Abstract Submitted
for the DPP00 Meeting of
The American Physical Society

Sorting Category: 6.6.2 (Experimental)

Thermal Stability of a Tokamak Plasma System with Classical Thermal Diffusivity[1] C.-L. HSIEH, B. BRAY, General Atomics — The formation of a transport barrier in the plasma interior, as shown in many Tokamak experiments, has lowered the observed ion thermal diffusivity to values comparable to neoclassical estimates. It is generally assumed that, once the plasma instabilities are suppressed, the plasma interior would have to rely upon the classical mechanism for heat transport. Hence, it is of interest to study how the classical diffusivity operate. For instance, could the mechanism establish and maintain a stable temperature profile? Different from Bohm diffusivity, the classical diffusivity has an inverse dependence on the plasma temperature. That is, an increase in temperature reduces the diffusivity and the local heat flow for a region with sufficiently weak temperature gradients; as a result, the local temperature may increase further, a potential situation for instability. Without thermal stability, it is doubtful that the classical system can ever come into existence. A model is being developed to study the stability problem in two steps: first to find the solution for the steady state temperature profile, then to apply small temperature perturbations. The resultant diffusion equations are highly nonlinear and are being studied using numerical techniques.

[1]Work supported by U.S. DOE Contract DE-AC03-99ER54463.

☐ Prefer Oral Session
☒ Prefer Poster Session

C.-L. Hsieh
hsieh@fusion.gat.com
General Atomics

Special instructions: Transport (Core), immediately following TC Jernigan

Date submitted: July 11, 2000

Electronic form version 1.4

INTRODUCTION

- In ITB (Internal transport barrier) plasmas.

  $\chi_{Exp} \lesssim \chi_{NC}$ (neoclassical thermal diffusivity)

  Hence, it is generally felt that the formation of ITB is the result of instability suppresion, which allows the classical thermal conduction to become the dominant transport process.

- An inspection of the heat transport equations,

$$-\nabla \bullet n\chi\nabla T + \gamma n \left(\frac{\partial T}{\partial t} + u \bullet \nabla T \right) + p\nabla \bullet u + P: \nabla u = Q$$

  Shows there are other possibilities for the formation of ITB, for instance, the pinch term or the cross terms like the one driven by the stress tensor

INTRODUCTION (Continued)

- For instance, in a model developed previously, we showed the ITB can be formed by a pinch process.

- The pinch model can be made to reproduce both the temperature and density profiles of JT-60U's ITB plasmas. Furthermore, it reproduces also

$$\chi_{Exp} < \chi_{NEO}$$

where $\chi_{Exp} = \dfrac{Q}{n\dfrac{\partial T}{\partial r}}$ as defined by the routine experimental data reduction

- Hence, for the formation of ITB there are a number of candidates. So how can we tell which one is more likely than others?

INTRODUCTION (Continued)

- The present study includes

1. Thermal stability considerations of χ_{NEO}.

2. Confinement scaling of tokamak plasma with χ_{NEO}.

3. Heat pulse propagation as a tool to gain more information.

THERMAL STABILITY CONSIDERATIONS FOR PLASMAS WITH χ_{NEO}

- Motivation: we suspect the inverse temperature dependence, $\chi_{NEO} \propto 1/T^{1/2}$ may make the temperature profile to become unstable to thermal fluctuation of long wavelength spatial modes. Hence impose a limit on the location and width of ITB region.

- The stability analysis involves first finding the steady state temperature profile T_0 (ρ), then applying the first order perturbation T_1 (ρ,t).

- For χ = constant, it can be shown that all temperature perturbations damp away in time, with the spatial mode of higher eigenvalue damping faster.

- For $\chi \propto 1/T^{1/2}$, the perturbation equation appears to have a term which tries to destabilize the temperature profile. The magnitude of the term appears to be affected by the width extent of ITB shell.

THERMAL STABILITY CONSIDERATIONS FOR PLASMAS WITH χ_{NEO} (Continued)

- In a discussion with Ming Chu, he showed a general proof that the system under our investigation will never become thermally unstable; that is, all perturbations will eventually be damped away. This is a consequence due to the intrinsic structure of the heat diffusion equation.

- So, we were wrong with our hunch. The width of ITB shell has nothing to do with the thermal stability consideration. The classical system with $\chi_{NEO} \propto 1/T^{1/2}$ is always thermally stable.

CONFINEMENT SCALING OF A TOKAMAK PLASMA WITH χ_{NEO}

- A direct comparison between local χ_{Exp} and χ_{NEO} can be difficult because estimations have to be made over various quantities, such as the temperature gradient, the power deposition profile, and the current density profile. On the other hand, even though only the global parameters are involved, the confinement scaling can be indicative as to the nature of the transport process.

- Question: Can the confinement scaling be found for $\chi = \chi_{NEO}$ in a tokamak plasma? Answer: Yes, it can be derived analytically for the χ_{NEO} to be in its simplest expression, no correction of any kind.

- We obtain

$$\tau_{NEO} \propto \frac{I^4 P}{n^3 a^3} \quad \text{or} \quad W_{NEO} \propto \frac{I^4 P^2}{n^3 a^3}$$

In comparing with

$$\tau_{ITER89P} \propto \frac{n^{0.1}\ I^{0.85}\ R^{1.2}\ a^{0.3}\ B^{0.2}\ M^{0.5}}{P^{0.5}}$$

CONFINEMENT SCALING OF A TOKAMAK PLASMA WITH χ_{NEO} (Continued)

τ_{NEO} has a much stronger power law dependence on the global plasma parameters. The strong dependence is the characteristic of neoclassical confinement. It is a natural consequence of χ_{NEO} that the temperature is sensitive to changes, while the temperature of L–mode plasmas is noted for its resistance to changes.

- The H factor can be approximated as $H_{NEO} = \dfrac{\tau_{NEO}}{\tau_{ITER89P}} \propto \dfrac{I^3\,P^{1.5}}{n^3\,a^{3.3}\,R^{1.2}\,B^{0.2}}$

  Clearly H_{NEO} has also a strong dependence on the global plasma parameters.

- Based on the results obtained in JT–60U ITB experiments [Nuclear Fusion 38 (1998), 218 by Fujita et al.], in which q_{min} was scanned by raising the current for a fixed B_T, the energy confinement time τ_E increases with I or with the decrease of q_{min}. The H factor is seen to increase with the decrease of q_{95}; a peak H of 3.3 is obtained with q_{95} coming down to about 3. The local transport analysis gives χ_{Exp} substantially less than χ_{NEO} predicted by the Chang-Hinton formula

- Even though τ_E or H is seen to increase with I, the dependence is certainly not as strong as the power law dependence given by τ_{NEO} and H_{NEO}, while $\chi_{Exp} \leq \chi_{NEO}$.

HEAT PULSE PROPAGATION

- A major difference between χ_{NEO} and χ_L is at their radial dependence. As the radius increases, χ_{NEO} and χ_L move in opposite ways – χ_{NEO} decreases while χ_L increases.
- How fast or slow the heat pulse propagates depends on χ. For an observation point to see the peak perturbation, the time it takes is

$$t \propto \frac{d^2}{\chi}$$

  Hence, how does the heat pulse propagates in a medium may give useful information to tell the nature of χ in the medium
- The heat pulse propagation with χ_{NEO} can be a complicated matter to analyze because χ_{NEO} can be broken down into two parts. One part gives its overall radial dependence and the other its non-linear linkage to temperature.
- Qualitatively speaking, the heat pulse picks up its speed as χ increases along its path and slows down as χ decreases. So with χ_{NEO}, we expect the outward going heat pulse to slow down and the inward going to speed up. χ_L does the opposite of χ_{NEO}.
- Even though χ_{Exp} is about the same level as χ_{NEO}, it is not necessary to mean that the ITB transport process is neoclassical. The information on the heat pulse propagation may be useful to know the true nature of the transport process in ITB plasmas.

SUMMARY

- It is not what we originally suspected that the ITB's shell structure may be due to the inverse temperature dependence in χ_{NEO} which drives the low value eigenmode thermally unstable. As a matter of fact, it can be shown that the classical system is as thermally stable as it can be; that is, all perturbations damp away in time.

- The confinement scaling for plasmas with χ_{NEO} is found analytically as

$$\tau_{NEO} \propto \frac{I^2 P}{n^3 a^3}$$

  The strong dependence on the global plasma parameters is its characteristic feature. ITB experiments in JT–60U show a similar trend in τ_{Exp} dependence on I, but not so strong a power law as given by τ_{NEO}. In general, it does not appear that τ_{Exp} or H_{Exp} ever become as sensitive to global changes as given by τ_{NEO} or H_{NEO}.

- Since the heat pulse propagation is related directly to χ, it may give us usable information to identify, or to distinguish among the possibilities, the transport process in ITB plasmas.

THERMAL STABILITY ANALYSIS (CONSTANT K)

$$-\frac{\partial}{\partial x} K \frac{\partial T}{\partial x} + C_V L^2 \frac{\partial T}{\partial t} = p_o L^2 , \qquad 0 < x < 1$$

with $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \quad T(x=1) = 0 \quad$ K, C_V and L are constants

Let $T(x,t) = T_0(x) + T_1(x,t) \qquad T_1 << T_0$

Steady state solution
$$\begin{cases} -\frac{\partial}{\partial x} K \frac{\partial T}{\partial x} = p_o L^2 \\ T_0 = \frac{p_o L^2}{2K}(1-x^2) \end{cases}$$

First order equation:

$$-\frac{\partial^2 T_1}{\partial x^2} + \frac{C_V L^2}{K} \frac{\partial T_1}{\partial t} = 0$$

$$T_{1n} = e^{\alpha_n t} \cos(k_n x)$$

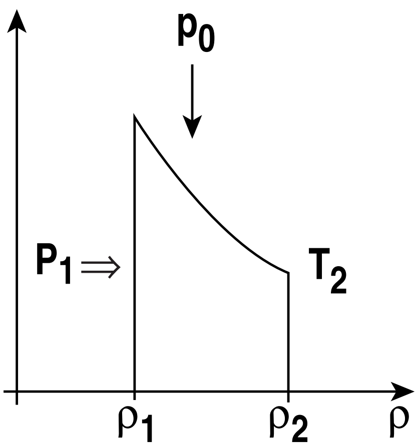
$$\alpha_n = -\left(\frac{K}{C_V L^2}\right) k_n^2 , \quad k_n = \frac{(2n+1)}{2}\pi \; ; n = 0, 1, 2, - - -$$

Since $\alpha_n < 0$, The perturbation T_{1n} alway damp away

DIII–D NATIONAL FUSION FACILITY SAN DIEGO

266-00/jy

THERMAL STABILITY ANALYSIS ($K \propto T^{-1/2}$)



$$-\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho K \frac{\partial T}{\partial \rho} + C_v a^2 \frac{\partial T}{\partial t} = a^2 p_0, \ \rho_1 < \rho < \rho_2$$

with $K = K_R \dfrac{T_R^{1/2}}{T^{1/2}}$

and
$$\begin{cases} -2\pi\rho K \left.\dfrac{\partial T}{\partial \rho}\right|_{\rho=\rho_1} = P_1 = \pi \rho_1^2 a^2 p_0 \\ T(\rho=\rho_2) = T_2 \end{cases}$$

$$T(\rho,t) = T_0(\rho) + T_1(\rho,t)$$

Steady state solution

$$T_0 = \left[T_2^{1/2} + \frac{a^2 p_0}{8K_R \ T_R^{1/2}} (\rho_2^2 - \rho^2) \right]^2$$

First order equation

$$\boxed{\frac{\partial T_1}{\partial t}} = A(\rho)\left[\frac{\partial^2 T_1}{\partial \rho^2} + F(\rho)\frac{\partial T_1}{\partial \rho} + \boxed{G(\rho)\, T_1}\right]$$

$$A(\rho) = \frac{K_R \ T_R^{1/2}}{C_v a^2} \ \frac{1}{T_0^{1/2}}$$

$$F(\rho) = \frac{1}{\rho} - \frac{1}{T_0}\frac{\partial T_0}{\partial \rho}$$

$$\boxed{G(\rho) = -\frac{1}{2\rho T_0}\frac{\partial T_0}{\partial \rho} + \frac{3}{4}\frac{1}{\rho T_0^2}\left(\frac{\partial T_0}{\partial \rho}\right)^2 - \frac{1}{2T_0}\frac{\partial^2 T_0}{\partial \rho^2}}$$

It can be shown $G(\rho) > 0$, Hence it is a destabilizing term.

DIII–D NATIONAL FUSION FACILITY SAN DIEGO

266-00/jy

THERMAL STABILITY OF A TOKAMAK SYSTEM WITH χ_{NEO} A PROOF BY MING CHU

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho n\chi_{NEO}\frac{\partial T}{\partial\rho} + na^2\frac{\partial T}{\partial t} = a^2 p(\rho) \qquad 0 < \rho < 1$$

With $\left.\frac{\partial T}{\partial t}\right|_{\rho=0} = 0$ and $T(\rho,1) = 0$

Let $\rho n\chi_{NEO} = F(\rho)/T^{1/2}$; $T(\rho,t) = T_0(\rho) + T_1(\rho,t)$, $T_1 << T_0$

Variable trasformation $V = T_1/T_0^{1/2}$

The first order equation is

$$\frac{\partial}{\partial\rho}F(\rho)\frac{\partial V}{\partial\rho} - G(\rho)\frac{\partial V}{\partial t} = 0, \text{ where } G(\rho) = na^2\rho\, T_0^{1/2}$$

Separation of varible: $V(\rho,t) = U(\rho)e\lambda t$

$$\frac{\partial}{\partial\rho}F(\rho)\frac{\partial U}{\partial\rho} - G(\rho)\lambda U = 0, \qquad \text{with } \left.\frac{\partial U}{\partial\rho}\right|_{\rho=0} = 0, \quad U(\rho=1) = 0$$

Integration by parts:

$$\int_0^1 U\left[\frac{\partial}{\partial\rho}F(\rho)\frac{\partial U}{\partial\rho} - \lambda G(\rho)U\right]d\rho = FUU'\Big|_0^1 + \int_0^1\left[-FU'^2 - \lambda GU^2\right]d\rho = 0$$

Hence,

$$\boxed{\lambda = \frac{-\int_0^1 F\left(\frac{\partial U}{\partial\rho}\right)^2 d\rho}{\int_0^1 GU^2\, d\rho}}$$

And $\lambda < 0$ if $F(\rho)$ and $G(\rho)$ are positive in $0 < \rho < 1$

DIII-D NATIONAL FUSION FACILITY SAN DIEGO

266-00/jy

NEOCLASSICAL ENERGY CONFINEMENT SCALING

$$\chi_{NEO} = c_\chi\, n^1\, T^{-1/2}\, B_p^{-2}\, r^{1/2}\, R^{1/2}$$

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\,\rho n \chi_{NEO}\frac{\partial T}{\partial \rho} = a^2 p(\rho), \quad 0 < \rho < 1$$

Let $p(\rho) = p_0 f(\rho)$, $n(\rho) = n_0 h(\rho)$, $j(\rho) = j_0 u(\rho)$, $X(\rho) = \int_0^\rho x(\rho) \bullet \rho d\rho$

Relations to globale parameters:

$$P = 2p_0 V\, F(\rho{=}1), \quad nv = 2n_0 H(\rho{=}1), \quad I = 2\pi a^2 j_0 U(\rho{=}1)$$

$$B_p(\rho) = \mu_o a j_0\, U(\rho)/\rho$$

Solve for $T(\rho)$:

$$T(\rho) = Y_{NC}\, Y_{GP}\, Y_{PF}$$

$$Y_{GP} = \left[\frac{I^2 P}{n_v^2 R^{1/2} a^{5/2}}\right]^2 \qquad Y_{NC} = \left[\frac{\mu_0^2}{2\pi^4 C_\chi}\right]^2$$

$$Y_{PF} = \left[\frac{H^2(r{=}1)}{U^2(\rho{=}1)\, F(\rho{=}1)} \int_\rho^1 \frac{FU^2}{y^{7/2} h^2} dy\right]^2$$

Solve for τ_E:

$$\tau_E = Z_{NC}\, Z_{GP}\, Z_{PF}$$

$$\boxed{Z_{GP} = \frac{I^4 P}{n_v^3 a^3}} \longleftarrow \text{Neoclassical confinement scaling}$$

A MODEL – HEAT PULSE PROPAGATION IN TOKAMAK PLASMA

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho n\chi\frac{\partial T}{\partial\rho} + na^2 = p(\rho)\,a^2 + \delta(\rho_1,t_1)\,a^2$$

Let $\rho n\chi = F(\rho)\,T^m$:

For L–mode: $m = 1$, $F(\rho)$ makes χ_L increases with ρ

For ITB: $m = -1/2$, $F(\rho)$ makes χ_{NEO} decreases with ρ

And $\delta(\rho_1,t_1)$, a delta function deliveries ΔQ at (ρ_1,t_1)

Assume $T = T_0(\rho) + T_1(\rho,t)$

The equation for heat pulse propagation is

$$\frac{\partial}{\partial\rho}F(\rho)\frac{\partial U}{\partial\rho} - G(\rho)\frac{\partial U}{\partial t} = H(\rho)\,\delta(\rho_1,t_1)$$

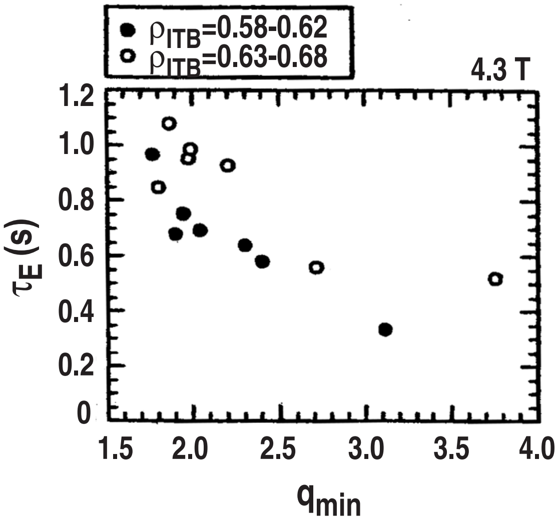
Where $G(\rho) = \dfrac{na^2\rho}{T_0^m}$ $H(\rho) = \dfrac{a^2\rho}{T_0^m}$

The above equation may be solved by Laplace tranformation for given $F(\rho)$, $G(\rho)$ and $T_0(\rho)$

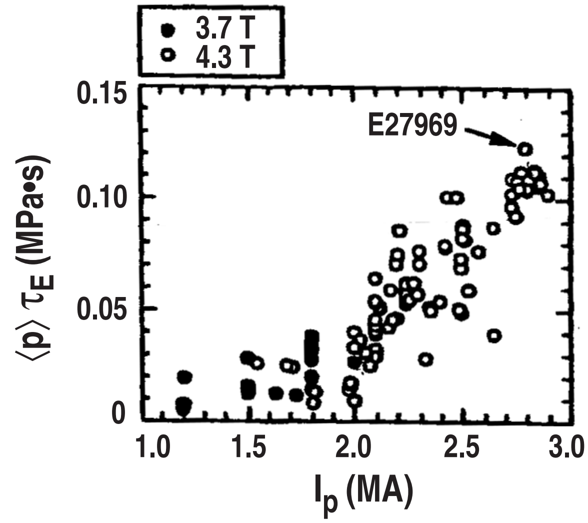
DIII–D NATIONAL FUSION FACILITY SAN DIEGO

266-00/jy

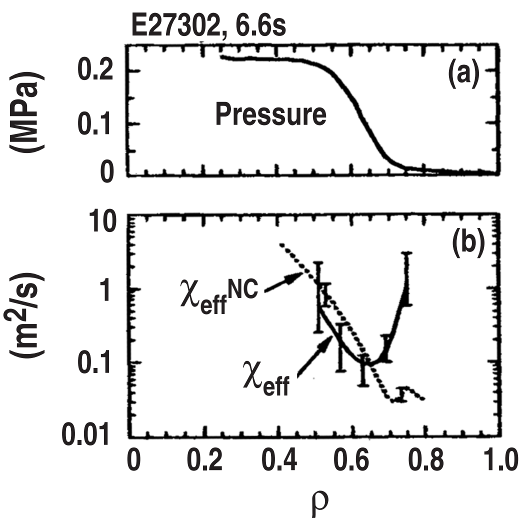
NUCLEAR FUSION 38 (1998) 207 BY FUJITA et al.



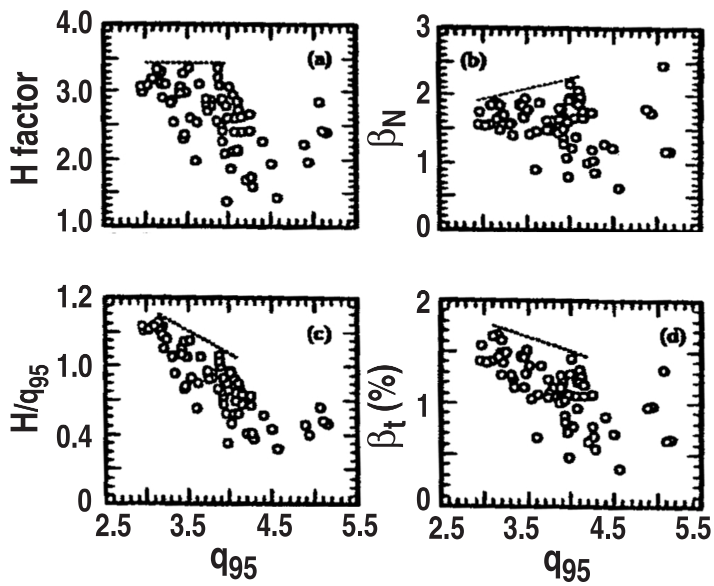
Energy confinement time τ_E versus q_{min}. The closed symbols denote plasmas with $0.58 < \rho_{ITB} < 0.62$ and the open symbols denote plasmas with $0.63 < \rho_{ITB} < 0.68$. BT = 4.3 T



Plot of $\langle p \rangle \tau_E$ versus plasma current. Closed circles denote plasmas with 3.7 T and open circles plasmas with 4.3 T



Radial profiles of (a) pressure and (b) effective one fluid thermal diffusivity, χ_{eff}. The effective one fluid thermal diffusivity predicted by the neoclassical transport using the Chang-Hinton formula, χ_{eff}^{NC}, is also shown (dotted curve).

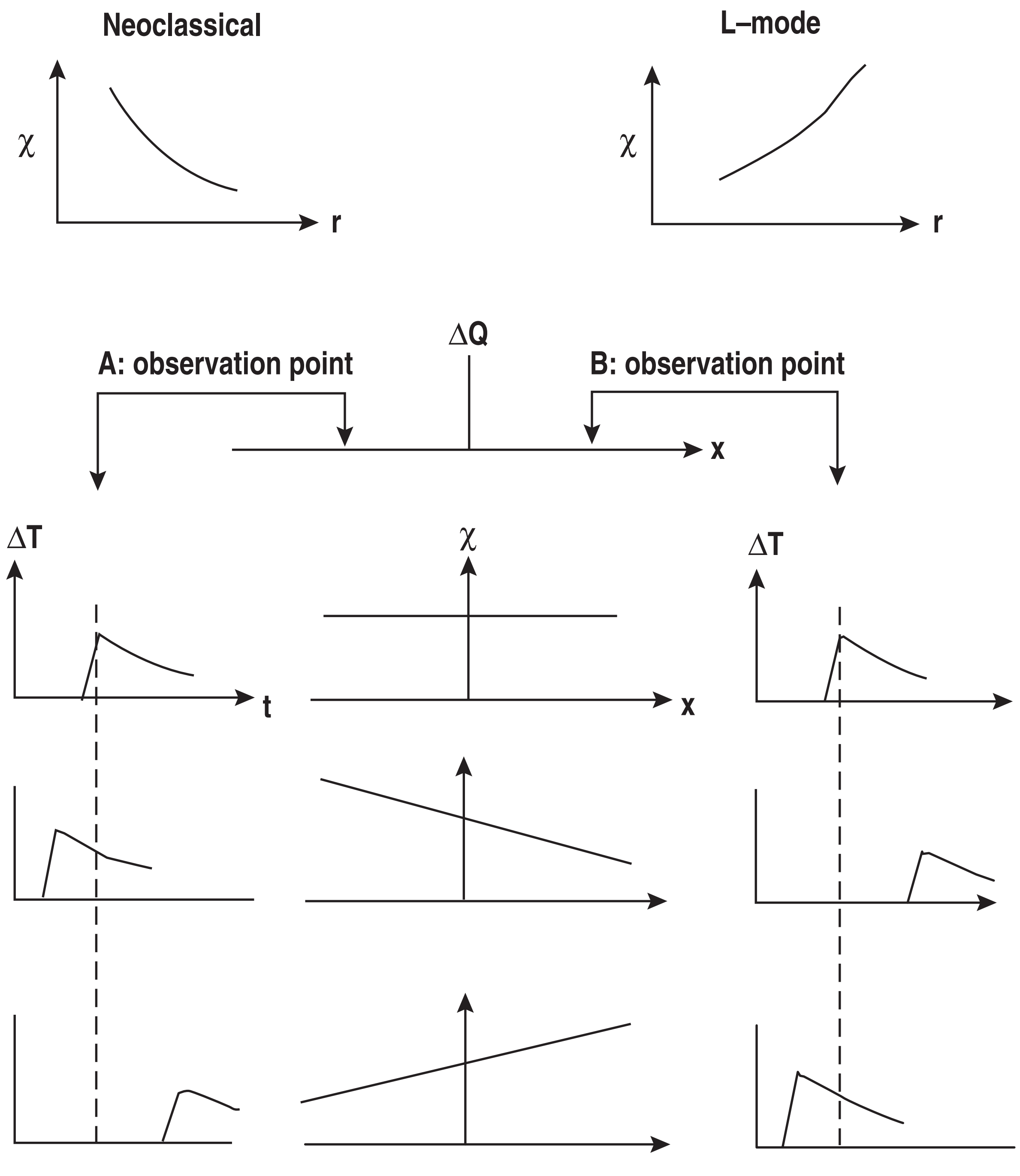


H factor, (b) β_N, (c) H/q95 and (d) β_t as functions of q95. B_T = 3.7 and 4.3 T

DIII-D NATIONAL FUSION FACILITY SAN DIEGO

266-00/jy

HEAT PULSE PROPAGATION IN THE MEDIUM WITH VARIABLE χ



DIII–D
NATIONAL FUSION FACILITY
SAN DIEGO

266-00/jy