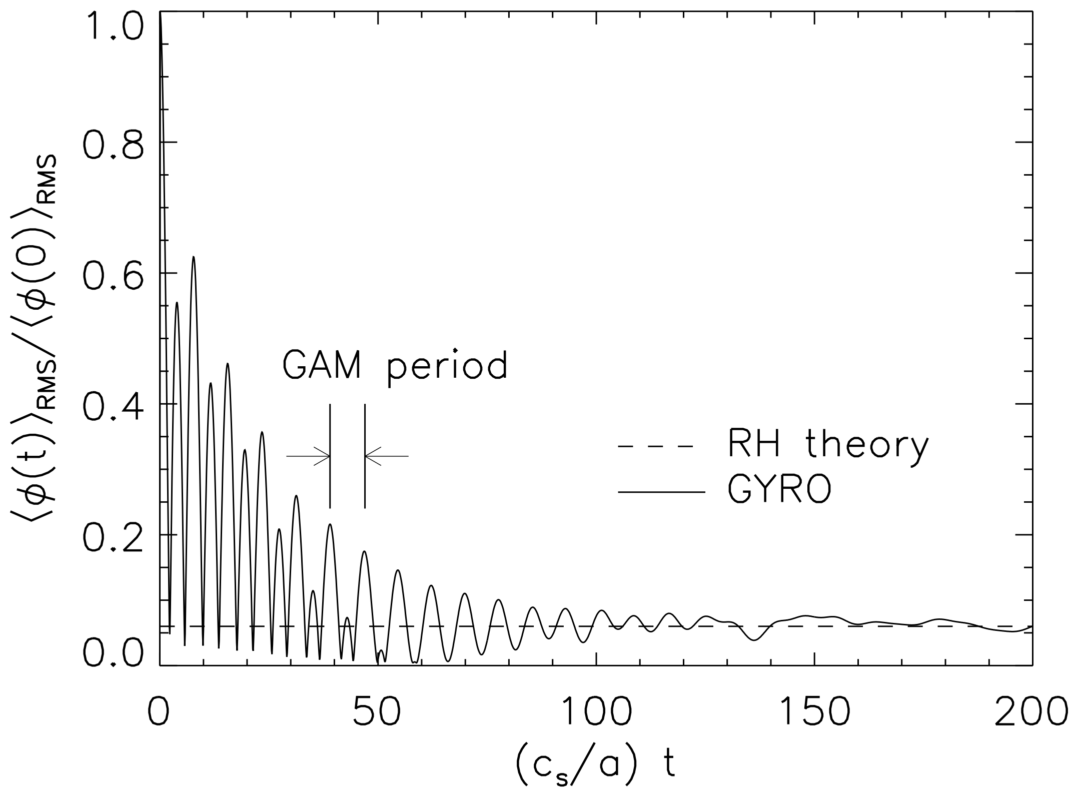
GENERAL ATOMICS

n=0 radial mode (Zonal Flow) decay to Rosenbluth-Hinton residual

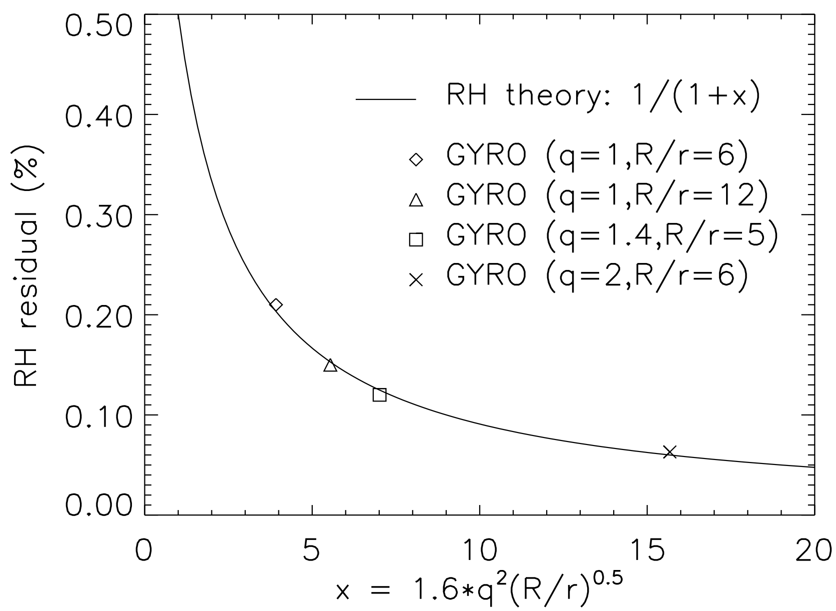
geodesic acoustic mode (GAM)

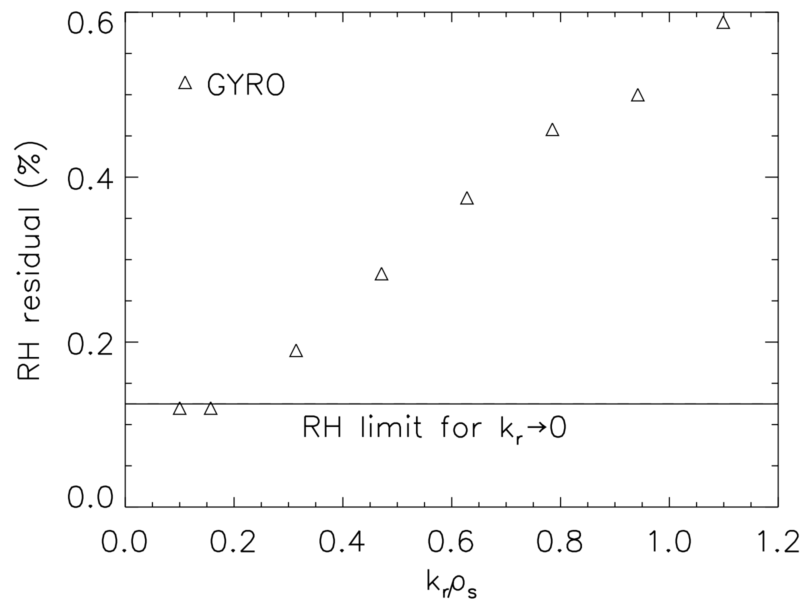


$R/r=6$, $r/a=0.5$, $q=2$, $T_e=T_i$, $(s=1)$, $k_x \rho_s = 0.078$ (lowest mode) RH residual = 6%

$n_e=5=(3,2)$, $n_\lambda=26$, $n_\theta=16/\pi$, $n_r=96$, $L_x= 80\ \rho_s$

Scaling of n=0 radial mode residuals at kχ->0 and vs. kχ





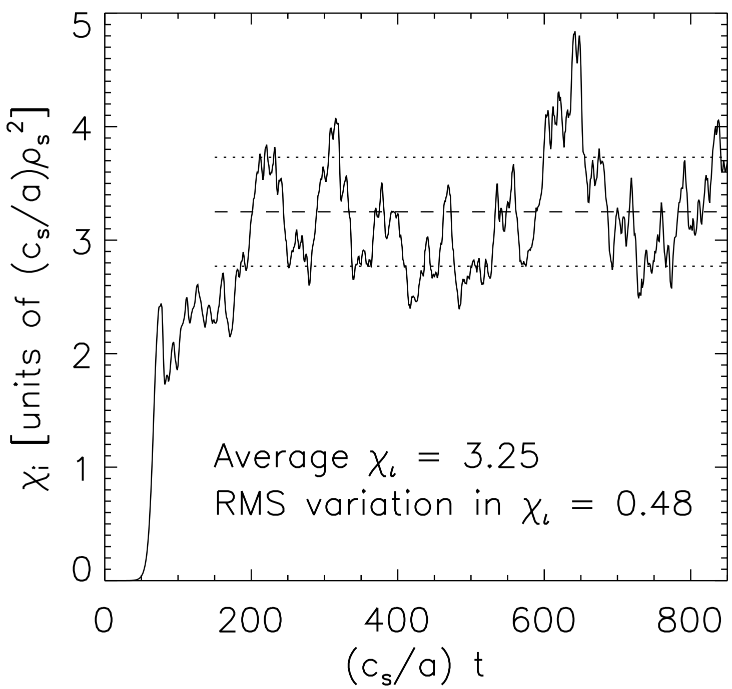
R/r=5, r/a=0.5, q=1.4, $T_e=T_i$, (s=1)

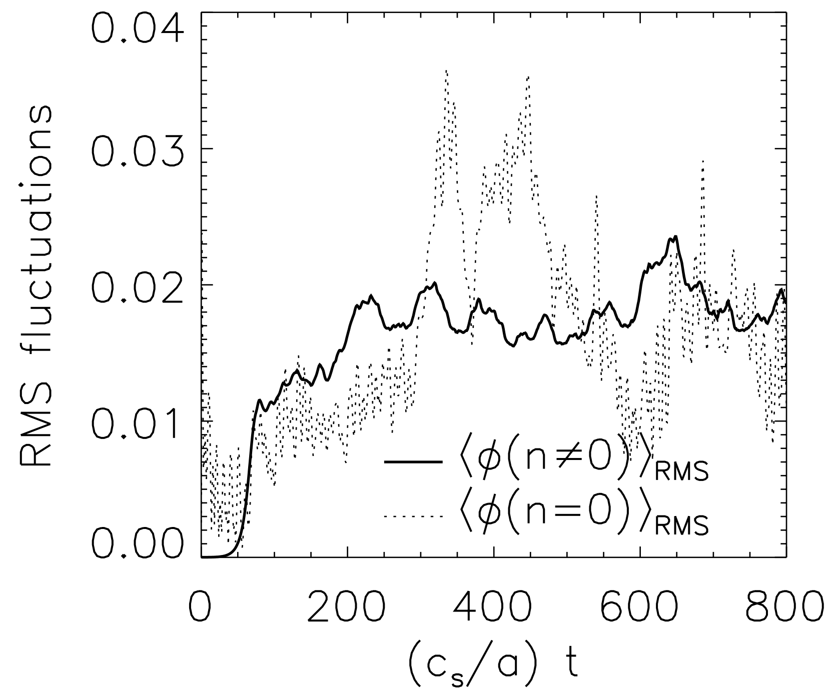
n_e=5 =(3,2), n_λ=26, $n_\theta=32/2\pi$

kχ-plot agrees with GS2 **F. Jenko, W. Dorland, et al Phys Plas. 7 (2000) 19(**

Nonlinear Simulation

ITG adiabatic electron standard case





$a/L_T=3$, $a/L_n=1$, $R/a=3$, $r/a=0.5$, $s=1$, $q=2$, $T_e=T_i$,

$n_e=5=(3,2)$, $n_\lambda=14$, $n_\theta=16/2\pi$, $dt=0.05a/c_s$: 60 hrs on 45ps GA Stella LINUX cluster 16000step

$n_r=96$, $L_x= 80\ \rho_s$, $n_n= 14$: $k_y\rho_s$ =0.0, 0.06,0.12,0.18....0.78, $k_x\rho_s$_min = 0.078, $k_x\rho_s$_max=3.6

GENERAL ATOMICS

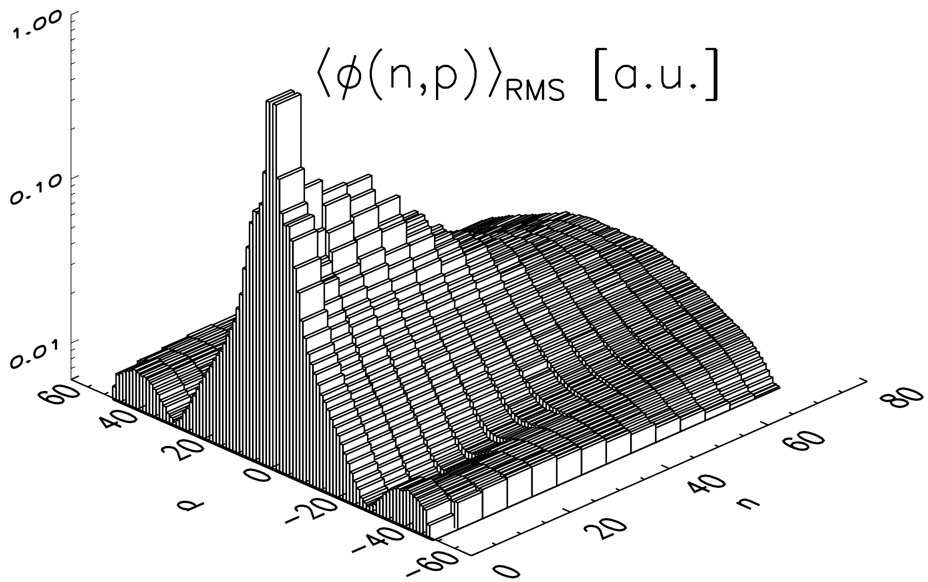
How to measure avalanche fraction

Hypothesis: $\chi_{rms\_variation}/\chi_{rms\_average}$

- Avalanches are intermittent events
- Spikes in χ are correlated in time with Flux(r,t) streamers interpreted as avalanches
- Example from standard case $\chi_{rms\_variation}/\chi_{rms\_average} = 0.48/3.25 = 14.7\%$

Nonlinear Simulation Spectrum

ITG adiabatic electron standard case.....continued



$k_x \rho_s = p\ 0.078$ $\quad$ p= - 48 ->+48

$k_x \rho_s = (n/70)\ 0.70$ $\quad$ n=0,6,12,18.....78

Nonlinear Simulation Visualization

ITG adiabatic electron standard case.....continued

- Annulus contour plot of physical $[\tilde{n}_e/n_{e0}](r,\theta)=\Phi(r,\theta)-\langle\Phi(r,\theta)\rangle_{\theta-ave}$ at ζ=0

reconstructed from $\phi_n(r,\theta)$:

$$\Phi(r,\theta)=\sum_n \exp[-inq(r)\theta_{fine\_scale}]\phi_n(r,\theta_{course\_scale})$$

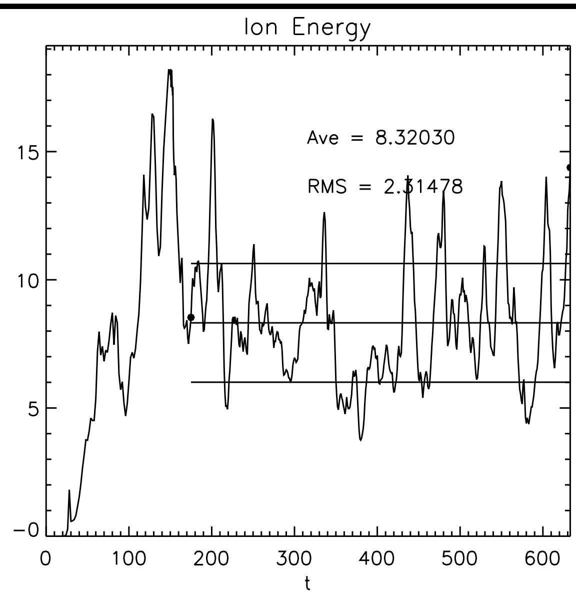
$\alpha = \zeta - q(r)\theta$; n=$\pm$0,5,10,15,....70 ; 80 ρ_s from r=0.4 to 0.6; ρ_s=2.5e-3

The simulation is done on a course scale θ-grid for ϕ_n, but reconstruction requires a fine scale grid for the eikonal. Otherwsie the eddies could not be seen. This illustrates the efficiency of the field line following (α,θ,r)-grid in place of the toroidal (ϕ,θ,r)-grid.

If we multiply n x 10 and ρ_s x 1/10, r -> 0.49 to 0.51 and eddies will scale down in size by 1/10, but χ normed to gyroBohm will be unchanged.

- Annulus contour plot of physical $\Phi(r,\theta)$ shows dominance of zonal flows over ballooning modes.

Preliminary nonadiabatic electron simulation χ_{Ei} =8 .3, χ_{Ee} = 2 .3, D= - 1.47 $(c_s/a)\rho_s^2$



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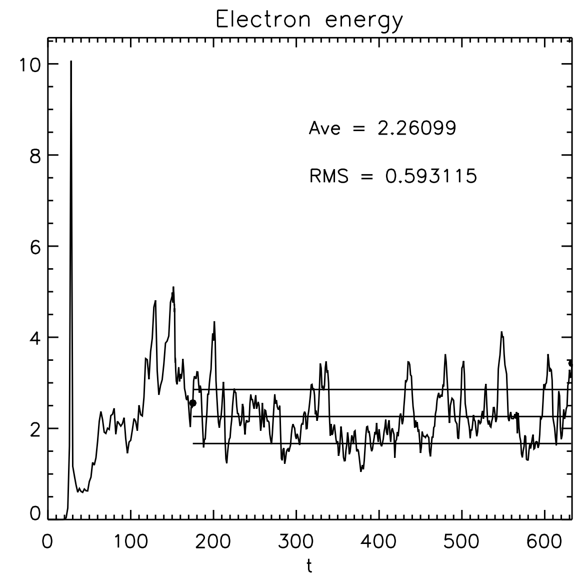

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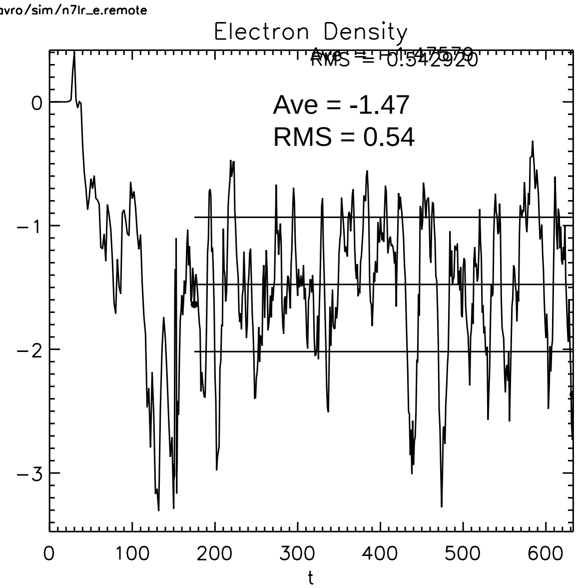

/u/waltz/avro/sim/n7lr_e.remote

$k_y\rho_s$ =0.0, 0.10,....0.70

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GENERAL ATOMICS

GYRO : right or wrong ? code checks and comparisons in progress

- **Linear stability of ballooning modes checked with GKS and spot checked with GS2**
- **Quasilinear flux / $|\phi|_{rms}^2$ with GS2** std case (except a/L_n=1.5) $k_y\rho_s$ = 0.3

GYRO / GS2	ions	electrons
energy	0.39 / 0.39	0.14 / 0.15
plasma	0.012 / 0.015	0.012 / 0.015

- **Nonlinear ky-pump mode test : γ_NL(kx) = $k_y\rho_s \cdot k_x\rho_s \cdot (e\phi(k_y)/T_e) / (\rho_s/a)\ c_s/a$ ± 7%**
- **Nonlinear ITG same grid resolution** std case GYRO / GS2

  _ **2 mode** $k_y\rho_s$ = (0, 0.3) : χ **= 1.3 / 1.5 (-14%)**

  _ **14 mode** $k_y\rho_s$ = (0, 0.06,0.120...0.78): χ **= [2.68,3.25] 3.40 / 4.2 (-19%)**

  dt = [0.1,0.05]0.025 0.01

- **For some collisionless cases, running beyond t=500 a/c_s(1500 L_T/c_s), and particulary at low q (1.4) we find the longest wave length zonal flow (n=0, lowest k_x) builds up to a large stationary value, causing the diffusion to collapse to nearly 0. Other codes do not see this.**

The CYCLONE case at R/L_T=9 is a good example. r/a=0.5, $a/L_T=3\Rightarrow3.29$, $a/L_n=1\Rightarrow0.8$, $R/a=3\Rightarrow2.77$, $s=1.0\Rightarrow0.78$, $q=2\Rightarrow1.4$ (This is far beyond Dimit's shift region R/L_T= 4 to 6)

With small level of ii-collisions GYRO has χ = 2 .72 (dt =0.025) **compared to Dimit's 4.0 (-32% po**

Progress and Conclusions

- We have how obtained good linear EM agreement with Kotschenreuther's GKS code

- We have found efficient MPI parallelization methods with better than linear scaling to 512ps.

- The use of banding and real space nonlinear 2 and 3-pt derivative allows N_r scaling.

- n=0 modes have excellent linear agreement with Rosenbluth-Hinton residules.

- Stable implicit linear methods well in hand

- Have EM real geometry linear runs on the 16ps GA LINUX Beowulf LUNA and moderate phase space resolution nonlinear runs ES adiabatic-e ITG and nonadiabatic-e runs on 45ps GA Stella LINUX cluster, and NERSC 512ps GSeaborg IBM-SP

- Immediate goals:

__Finish flux tube run benchmark with Dorland-Kotschenreuther GS2 code.

__Move to radial annulus operation with non-cyclic radial boundary conditions and profiles

References

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