Abstract Submitted
for the DPP00 Meeting of
The American Physical Society

Sorting Category: 6.2.0 (Th

Nonlinear Tearing Mode Driven by a Current Density Gradient[1] T.H. JENSEN, General Atomics — It has been suggested[2] that a linearly stable tearing mode may be nonlinearly unstable, driven by a current density gradient at the singular surface. This is under the assumptions of single fluid MHD. Qualitatively, this instability may have properties similar to those of "neoclassical" tearing modes,[3] driven by a pressure gradient. In order to find out if the suggestion[2] is correct, a numerical study was undertaken. The algorithm used for the study is a generalization of one used previously[4] for studying the saturated island width of linearly unstable modes. The algorithm allows construction of a sequence of forced equilibria sharing a set of "almost ideal MHD constraints." A sign of the forcing function reveals whether the island is growing or shrinking. The previous study[4] showed that the method is well suited for the study of nonlinear instabilities although neither growth rates nor decay rates are found. Results from the numerical study will be discussed.

[1]Work supported by U.S. DOE Grant DE-FG03-95ER54309.
[2]T.H. Jensen and W.B. Thompson, Phys. Fluids **30**, 3052 (1987).
[3]W.Q. Xu and J.D. Callen, UWPR 85-5, 1985; R. Carrera, R.D. Hazeltine, and M. Kotschenreuther, Phys. Fluids **29**, 899 (1986).
[4]T.H. Jensen, Bull. Am. Phys. Soc. **44**, 79 (1999).

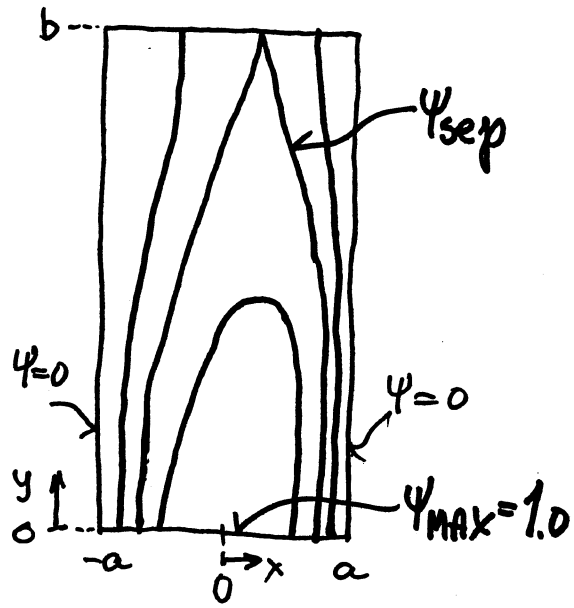
1. OVERVIEW

i) CIRCUMSTANCES ARE FOUND UNDER WHICH A PLASMA, LINEARLY STABLE TOWARD TEARING, IS NONLINEARLY UNSTABLE, DRIVEN BY A CURRENT DENSITY GRADIENT AT THE SINGULAR SURFACE (SEPARATRIX).

ii) WHEN A CURRENT DENSITY GRADIENT IS PRESENT AT THE SINGULAR SURFACE (SEPARATRIX), TEARING IS AN IRREVERSIBLE PROCESS.

iii) THE ISLANDS RESULTING FROM THIS NONLINEAR INSTABILITY WILL INITIALLY GROW AND MAY THEN (BECAUSE OF THE IRREVERSIBILITY) SHRINK (DISAPPEAR). THE INSTABILITY MAY DESCRIBE A WAY FOR A PLASMA TO APPROACH THE TAYLOR STATE.

2. MODEL



- SINGLE FLUID MHD ; $\bar{\nabla} p = 0$
- $\partial/\partial z = 0$; $\bar{B} = B_z \hat{z} + \bar{\nabla}\psi \times \hat{z}$
- EQUILIBRIUM : $(\bar{\nabla}\times\bar{B})\times\bar{B} = 0$

$\rightarrow \bar{\nabla} j \times \bar{\nabla}\psi = 0 \quad (j \equiv -\nabla^2\psi)$

SIMPLY NESTED FLUX SURFACES:

$j(\psi)$; IF NOT: $j_L(\psi), j_R(\psi) \ldots$

FOR $\psi_{sep} < \psi < \psi_{MAX} \; (= 1.0) \Rightarrow j(\psi)$

FOR $0 < \psi < \psi_{sep} \Rightarrow j_L(\psi), j_R(\psi)$

- CONSERVED QUANTITIES: $K_\nu = \int G_\nu(\psi)\, dA \quad ((\nabla\psi/B_z)^2 \ll 1)$

PERTURBATION OF EQUILIBRIUM (IDEAL MHD):

$\delta\psi + \bar{\xi}\cdot\bar{\nabla}\psi = 0 \Rightarrow \delta K_\nu = \delta \int G_\nu(\psi)\, dA = 0$

IF: $\bar{\nabla}\cdot\bar{\xi} = 0$ AND $\oint_{BOUNDARY} G_\nu\, \bar{\xi}\cdot\hat{n}\, dS = 0$

$\nu_{MAX} \rightarrow \infty$: IDEAL MHD; TEARING NOT POSSIBLE

ν_{MAX} FINITE: "ALMOST IDEAL MHD"*) ; TEARING POSSIBLE

- BOUNDARIES: $x = \pm a$; $y = 0, b$; SEPARATRIX

$\Rightarrow KL_\nu$; KR_ν ; KI_ν (LEFT, RIGHT, ISLAND)

*) JENSEN, LEONARD + CHU, PHYS. FLUIDS B3 1650 (1991)

- AN ALGORITHM HAS BEEN FOUND FOR FINDING THE EQUILIBRIUM (WITH CONTINEOUS $\sigma^2\psi$) DEFINED BY KL_ν's, KR_ν's, KI_ν's AND ψ_{sep}. AN 'EXTERNAL' CURRENT (AT $y=b$) IS NEEDED TO INSURE TOPOLOGY AND ψ_{sep} AS REQUIRED.

- THE SIGN OF THE "EXTERNAL" CURRENT DETERMINES WHETHER THE ISLAND WILL GROW OR SHRINK IF THE "EXTERNAL" CURRENT IS REMOVED.

- NO INFORMATION ON GROWTH OR DECAY RATES (OF ISLANDS) IS OBTAINED.

3. SPECIFICS

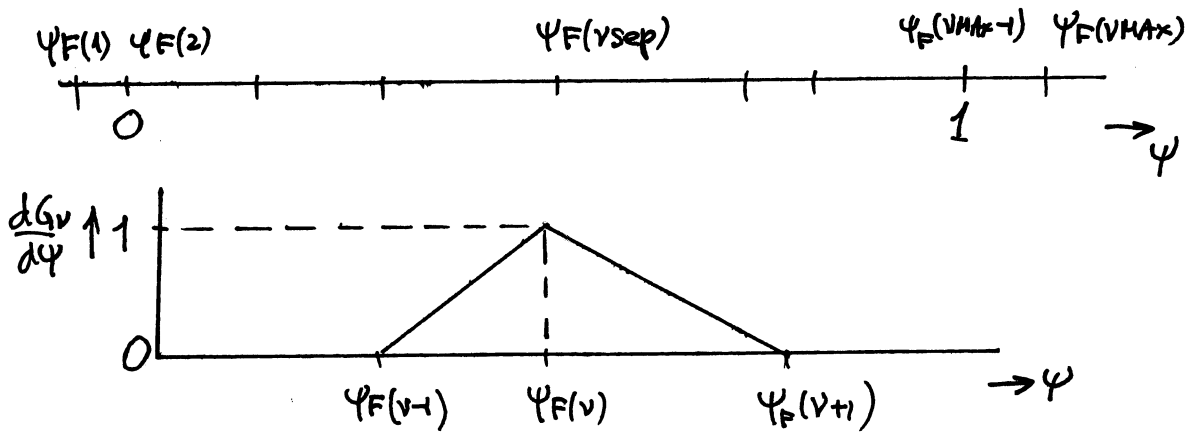
- AN "INITIAL", 1D EQUILIBRIUM IS DETERMINED BY TWO NUMBERS, ka AND kD, BY

$$\psi_I(x,y) = \frac{\cos\left(kx + kD\,\frac{\cosh kx - \cosh ka}{1 - \cosh ka}\right) - \cosh ka}{1 - \cosh ka}$$

FOR $kD = 0$, CURRENT DENSITY GRADIENT AT SINGULAR SURFACE (SEPARATRIX) VANISHES.

FOR THIS CASE ka AND b/a DETERMINES $\Delta' a$

- CHOOSE $\psi_F(\nu)$'S:



$G_\nu(\psi)$ NORMALIZED SO THAT:

$$G_\nu(\psi_F(\nu_{SEP})) = 0 \quad \text{FOR } \nu \neq \nu_{SEP}$$

$$G_\nu(\psi_F(\nu_{sep}-1)) = 0 \quad \text{FOR } \nu = \nu_{sep}$$

NOTE: $\psi_F(\nu_{sep}) \leq \psi_{sep}$

4. NUMERICS

$\psi(x,y) \rightarrow \psi(i,j)$; $i = 1, 2, \cdots i_{max}$; $j = 1, 2, \cdots j_{max}$

$\psi(1,j) = \psi(i_{max}, j) = 0$; $\psi(i,1) = \psi(i,3)$; $\psi(i, j_{max}) = \psi(i, j_{max}-2)$

$b/a = 2 \frac{j_{max}-3}{i_{max}-1} \frac{D_y}{D_x}$;

USUAL DIFFERENCE REPRESENTATION OF $\nabla^2\psi$.

DEFINE: $L(i,j) = 1$ FOR $\frac{\partial\psi}{\partial x} > 0$ AND $\psi < \psi_F(v_{sep})$; ELSE 0

$R(i,j) = 1$ FOR $\frac{\partial\psi}{\partial x} < 0$ AND $\psi > \psi_F(v_{sep})$; ELSE 0

$I(i,j) = 1$ FOR $\psi > \psi_F(v$; ELSE 0

$$KL(v) = \sum_{i,j} G_v(\psi(i,j))\, L(i,j)$$

$$KR(v) = \sum_{i,j} G_v(\psi(i,j))\, R(i,j)$$

$$KI(v) = \sum_{i,j} G_v(\psi(i,j))\, I(i,j)$$

- $KOL(v)$, $KOR(v)$, $KOI(v)$ ARE OBTAINED FROM INITIAL EQUILIBRIUM; THUS THEY DEPEND ON $\Delta'a$, kD, b/a, $\psi_F(v_{sep})$.

- INSTEAD OF kD USE $a(dy/dx)/y$ AT SINGULAR SURFACE

ITERATION:

$$\delta\psi = \frac{0.2}{1+(D_x/D_y)^2}\left\{\nabla^2\psi + \sum_\nu \alpha L(\nu)\frac{dG_\nu}{d\psi}L(i,j) + \sum_\nu \alpha R(\nu)\frac{dG_\nu}{d\psi}R(i,j) + \sum_\nu \alpha I(\nu)\frac{dG_\nu}{d\psi}I(i,j)\right\} \quad \text{(JACOBI)}$$

$$\delta\alpha L(\nu) = c\left[K_0 L(\nu) - K L(\nu)\right]$$

$$\delta\alpha R(\nu) = c\left[K_0 R(\nu) - K R(\nu)\right]$$

$$\delta\alpha I(\nu) = c\left[K_0 I(\nu) - K I(\nu)\right]$$

$$\psi_{NEW}(i, j_{max}-1) = \psi_{OLD}(i, j_{max}-1)\ \frac{\psi_{sep}}{MAX\{\psi_{OLD}(i, j_{max}-1)\}}$$

$$j_{ext} = \sum_i \left[\psi(i, j_{max}-1) - \psi(i, j_{max})\right] \times \frac{1}{i_{max}-1}$$

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OBJECT OF CALCULATION:

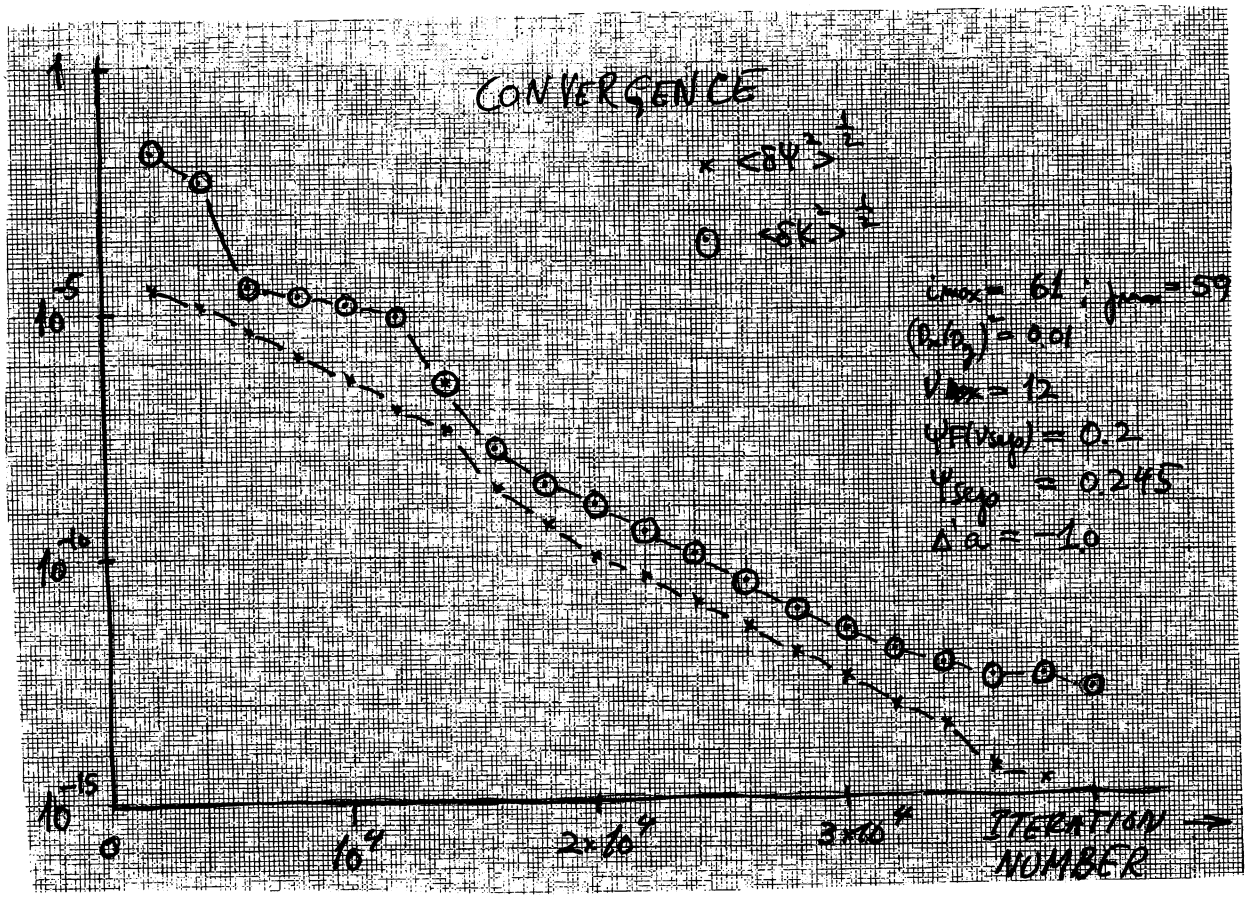
FIND $j_{ext}\{\Delta'a, R_D, b/a, \psi_F(\nu_{SEP}), \psi_{sep}\}$

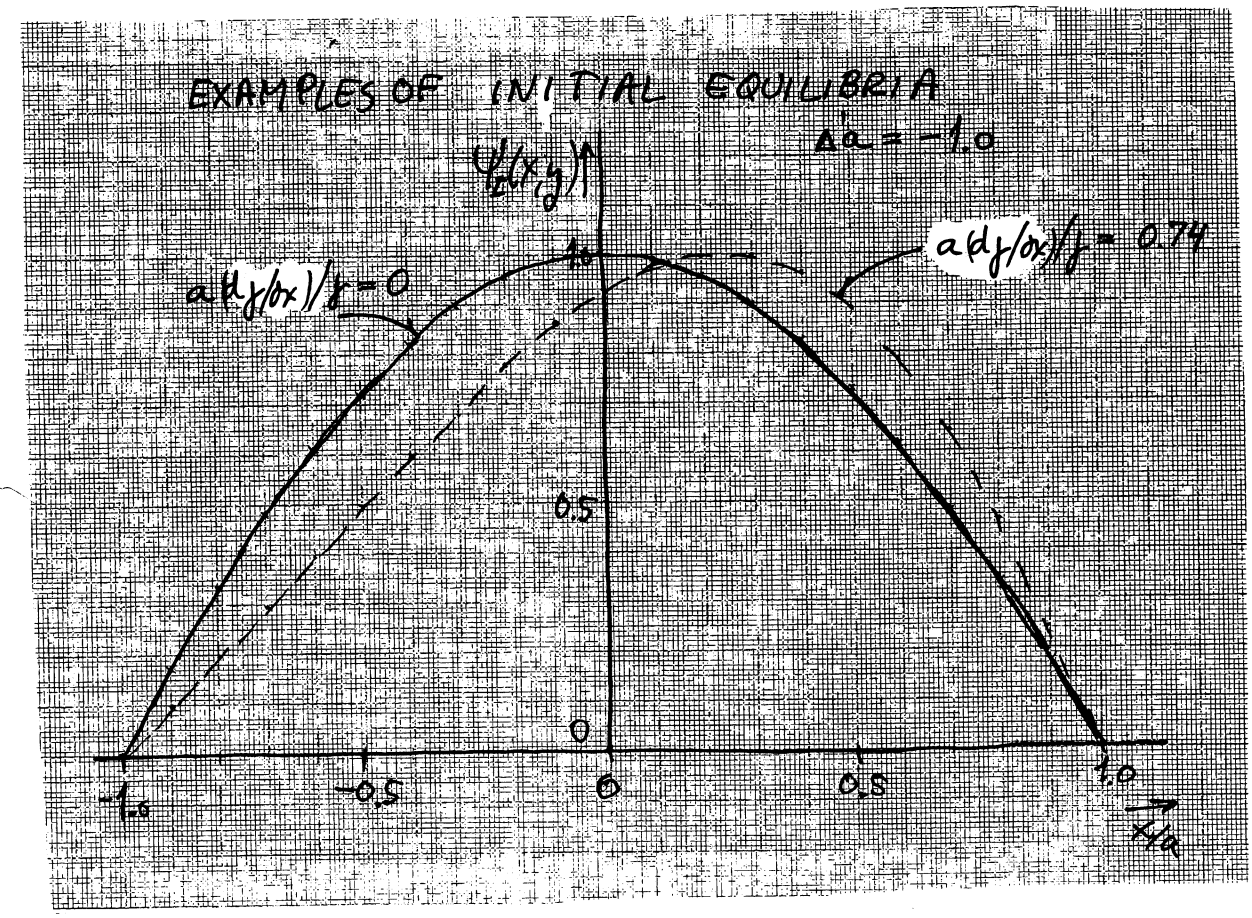
SENSITIVITY TO ν_{MAX}, $\psi_F(\nu)$'s, i_{max}, j_{max}

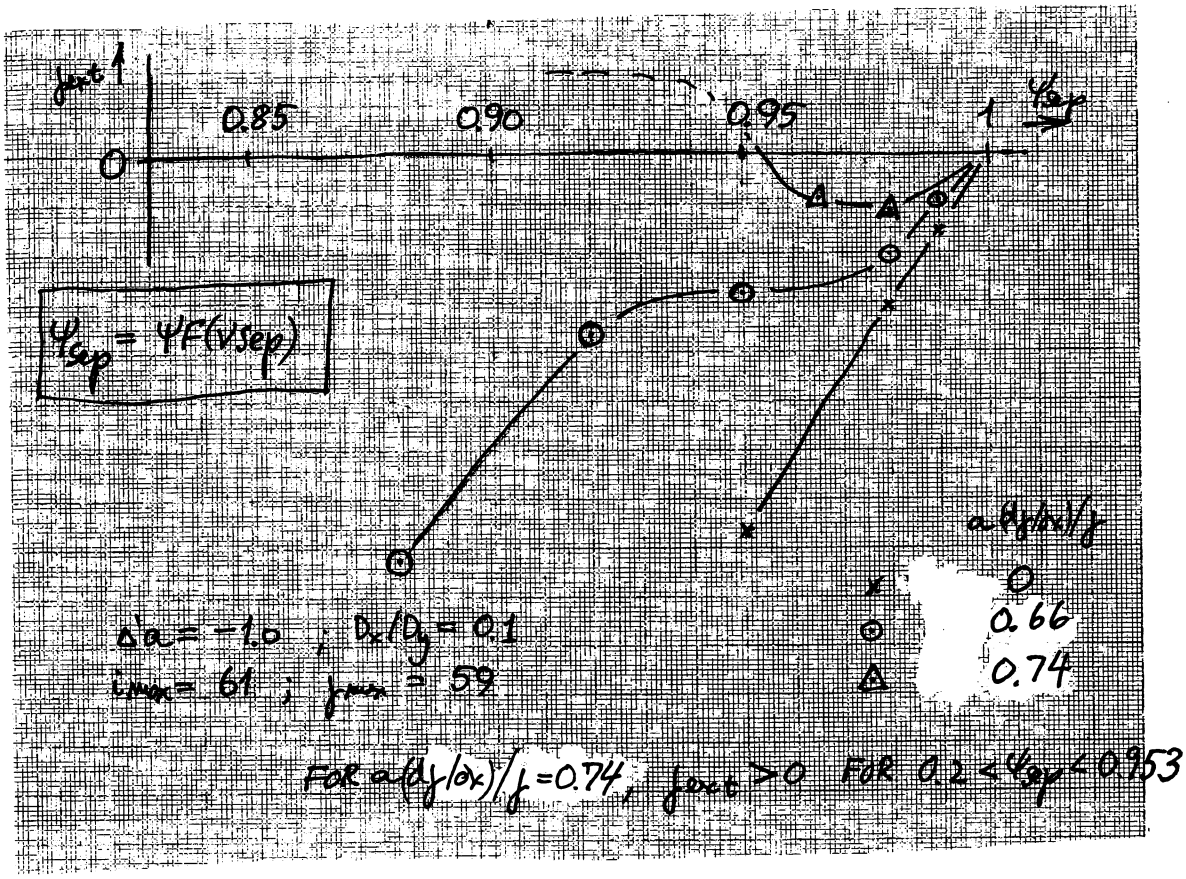
$j_{ext} > 0 \Rightarrow$ GROWING ISLAND

$j_{ext} < 0 \Rightarrow$ SHRINKING ISLAND

PARAMETER SPACE IS LARGE. ONLY SMALL PART HAS BEEN EXPLORED.


CONVERGENCE
× $\langle \delta\psi^2 \rangle^{\frac{1}{2}}$
⊙ $\langle \delta K^2 \rangle^{\frac{1}{2}}$
$i_{max} = 61$; $j_{max} = 59$
$(a/r_0)^2 = 0.01$
$V_{max} = 12$
$\psi_F(V_{sup}) = 0.2$
$\psi_{sep} = 0.245$
$\Delta' a = -1.0$
1
10^{-5}
10^{-10}
10^{-15}
0
10^4
2×10^4
3×10^4
ITERATION NUMBER


EXAMPLES OF INITIAL EQUILIBRIA
$\Delta' a = -1.0$
$\psi_E(x,y)$
$a(dy/dx)/y = 0$
$a(dy/dx)/y = 0.74$
1.0
0.5
0
-1.0
-0.5
0
0.5
1.0
x/a



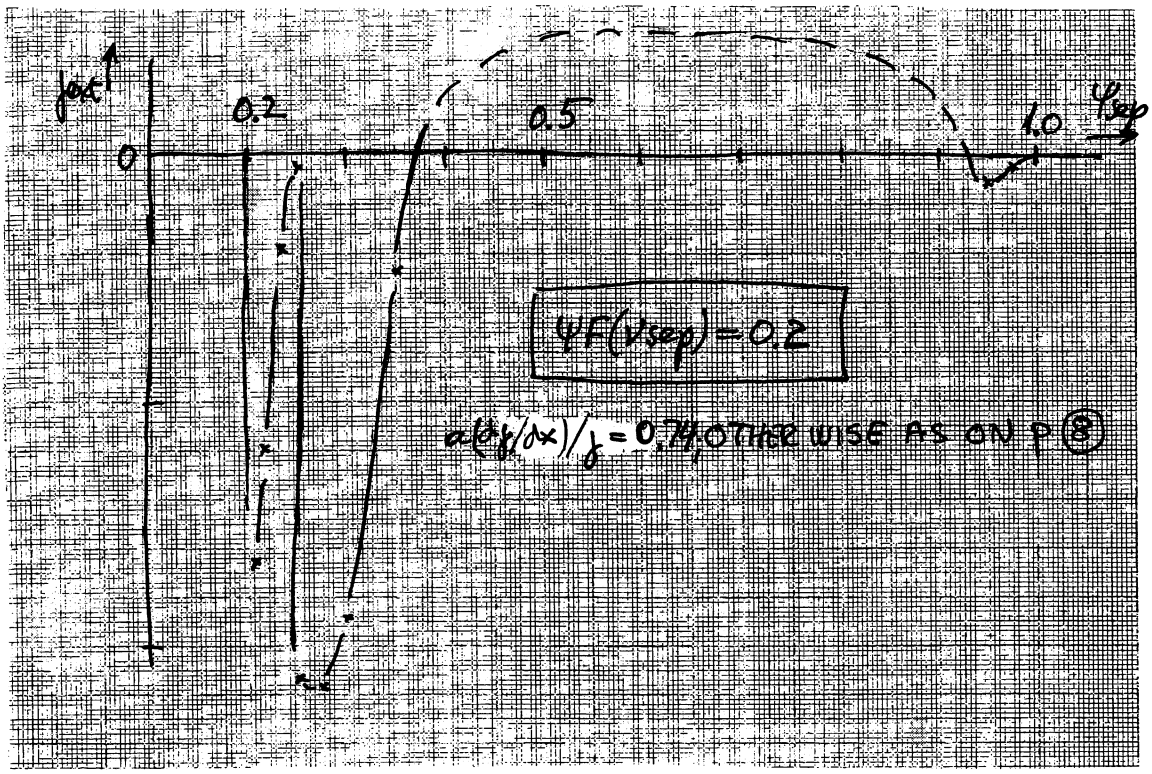
DEDUCTION:

i) INITIAL EQUILIBRIUM (NO ISLAND) IS LINEARLY STABLE

ii) FOR $a(dy/dx)/y < 0.66$, NONLINEARLY STABLE

iii) FOR $a(dy/dx)/y = 0.74$, INITIAL ISLAND (DRIVEN BY NOISE) WITH $\psi_{sep} < 0.953$ WILL GROW UNTIL $\psi_{sep} = 0.2$

CONVERGENCE WHEN $j_{ext} < 0$; ILLUSTRATION WHEN $j_{ext} > 0$



DEDUCTION:

THE SATURATED ISLAND AT $\psi_{sep} = 0.2$ IS UNSTABLE TOWARD SHRINKING; STABLE CONDITION AT $\psi_{sep} = 0.37$.

SPECULATION:

CASES MAY BE FOUND WITH $\psi_{sep} = 1.0$, i.e. ISLAND DISAPPEARS.

A MECHANISM WHEREBY TOKAMAKS ON AVERAGE MAINTAIN AXISYMMETRY WHILE EXHIBITING ANOMALOUS TRANSPORT?