Comparison of a Synthetic Phase Contrast Imaging Diagnostic with Experimental Measurements*

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Goal

Apply a synthetic PCI diagnostic to a simulation of a typical DIII–D plasma to improve the interpretation of experimental data.

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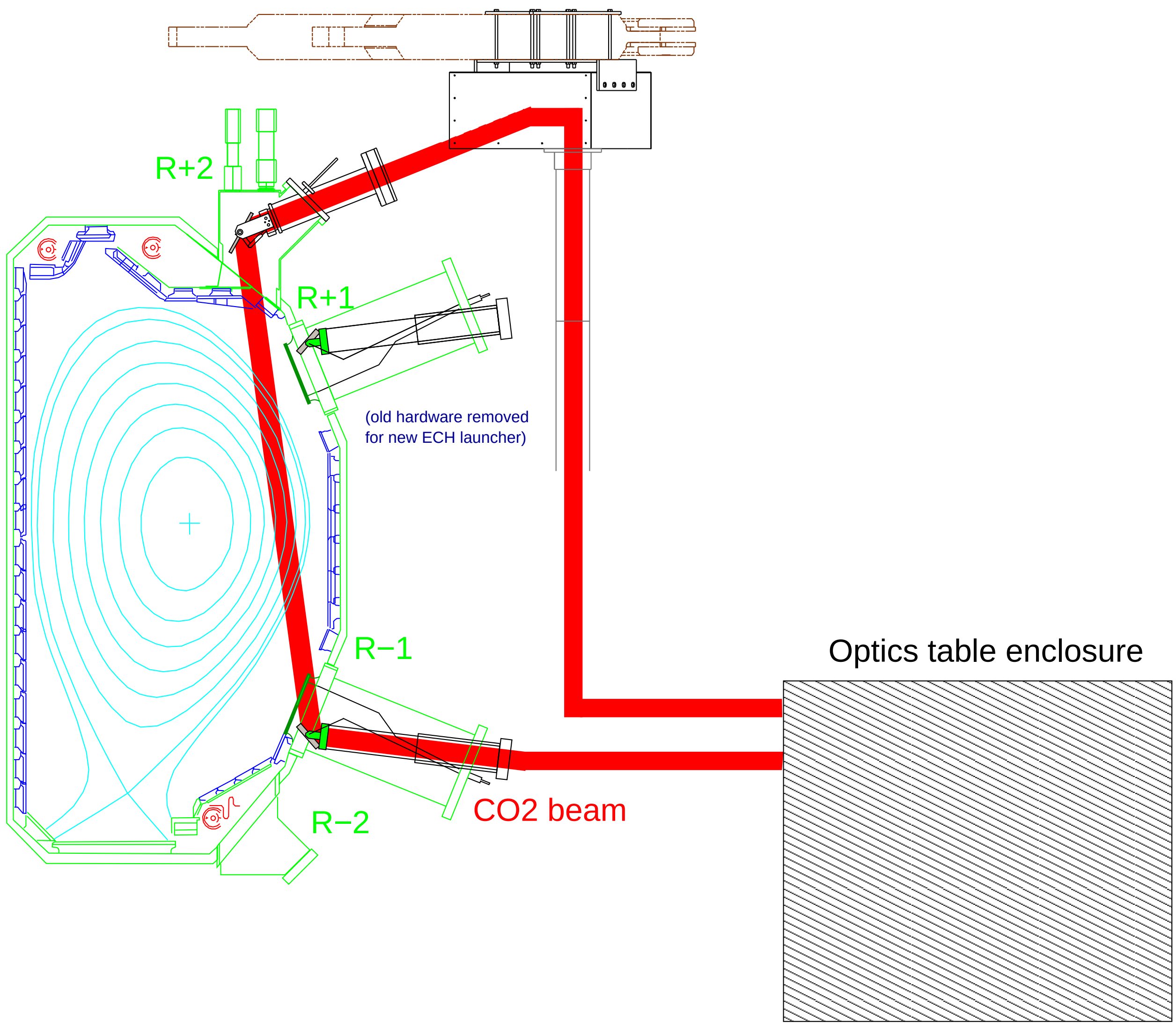
Outline

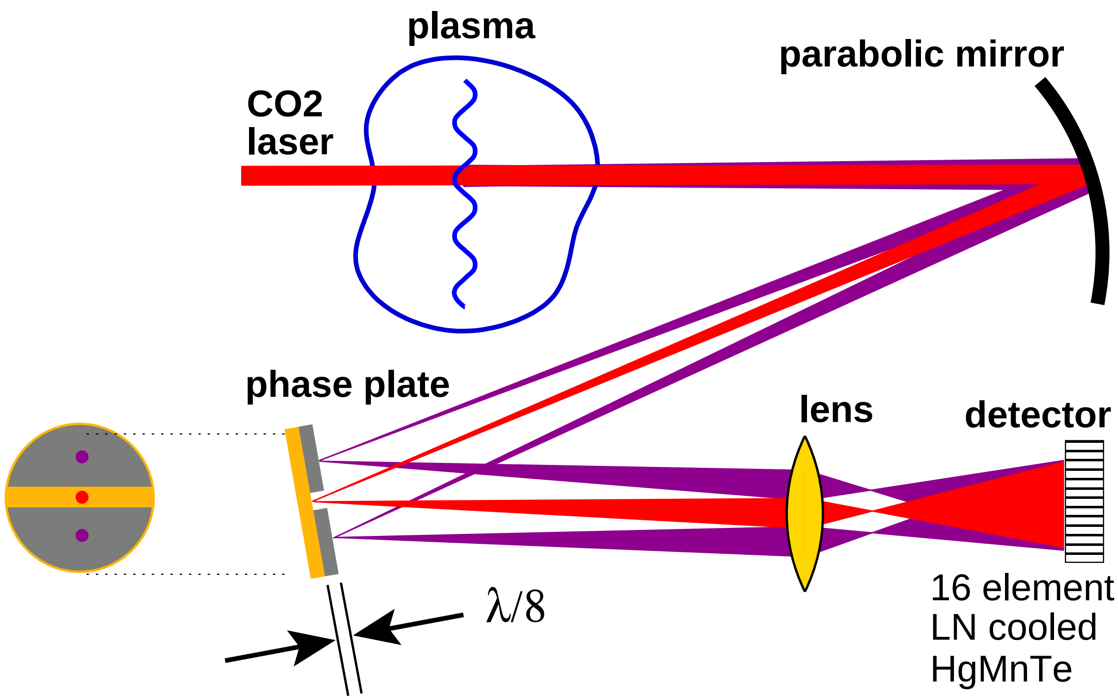
- PCI Experimental Apparatus
- PCI Instrumental Limits
- GYRO Simulation
- Description of GYRO Results
- Implementation of Synthetic PCI (SPCI)
- Results from SPCI
- New Interpretation of PCI Data

Phase Contrast Imaging Diagnostic on DIII–D



- 20 W cw CO_2 beam enters and exits vessel by ZnSe windows
- Beam steered by two in-vessel mirrors

"I" Stands for Imaging



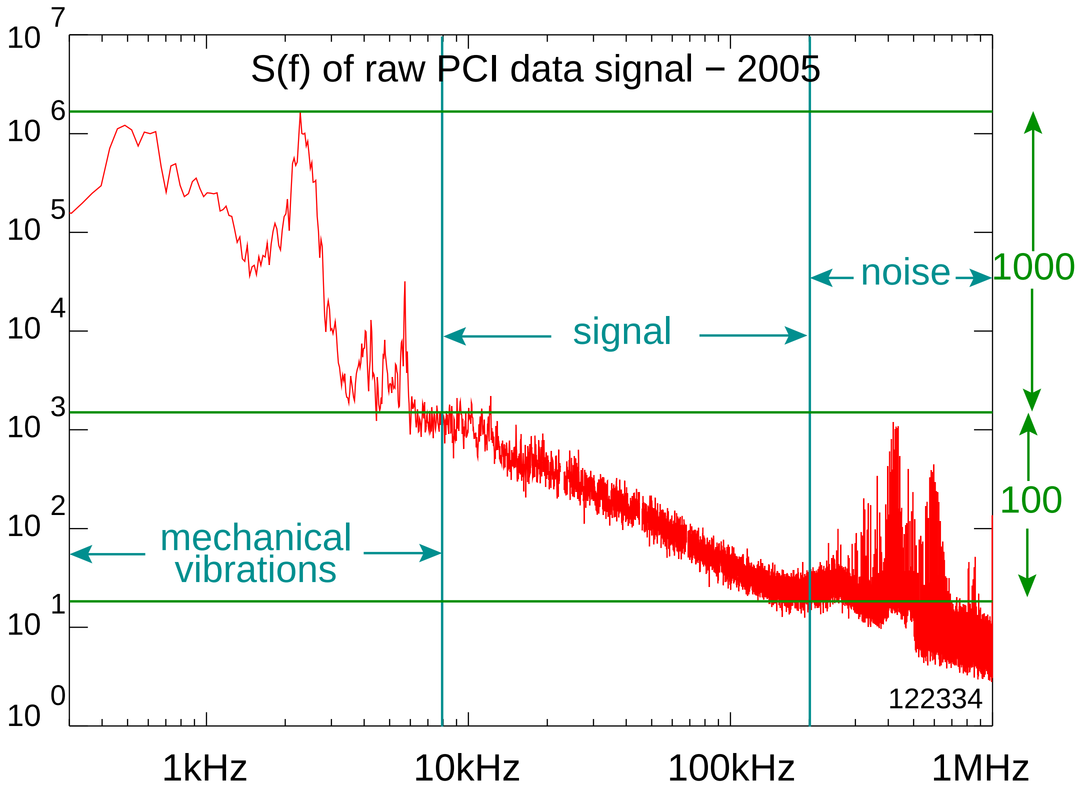
Beam passing through plasma acquires spatially dependent phase shift $\Delta\phi(R)$

- Before phase plate: $E_0 e^{i\Delta\phi} \simeq E_0(1 + i\Delta\phi)$
- After phase plate $E_0(i + i\Delta\phi)$
- $I \propto |E|^2 = E_0^2(1 + 2\Delta\phi)$

Phase shift represents plasma density

- Index of refraction for laser
  $N = (1 - \omega_{pe}^2/\omega^2)^{\frac{1}{2}} \simeq 1 - \omega_{pe}^2/2\omega^2$
- $\Delta\phi(R) \propto \int (N-1)dz \propto \int \tilde{n}(R,z)dz$
- Each detector channel j maps to a "radius" R_j ($\perp$ to beam). Each signal is $s_j = \int \tilde{n}(R_j, z)dz$

PCI Frequency Limits



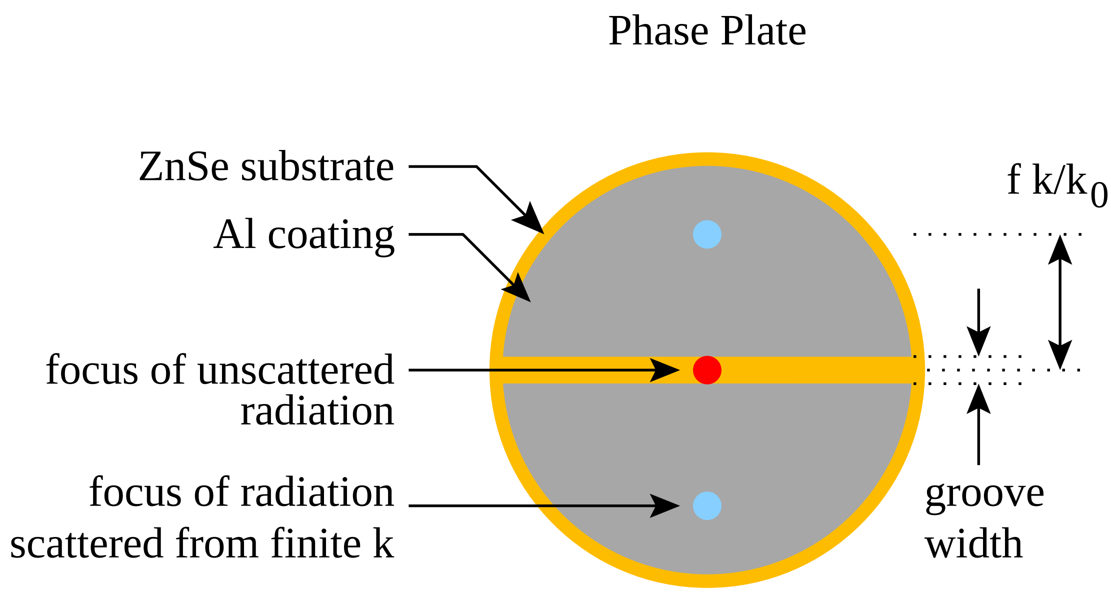
Minimum frequency f_{min}

- Mechanical vibrations at 2 kHz dominate PCI signal
- Limit frequency to $f > 10$ kHz
- Currently removed by analog filters

Maximum frequency f_{max}

- Maximum digitizer rate of 10 MHz (40 MHz with fewer channels) gives $f_{\text{max}} = 5$ MHz
- Limit from S/N may be below 1 MHz
- Plot above from before improved amplifier/filters installed

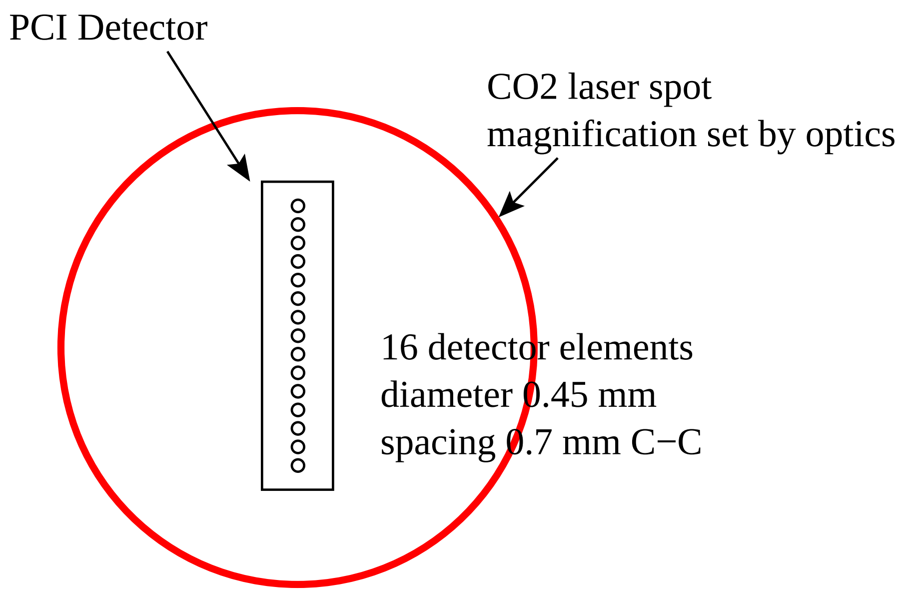
PCI Wavenumber Limits — $k_{\mathbf{min}}$



Minimum wavenumber k_{min}

- Lowest resolvable k set by detector size and magnification
- Lowest detectable k set by groove width
  - Scattered light must hit phase plate outside of groove
  - Groove must be wide enough to include unscattered radiation
  - Theoretical response verified in lab

PCI Wavenumber Limits — $k_{\mathbf{max}}$

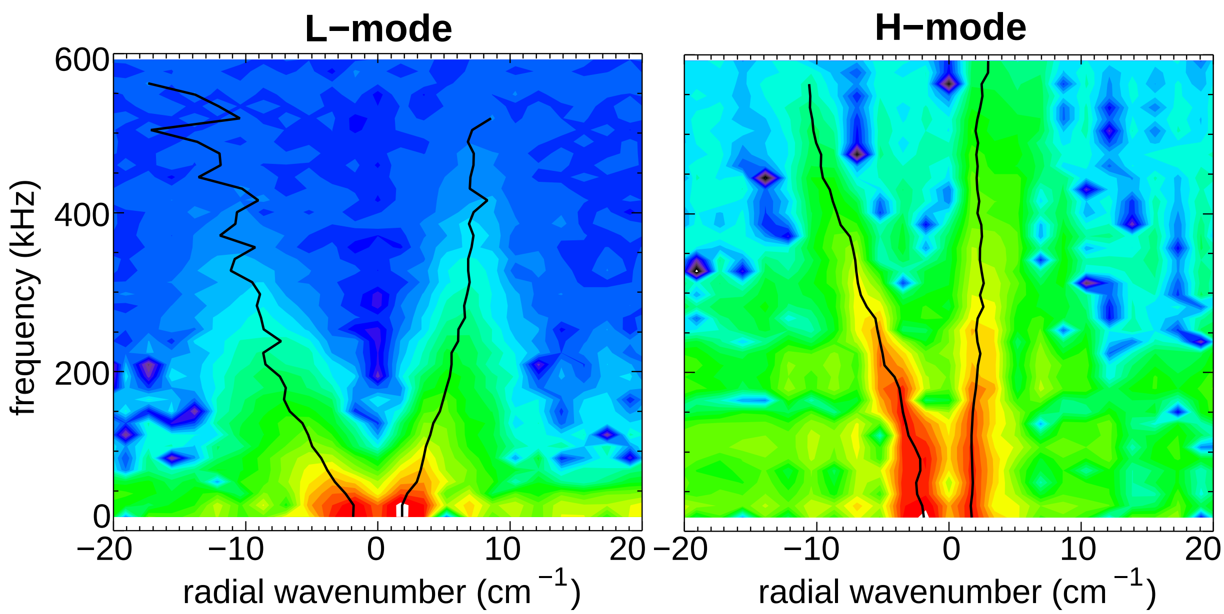


Maximum wavenumber k_{max}

- Set by optical magnification and spacing of detector elements
- Currently $k_{\text{max}} = 25$ cm^{-1} ($k\rho_i \sim 10$, $k\rho_e \sim 0.1$ for typical parameters with wide variation)
- Have operated with k_{max} 7–30 cm^{-1}
- Limit in k_{max} from physical apertures about 40 cm^{-1} without modifying diagnostic
- Spectral power decreases at higher k, so effective k_{max} is lower

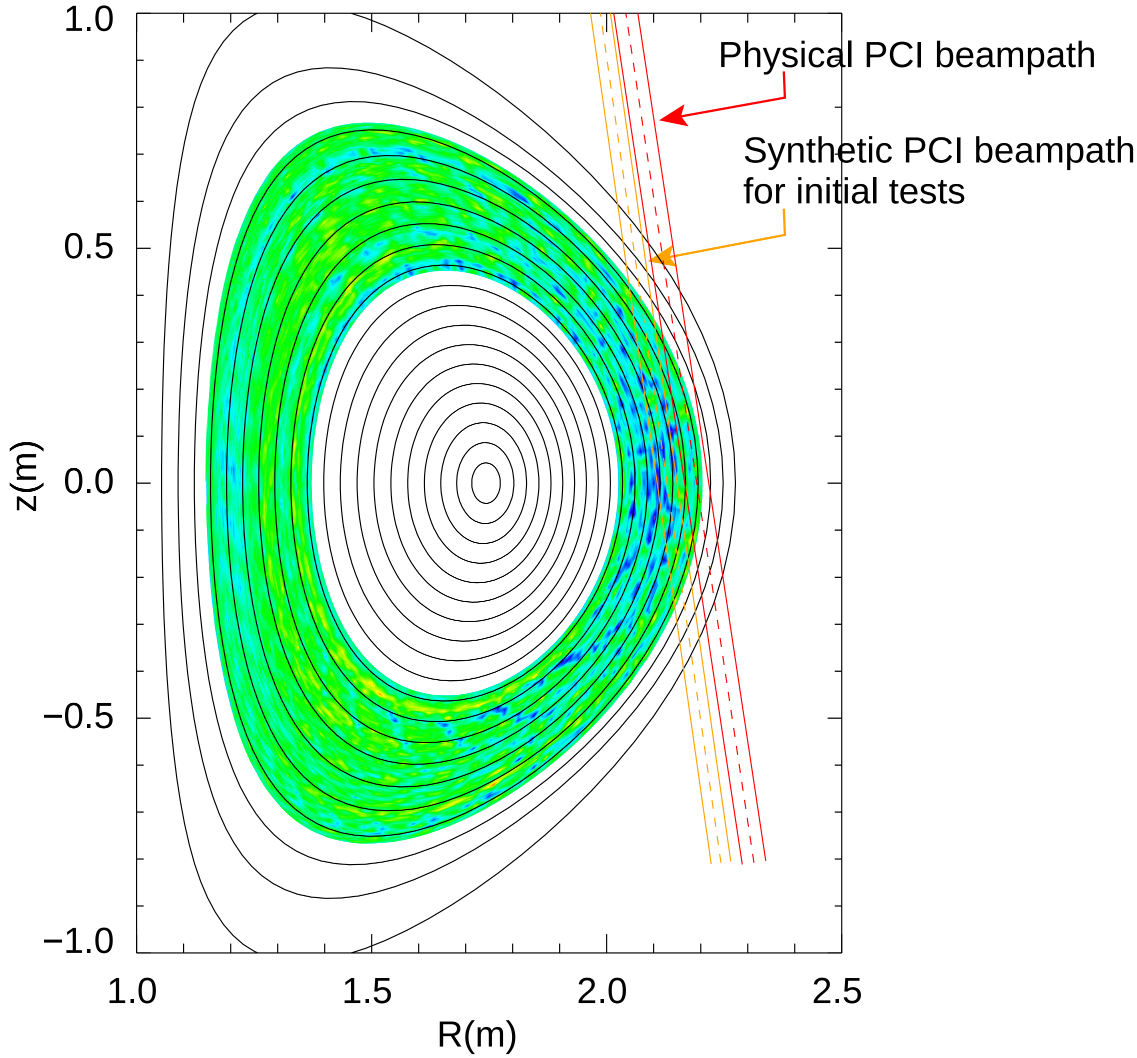
Within these limits, PCI is sensitive to portion of fluctuation spectrum with finite $k_\perp$ (perpendicular to PCI probe beam), $k_\parallel = 0$ (parallel to PCI probe beam)

Typical PCI S(k,f) results show modes with positive and negative wavenumbers



- PCI data always shows positive and negative k_R modes
- Relative amplitude varies depending on plasma
- Slope of peak has units of velocity
  - "Velocity" of positive and negative k_R branches not the same
  - Higher in H-mode
  - Increases roughly with $\sqrt{T_i}$

GYRO simulations used to model plasma fluctuations



- Nonlinear gyrokinetic turbulence simulation
- Shaped plasmas
- Full toroidal simulation
- Developed at General Atomics

Fluctuating density calculated from GYRO output

GYRO calculation performed in Miller Geometry

- $R = R_0 + r\cos(\theta + \sin^{-1}\delta\sin\theta)$
- $z = \kappa r \sin\theta$
- R_0, δ, κ all functions of r
- Cannot directly calculate (r, θ) from (R, z) — developing fast, accurate numerical coordinate conversion is a major step in creating synthetic diagnostic

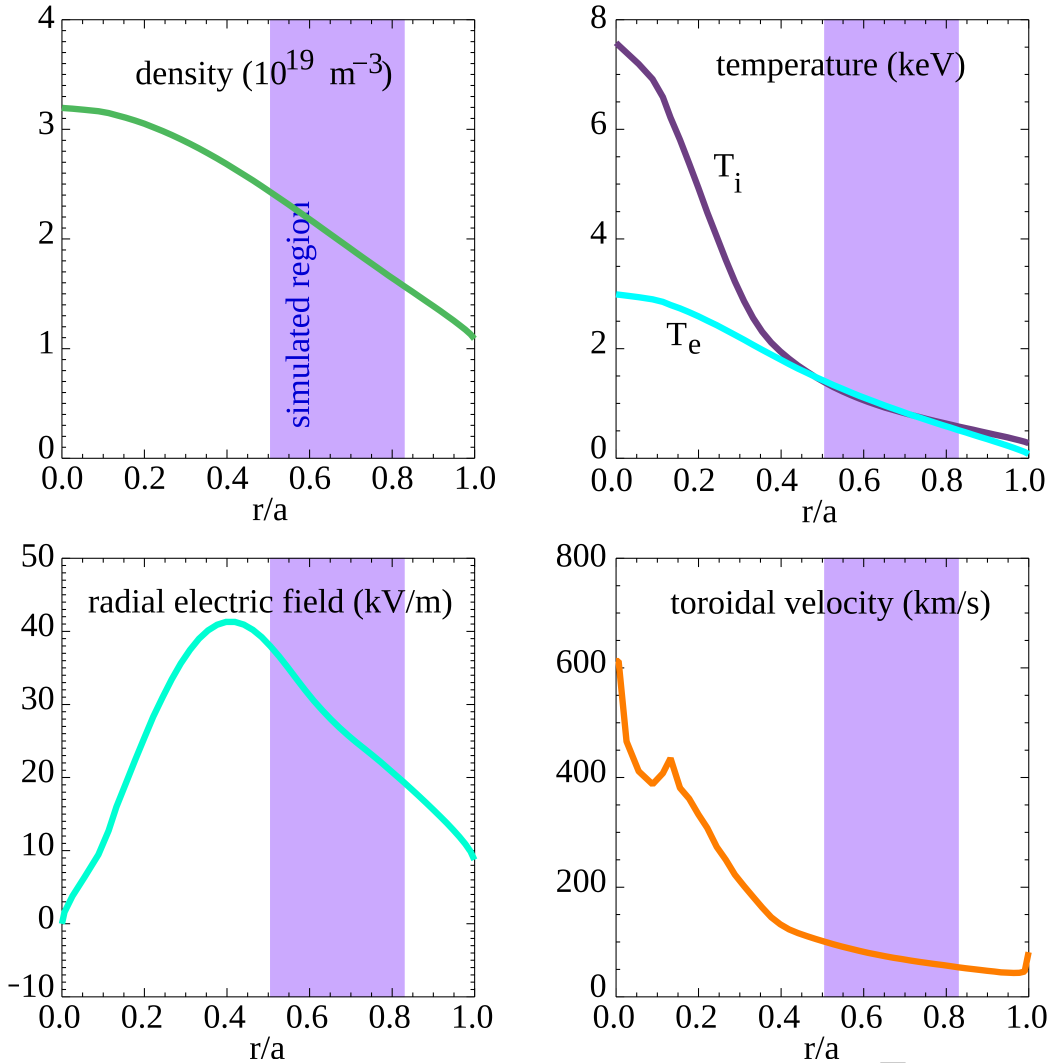
GYRO outputs combined to form physical quantities

- Output records density perturbation δn_j for each toroidal mode number n_j as a function of time

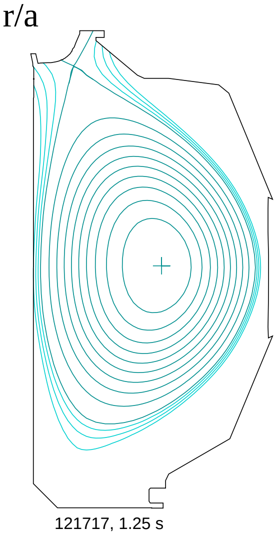
$$\delta n(r, \theta, \phi, t) = \mathrm{Re}\sum_{j=0}^{N-1} \delta n_j(r, \theta, t) e^{-in_j(\phi+\nu(r,\theta))}$$

- $\delta n_j \in \mathbb{C}$, $\nu \in \mathbb{R}$
- δn_k, ν recorded on a grid, interpolated to arbitrary (r, θ)

GYRO uses experimental profiles



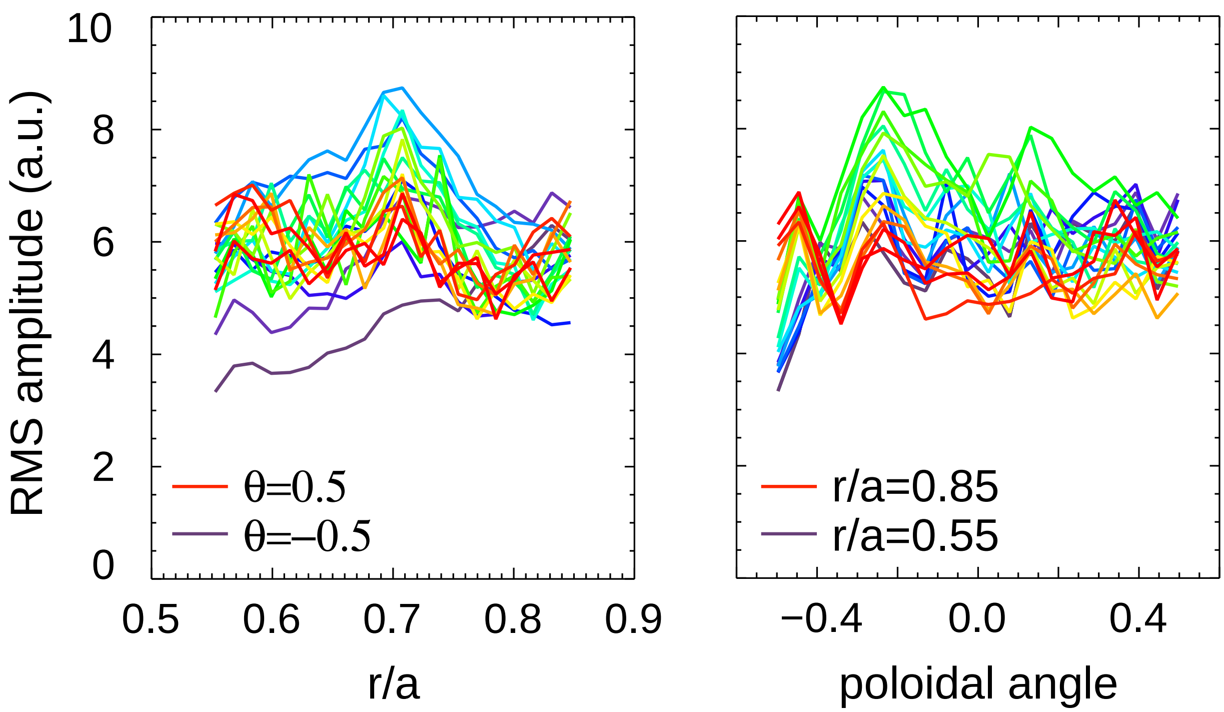
- $\rho_e = 0.1$ mm, $1/\rho_e = 100$ cm^{-1} (with reduced mass ratio)
- $\rho_i = 3$ mm, $1/\rho_i = 3$ cm^{-1}
- $\rho_s = 2$ mm, $1/\rho_s = 5$ cm^{-1}
- Drift velocity $v_{\star e} = 1$ km/s



Limited GYRO simulation used for initial tests

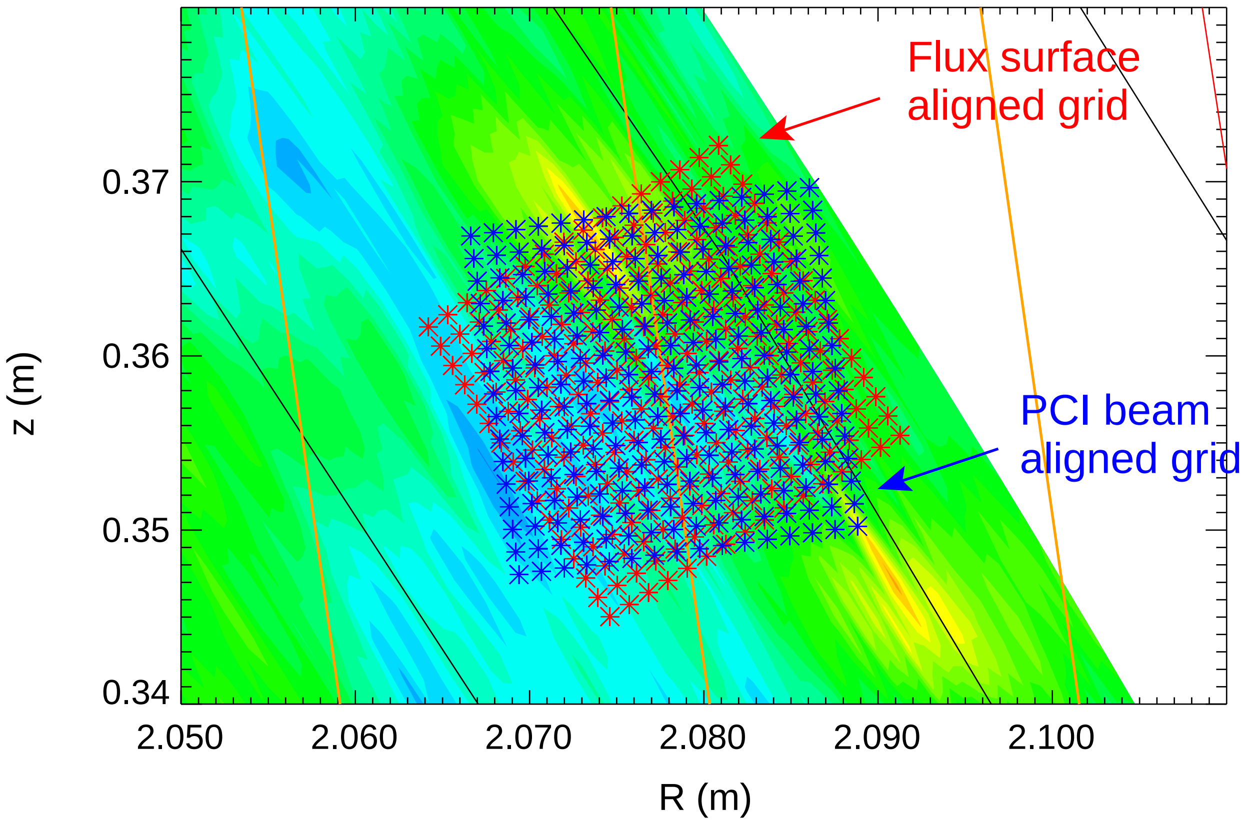
- Simulation covers $0.54 < r/a < 0.86$
  - Not adequate for comparison with experimental PCI data
  - Synthetic PCI moved inward to study response
  - Gyrokinetic equations may fail at some r/a near 1
- 400 radial grid points, $\Delta r \sim 0.5$ mm
- Toroidal modes modeled $n = 0, 10, 20 \ldots 150$
- Data time step 3 μs
- Simulation time step 0.03 μs
- Gyrokinetic ions, drift-kinetic electrons
- $\sqrt{m_i/m_e} = 30$, reduced for efficiency
- Synthetic PCI $k_{\text{max}} = 12$ cm^{-1} ($k_{\text{max}}\rho_i \sim 4$) because this GYRO run was not optimized for high k

Sample GYRO result shows little poloidal, radial amplitude variation



- Results sampled over $r/a =$0.55–0.85, $\theta = -0.5$–0.5
- Expect full simulation to show
  - Larger fluctuations at larger r
  - Ballooning structure (larger amplitude at $\theta = 0$)

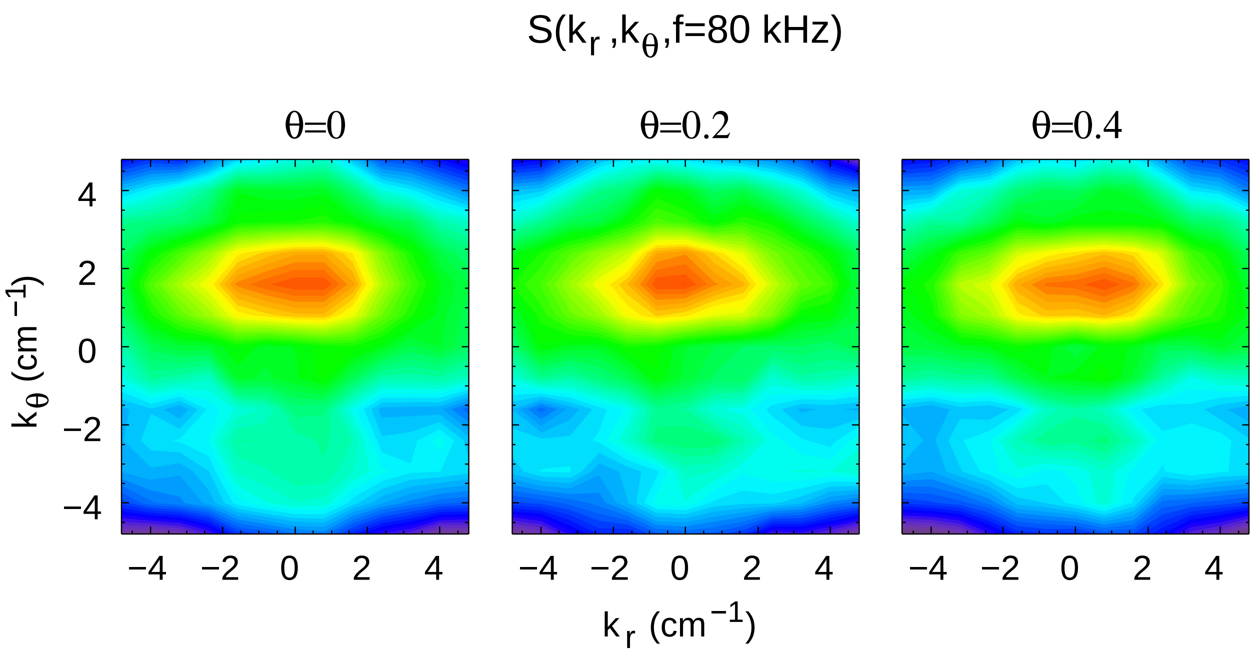
GYRO results characterized by sampling on grid



Sample GYRO results on a grid of points

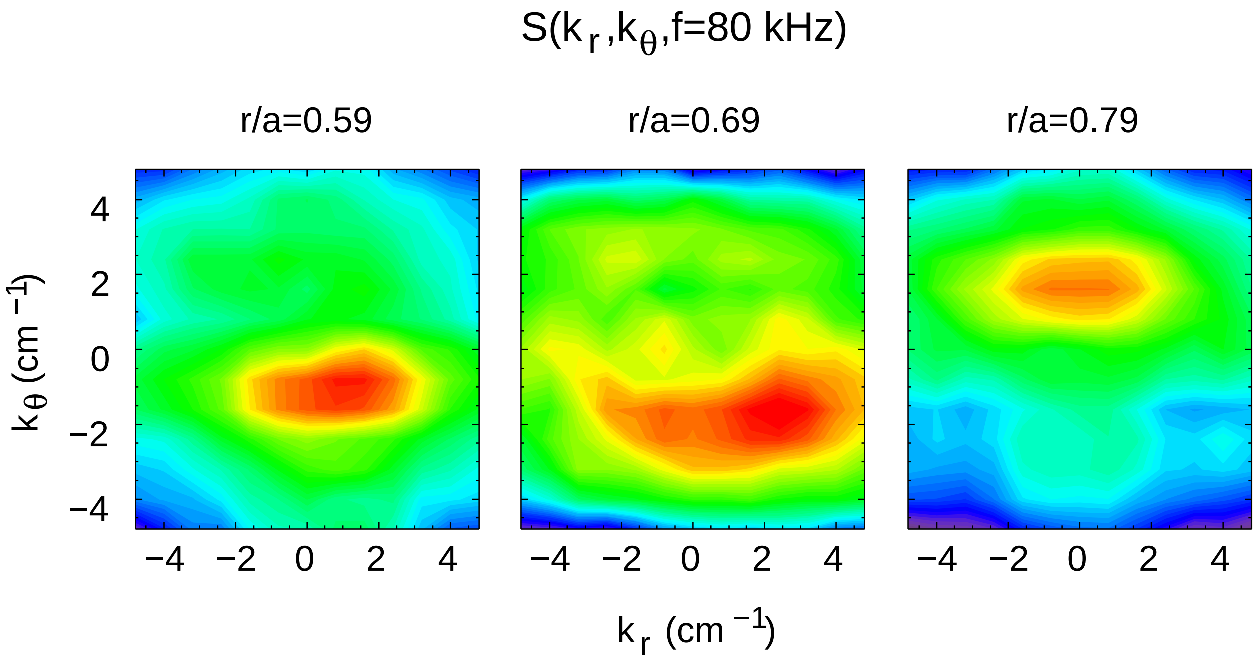
- At least 16×16
- Align grid with flux surface or PCI beam
- First calculate $S(\vec{k}, f)$, then simplified parameters that can be plotted
- Note that $S(\vec{k})$ is $\int_0^\infty df\ S(\vec{k}, f)$, not $\int_{-\infty}^\infty df\ S(\vec{k}, f)$, so it is not symmetric under $\vec{k} \rightarrow -\vec{k}$

Sample GYRO result shows little poloidal variation in spectrum



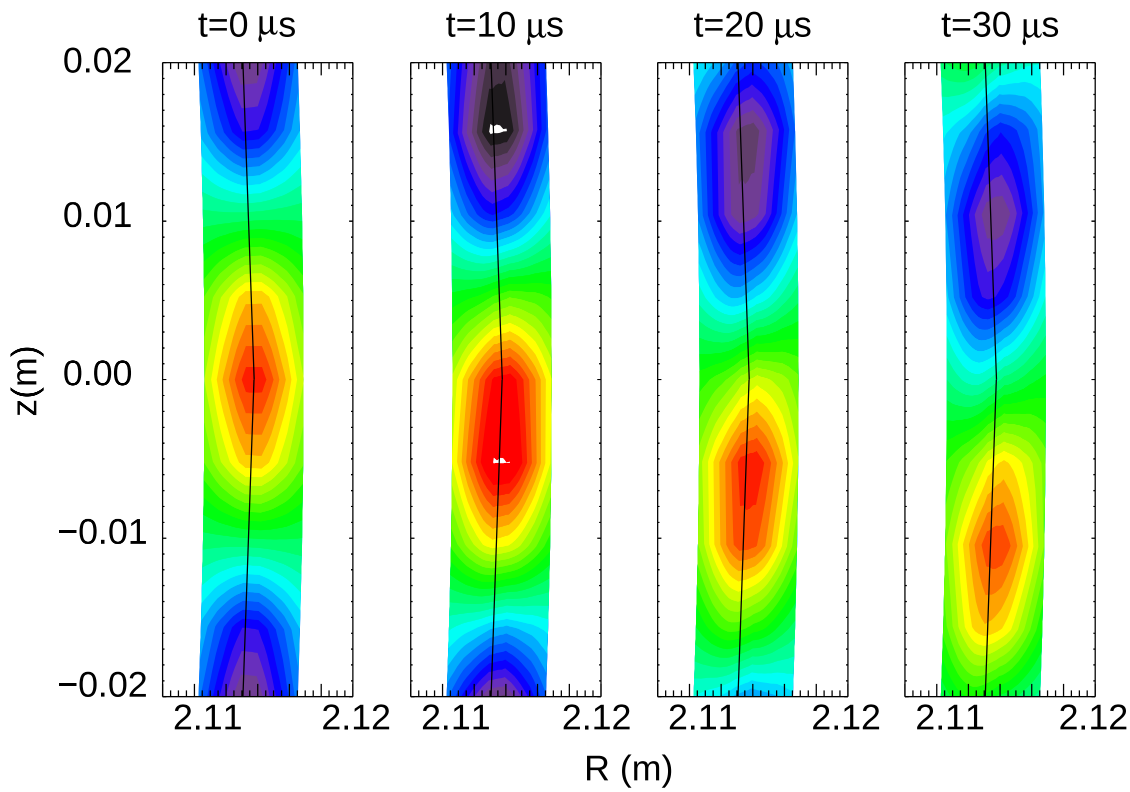
- Spectrum $S(k_r, k_\theta)$ is integrated from $f = 10$ kHz to 160 kHz
- Peak of turbulence near $k_\theta = 1.5$ cm^{-1} $(k_\theta \rho_i \sim 0.5)$
- Avoid the Triangularity Trap
  - Creating geometric parameters (especially δ) from experiment results in $noisy$ profiles
  - Noisy profiles do not affect turbulence
  - Noisy profiles have a large effect on $(\rho, \theta) \longrightarrow (R, z)$ mapping
  - This creates magnetic shear changing spectrum, especially in k_r
  - Mappings here have corrected triangularity, but noisy profiles used in simulation

Sample GYRO result shows fluctuations reverse across radius



- Spectrum $S(k_r, k_\theta)$ is integrated from $f = 10$ kHz to 160 kHz
- Not clear which nonlinear effects causes propagation direction to differ from linear instability
- Simulated plasma has no poloidal velocity flow

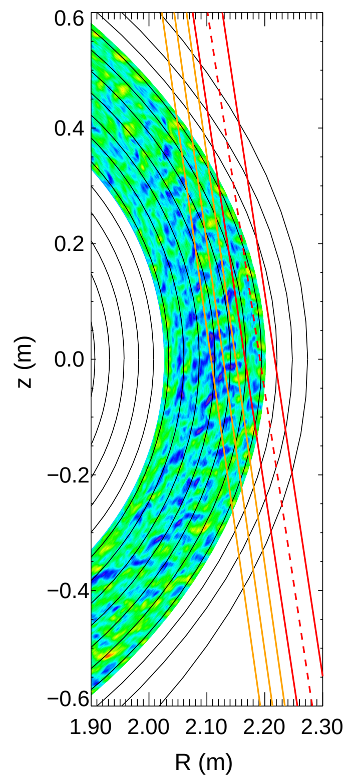
Primary Low-k Instability in this Regime is ITG



- Largest mode in nonlinear simulation is $n = 60$
- Linear GYRO run at $n = 60$, $r/a = 0.7$
- Time steps above separated by 10 μs
- Linear mode propagates in ion direction with $v_\theta = 3.6$ km/s

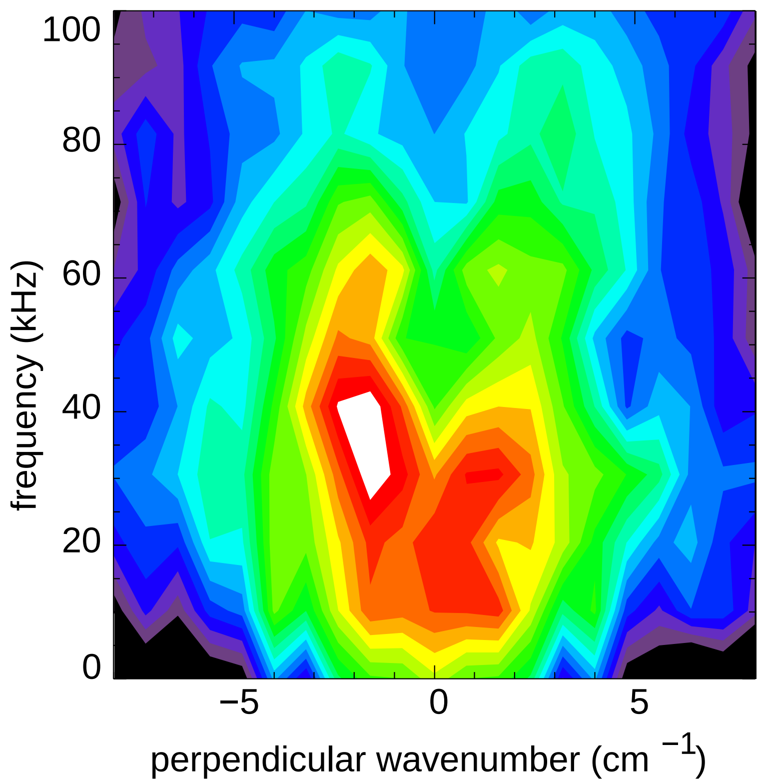
Synthetic PCI Implemented

- Implemented as post-processor analyzing GYRO output
- Line integrate along PCI beam path
  1. Find (r, θ) for (R, z) coordinates along PCI beam (point spacing < 1 mm)
     - Simplified: one chord for each of 16 detector elements
     - Full k response: space chords over entire width of beam with several chords per detector channel
  2. Interpolate to find δn at each point
  3. Sum to perform integration



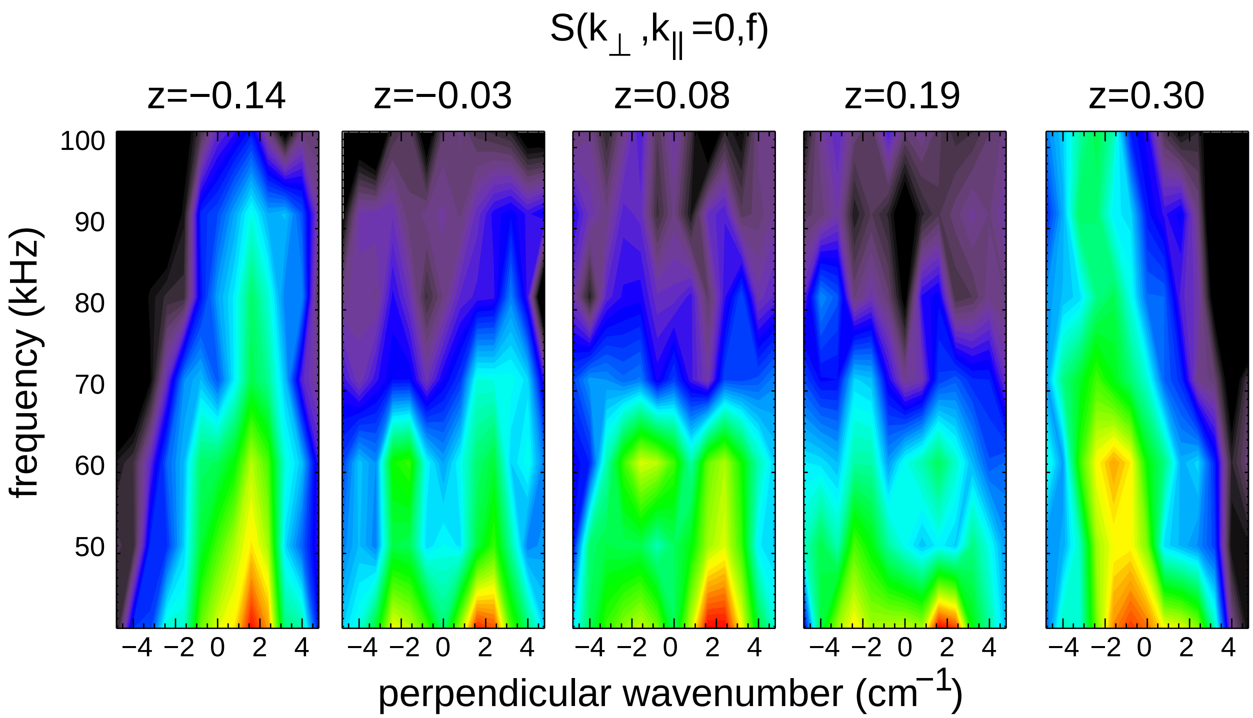
- To model full k response, perform high pass filtering in k space and combine sampled chords to represent detector element shape.
- Use PCI data analysis routines to analyze data

Synthetic PCI generates signals similar to experiment

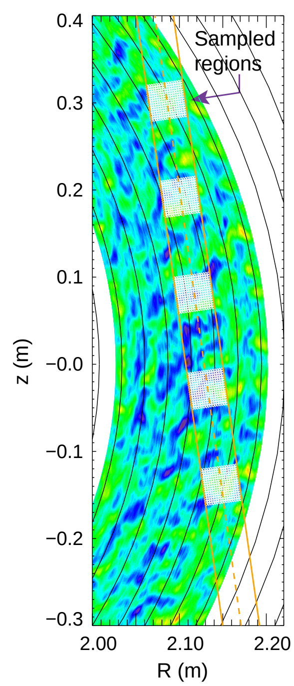


- Modes at positive and negative wavenumbers seen with different amplitudes
- Wavenumber increases with frequency

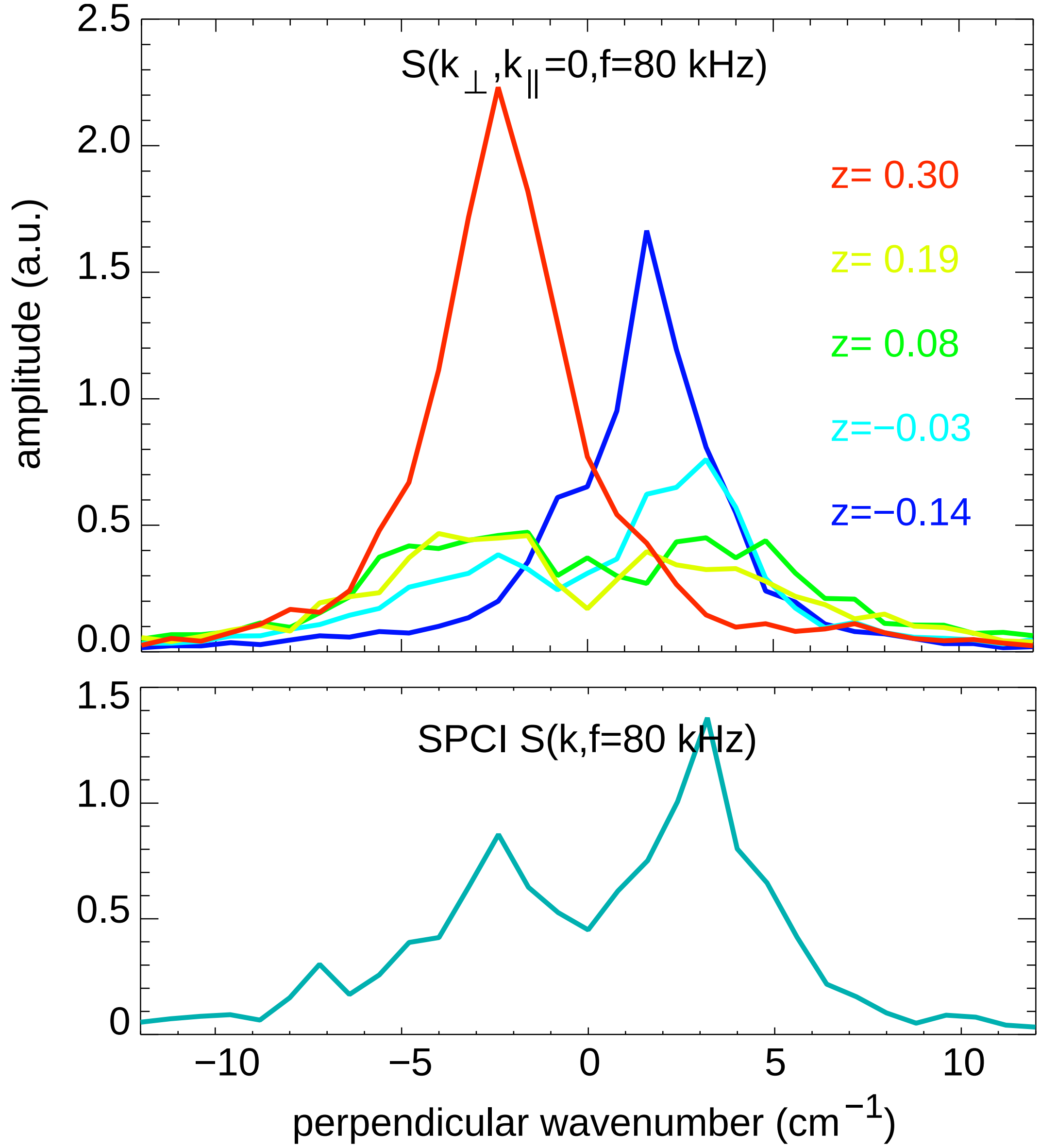
Different sections of beampath contribute to different parts of spectrum



- Sample turbulence in a 16×16 along beampath, aligned with beam
- Component of spectrum $S(k_\perp, k_\| = 0, f)$ represents portion of turbulence observed by PCI
- Total SPCI signal is equivalent to sum of these 5 spectra

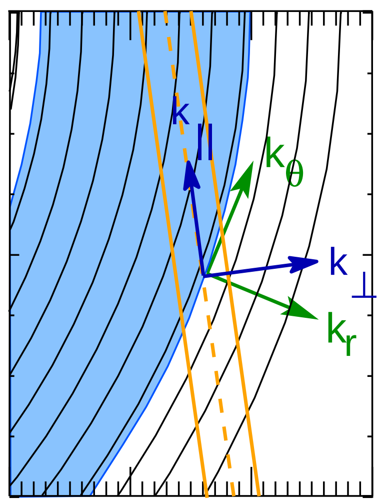


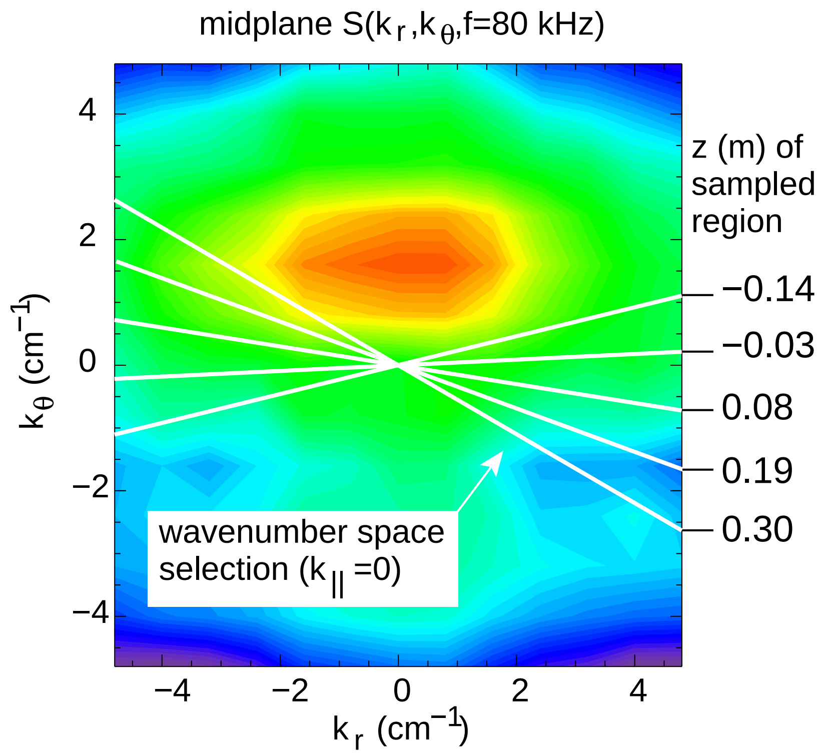
Region near LCFS contribute most to spectrum



- Spectra shown are $S(k_\perp, k_\| = 0, f = 80$ kHz$)$
- One edge contributes to positive wavenumber, opposite edge contributes signal at negative wavenumber
- Magnitude of dominant wavenumber shifts slightly along beampath

Variation along beampath results from change of angle between beam and flux surface





- Characteristic spectrum $S(k_r, k_\theta, f = 80$ kHz$)$ shown
- Angle between k_r and beampath changes along beampath, hence direction of $k_\perp$ changes in $(k_r, k_\theta$ coordinate system
- PCI detects edge of peak at $k_r = 0$, $k_\theta = 1.5$ cm^{-1}
- At $z < 0$, PCI sees edge at positive k; at $z > 0$, PCI sees edge at negative k

Conclusions

- Synthetic PCI has been implemented for analyzing output of GYRO simulation
  - Includes proper response at high and low wavenumber limits
  - Required techniques that will be valuable for development of other synthetic diagnostics
- Simulation of typical plasma leads to improved understanding of PCI measurement
  - Modes detected by PCI where mode propagates perpendicular to PCI beam
  - Direction perpendicular w.r.t. PCI beam depends on poloidal, not radial, propagation of turbulence
  - Apparent perpendicular velocity of PCI modes depends on ω/k_θ and geometry
  - Variation in sampling of k_r, k_θ space results in fluctuations near LCFS contributing most to PCI signal (in ITG range)
- Synthetic PCI now used for Alcator C-Mod as well (see poster by L. Lin)

Future Work

- Run GYRO simulations for DIII–D plasmas with good PCI data for comparisons with experiment
- Examine GYRO simulations optimized to record high k modes in output
- Extend simulation to as large r/a as possible
- Evolve synthetic PCI to include localization via rotating mask