CRITERIA FOR CURRENT DRIVE STABILIZATION OF NEOCLASSICAL TEARING MODES

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OUTLINE

- Island Evolution Equation
- Bootstrap and Current Drive Terms
- Four Criteria
- Projections for Reactor-Scale Devices
- Conclusions

ISLAND EVOLUTION EQUATION: MAGNETIC FIELDS

1. Assume magnetic field which has good helical flux surfaces χ = constant.

$$B = \frac{g}{R}\boldsymbol{\phi} + \frac{q}{q_o}\frac{\boldsymbol{\phi} \times \nabla\psi}{R} + \frac{\boldsymbol{\phi} \times \nabla\chi}{R}$$

$$\chi = \chi\left(\psi, \alpha\right) \qquad\qquad \alpha = \theta - \phi / q_o$$

2. Assume form for helical flux function χ - Not a complete calculation

$$\Phi = \frac{\chi}{\chi_o(\psi,t)} = \text{nondimensional helical flux function} = \frac{x^2}{w^2} - \left[\frac{1 + \cos(m\alpha)}{2}\right]$$

3. Island width w is really the helical flux function χ

$$w^2 = 2\,\chi_o\, r_s\, (\hat{s}\;\nabla\psi)^{-1}$$

ISLAND EVOLUTION EQUATION: BACK EMF

1. Starting point is back emf equation for helical flux

$$\frac{\partial \chi}{\partial t} = \eta \left[j_h - j_{bs} - j_{cd} \right]$$

- **j_h is helical current density; j_{bs}, j_{cd} are emfs**

2. Mechanical equilibrium requires that j_h be a function of helical flux

- Analogous to FF´ term in Grad-Shafranov equation — $\frac{\partial^2 \chi}{\partial x^2} = \mu_o \, j_h(\chi)$

3. Form flux surface average of back-emf equation.

- Only flux surface averages $\langle j_{bs} \rangle$ $\langle j_{cd} \rangle$ **enter**

- Equation is not balanced in detail; only in large-scale properties matter

ISLAND EVOLUTION EQUATION - 2

4. Recast flux-surface average back-emf equation into Rutherford island evolution equation.

$$\frac{\mu_o}{\eta}\left(\frac{2\,C_1}{\pi}\right)\frac{\partial w}{\partial t} = \Delta_o' + \Delta'$$

Δ_o' is usual tearing mode term **;** Δ' comes from current drive**;** $w^2 = 2q\chi_o/B\hat{s}$

4. Find formal expression for Δ' in terms of a helical flux function average current density $J(\psi)$

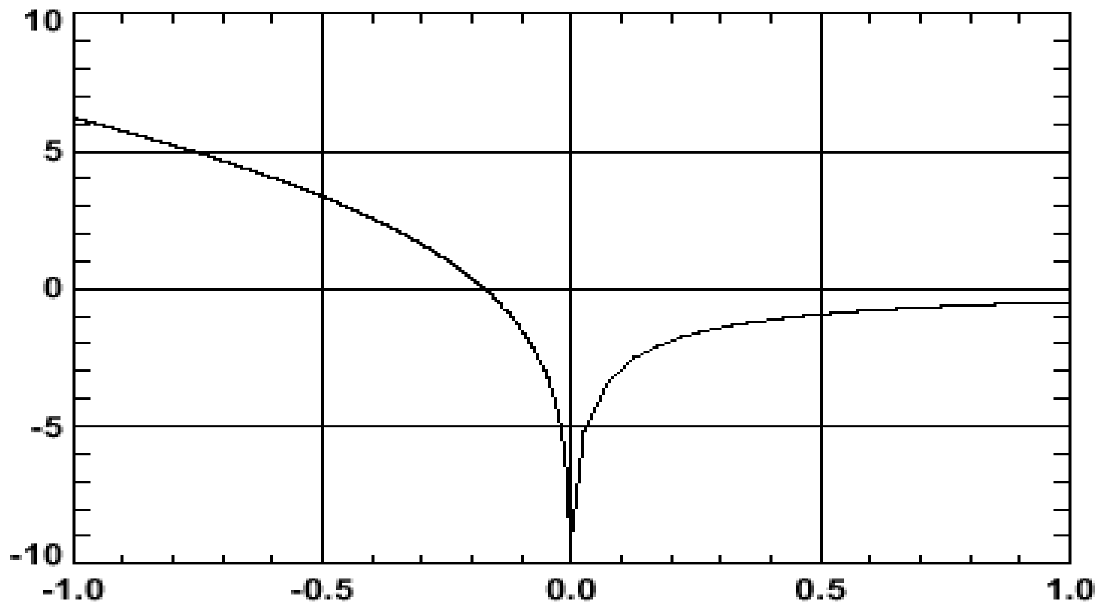
$$\Delta' = -\frac{4\,\mu_o\,Rq}{\pi\,w\,\hat{s}\,B}\int_{-1}^{\infty} d\psi\, W(\psi)\, J(\psi)$$

- **Mechanical equilibrium requires $j = J(\psi)$ - just like Grad-Shafranov Eq.**

WEIGHTING FUNCTION

5. W(ψ) is a weighting function for helical average current density and emfs.

- **Zero response for uniform current density** $\int_{-1}^{\infty} d\psi\, W = 0$



DRIVEN CURRENT DENSITY

1. **Gaussian Profile for driven current density is centered on rational surface and modulated by $M(\tau)$**

$$j_d = j_{cd} \exp\left\{-\frac{(r - r_s)^2}{w_{cd}^2}\right\} M(\tau) \qquad j_{cd} = \frac{I_{cd}}{2\,\pi^{3/2}\, r_s\, w_{cd}}$$

2. **Form helical averages for island evolution equation:**

$$J(\psi, \tau, w/w_{cd}) = \frac{j_{cd}}{V(\psi)} \oint d\alpha \, \frac{M(\alpha,\tau)}{\sqrt{\psi + \cos^2(\alpha)}} \exp\left\{-\frac{w^2}{w_{cd}^2}\left(\psi + \cos^2(\alpha)\right)\right\}$$

$$V(\psi) = \oint \frac{1}{\sqrt{\psi + \cos^2(\alpha)}}\, d\alpha$$

ISLAND EVOLUTION EQUATION

1. Island Evolution Equation with polarisation stabilization.

$$\frac{\mu_o}{\eta}\left(\frac{2\,C_1}{\pi}\right)\frac{\partial w}{\partial t} = \Delta'_o + \frac{32\,\mu_o\,Rq\,j_{bs}}{3\,\pi\,w\,\hat{s}\,B}\left\{1 - \left(\frac{x_o}{x}\right)^2 - \Lambda\,K_1(\tau,\,x)\right\}$$

$\Lambda = j_{cd}/\,j_{bs}$ $\quad$ $x = w/w_{cd}$ $\quad$ w = island half-width

w_{cd} = half width of current drive layer $\quad$ τ = modulation duty factor

w_{pol} = polarization stabilization half-width ª 1.2 ρ_{tor} $\quad$ $x_o = w_{pol}/w_{cd}$ ª 0.3

2. Function $K_1(\tau,x)$ represents stabilizing current- drive effects

3. Saturated island half-width $\quad w_{sat} = \left(\frac{32}{3\pi\,\hat{s}}\right)\left(\frac{j_{bs}\,R\,q\,\mu_o}{2B}\right)\frac{1}{(-\Delta_o')}$

- $(-\Delta_o')$ ª m/r **in absence of current drive**

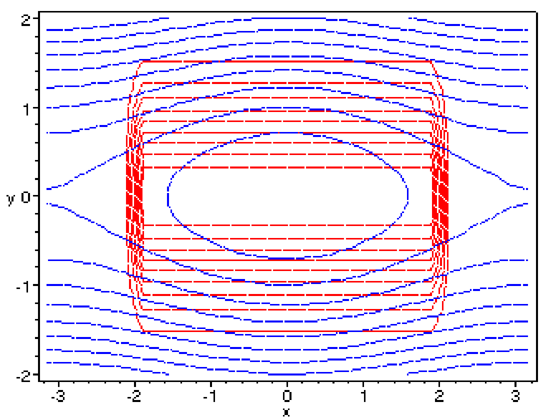
4. Island size comparable to minor radius when $\beta(0) \sim 0.05$ **.**

CALCULATION OF STABILIZING TERM $K_1(x, \tau)$

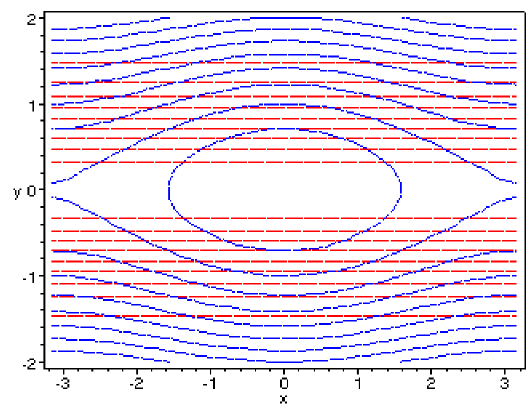
- **Model for driven current layer**

$$j_d = j_{cd} \exp\left\{-\frac{(r - r_s)^2}{w_{cd}^2}\right\} M(\tau) \qquad j_{cd} = \frac{I_{cd}}{2\,\pi^{3/2}\, r_s\, w_{cd}}$$

- **First, average current density over helical flux surface**
- **Second, evaluate weighted integral over helical flux**



$\tau = 0.64$



$\tau = 1.0$

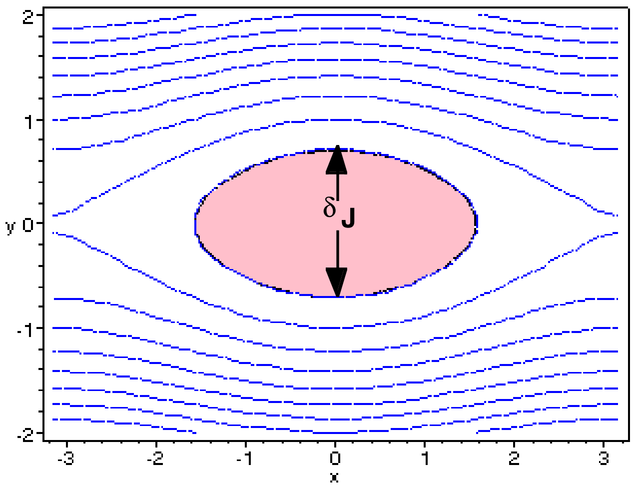
Do this as a function of island width and τ .

ANALYTIC CURRENT DRIVE MODEL

- **Driven current density model of Hegna and Callen for $\delta_J/2w = 0.707$ in the $m\alpha$ - $(r-r_s)/w$ plane.**

- **Current density assumed uniform in pink region**

- **Realization requires both spatial localization and modulation**



FOUR STABILITY CRITRIA

1. Stabilization of arbitrarily small islands ($w \ll w_{cd}$)

2. Limitation of growing island size to w ª w_{cd} .

a) Δ' independent of current drive

b) Current drive layer changes Δ'

3. Complete stabilization of NTMs in present devices

4. Reduction of already-established saturated islands to wª w_{cd}

- $w_{sat} \gg w_{cd}$ **assumed**

STABILIZATION OF SMALL ISLANDS

1. Results for $K_1(x, \tau)$ show that it has a finite value for $x \to 0$, assuming a modulated source.

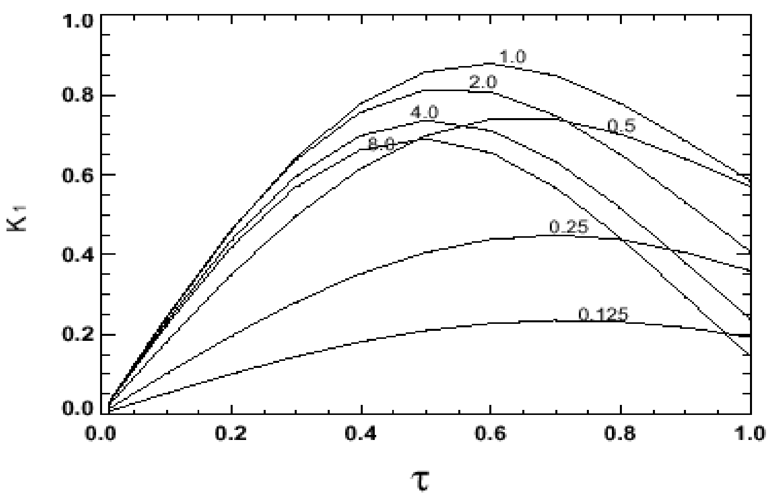


Fig. 4. K1 versus "on"-time τ for the various island widths w_{cd}/w marked on the diagram.

- Maximum value of $K_1(0,\tau)$ is $K_1 = 0.65$ with 50% "on" 50% "off"

- Stabilization will occur when $\Lambda = j_{cd}/ j_{bs} > 1.6$

ISLAND EVOLUTION WITH NO MODULATION

1. Evaluate $K_1(x,1)$ with no modulation and represent by analytic fit. (±5%)

$$K_1(x,1) = \frac{x}{1 + \left(\frac{2}{3}\right)x^2}$$

- **Correct power series and asymptotic forms**

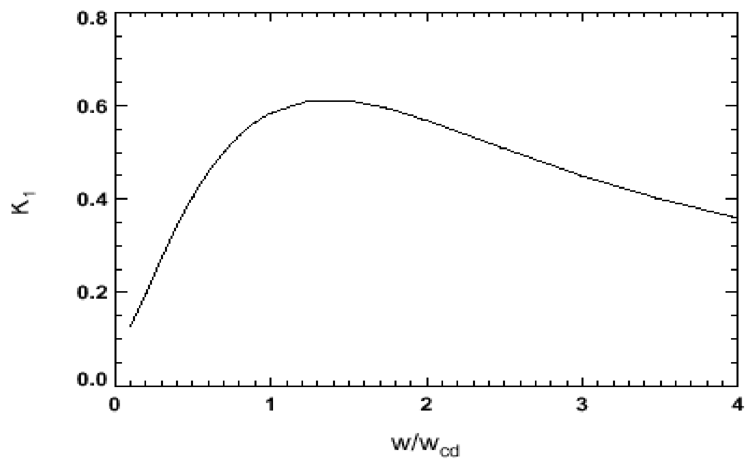


Fig. 5. K1 versus island width, for unmodulated ECCD
(τ = 1.0).

NONDIMENSIONAL ISLAND EQUATION

- **Non dimensional island evolution equation with X = (w_{sat}/w_{cd}) >> 1.**

$$\frac{dx}{dT} = -1 + X\left(\frac{1}{x}\left(1-\left(\frac{x_o}{x}\right)^2\right) - \frac{\Lambda}{1+\left(\frac{2}{3}\right)x^2}\right)$$

- **Usual saturated island is given by x=X.**

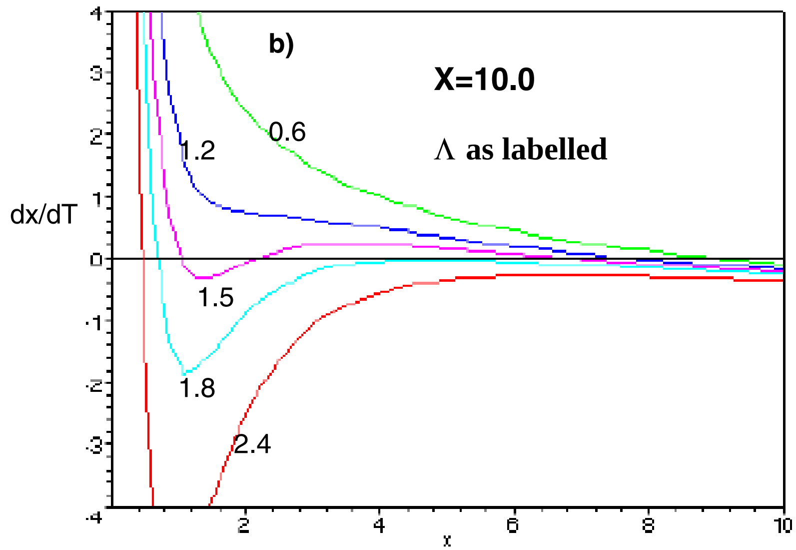
- **New roots possible when coefficient of X vanishes** (ignoring x_o)**.**

$$x = 0.75\left(\Lambda \pm \sqrt{\Lambda^2 - (8/3)}\right)$$

- **Criterion for two roots is** $\Lambda > \sqrt{8/3} = 1.63$ **when x_o << 1**

NEW ROOTS LIMIT ISLAND GROWTH

- **No Modulation**
- **No current drive effect on Δ'**
- **No polarization stabilization**

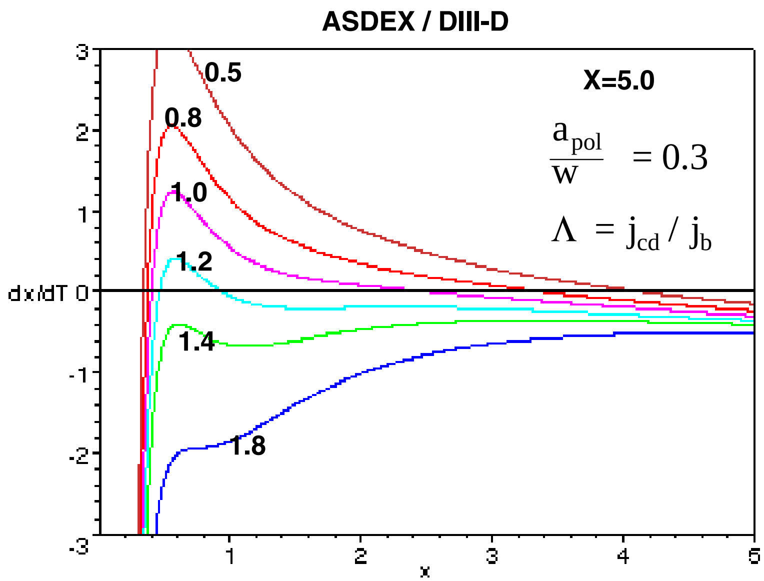


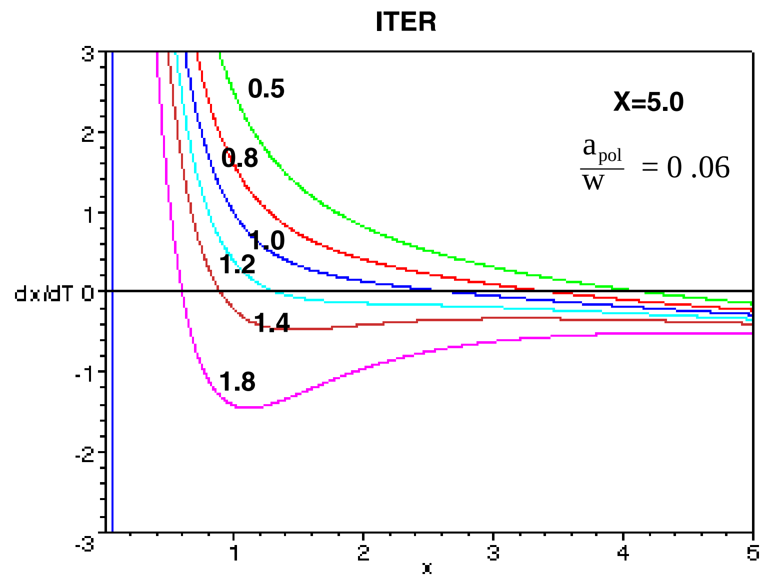
- **Current drive reduces saturated Island Size; (use asymptotic form for K_1).**

$$x = 0.5 \{ X \pm (X^2 - 6 \Lambda X)^{1/2} \}$$

- **Criterion for elimination of saturated islands $\Lambda > X/6$; $I_{cd} > 0.23\, I_{bs}$**

POLARIZATION STABILIZATION COMPLETELY SUPRESSES MODE





- **Suppression criterion: Λ > 1.3 for ASDEX/DIII-D and is sensitive to x_o .**

$$\Lambda > \frac{1}{3\sqrt{3}x_o} \quad \text{(with } \Delta'\text{)} \qquad \Lambda > \frac{2}{3\sqrt{3}x_o} \quad \text{(without } \Delta'\text{)}$$

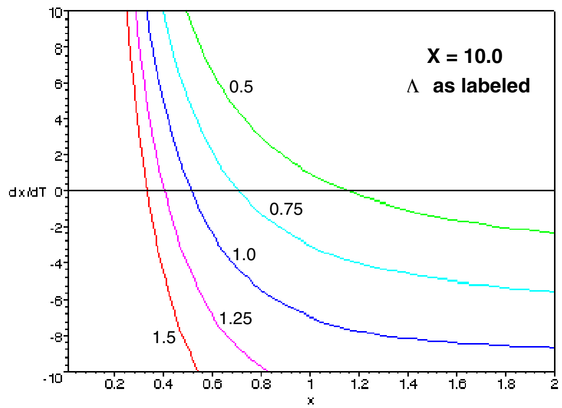
- **Supression criterion has unfavorable scaling with ρ^***

EFFECT OF CURRENT DRIVE LAYER ON Δ'

- **Current Drive Layer centered on rational surface**

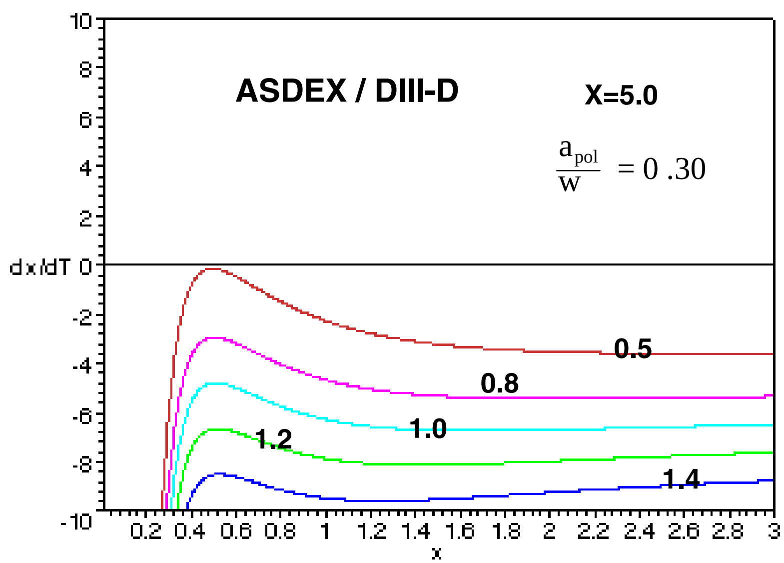
$$(-\Delta')_{cd} = (-\Delta'_o)\, X\, \Lambda \qquad \frac{dx}{dT} = -1 + X\left\{\frac{1}{x} - \Lambda\,\frac{2 + \left(\frac{2}{3}\right)x^2}{1 + \left(\frac{2}{3}\right)x^2}\right\}$$

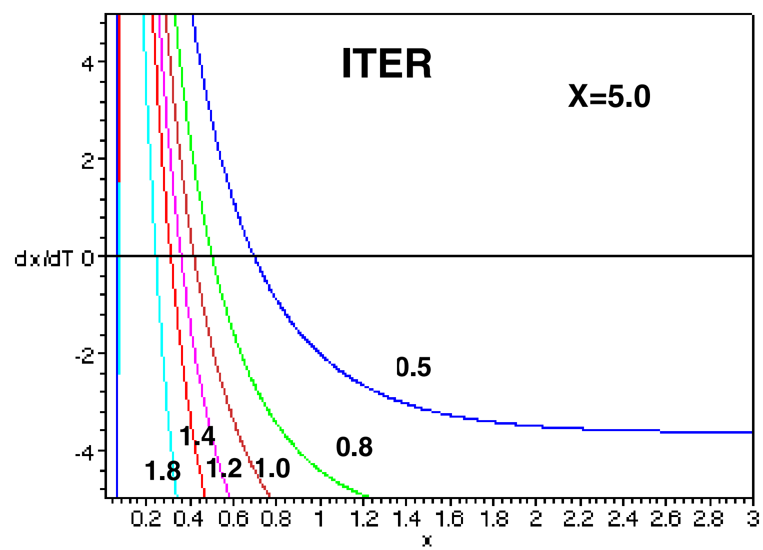
- **Only one real root; growth to saturated level is prohibited**



- **Island size limited to** $w < w_{cd}$ **when** $\Lambda > 0.6$ **.**

POLARIZATION DRIFT PLUS Δ′ ELIMINATES NTMs IN PRESENT FACILITIES FOR MODEST CURRENT DRIVE

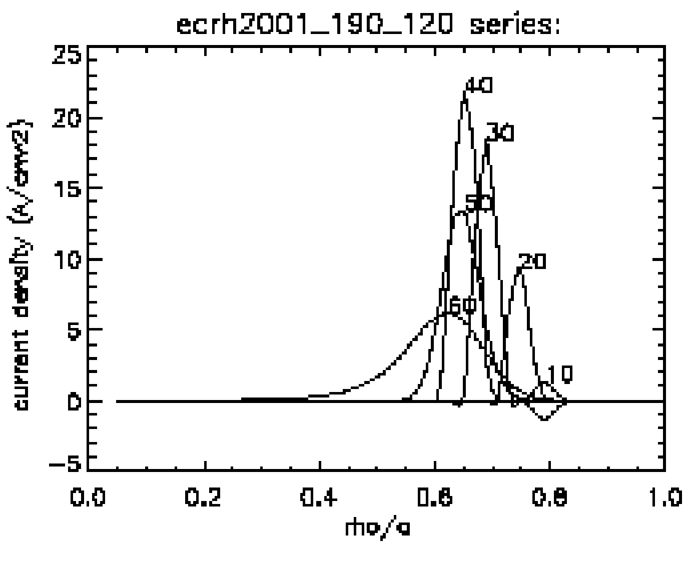


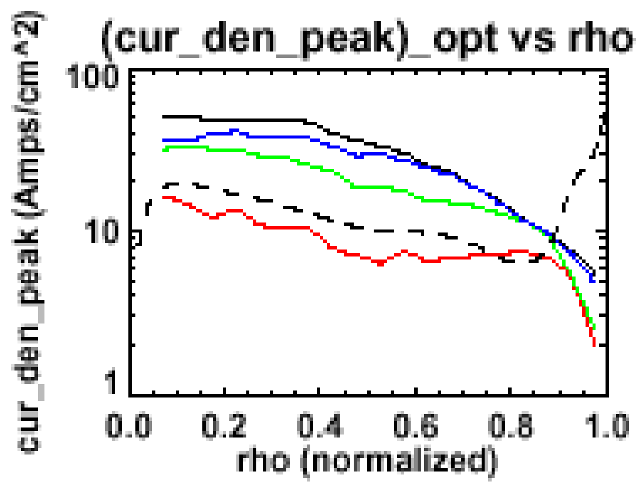


- **Suppression criterion Λ > 0.5 in ASDEX/DIII-D**
- **Polarization stabilization ineffective in an ITER-class device**

APPLICATION TO ITER FEAT

- **ECCD maximized by off-axis launch location**
- **For ITER FEAT with 20 MW ECCD; j_{bs} ª 0.07 MA/m^2 j_{cd} = 0.1 MA/m^2**
- **Wide range of gyrotron frequencies work; above midplane lauch location**





155 Ghz (red), 170 (green), 190 (blue)
Dashed= bootstrap

EFFECT OF Δ' IS NOT DELAYED (MUCH) BY BACK EMF.

1. Back-emf analysis for abrupt initiation of current drive carried out for contribution to island evolution equation

$$\frac{dx}{d\tau} = \left[\frac{j_{bs}}{j_q\,\hat{s}}\right]\left\{-\frac{1}{X} + \frac{1}{x} - \Lambda\left(\frac{1}{1+\left(\frac{2}{3}\right)x^2} + \frac{\tau-\tau_o}{1+\tau-\tau_o}\right)\right\}$$

$$j_q = 2B / (R\,\mu_o q)$$

2. Factor in [..] brackets is generally less than unity

- **Evolution of NTM slow compared to development of Δ'**

PHYSICS CONCLUSIONS

1. **Correct Figure-of-Merit for ECCD stabilization of Neoclassical Tearing Modes is $\Lambda = j_{cd} / j_{bs}$**

   - **$\Lambda > 1.3$ reduces island size to driven current layer thickness (no Δ')**
     $\Lambda > 0.6$ (with Δ')

   - **$\Lambda > 1.6$ (with modulation) completely stabilizes modes**

2. **Polarization stabilization completely suppresses NTMs in present devices but appears unimportant for ITER-FEAT. Do other stabilizing effects exist?**

3. **Island sizes comparable to current drive width expected for continuous ECCD in ITER-FEAT.**

   - **Modulation will still supress all islands for** $j_{cd} > 1.6\ j_{bs}$

   - **Retain modulation capability in ITER-FEAT design.**