Abstract Submitted
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Nonlinear Tearing Mode Driven by a Current Density Gradient\textsuperscript{1} T.H. JENSEN, General Atomics — It has been suggested\textsuperscript{2} that a linearly stable tearing mode may be nonlinearly unstable, driven by a current density gradient at the singular surface. This is under the assumptions of single fluid MHD. Qualitatively, this instability may have properties similar to those of "neoclassical" tearing modes,\textsuperscript{3} driven by a pressure gradient. In order to find out if the suggestion\textsuperscript{2} is correct, a numerical study was undertaken. The algorithm used for the study is a generalization of one used previously\textsuperscript{4} for studying the saturated island width of linearly unstable modes. The algorithm allows construction of a sequence of forced equilibria sharing a set of "almost ideal MHD constraints." A sign of the forcing function reveals whether the island is growing or shrinking. The previous study\textsuperscript{4} showed that the method is well suited for the study of nonlinear instabilities although neither growth rates nor decay rates are found. Results from the numerical study will be discussed.

\textsuperscript{1}Work supported by U.S. DOE Grant DE-FG03-95ER54309.
\textsuperscript{3}W.Q. Xu and J.D. Callen, UWPR 85-5, 1985; R. Carrera, R.D. Hazeltine, and M. Kotschenreuther, Phys. Fluids 29, 899 (1986).
1. Overview

i) Circumstances are found under which a plasma, linearly stable toward tearing, is nonlinearly unstable, driven by a current density gradient at the singular surface (separatrix).

ii) When a current density gradient is present at the singular surface (separatrix), tearing is an irreversible process.

iii) The islands resulting from this nonlinear instability will initially grow and may then (because of the irreversibility) shrink (disappear). The instability may describe a way for a plasma to approach the Taylor state.
2. Model

- Single fluid MHD; \( \nabla p = 0 \)
- \( \partial \phi / \partial z = 0 \); \( \vec{B} = B \hat{z} + \nabla \psi \times \hat{z} \)
- Equilibrium: \( (\nabla \times \vec{B}) \times \vec{B} = 0 \)
  \( \Rightarrow \nabla j \times \nabla \psi = 0 \) \( (j = -\nabla \psi) \)

Simply nested flux surfaces:
\( j(\psi) \); if not: \( j_L(\psi), j_R(\psi) \)...

For \( \psi_{sep} < \psi < \psi_{max} (=1.0) \) \( \Rightarrow j(\psi) \)

For \( 0 < \psi < \psi_{sep} \Rightarrow j_L(\psi), j_R(\psi) \)

- Conserved quantities: \( K_v = \int \delta v(\psi) dA \) \((\nu \psi / B)^2 \ll 1)\)

Perturbation of equilibrium (Ideal MHD):
\( \delta \psi + \vec{J} \cdot \nabla \psi = 0 \) \( \Rightarrow \delta K_v = \delta \int \delta \psi(\psi) dA = 0 \)

If: \( \nabla \cdot \vec{J} = 0 \) and \( \int_{\text{boundary}} \delta v \vec{J} \cdot \hat{n} ds = 0 \)

\( \psi_{max} \to \infty \): Ideal MHD; tearing not possible

\( \psi_{max} \) finite: "Almost ideal MHD"; tearing possible

- Boundaries: \( x = \pm a \); \( y = 0, b \); separatrix
  \( \Rightarrow K_L v, K_R v, K_I v \) (left, right, island)

• An algorithm has been found for finding the equilibrium (with continuous \( \partial \psi \)) defined by \( K_{v}', K_{v} \), \( K_{v}' \), and \( \psi_{\text{sep}} \). An "external" current (at \( y = b \)) is needed to insure topology and \( \psi_{\text{sep}} \) as required.

• The sign of the "external" current determines whether the island will grow or shrink if the "external" current is removed.

• No information on growth or decay rates (or islands) is obtained.
3. SPECIFICS

- AN "INITIAL", 1D EQUILIBRIUM IS DETERMINED BY TWO NUMBERS, $k_a$ AND $k_D$, BY

$$
\psi_i(x, y) = \frac{\cos(k_a + k_D) \frac{\cosh(k_a + k_D) - \cosh(k_a)}{1 - \cosh k_a}}{1 - \cosh k_a}
$$

FOR $k_D = 0$, CURRENT DENSITY GRADIENT AT SINGULAR SURFACE (SEPARATRIX) VANSHES.

FOR THIS CASE $k_a$ AND $b/a$ DETERMINE $d'/a$.

- CHOOSE $\psi_F(\nu)'s$:

\[ \begin{align*}
&\psi_F(1) \quad \psi_F(2) \\
&\psi_F(\nu_{sep}) \\
&\psi_F(\nu_{sep}^{-1}) \quad \psi_F(\nu_{max})
\end{align*} \]

\[ \begin{align*}
\frac{dG_F}{dy} &\uparrow 1 \\
0 &\quad \psi_F(\nu_1) \quad \psi_F(\nu) \quad \psi_F(\nu_{sep}) \\
&\quad \psi_F(\nu_{sep}^{-1}) \quad \psi_F(\nu_{sep}^{-1}) \quad \psi_F(\nu_{max}) \quad \rightarrow \psi
\end{align*} \]

$G_F(\nu)$ NORMALIZED SO THAT:

\[ G_F(\nu_F(\nu_{sep})) = 0 \quad \text{FOR} \quad \nu \neq \nu_{sep} \]

\[ G_F(\nu_F(\nu_{sep}^{-1})) = 0 \quad \text{FOR} \quad \nu = \nu_{sep} \]

\[ \text{NOTE:} \quad \psi_F(\nu_{sep}) \leq \psi_{sep} \]
4. NUMERICS

\[ \Psi(x, y) \rightarrow \Psi(i, j) ; \ i = 1, 2, \ldots, i_{\text{max}} ; \ j = 1, 2, \ldots, j_{\text{max}} \]
\[ \Psi(1, j) = \Psi(i_{\text{max}}, j) = 0 ; \ \Psi(i, 1) = \Psi(i, 3) ; \ \Psi(i, j_{\text{max}}) = \Psi(i, j_{\text{max}} - 2) \]
\[ b/a - 2 \frac{j_{\text{max}} - 3}{c_{\text{max}} - 1} \frac{\partial \Psi}{\partial x} ; \]

**USUAL DIFFERENCE REPRESENTATION OF \( \partial^2 \Psi \)**

**DEFINE:**
- \( L(i, j) = 1 \) for \( \frac{\partial \Psi}{\partial x} > 0 \) and \( \Psi < \Psi_F(\text{TR}) \); ELSE 0
- \( R(i, j) = 1 \) for \( \frac{\partial \Psi}{\partial x} < 0 \) and \( \Psi > \Psi_F(\text{TR}) \); ELSE 0
- \( I(i, j) = 1 \) for \( \Psi > \Psi_F(\nu) \); ELSE 0

\[ K_L(v) = \sum_{i, j} G \Psi(i, j) L(i, j) \]
\[ K_R(v) = \sum_{i, j} G \Psi(i, j) R(i, j) \]
\[ K_I(v) = \sum_{i, j} G \Psi(i, j) I(i, j) \]

* K_L(v), K_R(v), K_I(v) ARE OBTAINED FROM INITIAL EQUILIBRIUM; THUS THEY DEPEND ON \( \Delta^1 a, \text{TR}, b/a, \Psi_F(\text{VSEP}) \).

* INSTEAD OF TR USE \( a(\text{dy}/\text{dx})/f \) AT SINGULAR SURFACE.
ITERATION:

\[ \delta \psi = \frac{e_1 \psi}{r(u, b_u)} \left\{ \sum_i \delta L(v) \frac{\partial L(i, j)}{\partial \psi} + \sum_i \delta R(v) \frac{\partial R(i, j)}{\partial \psi} + \sum_i \delta I(v) \frac{\partial I(i, j)}{\partial \psi} \right\} \]  \hspace{1cm} \text{(JACOBI)}

\[ \delta \delta L(v) = \left[ K_0 L(v) - K L(v) \right] \]
\[ \delta \delta R(v) = \left[ K_0 R(v) - K R(v) \right] \]
\[ \delta \delta I(v) = \left[ K_0 I(v) - K I(v) \right] \]

\[ \psi_{new}(i, j_{\text{max}}) = \psi_{old}(i, j_{\text{max}}) \frac{\psi_{\text{exp}}}{\max \{ \psi_{old}(i, j_{\text{max}}) \}} \]

\[ f_{\text{inc}} = \sum_i [\psi(i, j_{\text{max}}) - \psi(i, j_{\text{max}})] \times \frac{1}{i_{\text{max}} - 1} \]

OBJECT OF CALCULATION:

Find \[ \{a, b, b_u, \psi_{\text{exp}}, \psi_{\text{exp}}\} \]

Sensitivity to \[ \psi_{\text{max}}, \psi_{\text{F}}(v)'s, i_{\text{max}}, j_{\text{max}} \]

\[ f_{\text{inc}} > 0 \Rightarrow \text{GROWING ISLAND} \]
\[ f_{\text{inc}} < 0 \Rightarrow \text{SHRINKING ISLAND} \]

Parameter space is large. Only small part has been explored.
DEDUCTION:

i) INITIAL EQUILIBRIUM (NO ISLAND) IS LINEARLY STABLE

ii) FOR $a(dy/dx)/f < 0.66$, NONLINEARLY STABLE

iii) FOR $a(dy/dx)/f = 0.74$, INITIAL ISLAND (DRIVEN BY NOISE) WITH $\psi_{sep} < 0.953$ WILL GROW UNTIL $\psi_{sep} = 0.2$

CONVERGENCE WHEN $\not x = 0$, ILLUSTRATION WHEN $\not x > 0$
DEDUCTION:

THE SATURATED ISLAND AT $\psi_{sep} = 0.2$ IS UNSTABLE TOWARD SHRINKING; STABLE CONDITION AT $\psi_{sep} = 0.37$.

SPECULATION:

CASES MAY BE FOUND WITH $\psi_{sep} = 1.0$, i.e. ISLAND DISAPPEARS.

A MECHANISM WHEREBY TOKAMAKS ON AVERAGE MAINTAIN ASYMMETRY WHILE EXHIBITING ANOMALOUS TRANSPORT?