SENSITIVITY STUDIES OF TEARING MODE STABILITY CALCULATIONS

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Abstract

For high β, highly shaped plasmas in the DIII-D tokamak, the value of the tearing mode stability index Δ' calculated at a rational surface can depend sensitively on the pressure and current profiles when an ideal mode is near marginal stability in the equilibrium current profile parameter space. Using a single time slice of experimental data and fitting equilibria around a minimum in χ^2, we show that an estimate of the error in Δ' will be low when no ideal mode is present. Also, the Δ' calculation will systematically indicate linear stability to tearing modes when a global ideal mode is present. Between these regions, near marginal stability for the global ideal mode, a pole in Δ' exists as predicted by analytic theory [$\mu=(-D_I)^{1/2}$ is near 0.5 at the rational surface], and the proximity of the best equilibrium fit to this pole in parameter space is crucial to the accuracy of the tearing mode stability calculation.

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OUTLINE

- The linear stability of resistive modes is calculated numerically and analytically, and the theoretical methods used are outlined.
- The equilibrium fitting parameters are varied around a minimum in χ^2 , and Δ' is calculated at each rational surface individually, using PestIII and Tear codes.
- A low q_{min} sawtoothing ELMing H-Mode shot and an Advanced Tokamak (AT) high q_{min} ELMing H-Mode shot are analyzed in this way, and the results are compared.
- Poles in Δ' exist in parameter space for the low q_{min} shot while the calculation is more robust for the high q_{min} shot.
- The effect of residual error from the iterative numerical solution is also studied.
- Stability analyses of highly accurate kinetic efits are compared to experimental measurements near the onset of tearing modes, and when no tearing modes exist.

THE LINEAR TEARING STABILITY CALCULATION METHOD

In cylindrical geometry, the stability equation becomes
Newcomb '60

$$L\xi \equiv \frac{d}{d\psi} f \frac{d\xi}{d\psi} - g\xi = 0 \qquad (1)$$

In toroidal geometry, the general form for the PestIII numerical method is
Dewar and Pletzer '90,'91

$$L\xi \equiv -(\partial_\psi \mathcal{D}_\theta + \mathcal{Q}^\dagger)\mathcal{G}(\mathcal{Q} + \mathcal{D}_\theta \partial_\psi)\xi + \mathcal{K}\xi = 0 \qquad (2)$$

$$\text{where} \qquad \mathcal{D}_\theta \equiv \partial_\theta - inq \qquad (3)$$

For a $m \gg 1$ approximation, an analytical approach can be applied
Hegna '94

$$\lambda = -\frac{Iq\mu_0\tilde{\sigma}'}{2mq'} \frac{1}{\sqrt{\tilde{g}^{\chi\chi}\tilde{g}^{\psi\psi}}} \qquad (4)$$

$$\triangle_0' = 2m\sqrt{\tilde{g}^{\chi\chi}}\lambda\pi \cot(\lambda\pi) \qquad (5)$$

In general these methods are highly sensitive to the equilibrium profiles near the rational surface, and their stability predictions can differ.

- We aim to determine the uncertainty in Δ' in these methods, and to compare our best results to experimental data.

FITTING EQUILIBRIA AROUND A MINIMUM IN χ^2 USING EFIT

- We estimate the uncertainty in Δ' by varying profiles within the constraints of the experimental data.

- Constructing a single parameter family of equilibria by varying the location of the intermediate knot in a 3 knot cubic spline representation of the current profile, minimizing χ^2 each time.

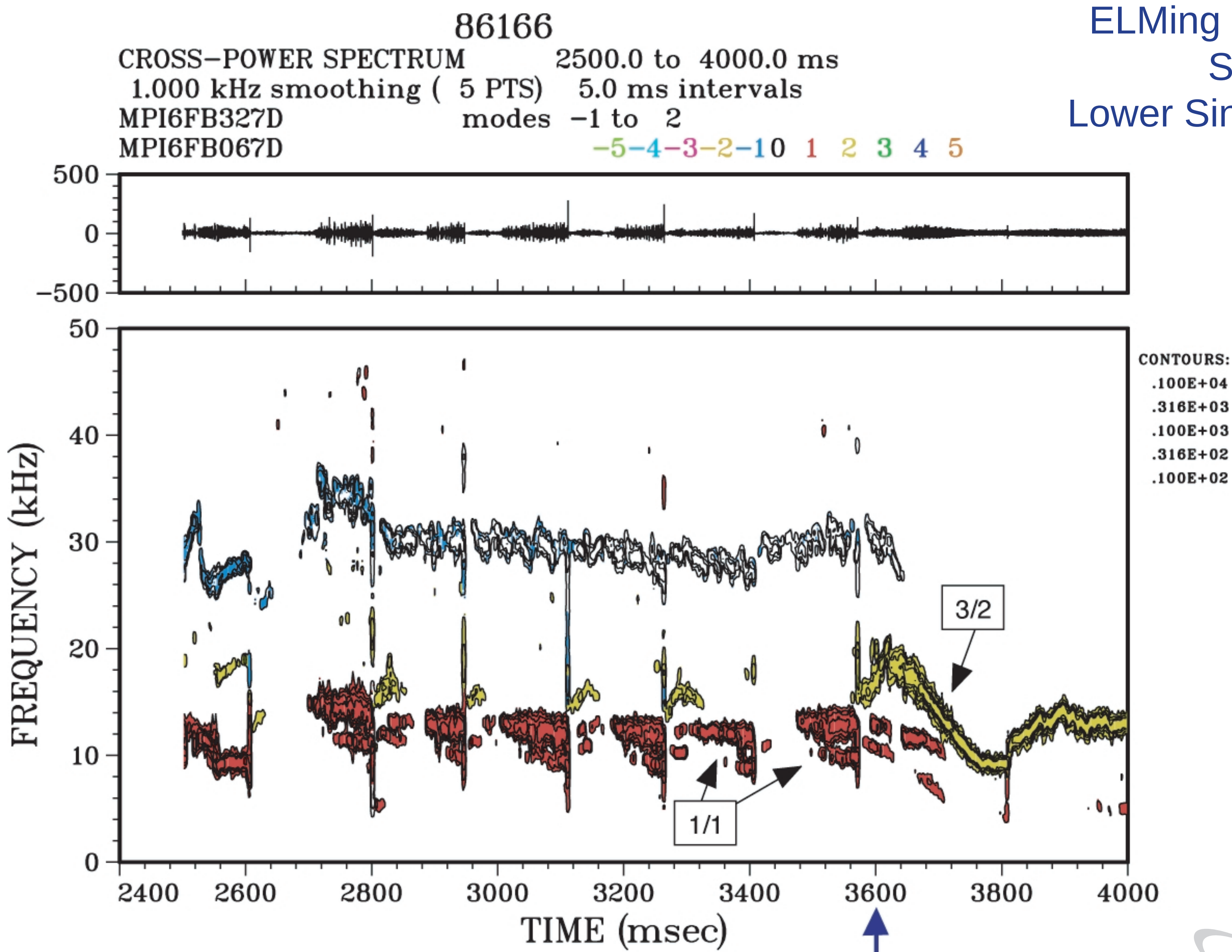
- Terms that are critical to the uncertainty and stability analysis are calculated, as well as diagnostic information, such as the differential change in equilibria with the fitting parameter,

$$\frac{\triangle\psi_{total}}{\triangle\psi_{kp}} = \frac{\sum_{i,j}\psi(r_i(k-1), z_j(k-1), k) - \psi(r_i(k-1), z_j(k-1), k-1)}{\psi_k^{kp} - \psi_{k-1}^{kp}}$$

  the ratio j'/q' at the rational surface, and the location of the rational surface.

THE ORIGINS OF 3/2 TEARING MODE IN ELMING H MODE SHOTS IS THOUGHT TO BE NEOCLASSICAL

ELMing H-Mode
Sawteeth
Lower Single Null
β_N=3.1



Analysis at 3600

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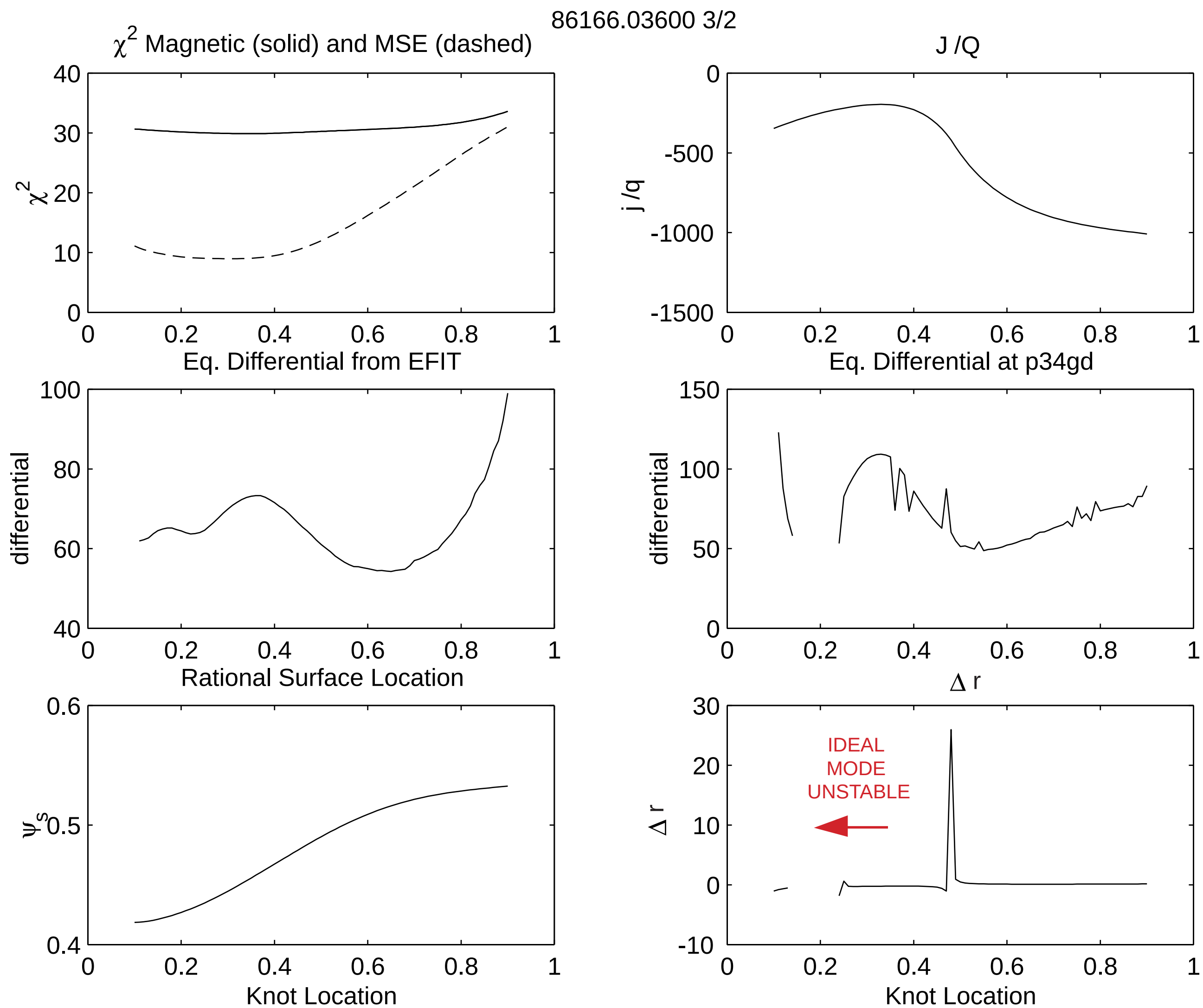
Δ' HAS A POLE (IN PESTIII) IN THE KNOT LOCATION SPACE FOR 3/2 (ONSET) IN AN ELMING H-MODE SHOT

q_0~1.2
q_{min}~1.0



With q_{min}~1, although the MSE improves the accuracy of the equilibrium construction, Δ' is indeterminate around the minimum in χ^2 because of ideal instability and the inertia free assumption in the theoretical method.

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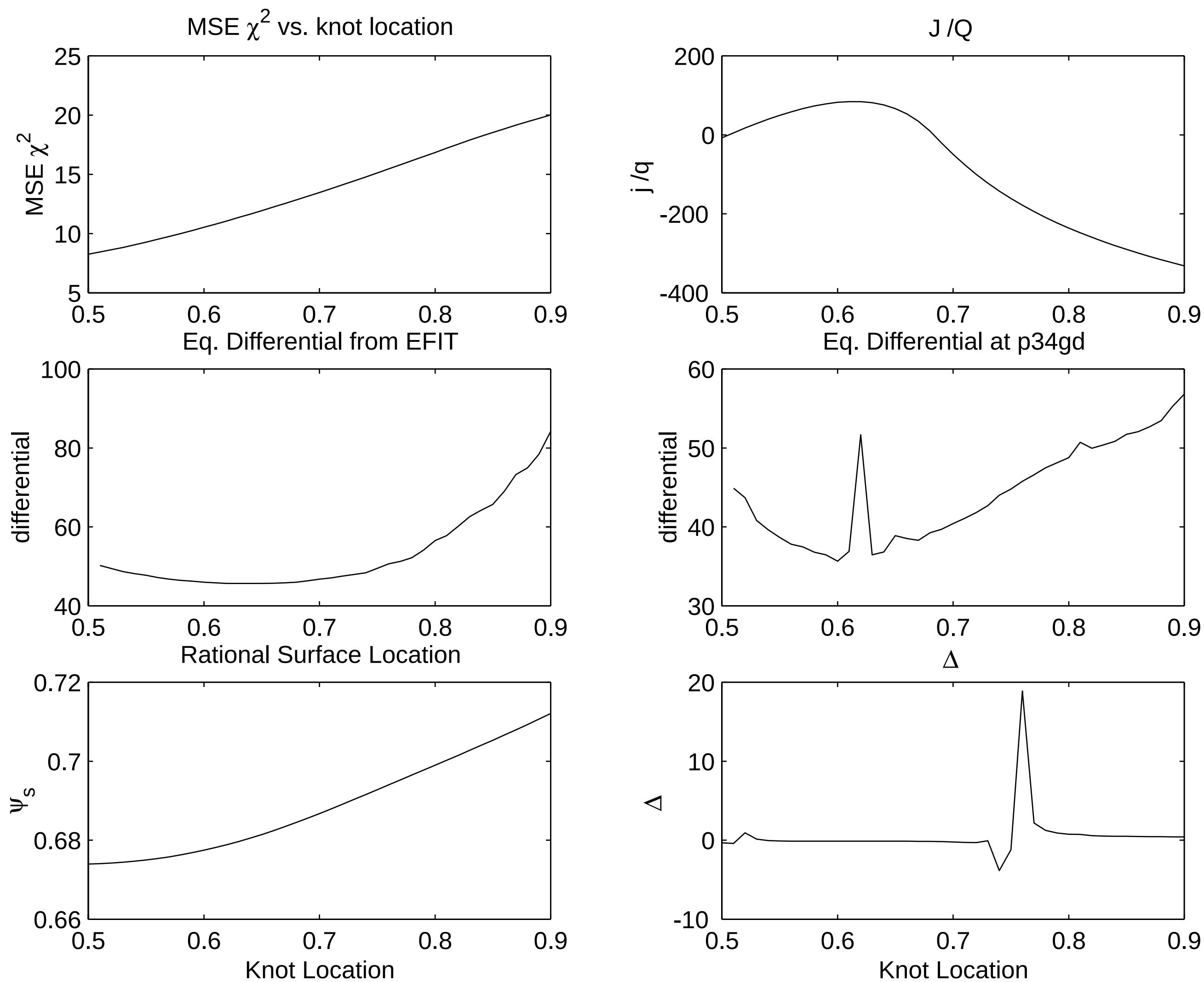
Δ' HAS POLES (IN PESTIII) IN THE KNOT LOCATION SPACE ALSO FOR 2/1 (NOT OBSERVED) IN AN ELMING H-MODE SHOT

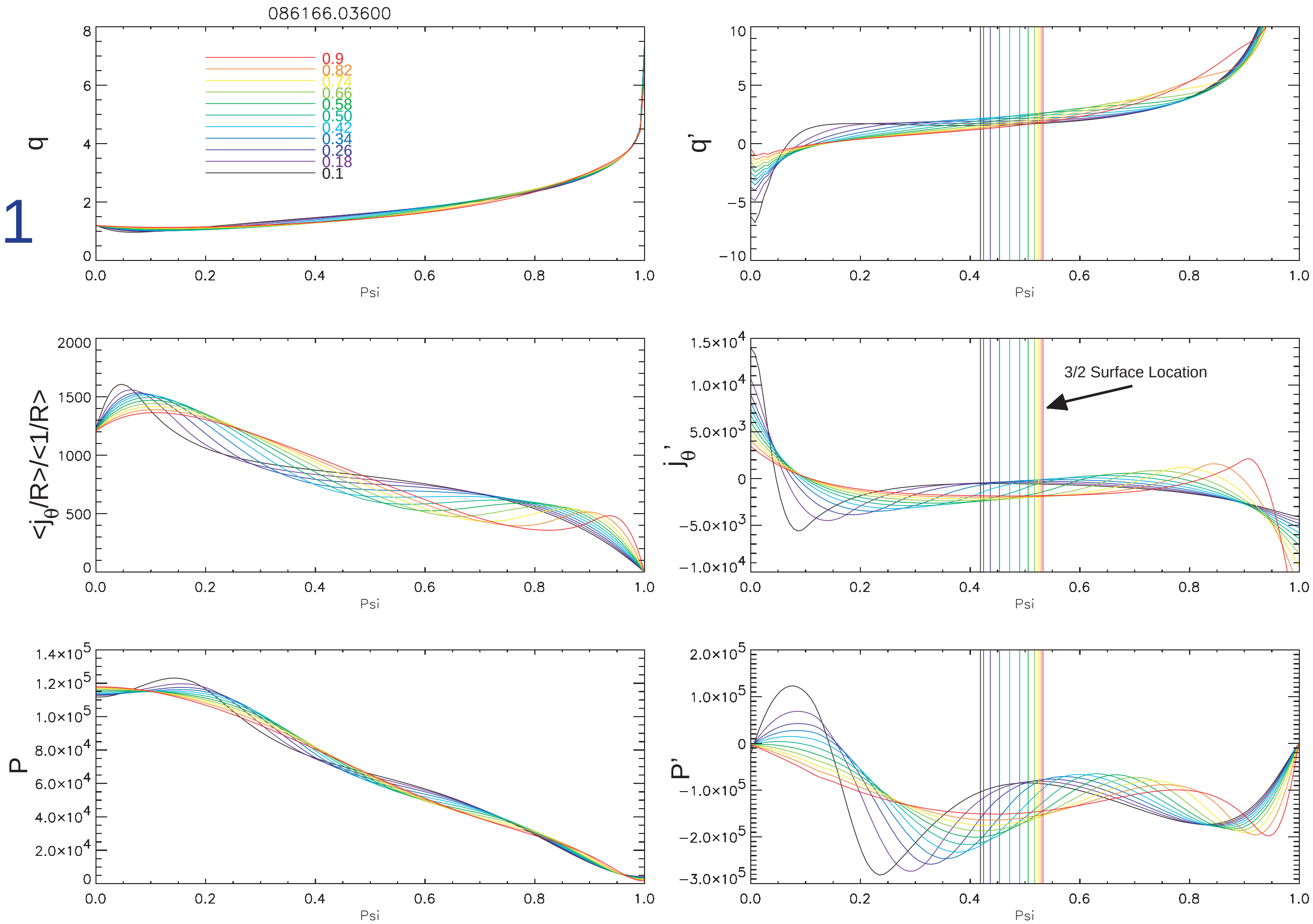
q_0~1.05
q_{min}~1.0



With q_0~1.05, an D_I becomes positive at a rational surface where the best equilibrium fit exists, preventing a stability analysis there

PROFILES CHANGE MODERATELY AT RATIONAL SURFACE WITH THE CHANGE IN KNOT POSITION

q_{min}~1



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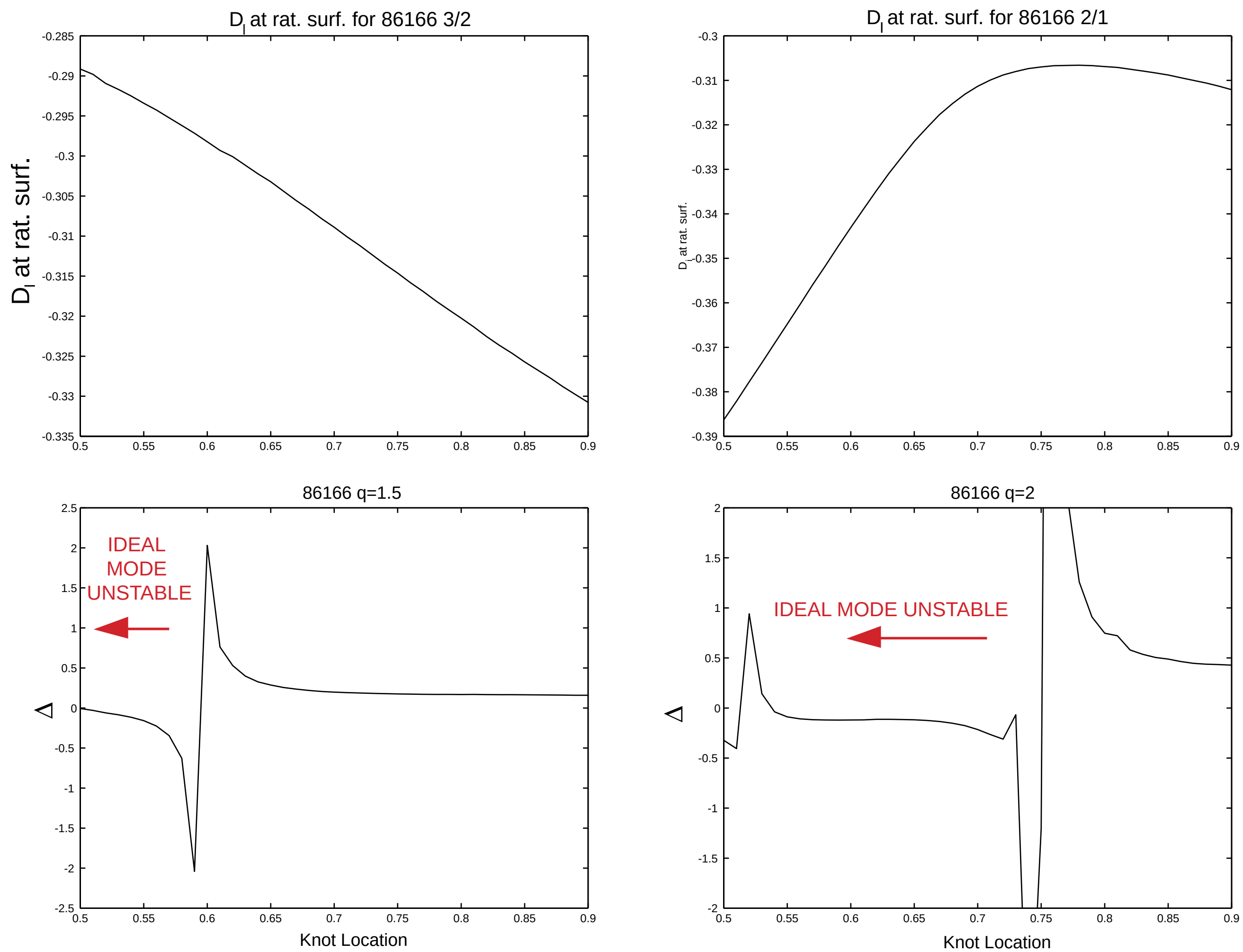
POLE LOCATIONS MARK THE POINT WHERE IDEAL MODES BECOME UNSTABLE, MAKING THE TEARING MODE CALCULATION INDETERMINATE



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THE D_I AND D_R PROFILES CHANGE WITH FITTING PARAMETERS CAUSING D_I >0 AT A RATIONAL SURFACE

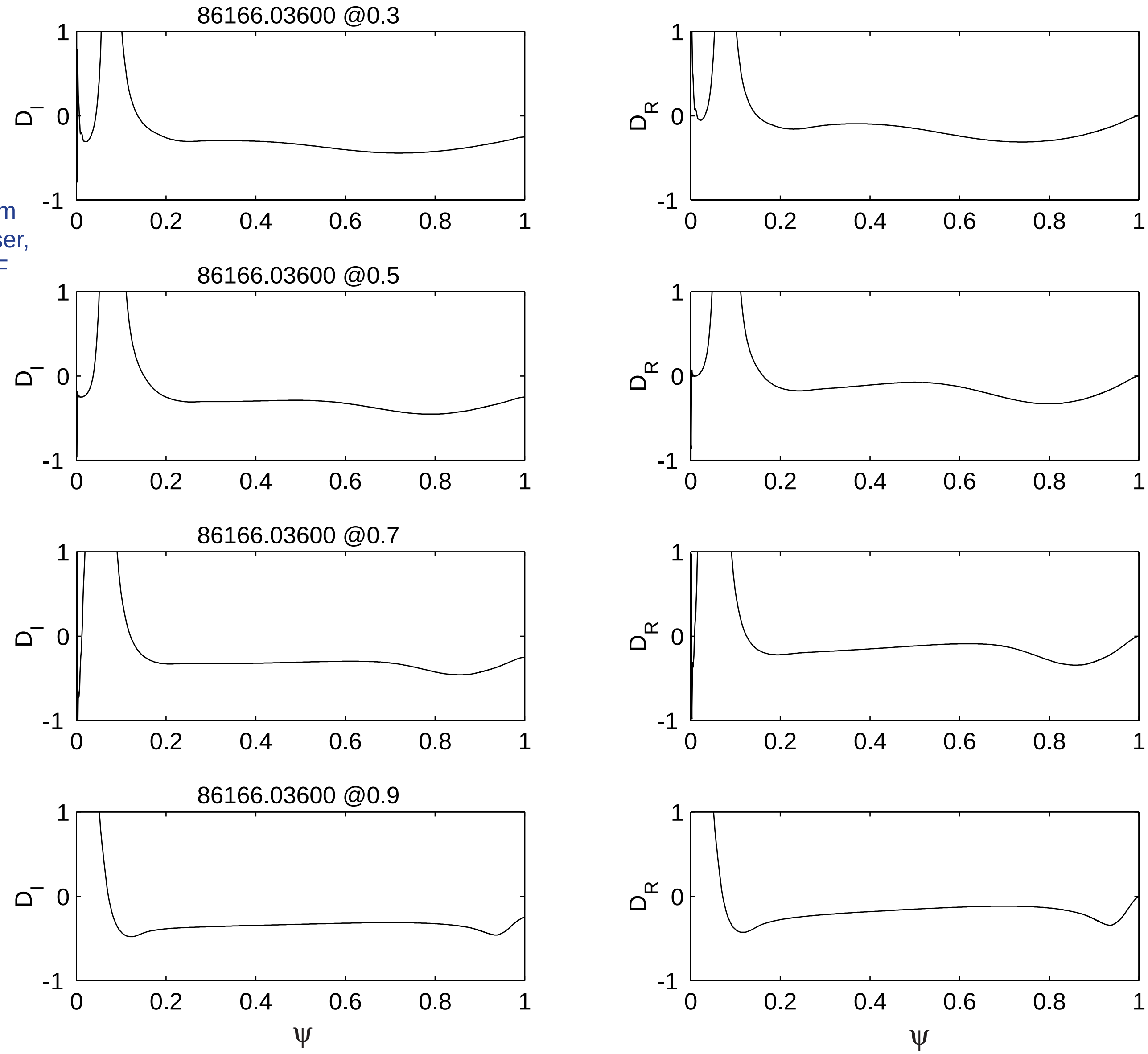
D_I is the ideal Mercier index:

$D_I = E+F+H-1/4$
$D_R = E+F+H^2$

where E,F,H are equilibrium quantities defined in Glasser, Greene and Johnson, POF 875 (1975)

The asymptotic solutions are
$\Psi \sim X^{1/2 +/- \mu}$
$\Xi \sim \Psi / X$

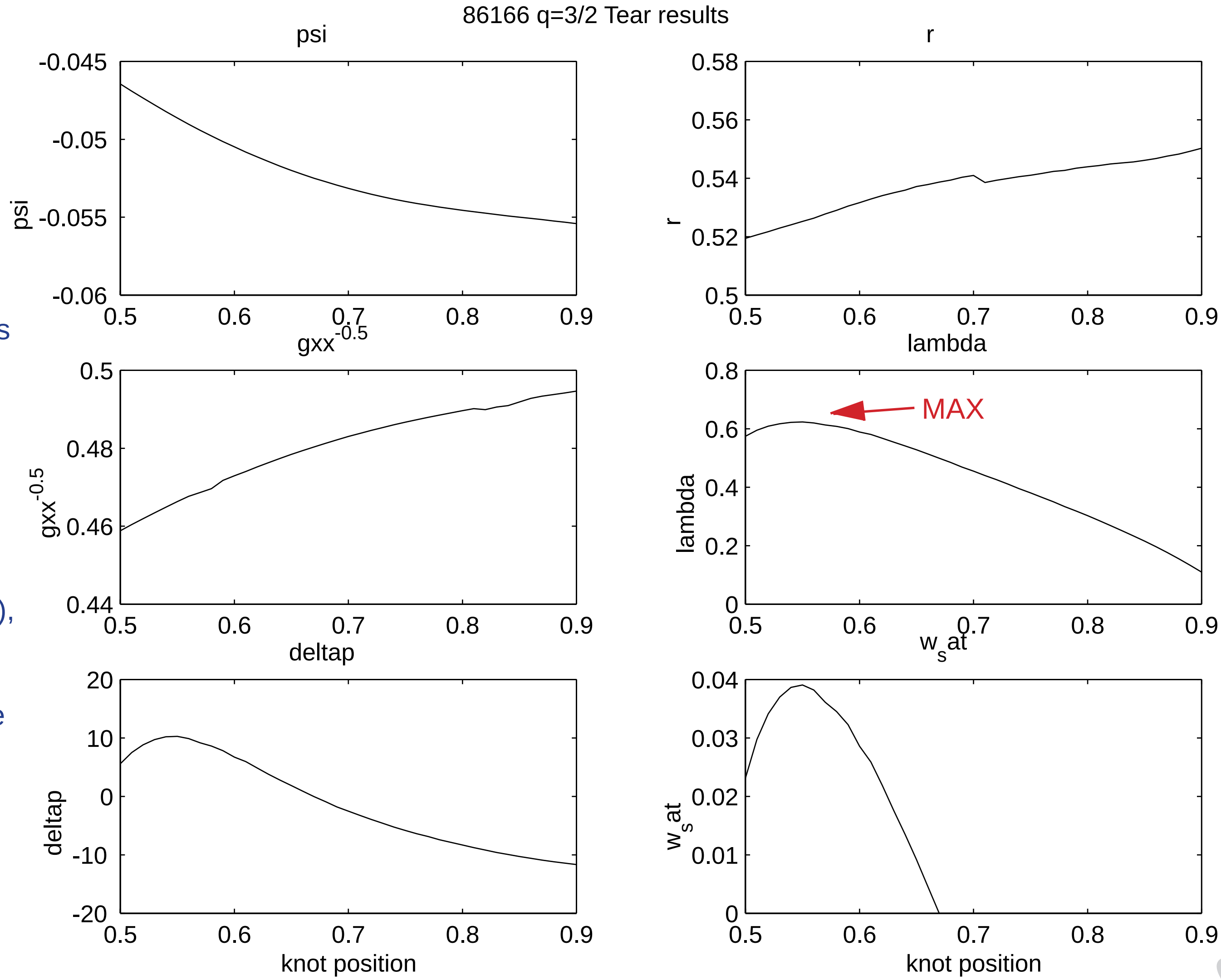
where $\mu = (-D_I)^{1/2}$ is required to be real for ideal stability, and the calculation of the tearing mode stability.



TEAR RESULTS SHOW THAT THE MAXIMUM IN λ CORRESPONDS TO PESTIII POLE LOCATION (HEGNA '94 TANH POLE)

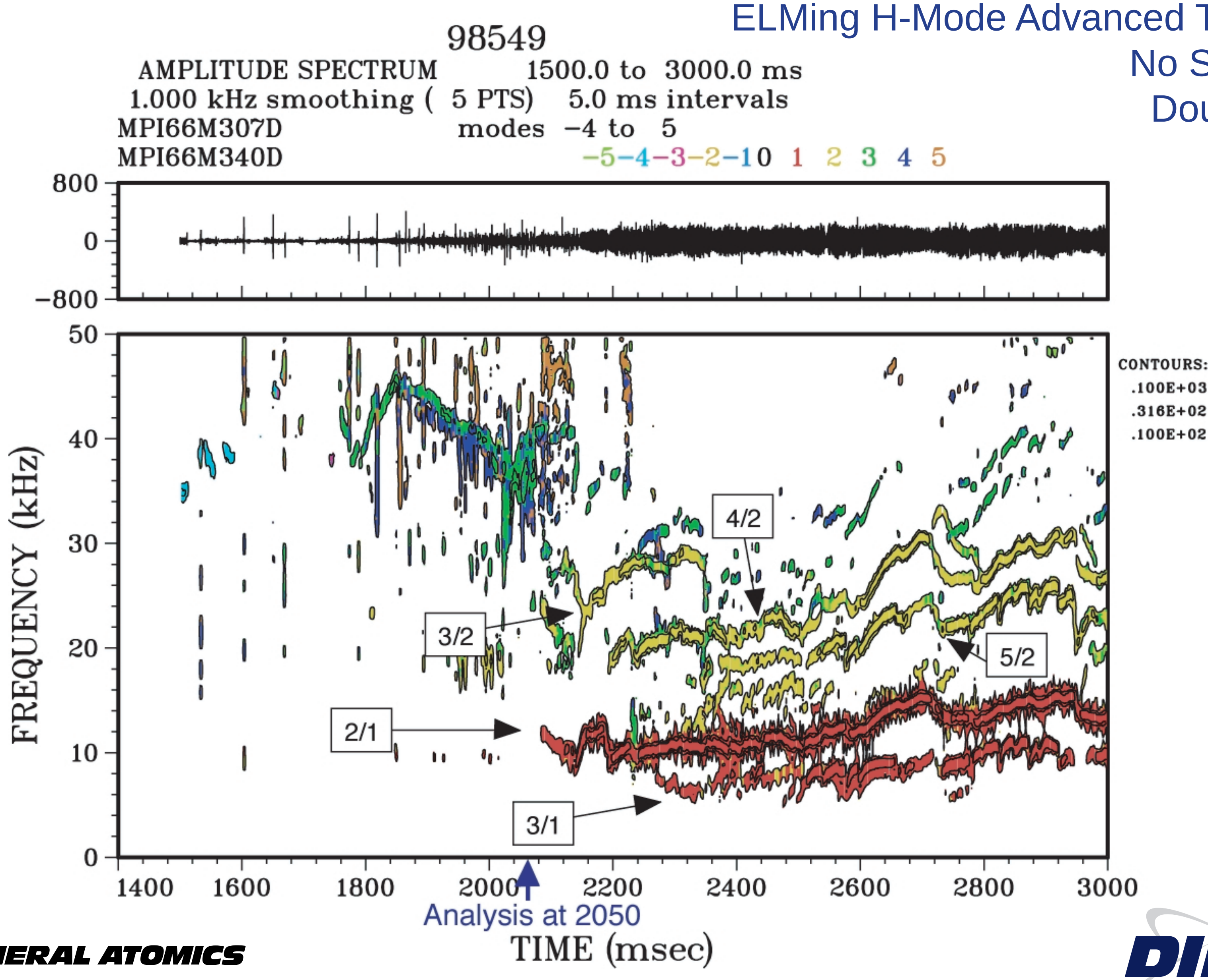
The analytical m>>1 calculation of Δ' has a tanh(πλ) factor, which has poles at λ=0,1,2... due to ideal instability.

Although this calculation does not show a pole (probably because of the assumptions in the theory or numerical error), the maximum in 0<λ<1 from the TEAR code occurs very near the pole location in PestIII.



ACCURATE TEARING MODE CALCULATIONS AGREE WITH EXPERIMENT IN RELATIVE AMPLITUDES

ELMing H-Mode Advanced Tokamak
No Sawteeth
Double Null
β_N=3.4



IN HIGH QMIN AT SHOTS, NO POLES ARE ENCOUNTERED AND THE CALCULATIONS ARE MORE ROBUST

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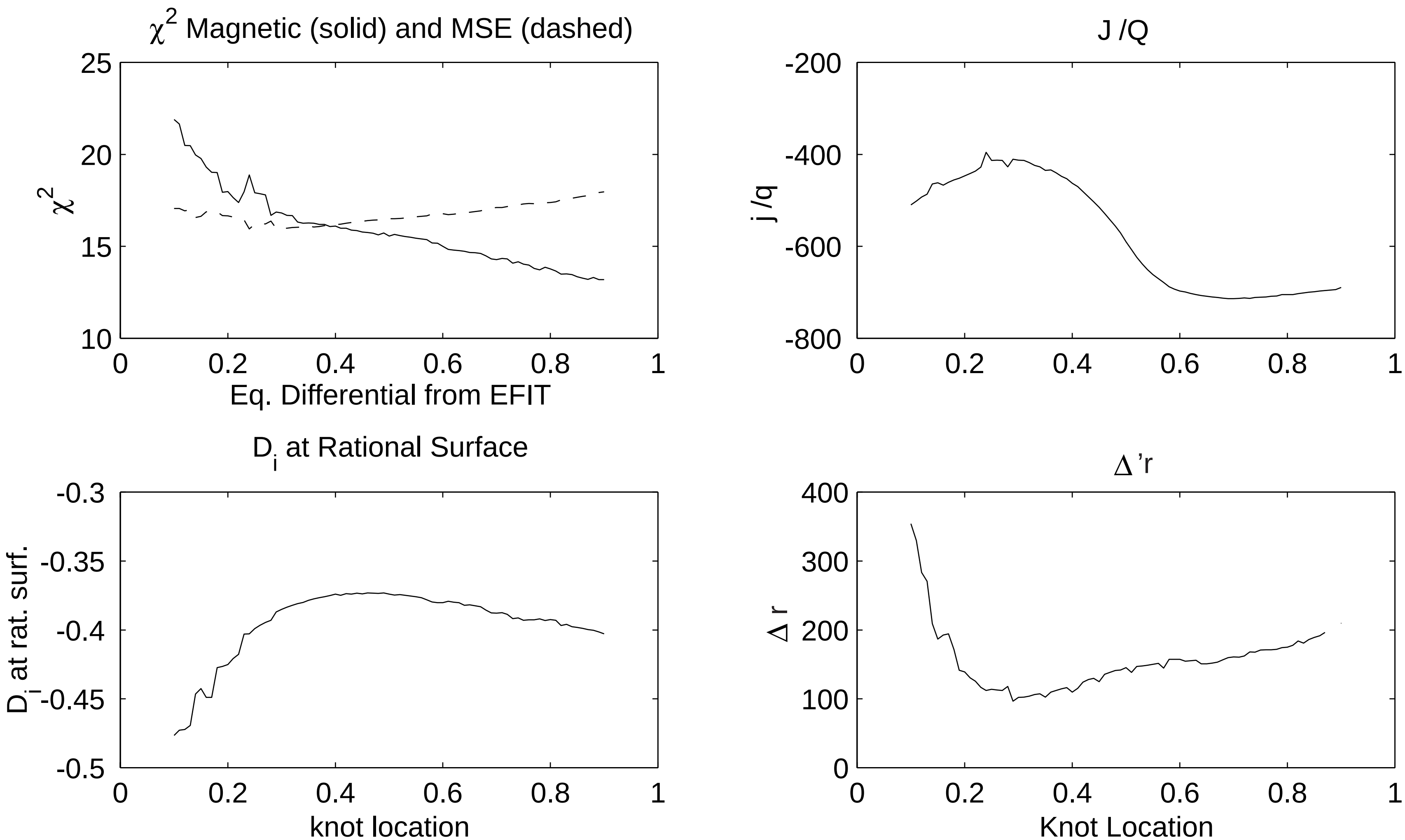

Δ' is robustly positive and has a definite value at χ^2_{min}

HOWEVER, EVEN IN HIGH q_{min} AT SHOTS, MARGINAL STABILITY CALCULATIONS CAN BE INNACURATE

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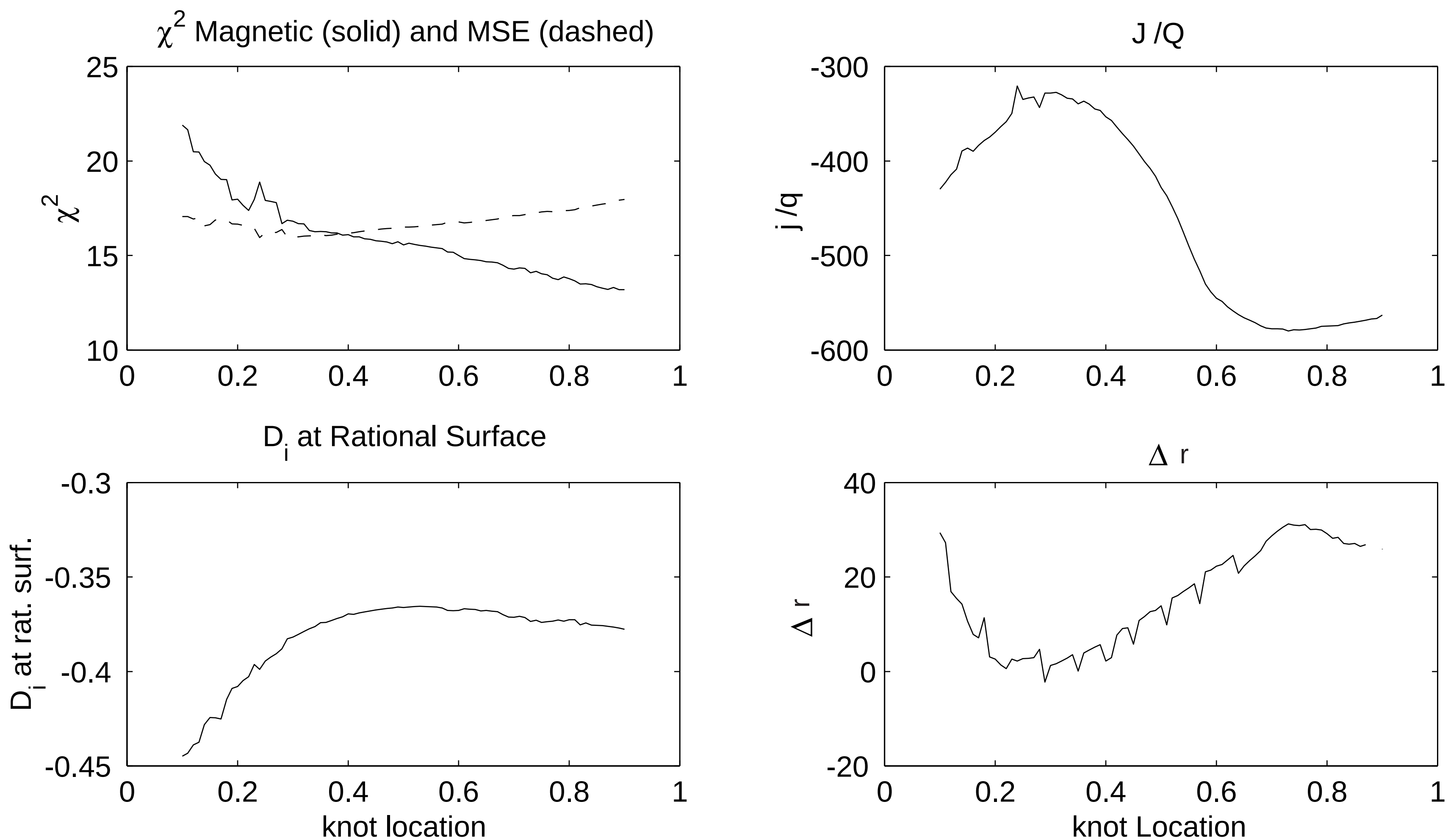
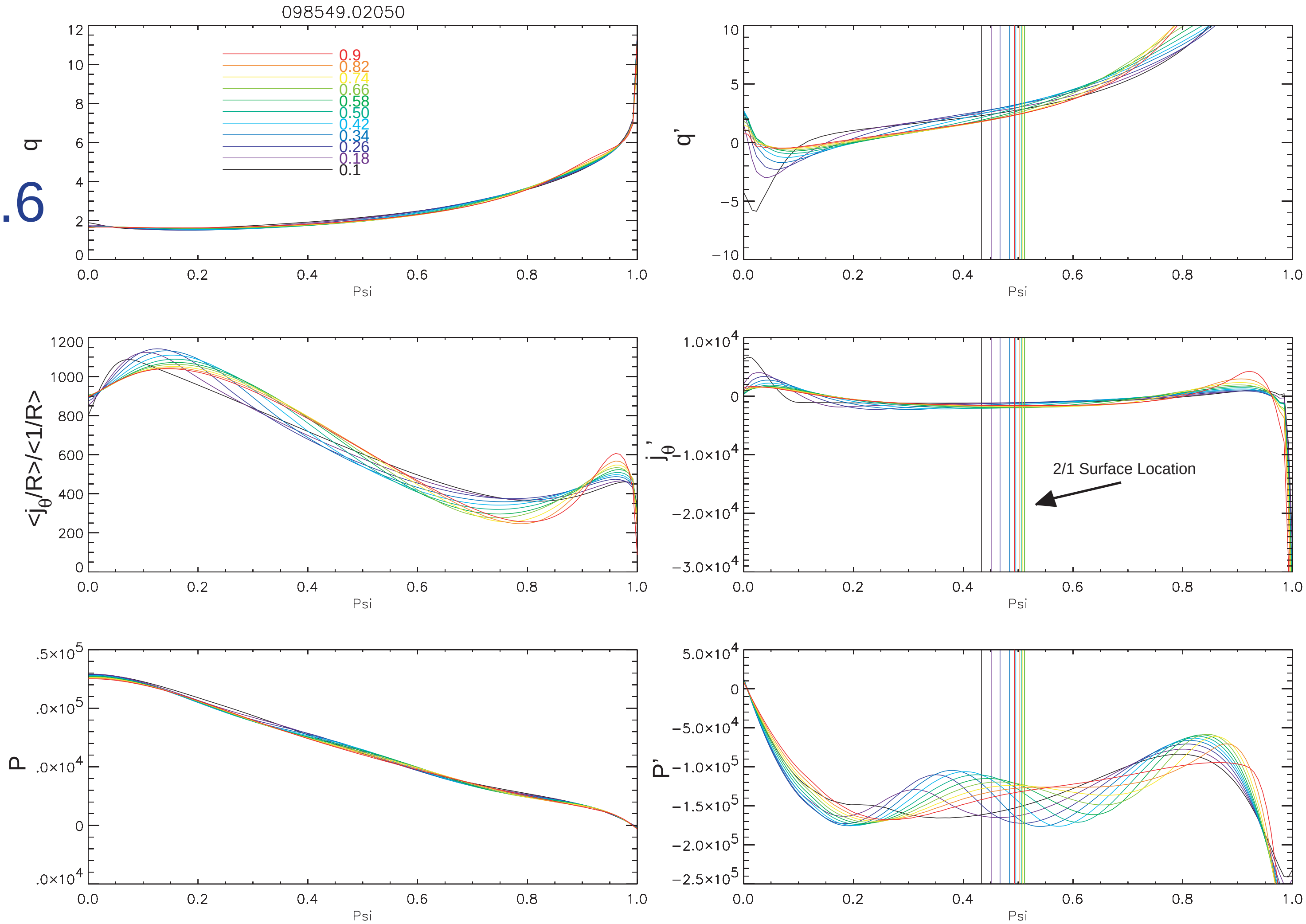


Δ' for 3/1 is marginal at χ^2_{min}

HIGH q_{min} AT SHOTS SHOW SIMILAR PROFILE DEPENDENCE, WITH VERY DIFFERENT RESULT

q_{min}~1.6



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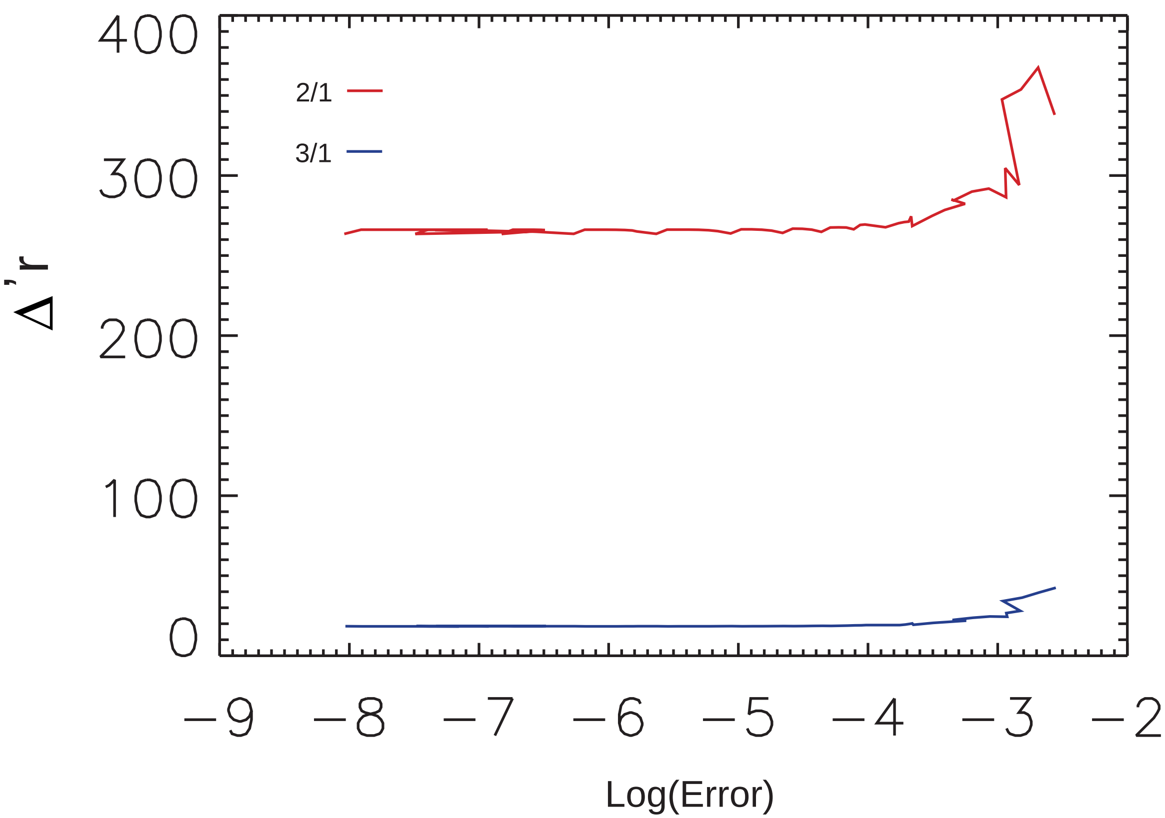
RESIDUAL ERROR MUST BE BELOW 1E-4 FOR RELIABLE STABILITY RESULTS

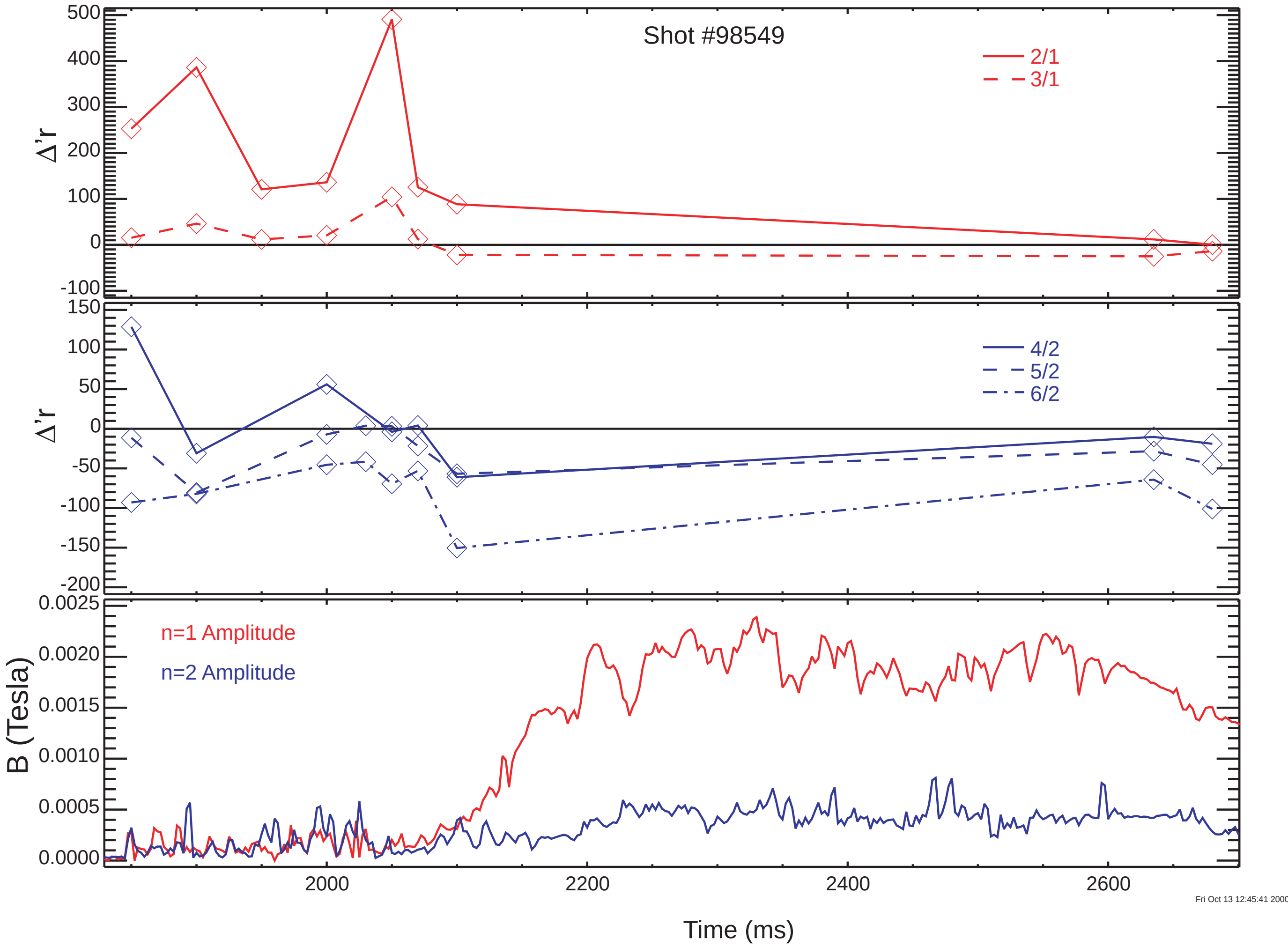
The Grad Shafranov residue of the numerical solution in EFIT is the residual error, which can cause deviations in Δ' above 1e-4.



Little variation in Δ' for Log(error) less than 1e-4 in EFIT calculation.

TEARING MODES IN AN AT PLASMA ARE DETERMINED TO BEGIN CLASSICALLY

* Early times show saturated low amplitude islands and positive deltaprime
* As the shot progresses, qmin decreases, large 2/1 deltaprime causes island growth
* The saturation may depend on the reduced deltaprime and helically perturbed J_{bs}



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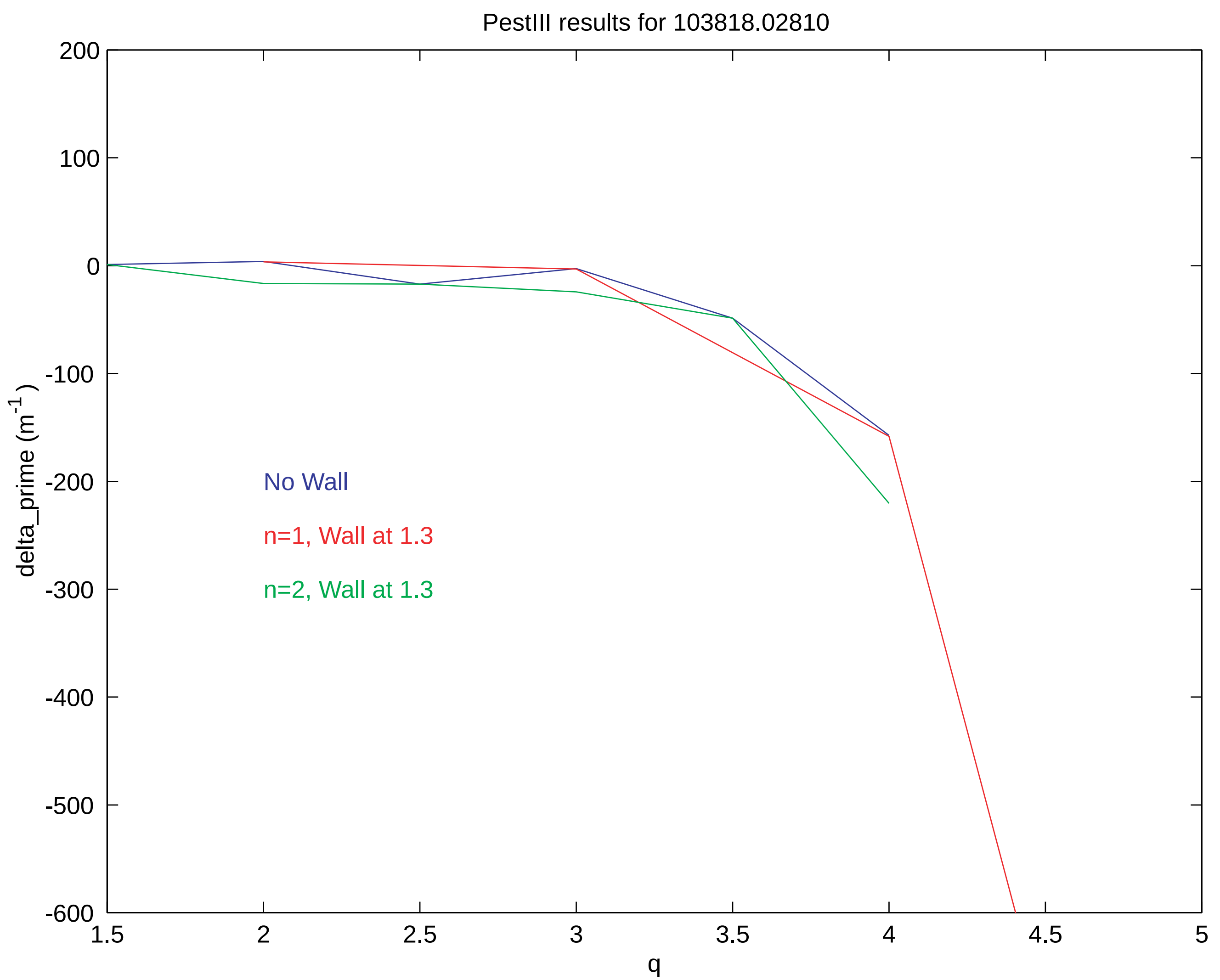
ACCURATE CALCULATIONS CAN BE USED TO CONFIRM THE STABILITY OF TEARING MODES



CONCLUSIONS

- Local pole increases sensitivity of Δ' to equilibrium fitting parameters in a high Beta, low q_{min} elming H-mode shot
  - Two types of poles exist:
    - Parity selection of the eigenvectors where Δ' OR $\Gamma' \rightarrow \infty$
    - Ideal marginal stability,where Δ' AND $\Gamma' \rightarrow \infty$.
  - The poles shown are from the ideal modes due to low q_{min}.
  - Equilibrium profiles of these cases show small, continuous variation
  - Pseudo analytical calculations for 3/2 mode shows qualitative agreement with numerical calculations.
- The need for good kinetic equilibrium fits with MSE and Thompson data is clear
  - The degree of error and resolution necessary in these solutions is only 1e-4 as indicated by the reduction of standard deviation.
  - Most of the error in Δ' calculations originates from fitting errors and not from residual error.
- A high q_{min} AT shot shows no pole in the equilibrium parameter space and the best calculations on this shot are in agreement with experiment
  - The Δ' calculations are robustly positive for the 2/1 and 3/1 modes, although the magnitude depends strongly on the fitting parameters.
  - Δ' calculations using a time series of high resolution kinetic efits indicate that tearing modes in this AT shot are linearly unstable and not Neoclassical.
  - A comparison to the time dependent frequency spectrum details of this shot agrees well with which modes are unstable, and when.

FUTURE WORK

- Non-Linear Studies with NIMROD
  - Saturation amplitudes will be compared to linear predictions
  - Resistive wall mode seeding of tearing modes
- Linear Studies with NIMROD
  - Growth rate calculations, and comparisons with Δ'
- Linear Studies with TWISTR
  - The new TWISTR removes the singularities from the asymptotic matching methods, and should be more robust, stable and accurate.
- ECCD mode stabilisation
  - Must Δ' change, or is it enough to fill the J_{bs} deficit.