SIMULATIONS of HIGH betaN*H
ADVANCED TOKAMAK
DISCHARGES IN DIII-D WITH
THE 3D NONLINEAR CODE NFTC .

A.M.Popov, N.N.Popova
Moscow State University

V.S.Chan, R.J. La Haye, A.D. Turnbull
General Atomics

M.Murakami
Oak Ridge National Laboratory

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Nonlinear self-consistent MHD stability simulations of neoclassical tearing modes (NTM) in high betaN Advanced Tokamak (AT) are presented. Radially localized electron cyclotron current drive (ECCD) current profile control is considered based on DIII-D discharge # 99411 in which $\beta_N = 3.9$ and H89P-factor=2.9 are reached. Simulations were performed with the full 3D nonlinear code NFTC. Neoclassical terms are included in the basic equations for the magnetic field and pressure. An effective fully implicit numerical scheme allows the transport profile to evolve self-consistently with the nonlinear MHD instabilities and an externally applied source such as ECCD. NTM activity with m/n=2/1 is found in simulations to correspond to experiment. It is shown that magnetic islands can be quickly suppressed by localized ECCD at q=2 before the mode grows substantially. The time response and nonlinear evolution of the 2/1 island width for ECCD and required modulation phasing, the CD location with respect to the q=2 surface, and the width of the spatial distribution are determined. The possibility of q-profile modification by ECCD well before MHD activity to keep $q_{min} > 2$, so that the discharge evolves stably is also discussed.
GOAL

- Simulations of the nonlinear MHD stability of high $\beta_N$ AT discharge in DIII-D by three-dimensional MHD code NFTC.
  - Discharge # 99411 at the moment $t=1800\text{ms}$ ($q_{\text{min}} = 1.67$)
  - The problem of critical seed island necessary to trigger NTMs.
  - Comparison with experiment.
  - Suppression of NTM instability by radially localized ECCD around $q=2$. (Magnitude of ECCD, width of driven current, location)

- **Motivation.** Discharges with $q_{\text{min}} > 1.5$ and NCS (Negative Central Shear) are the leading scenario for AT (Advanced Tokamak) operation in DIII-D.
  - NTM modes still remain a significant obstacle to approaching the ideal $\beta$ limit.
  - High $\beta_N$ ($\beta_N \sim 3$) is observed in long pulse NCS discharges. No sawteeth or fishbons to provide the seed island.
The nonlinear three dimensional evolution of a tokamak plasma is described by the full (nonreduced, compressible) MHD system of equations in general toroidal geometry.

We seek the solutions \( \{ \mathbf{V}, \mathbf{B}, P \} \) by decomposition:

\[
\mathbf{V}(\rho, \theta, \varphi, t) = \mathbf{V}_0(\rho, \theta) + \sum_{n=0}^{\infty} \mathbf{V}(\rho, \theta, \varphi, t) \\
\mathbf{B}(\rho, \theta, \varphi, t) = \mathbf{B}_{eq}(\rho, \theta) + \sum_{n=0}^{\infty} \mathbf{B}(\rho, \theta, \varphi, t) \\
P(\rho, \theta, \varphi, t) = P_{eq}(\rho) + \sum_{n=0}^{\infty} P(\rho, \theta, \varphi, t)
\]

Our problem is to find \( \{ \mathbf{V}, \mathbf{B}, P \} \) for a given \( \{ \mathbf{V}_0, \mathbf{B}_{eq}, P_{eq} \} \).

The solutions satisfy the fixed boundary conditions:

\[
\mathbf{B}_n = 0, \quad \mathbf{V}_n = 0, \quad P = 0.
\]

Neoclassical terms in the model are included:

- Helical and axi-symmetric bootstrap current terms are added into the magnetic field equation.
- Polarization current effect is included in the magnetic field equation as a helical current perturbation.
- Radially localized toroidal current density from ECCD is included in the magnetic field equation.
- The equation for pressure is modified by incorporated the perpendicular transport across a seed magnetic island and along magnetic field lines.
\[ \frac{\partial \mathbf{V}}{\partial t} = -\nabla \frac{P}{\rho} + (\mathbf{V} \cdot \nabla) \mathbf{V} - \beta_0 \nabla P + \]
\[ + \left[ [\nabla \times \mathbf{B}_{eq}] \times \mathbf{B} + [\nabla \times \mathbf{B}] \times \mathbf{B}_{eq} \right] + \nu \triangle \mathbf{V} - \]
\[ - \nabla \left( \mathbf{V} \cdot \nabla \mathbf{V} \right) + \mathbf{B}_{eq} \times \mathbf{B} \]
\[ + \left[ [\nabla \times \mathbf{B}] \times \mathbf{B} \right] - \rho \left( \nabla \cdot \mathbf{V} \right) + \frac{\nabla}{\nabla} \mathbf{E}_{eq}; \]  
(1)

\[ \frac{\partial \mathbf{B}}{\partial t} = [\nabla \times [\mathbf{V} \times \mathbf{B}_{eq}]] + [\nabla \times [\mathbf{V}_0 \times \mathbf{B}]] - [\nabla \times (\eta \nabla \times \mathbf{B})] + \]
\[ + \left[ [\nabla \times \mathbf{V}] \times \mathbf{B} + [\nabla \times \mathbf{V} \times \mathbf{B}] \right] + \]
\[ + \left[ \nabla \times \mathbf{B} \right] + \mathbf{E}_{eq}; \]  
(2)

\[ \frac{\partial \mathbf{B}}{\partial t} = [\nabla \times [\mathbf{V} \times \mathbf{B}_{eq}]] + [\nabla \times [\mathbf{V}_0 \times \mathbf{B}]] - [\nabla \times (\eta \nabla \times \mathbf{B})] + \]
\[ + \left[ [\nabla \times \mathbf{V}] \times \mathbf{B} + [\nabla \times \mathbf{V} \times \mathbf{B}] \right] + \]
\[ + \left[ \nabla \times \mathbf{B} \right] + \mathbf{E}_{eq}; \]  
(3)

\[ \frac{\partial \mathbf{B}}{\partial t} = [\nabla \times [\mathbf{V} \times \mathbf{B}_{eq}]] + [\nabla \times [\mathbf{V}_0 \times \mathbf{B}]] - [\nabla \times (\eta \nabla \times \mathbf{B})] + \]
\[ + \left[ [\nabla \times \mathbf{V}] \times \mathbf{B} + [\nabla \times \mathbf{V} \times \mathbf{B}] \right] + \]
\[ + \left[ \nabla \times \mathbf{B} \right] + \mathbf{E}_{eq}; \]  
(4)

\[ \frac{\partial \mathbf{B}}{\partial t} = [\nabla \times [\mathbf{V} \times \mathbf{B}_{eq}]] + [\nabla \times [\mathbf{V}_0 \times \mathbf{B}]] - [\nabla \times (\eta \nabla \times \mathbf{B})] + \]
\[ + \left[ [\nabla \times \mathbf{V}] \times \mathbf{B} + [\nabla \times \mathbf{V} \times \mathbf{B}] \right] + \]
\[ + \left[ \nabla \times \mathbf{B} \right] + \mathbf{E}_{eq}; \]  
(5)

\[ \frac{\partial \mathbf{B}}{\partial t} = [\nabla \times [\mathbf{V} \times \mathbf{B}_{eq}]] + [\nabla \times [\mathbf{V}_0 \times \mathbf{B}]] - [\nabla \times (\eta \nabla \times \mathbf{B})] + \]
\[ + \left[ [\nabla \times \mathbf{V}] \times \mathbf{B} + [\nabla \times \mathbf{V} \times \mathbf{B}] \right] + \]
\[ + \left[ \nabla \times \mathbf{B} \right] + \mathbf{E}_{eq}; \]  
(6)

\[ P = f(\rho, T). \]  
(7)
The sources of the current density
The total parallel current density is the sum of Ohmic current and the non-inductive current:

\[ \tilde{j} = j_{eq} + \hat{j} + \tilde{j} = j_{\Omega} + j_{BS} + j_{pol} + j_{cd}, \]

Axisymmetric and helical components of current density perturbation:

\[ \hat{j} + j_{n} = \left( -j_{eq} + j_{\Omega} + j_{BS} + j_{cd} \right)_{n=0} + j_{n} + j_{BS} + j_{pol} + j_{cd} \]

Source term for the equations is included as \( \nabla \times E_{P} \), where

\[ E_{P} = \tilde{E}_{P} = \eta(-j_{eq} + j_{BS} + j_{cd}) \quad \text{for } n = 0 \]

\[ E_{P} = \eta(j_{BS} + j_{pol} + j_{cd}) \quad \text{for } n \neq 0 \]

The bootstrap current \( j_{BS} \) is included in the simplest model form:

\[ \hat{j}_{BS} = Ag \cdot 1.46 \sqrt{\varepsilon}[ - \frac{\partial \hat{P}/\partial \rho}{B_{pol}} ], \]

\[ \tilde{j}_{BS} = Ag \cdot 1.46 \sqrt{\varepsilon}[ - \frac{\partial P_{n \neq 0}/\partial \rho}{B_{pol}} ], \]

where \( Ag = O(1) \) is the geometric parameter of the model.

The polarization current \( \tilde{j}_{pol} \) is included in the model form:

\[ \tilde{j}_{pol} = - \frac{W_{th}^{2}}{W_{th}^{2}} j_{BS} \]

where \( W_{th} \) is the parameter of the model.
• **Representation of current density from ECCD**

Current density \( j_{cd} \) from ECCD is represented as a radially localized toroidal current

\[
\mathbf{J}_{cd} = j_{cd} e_{\varphi}, j_{cd} = j(\psi^*)
\]

on the perturbed helical magnetic flux \( \psi^* \).

A normalized helical flux function:

\[
\tilde{\psi}^* = -(\rho - \rho_s)^2 + \frac{W_{\text{nm}}^2}{8} \cos(m\theta - n\varphi + \alpha\pi)
\]

represents the local behaviour near the resonant surface \( \rho_s \).

The island width \( W \) is defined as

\[
W = 4\sqrt{\frac{g^2 Y^1(\rho_s)}{m\rho_s q'^*}},
\]

where \( Y^1 = \sqrt{g}B^1 \) and \( g \) is a metric element.

Define

\[
j_{cd}(\tilde{\psi}^*) = I_{cd} \frac{1}{2\pi^{3/2}} \frac{1}{W_{cd}} e^{\tilde{\psi}^* / W_{cd}^2}
\]

The following simplified form of \( j_{cd}(\tilde{\psi}^*) \) is used in NFTC code.

\[
j_{cd} = I_{cd} \frac{1}{2\pi^{3/2}} \frac{1}{W_{cd}} e^{-(\rho - \rho_0)^2 / W_{cd}^2} [1 \pm \frac{1}{8} (\frac{W_{\text{nm}}}{W_{cd}})^2 \cos(m\theta - n\varphi + \alpha\pi)]
\]

Parameters:

- \( I_{cd}(t||W) \) is the total ECCD current. This can have any arbitrary dependence on time. A square wave is used in the simulations presented here.

- \( W_{cd} \) is the width of radial distribution,

- \( \Delta \rho = \rho_0 - \rho_s \) denotes the offset of the driven current layer from the rational surface.

- \( \Phi(t) = \alpha\pi \) is the phase fitted to the magnetic island rotation.
• Modification of the transport equations

\[
\frac{\partial P}{\partial t} = \cdots + \nabla \cdot (\chi_{\perp} \nabla P) + \nabla \cdot (\chi_{\parallel} \nabla P);
\]

\[
\frac{\partial \hat{P}}{\partial t} = \cdots + \nabla \cdot (\chi_{\perp} \nabla \hat{P}) + \nabla \cdot (\chi_{\parallel} \nabla \hat{P}) + [\nabla \cdot (\chi_{\parallel} \nabla P)]_{n=0} + Q;
\]

The unit vector along magnetic field line is defined as:

\[
b_{\text{tot}} \equiv \frac{B_{\text{tot}}}{|B_{\text{tot}}|} \approx \frac{B_{\text{eq}} + \hat{B} + B}{|B_{\text{eq}}|} \equiv b_{\text{eq}} + \hat{b} + b.
\]

The differential operator along magnetic field line is:

\[
\nabla_{\parallel} P_{\text{tot}} = (b_{\text{tot}} \cdot \nabla) P_{\text{tot}} = (b_{\text{eq}} \cdot \nabla + \hat{b} \cdot \nabla + b \cdot \nabla)(P_{\text{eq}} + \hat{P} + P)
\]

The total heat conductivity operator is:

\[
\nabla_{\parallel}^2 P_{\text{tot}}\big|_{n \neq 0} = \frac{b_{\text{eq}} \cdot \nabla (b_{\text{eq}} \cdot \nabla P + b \cdot \nabla P_{\text{eq}})}{|B_{\text{tot}}|} + \frac{b_{\text{eq}} \cdot \nabla (b \cdot \nabla P) + b \cdot \nabla (b_{\text{eq}} \cdot \nabla P) + b \cdot \nabla (b \cdot \nabla P_{\text{eq}})}{|B_{\text{eq}}|} + \frac{b \cdot \nabla (b \cdot \nabla P)}{B_{\text{eq}}}
\]

\[
+ [\hat{b} \cdot \nabla (b_{\text{eq}} \cdot \nabla P + b \cdot \nabla P_{\text{eq}}) + b \cdot \nabla (b_{\text{eq}} \cdot \nabla \hat{P} + \hat{b} \cdot \nabla P_{\text{eq}}) + b_{\text{eq}} \cdot \nabla (\hat{b} \cdot \nabla P) + b_{\text{eq}} \cdot \nabla (b \cdot \nabla \hat{P}) + \hat{b} \cdot \nabla (b \cdot \nabla P + b \cdot \nabla \hat{P}) + \hat{b} \cdot \nabla (\hat{b} \cdot \nabla P + b \cdot \nabla \hat{P})];
\]

\[
\nabla_{\parallel}^2 P_{\text{tot}}\big|_{n=0} = b_{\text{eq}} \cdot \nabla (b \cdot \nabla P) + b \cdot \nabla (b_{\text{eq}} \cdot \nabla P) + b \cdot \nabla (b \cdot \nabla P_{\text{eq}});
\]

\[
\nabla_{\parallel}^2 \hat{P}\big|_{m=0} = (b_{\text{eq}} + \hat{b}) \cdot \nabla ((b_{\text{eq}} + \hat{b}) \cdot \nabla \hat{P}).
\]

Underlined terms in the cylindrical approximation:

\[
\nabla_{\parallel}^2 P_m\big|_{n \neq 0} \cos (m\theta - n\varphi) = \left\{ \frac{1}{|B_{\text{eq}}|^2} \left[ -F^2 P + F P_{\text{eq}} \frac{1}{\sqrt{g}} Y^1 \right] + \left[ Y^1 \frac{1}{\sqrt{g}} \frac{\partial}{\partial \rho} \left( \frac{Y^1}{|B_{\text{eq}}|^2} \frac{\partial P}{\partial \rho} \right) \right]_{mn} \right\} \cos (m\theta - n\varphi); F_{mn} = -B_{\text{eq}}^3 \left( \frac{m}{q} - n \right).
\]
ADVANCED TOKAMAK DISCHARGE IN DIII–D
(AT DURATION ENDED BY 2/1 TEARING MODE)
SHOT 99411 BT(set) = –1.60 T

\[ IP(A) \]

\[ P_{\text{INJ}}(\text{KW}) \]

\[ D_{\alpha} \]

\[ \beta_N \]

\[ 4 \times \ell_i \]

\[ q_{0\text{mse}} \]

\[ q_{\text{minmse}} \]

\[ n=2 \text{ rms (T)} \]

\[ n=1 \text{ rms (T)} \]
• First splash at $t=1300\text{ms}$ after $q_{\text{min}} \sim 2.0$ and $\beta_N \sim 3.0$
• Second low level splash when $q_0$ decreases to the value 2.0.
• The third, basic splash at $t=1800\text{ms}$. Perturbation increases by 20 times. $\beta_N = 3.78; q_{\text{min}} = 1.67$. 

**Figure a**
Mirnov signal, rms Btheta. n=1

**Figure b**
Mirnov signal, rms Btheta. n=2
- At $t=1800\text{ms}$ only one rational surface $q=2$ located on $\rho_s = 0.74a_H$ and $\Delta q = q_0 - q_{min} = 0.13$, Shear $S=1.28$.

- At $t=1915\text{ms}$ two rational surfaces located on $\rho^1_s = 0.42$ and $\rho^2_s = 0.68.S^{(1)} = -0.16, S^{(2)} = 0.43$. 

![q-profile](image)

![Pressure Peq](image)
Nonlinear calculations of NTM for eq.1800ms.

- Find $W_{th}$ for $m/n=2/1$ as amplitude of $j_{pol}$ with “Eq.1.8s”.
- Two parameters of the model: Ampl and $W_{th}$ with $W_d=0$.
  - First, increase Ampl • eigefunction (with $W_{th}=0$ until NTM is exited. Ampl, $W_{th}$.
  - Second, with fixed $W_{cr}$ change $W_{th}$ until $W(t), W_{sat}$ saturation will be found.
- Then found $\frac{W_{th}}{a} = 0.0235$ and this is used for all equilibriua.
- $\frac{W_{cr}}{a} = 0.045$ and $\frac{W_{sat}}{a} = 0.2$

Islands evolution , Eq.1800 msec, qmin=1.67

(dW,W)-Diagram

Jbs Evolution t=1800
- Helical perturbations of magnetic field $\rho \cdot B^\rho(\rho, t)$ and pressure $P(\rho, t)$ for $m/n=2/1$.
- Axisymmetric component of quasilinear corrections for poloidal field and pressure.
- Profiles alter from green to red.
Critical island width evaluation.

- Magnetic islands $m/n=2/1$ evolution with different seed islands for eq.1.8s.
- Bootstrap current density for $m/n=2/1$ in the fixed radial point.
- m/n=2/1 NTM instability excites toroidal modes n=2 with same helicity.
- Harmonic m/m=4/2 increases to the level \( W_{4/2} \sim 0.09a_H \) and m/n=6/3 has \( W_{6/3} \sim 0.07a_H \).
- The double NTM 5/3 has the large island width \( W_{5/3}^1 = 0.1a_H \) and \( W_{5/3}^2 = 0.07a_H \) located in the middle of plasma area with low shear on \( q=5/3 \). Low toroidal coupling with 2/1.

Islands evolution, Eq.1800 msec, qmin=1.67
Magnetic islands due to NTM instability.

Magnetic islands $t=200t_{0A}$ Eq."1800msec"

$q$-profile $t=200t_{0A}$  (Eq.1800)
Magnetic islands due to NTM instability.

Magnetic islands \( t=600\tau_{0A} \) Eq."1800msec"

\[
\begin{align*}
\rho & = 0.4, 0.6, 0.8, 1.0 \\
\theta & = 0, 1, 2, 3, 4, 5, 6
\end{align*}
\]

\[b\]

\[\theta\]

\[q\]

q-profile \( t=600\tau_{0A} \) (Eq.1800)
Magnetic islands due to NTM instability.
- Amplitude of magnetic perturbation $n=1$ and magnetic island width $W_{2/1}$ as a function of time.
- The $m/n=2/1$ harmonic dominates in magnetic perturbation.
- The level and radial profile of quasilinear correction of equilibrium poloidal magnetic field Eq.1.8s corresponds to MSE diagnostic.

Comparison with experiment. Signal on Mirnov coils $B_p$.

Comparison with MSE diagnostic. $t=1.80-1.84s$. 

![Graph showing comparison with experiment and simulation results](image-url)
Conventional double tearing instability when $q_{\text{min}}$ crosses 2.0.

- Successive changes of eigenfunctions for decreasing $q_{\text{min}}$.
- When $q_0 = 2.25$ ($q_{\text{min}} = 1.993$) and $q_0 = 2.2$ ($q_{\text{min}} = 1.95$) conventional double tearing mode 2/1 is weakly unstable. Corresponds to $t=1300\text{ms}$ in discharge. Nonlinearly stabilizes.
- When $q_0 = 1.87$ ($q_{\text{min}} = 1.67$) conventional tearing mode is stable. Nonlinear NTM unstable. Corresponds to $t=1800\text{ms}$ in discharge.
- Comparison of theoretical limit $\beta_N^{cr} = C \cdot \sqrt{S}$ with experimental data for # 99411 discharge.

- Equilibrium profiles and parameters are fitted to the moment $t=1.8s$ in experiment. Linear approximation of $q_{min}(t) = 2.075 - 0.225t$ in the period of instability.

- When $t=1.3$ parameter $q_0 = 2.0$ and $q_{min} = 1.78$ the first MHD burst happens. When $t=1.8$ the main NTM instability is excited.
- Time evolution of quasilinear correction of equilibrium pressure profile shows the decreasing of central pressure due to NTM instability.

- Heat conductivity is artificially increased by two orders in the area of magnetic islands overlapping ($4/2, 6/3, 5/3, ..$).
• a) $W(t)$ with different magnitude of $I_{cd}/I_p$. $W_{cd} = 0.06a_H$. Island width begins to oscillate. Critical seed island $W_{2/1}^{cr} = 0.045a_H$. If $I_{cd}/I_p > 0.055$ then $W_{sat} = W_{max} = 0.02a_H$.

• b) $W_{2/1}^{sat}$ and $W_{2/1}^{max}$ vs. $I_{cd}/I_p$.

![Islands evolution, Eq. 1800 msec, qmin=1.67](image)

![Island Suppression vs. Icd.](image)
• The effectiveness of magnetic islands suppression increases with decreasing of driven current layer $W_{cd}$, when the magnitude of $I_{cd}/I_p$ is constant (here 0.023).

• The less $W_{cd}$ the less period of oscillations. $W_{max} \approx W_{cd}$. The best conditions: if $W_{cd} > 0.5W_{cr}$ then no oscillations during period of suppression.

![Islands evolution, Eq.1800 msec, qmin=1.67](image)

![Island Suppression vs. Wcd.](image)
The effectiveness of magnetic islands suppression.

- Area of magnetic islands suppression on the planes: \( \left( \frac{I_{cd}}{I_p}; \frac{W_{cd}}{a_H} \right) \) and \( \left( \frac{I_{cd}}{I_p}; \frac{W}{W_{cd}} \right) \).

- Each point on curves corresponds to the same maximal magnetic islands width equal \( W_{2/1} = 0.06a_H \) obtained during calculations.
- Maximum of stabilization effect occurs when $\Delta \rho = 0$. When $\Delta \rho \geq \frac{W_{cd}}{a_H}$, then the stabilization effect is lost. $W_{cd} = 0.06a_H$ for these cases.
- Magnetic islands width during ECCD suppression with different islands rotation.

- Comparison of ECCD efficiency for only axi-symmetric component and axi-symmetric and helical components of $j_{cd}$.
• Stability calculations by NFTC of high $\beta_N \times H$ AT in DIII-D (# 99411 discharge) were performed. Comparison of nonlinear NTM calculations with experimental data shows
  - correspondence with the level of the helical magnetic perturbations with Mirnov signal
  - correspondence of quasilinear correction of equilibrium poloidal field with MSE diagnostic.

• Explanation of critical seed island were done as aftereffect of conventional tearing instability when $q_{min}$ crosses the value $q_{min} = 2$.

• A radially localized current from ECCD is included for NFTC code as a source term. Driven current $j_{cd}$ depends on perturbed magnetic surfaces and has helical and axisymmetric components in its representation. Self-consistent calculations using the NFTC code show the effectiveness of ECCD suppression of neoclassically destabilized magnetic islands.

• ECCD current which is needed for the island suppression increases parabolically with the driven current layer width $W_{cd}$. The less $W_{cd}$ is, the less total current is needed for suppression. Magnetic island can be suppressed to the level of order $W_{cd}$. For $W_{cd} < 0.05a_H$ and $I_{cd} = 0.05I_p$ the seed magnetic islands are suppressed and NTM stabilizes at $t=1800\text{ms}$ in discharge # 99411.

• Localization of $max(j_{cd})$ relating rational surface of mode is very important for stabilization. Maximum shift is of order $\Delta \rho_{max} \sim W_{cd}/a_H$, which is equal to $0.05a_H$ for # 99411 discharge.