Nonlinear Excitation of Damped Eigenmodes in Microturbulence Simulations

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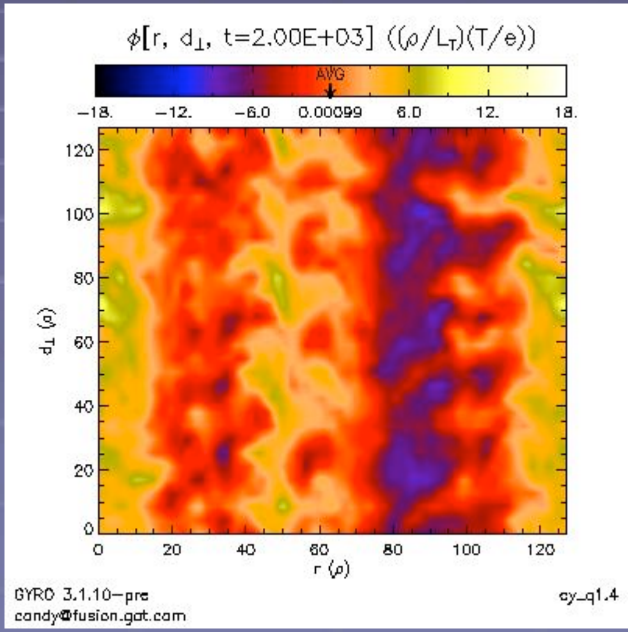

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Outline

Reduced ITG-like model

- Simple fluid model used to illustrate concepts
- Damped eigenmodes affect on energy, fluxes

Gyrokinetic Simulations (cyclone base case ITG)

- Are damped eigenmodes necessary to account for saturation?
- Deviation from pressure/potential phase of ITG mode.
- Damped eigenmodes – affect on fluxes
  - Quasilinear flux overestimates true flux.
- GLF23 transport code eigenvalue solver - eight mode frequencies growth rates (can only get fastest mode from initial value codes).
- Damped eigenmode frequencies appear in nonlinear frequency spectrum.

Importance of Damped Modes

Excitation of Damped Modes

Affect on energy transfer and saturation:

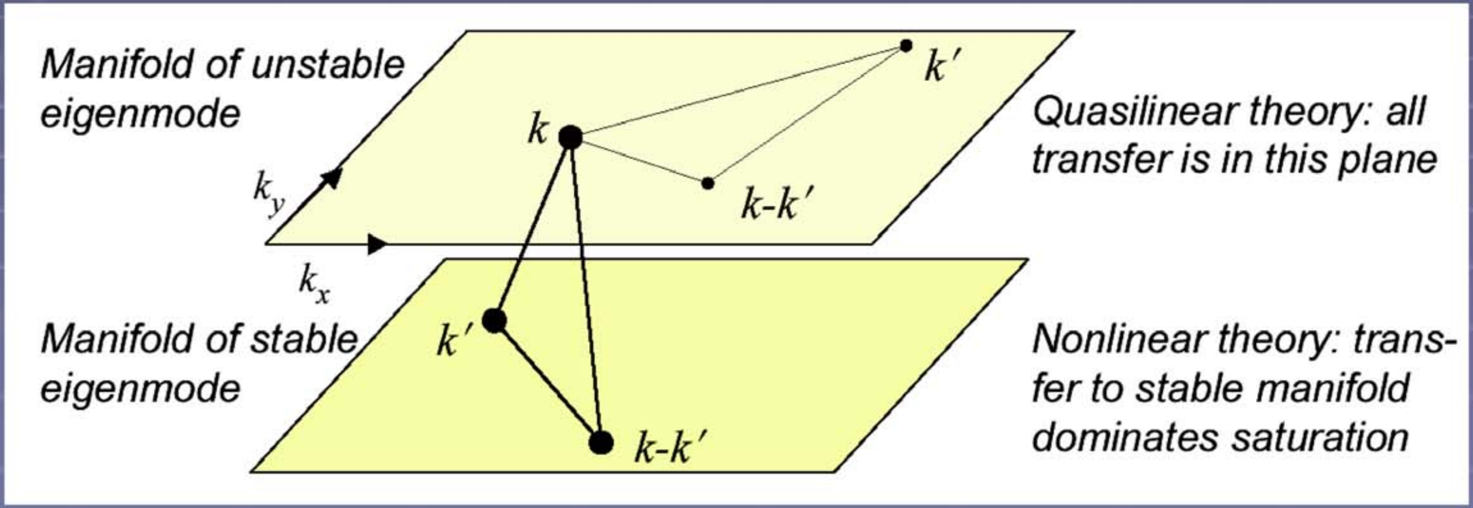
- Dissipates energy.
- Different from viscous damping at high k.
- Additional saturation mechanism.

Affect on Transport:

- Contributes to an inward component of flux.
- Reduces net transport.

Damped Modes - Energy Transfer

Two eigenmode energy transfer



Excitation of damped eigenmodes provides an alternative channel for energy transfer => additional saturation mechanism.

Reduced ITG-like Model

Three field local model.
Nonlinearites and linear coupling capture ITG physics.
ITG -- Slab Branch, Fluid Limit

Used to illustrate concepts of damped eigenmodes.

$$(1+k^2)\frac{\partial\phi}{\partial t} - ik_y v_D \phi(\hat{\eta}k^2 - 1) + ik_z u_\parallel$$

$$= -\sum_{k'} (k' \times \hat{z}\cdot k)\phi_{k'}\phi_{k-k'}k'^2$$

$$\equiv (1+k^2)N_\phi,$$

$$\frac{\partial u_\parallel}{\partial t} + ik_z\phi + ik_z p = -\sum_{k'} (k' \times \hat{z}\cdot k)\phi_{k'}u_{\parallel k-k'} \equiv N_{u_\parallel},$$

$$\frac{\partial p}{\partial t} + ik_y v_D \hat{\eta}\phi = -\sum_{k'} (k' \times \hat{z}\cdot k)\phi_{k'}p_{k-k'} \equiv N_p,$$

Linear Eigenmode Basis

$$\begin{pmatrix} p \\ u_{\parallel} \\ \phi \end{pmatrix} = \beta_1 \begin{pmatrix} b_1 \\ a_1 \\ 1 \end{pmatrix} + \beta_2 \begin{pmatrix} b_2 \\ a_2 \\ 1 \end{pmatrix} + \beta_3 \begin{pmatrix} b_3 \\ a_3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

$$\equiv \mathbf{M} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}.$$

Linearize system of equations.

Solve dispersion relation for mode frequencies.

Solve for eigenvectors associated with each frequency.

Linear transformation changes to eigenmode basis.

Eigenmode basis diagonalizes linear coupling.

Nonlinear Excitation of Damped Modes

How are damped modes excited, and when are they important in the dynamics?

x_1 is an unstable eigenmode, x_2 is a damped or marginally stable mode.

$$\dot{x}_1 = \gamma_1 x_1 + C_1 x_1^2 + D_1 x_1 x_2,$$

$$\dot{x}_2 = -\gamma_2 x_2 + C_2 x_1^2 + \ \ldots .$$

Evolution equations: linear growth and important coupling terms.

$$x_1 = x_i \exp[\gamma_1 t],$$

$$x_2 = \frac{C_2 x_i^2}{\gamma_2 + 2\gamma_1}[\exp(2\gamma_1 t) - \exp(-\gamma_2 t)] + x_i \exp(-\gamma_2 t)$$

Initial behavior of growing (x1) and damped (x2) modes.

Eventually coupling to growing mode will <u>always</u> excite mode to some extent.

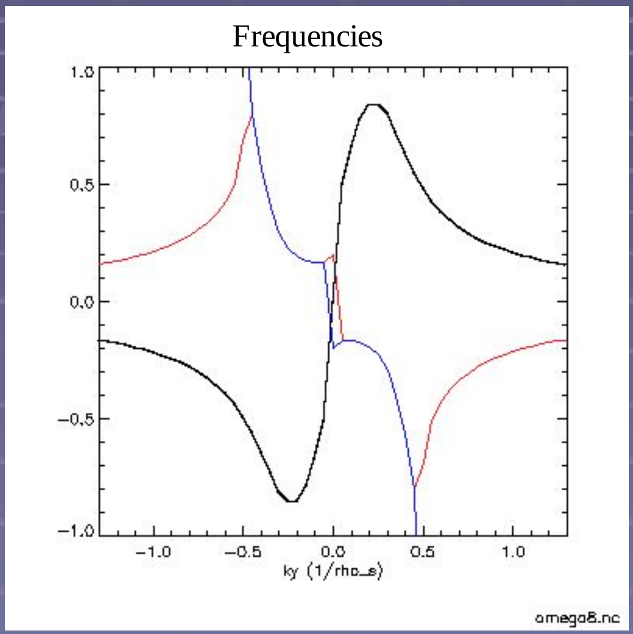
Damped mode affects saturation if:

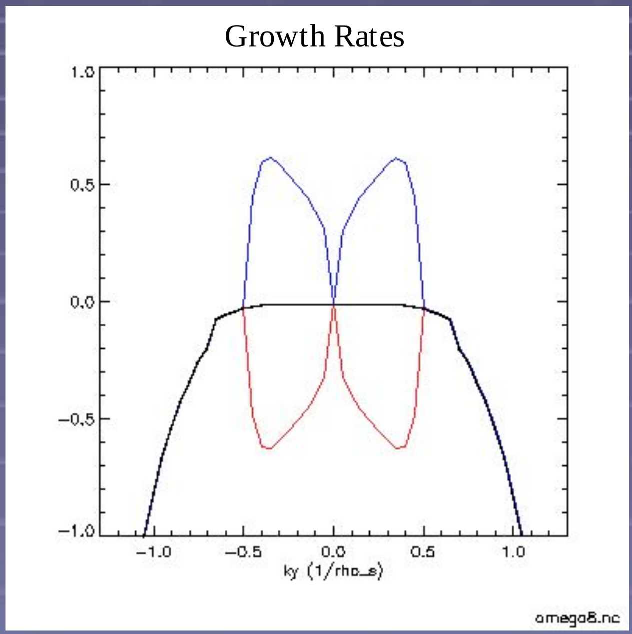
$$D_1 x_1 x_2 \approx C_1 x_1^2.$$

$$P_t \equiv \frac{D_1 C_2}{C_1^2(2 + \gamma_2/\gamma_1)} \approx 1$$

Criterion for damped modes influencing saturation.

Reduced Model – Dispersion Relation





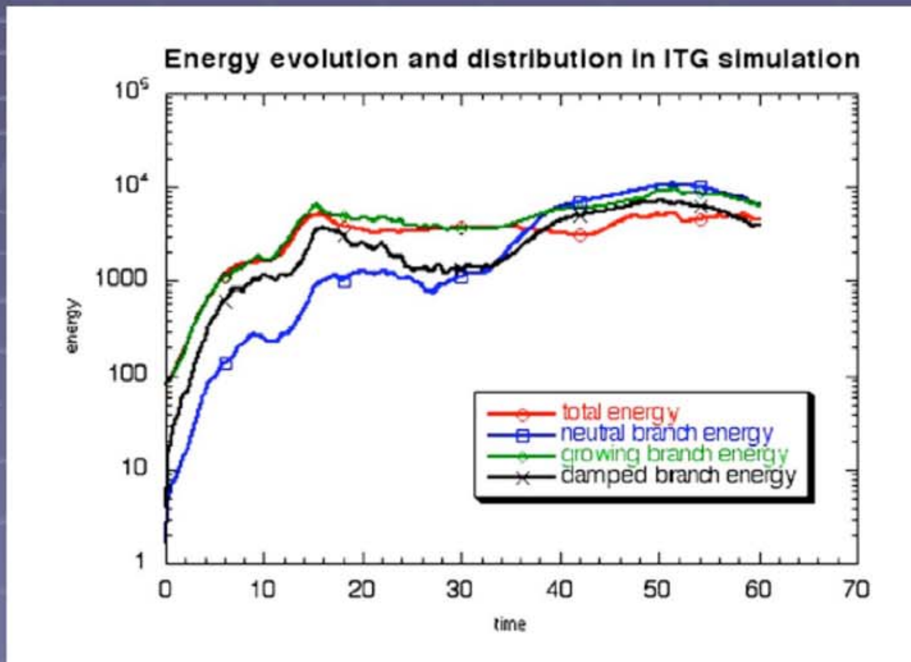
Blue: Growing Mode
Red: Damped Mode
Black: Marginal Mode

Three modes:
1. Marginal
2. Growing
3. Damped

Region of linear growth is the same as stability region of the stable mode.

Hyper-viscous damping at high k.

Reduced Model - Energy



$$U = \sum_k U_k = \sum_k \left[(1+k^2)|\phi|^2 + |u_\parallel|^2 + |p|^2\right].$$

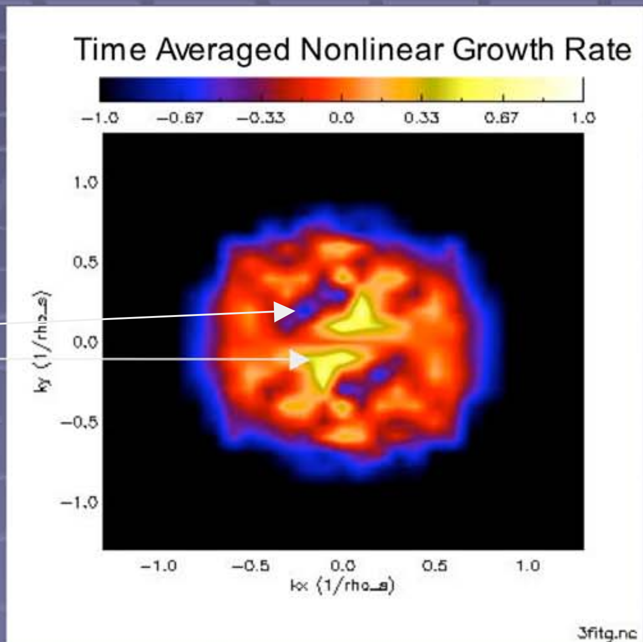
Total Energy

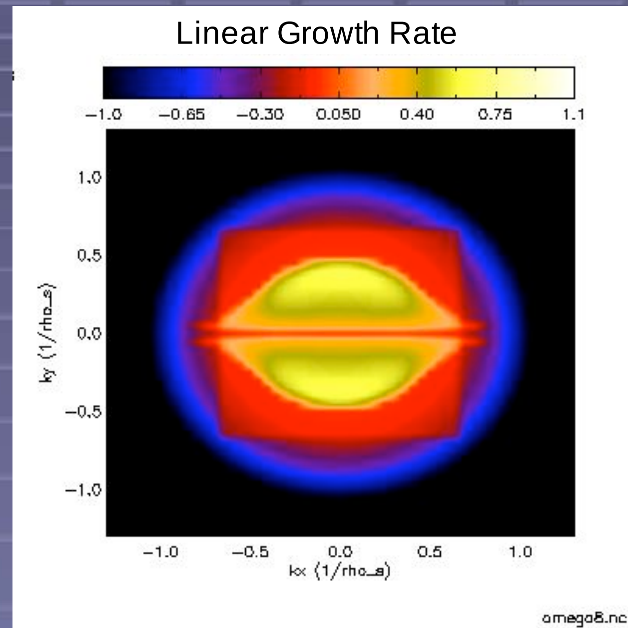
$$U = \sum_k \Bigg\{ \sum_{m=1}^{3} \left[(1+k^2)|a_m|^2 + |b_m|^2 + |c_m|^2\right]|\beta_m|^2 + 2\,\mathrm{Re} \sum_{m=1}^{2} \sum_{n>m}^{3} \left[(1+k^2)(a_m a_n^*) + (b_m b_n^*) + (c_m c_n^*)\right] \times \langle \beta_m \beta_n^* \rangle \Bigg\}.$$

Total energy expressed in eigenmode basis

Damped and marginal mode energies excited to significant amplitudes.

Reduced Model – Growth Rates





Damping in region of linear excitation.
Fastest growing Fourier modes at lower k than peak linear growth rate.

$$\frac{\partial U}{\partial t} = \sum_k \left[k_z \,\mathrm{Im}\langle p u_\parallel^* \rangle + v_D k_y \hat{\eta}_e \,\mathrm{Im}\langle \phi p^* \rangle - \gamma_D U_k\right],$$

→

$$\gamma_k^{\mathrm{nl}} = \frac{k_z \,\mathrm{Im}\langle p u_\parallel^* \rangle + k_y \hat{\eta}_e \,\mathrm{Im}\langle \phi p^* \rangle}{U_k} - \gamma_D .$$

Nonlinear growth rate shows where <u>actual finite amplitude</u> damping and excitation occur.

Reduced Model - Heat Flux

$$Q = \sum_k k_y \,\mathrm{Im}\langle \phi^* p \rangle .$$

Heat Flux

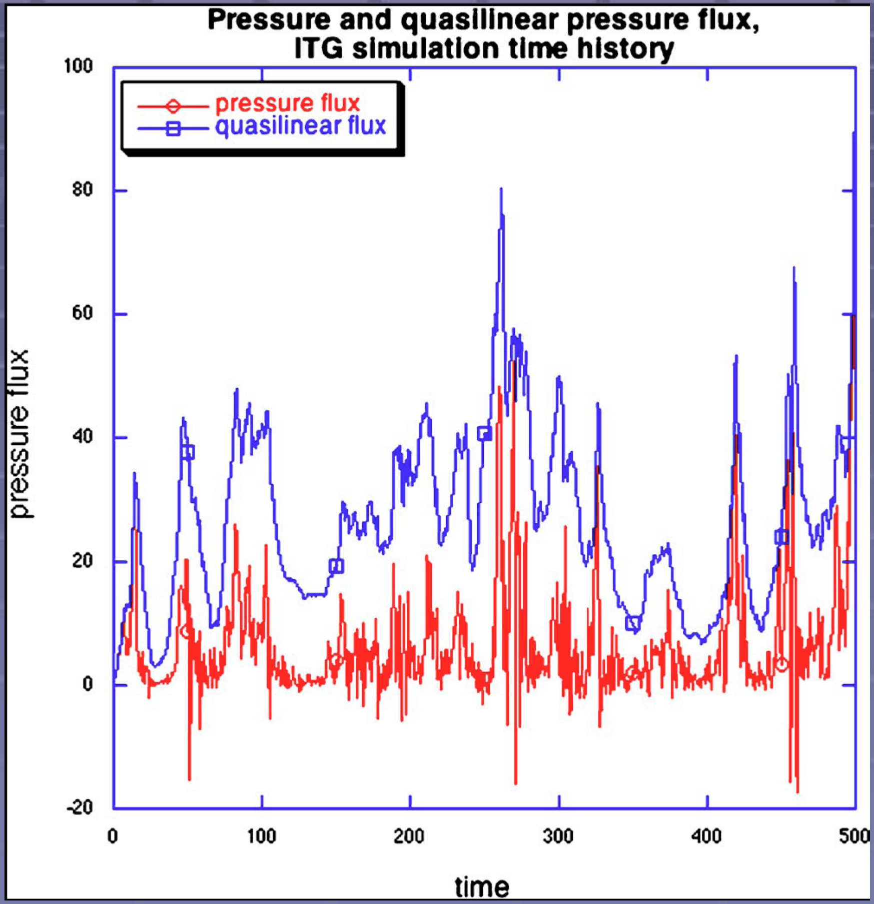
$$Q = \sum_k k_y \hat{\eta}_e \,\mathrm{Im}\{|\beta_1|^2 b_1^* + |\beta_2|^2 b_2^* + |\beta_3|^2 b_3^*$$
$$+ \langle \beta_1 \beta_2^* \rangle b_2^* + \langle \beta_1^* \beta_2 \rangle b_1^*$$
$$+ \langle \beta_1 \beta_3^* \rangle b_3^* + \langle \beta_1^* \beta_3 \rangle b_1^* + \langle \beta_2 \beta_3^* \rangle b_3^* + \langle \beta_2^* \beta_3 \rangle b_2^* \} .$$

Heat Flux in eigenmode basis.

b_3 (coefficient of damped branch amplitude) is negative

→ Damped modes contribute an inward flux component.

Reduced Model - Heat Flux



Quasilinear Heat Flux accounts only for contribution from fastest growing mode.

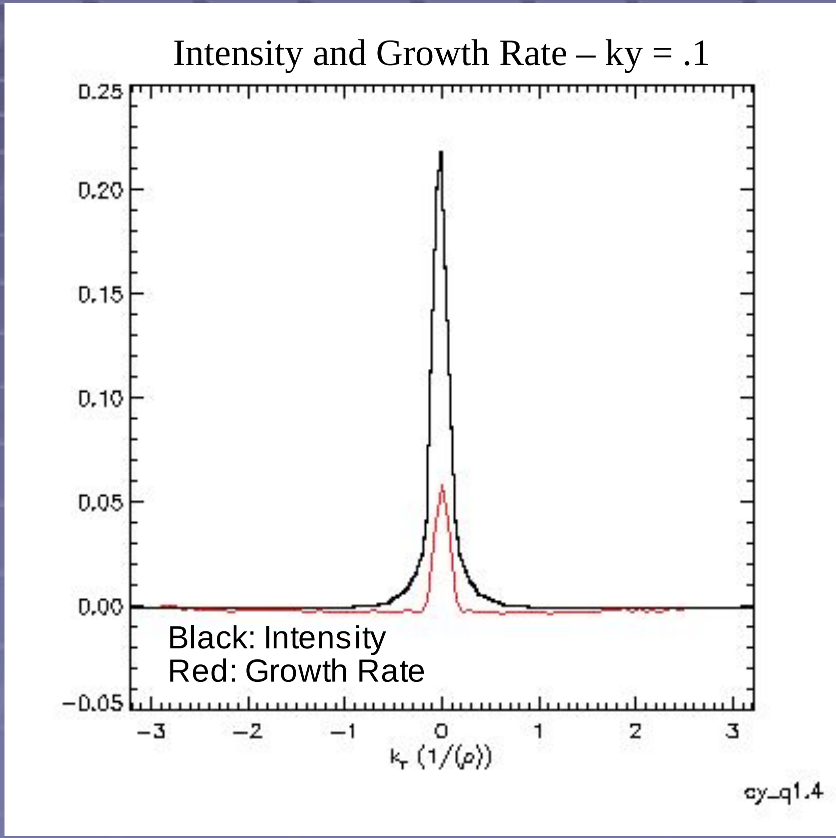
Inward flux from damped modes reduces true value in comparison to quasilinear value.

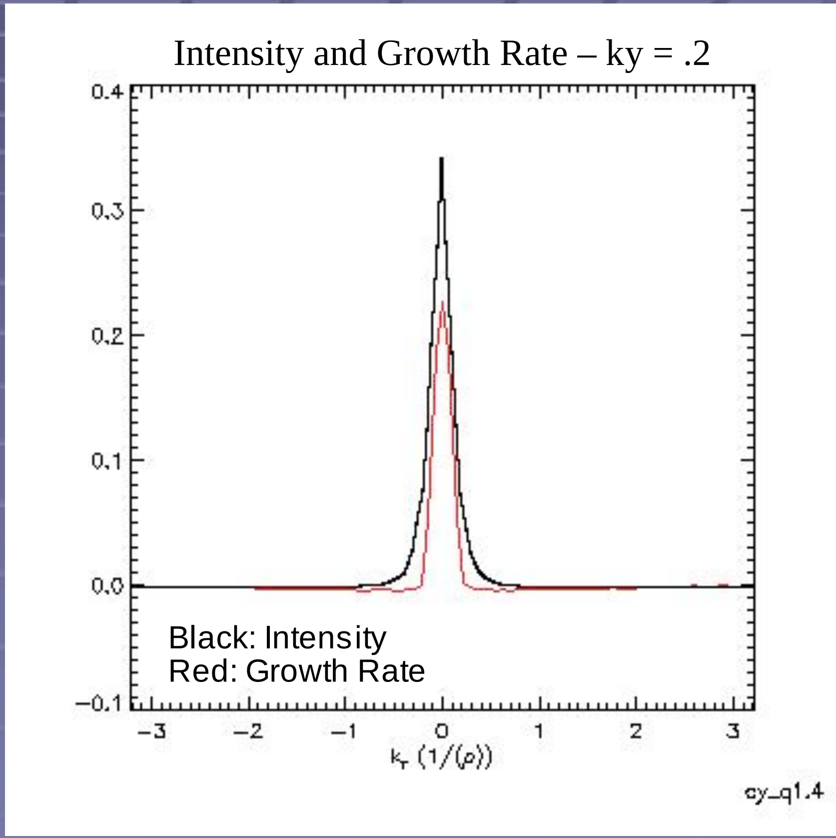
Reduction from quasilinear flux implies excitation of damped modes.

Damped Eigenmodes in Gyrokinetic Simulations

- Is damped eigenmode excitation necessary to account for saturation?
- How do damped eigenmodes affect flux?
  - Quasilinear vs. true nonlinear flux
- Deviation from phase angle defined by unstable eigenmode.
- Frequencies of damped eigenmodes seen in the nonlinear spectrum.

Gyrokinetics: Growth Rates and Intensity: k_x



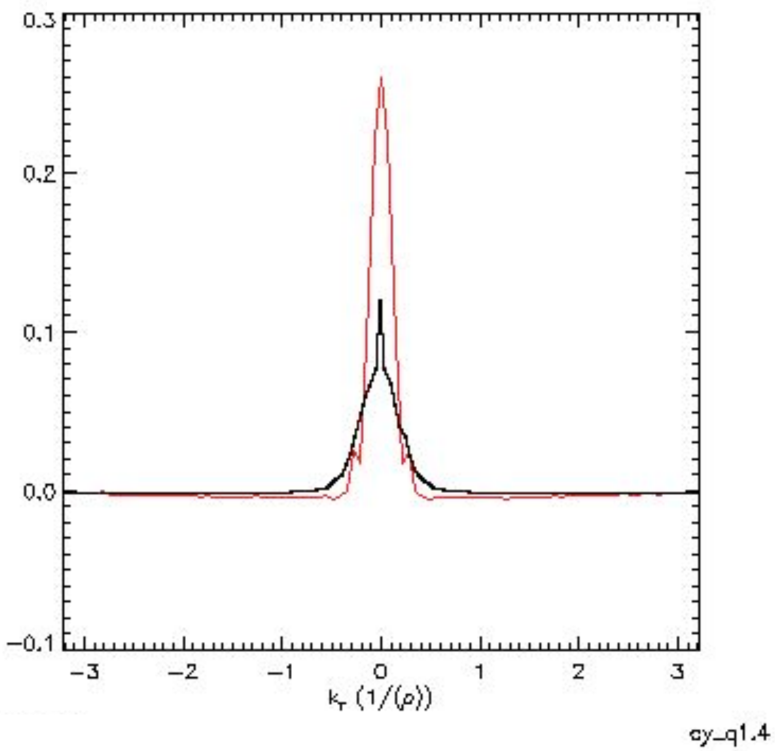


If only the unstable mode affects dynamics, fluctuation amplitude must extend past the region of instability in order to dissipate energy and achieve saturation.

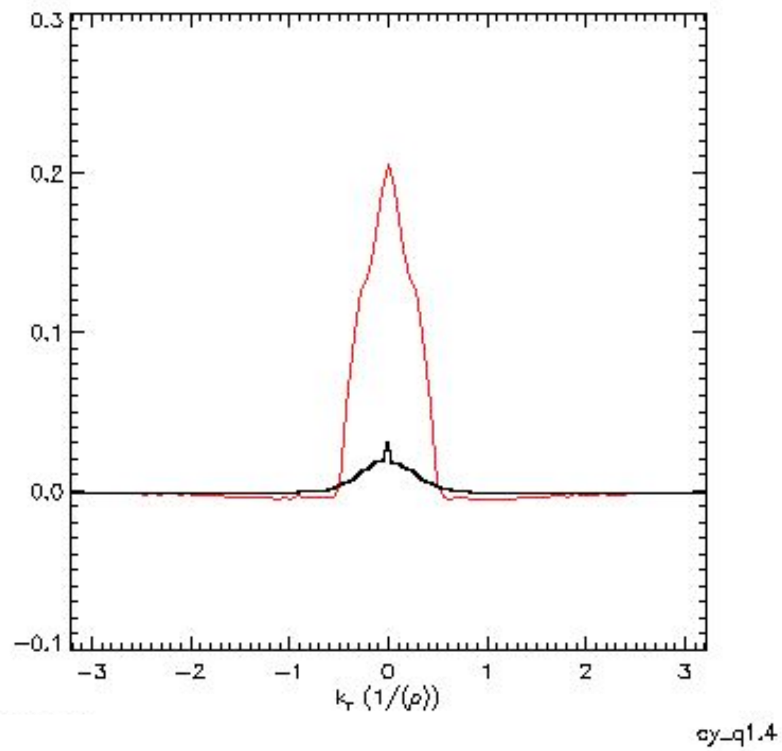
Fluctuation amplitude is small beyond the instability region.

Gyrokinetics: Growth Rates and Intensity: k_x

Intensity and Growth Rate – ky = . 3

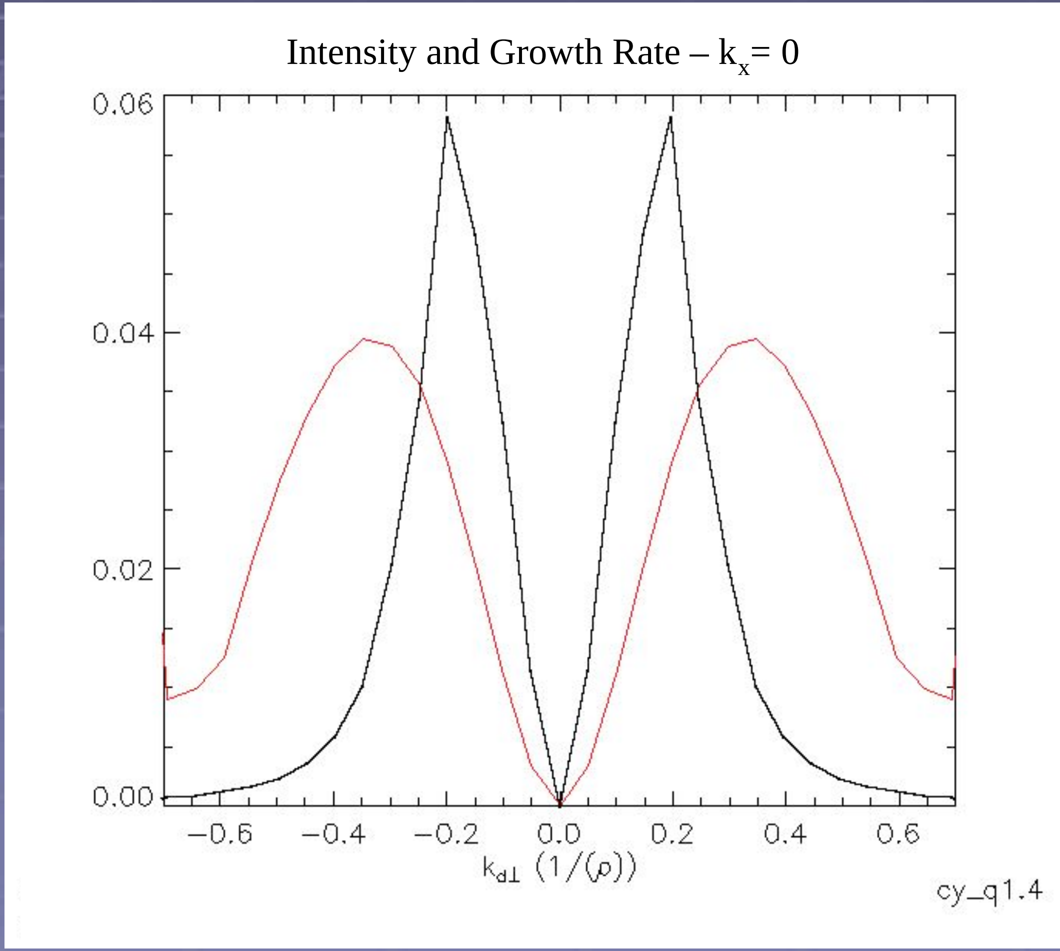


Intensity and Growth Rate – ky = .4



In all cases fluctuation amplitude is very small beyond the instability region.

Gyrokinetics: Growth Rates and Intensity: k_y



Spectrum peaks at lower k_y than linear growth rate.

Damped modes dissipating Energy at k_y=.35?

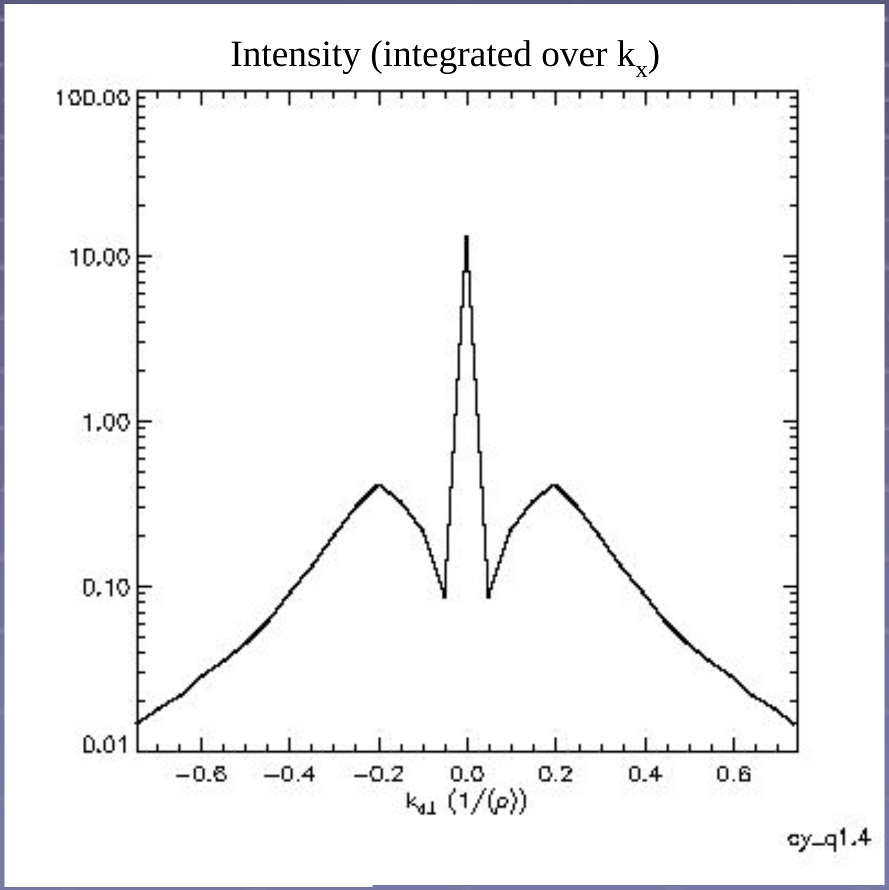
Or

Efficient nonlinear energy transfer?

Nonlinear growth rate diagnostic would answer the question.

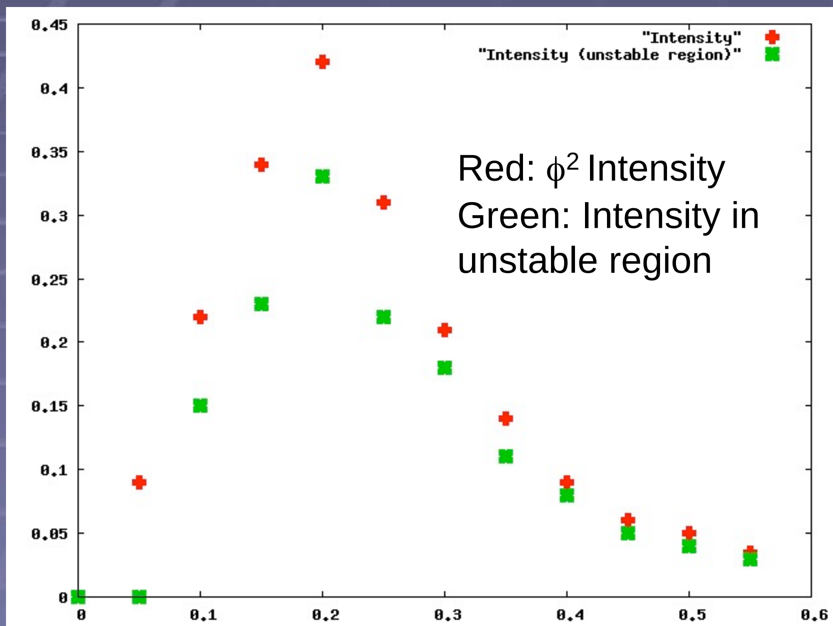
Again, fluctuation amplitude is contained in the region of instability.

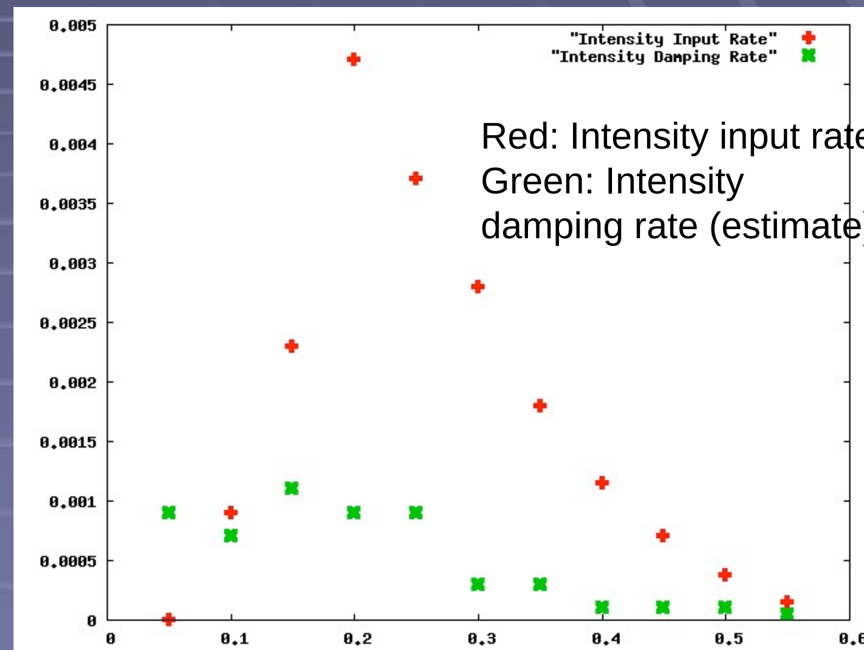
Gyrokinetics: Energy Balance



Intensity spectrum dominated by zonal modes which are not linearly damped with the exception of GAMs.

Gyrokinetics: Energy Balance





Very little intensity extends into regions of linear damping.

Intensity input rate much larger than damping rates at all k_y.

Gyrokinetics: Energy Balance

Intensity Balance:

Total intensity input rate (assuming only unstable mode)

$.01857\ (\rho/L_T\ \ T/e)^{2}(v_{Ti}/L_T)$

Total high k damping rate

-.00545 (estimate)

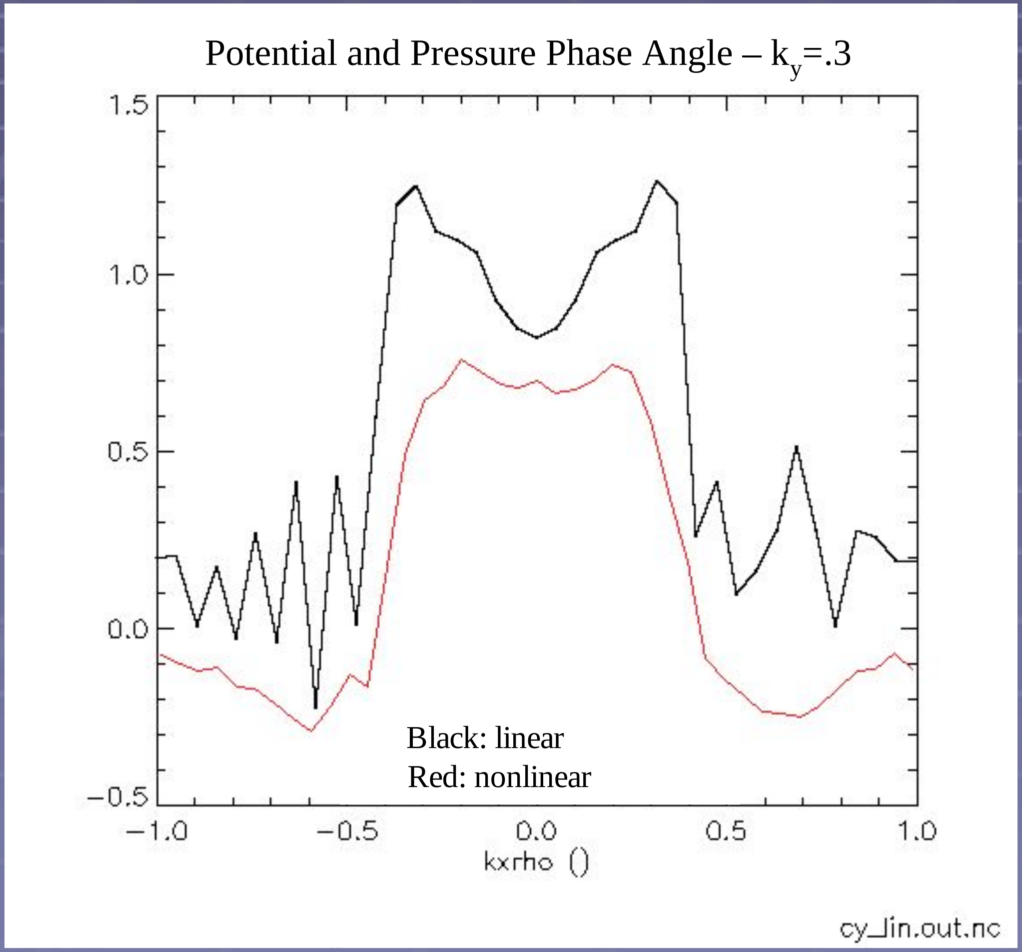
GAM contribution

What's left?:

Damping on parallel modes

Damping from excitation of damped eigenmodes

Phase Angles Deviate

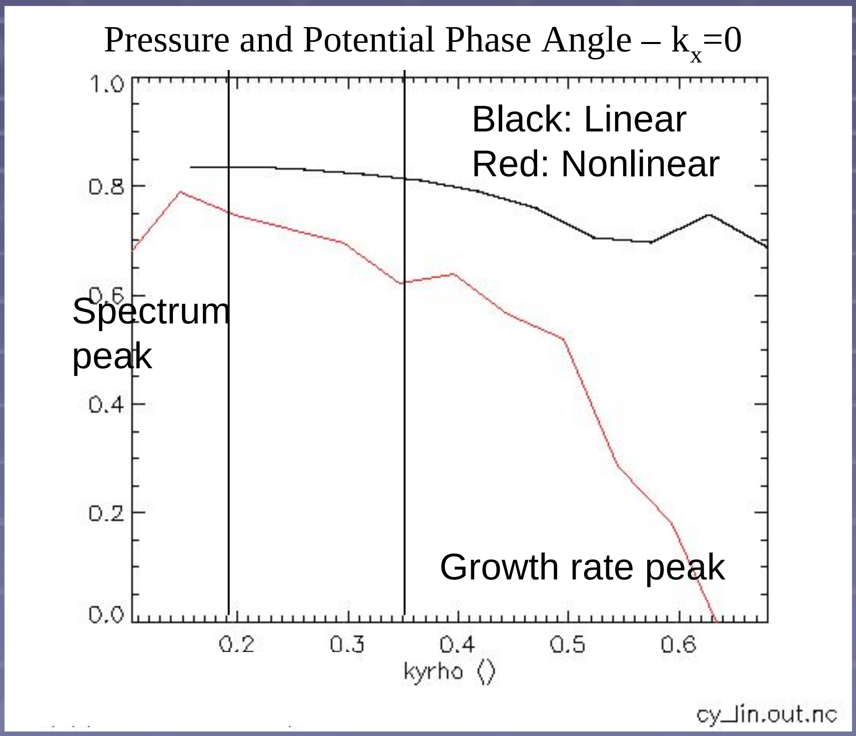


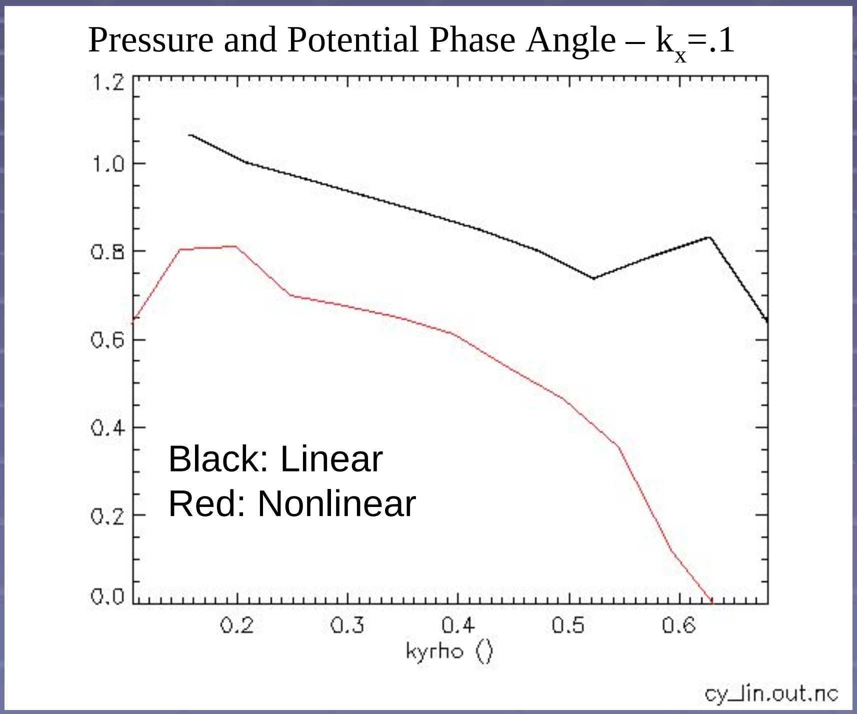
Phase angle is defined by linear eigenmode.

Deviation ➔ Superposition of unstable and damped modes

Deviation increases at finite k_x

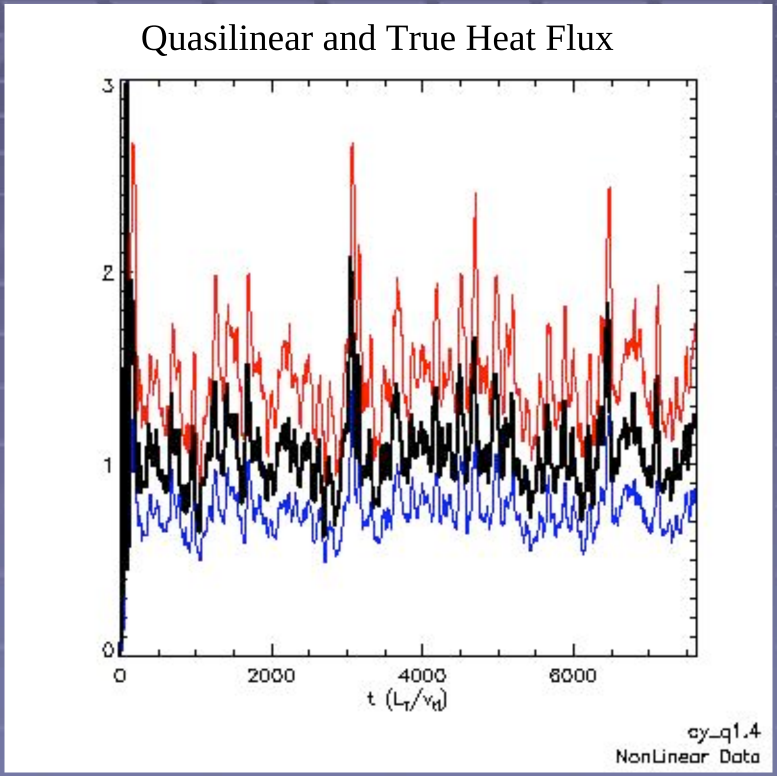
Phase Angles Deviate

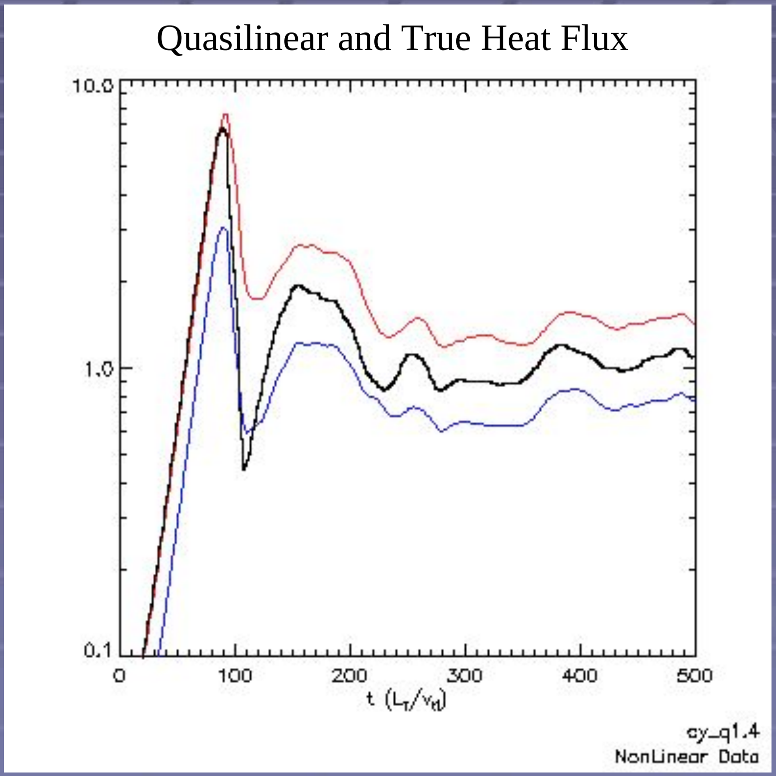




Difference in phase angle increases with k_y. Do damped eigenmodes cause shift in spectrum peak by damping energy at peak growth rate (k_y=.35)?

Quasilinear and Nonlinear Flux



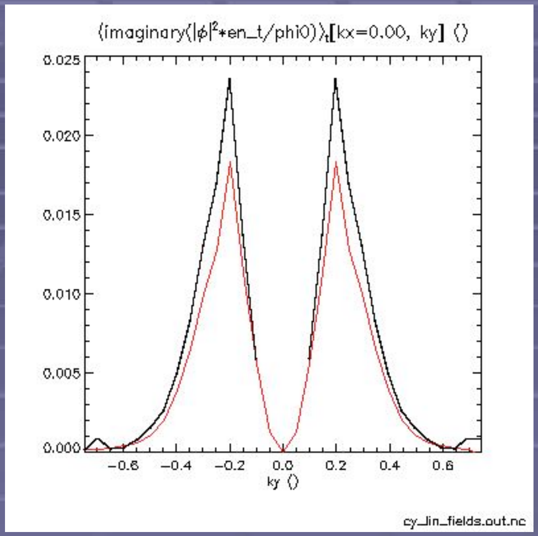


Red: Quasilinear Heat flux (midplane)
Black: Heat flux (midplane)
Blue: Heat flux (flux surface averaged)

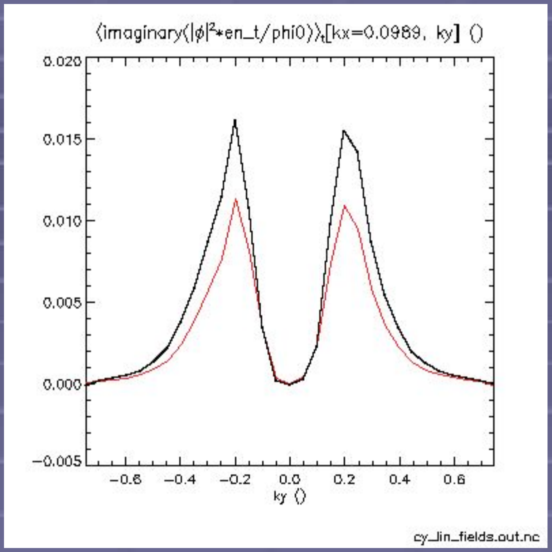
Quasilinear flux tracks true flux in linear regime.

Quasilinear estimate larger than true flux ~ 36%.

Quasilinear and Nonlinear Flux

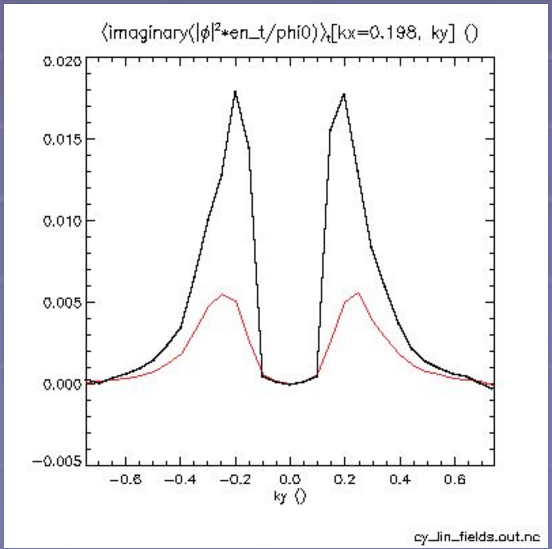


k_x=0

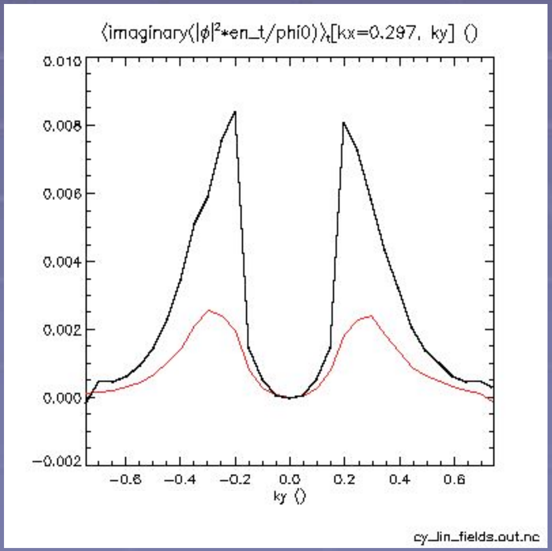


k_x=0.1

k_x=0.2



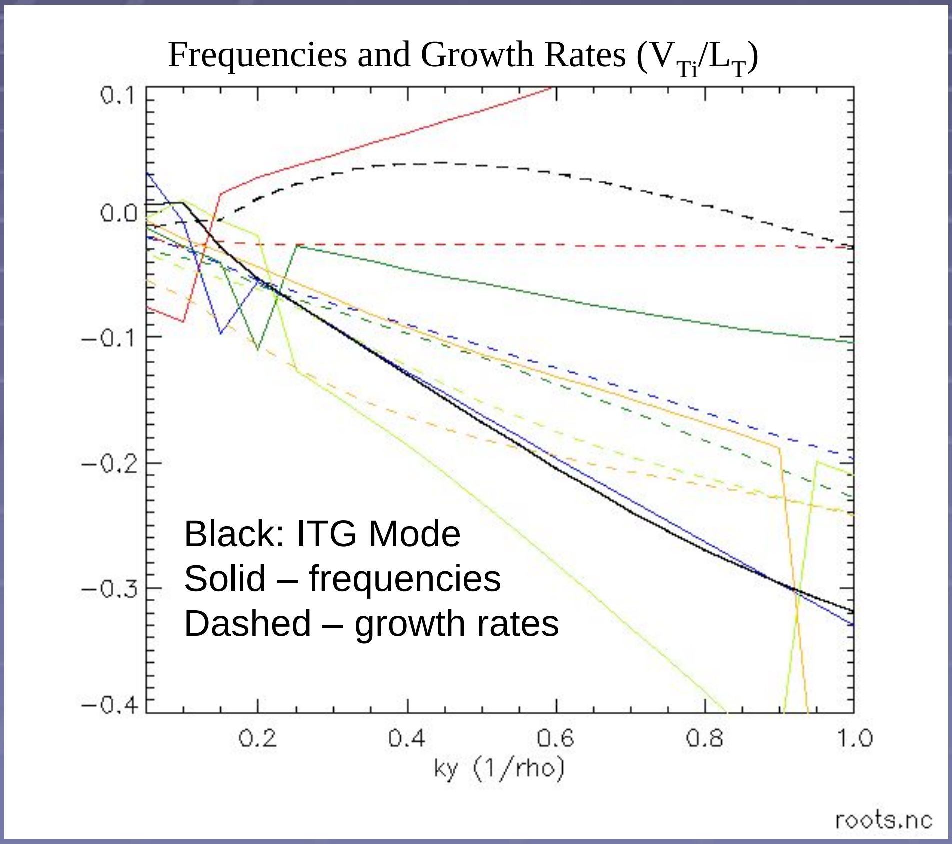
k_x=0.3

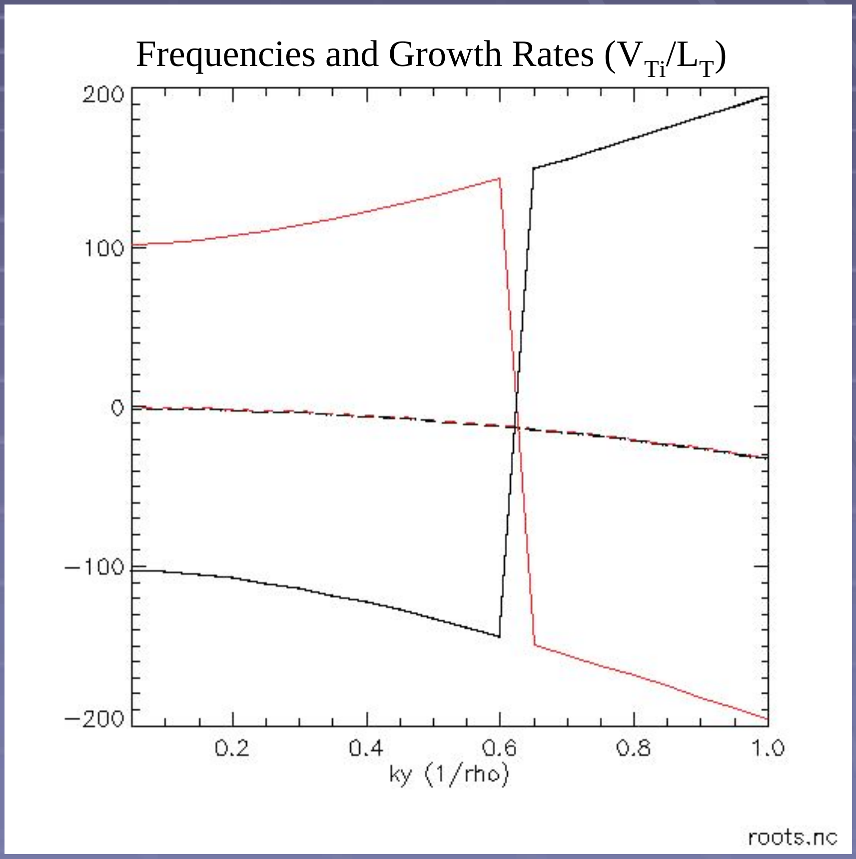


Quasilinear estimate worsens with increasing k_x.

GLF23 Frequencies and GR's

GLF23 – Transport model based on eight field gyro-landau fluid model. Eigenvalue solver – eight modes.

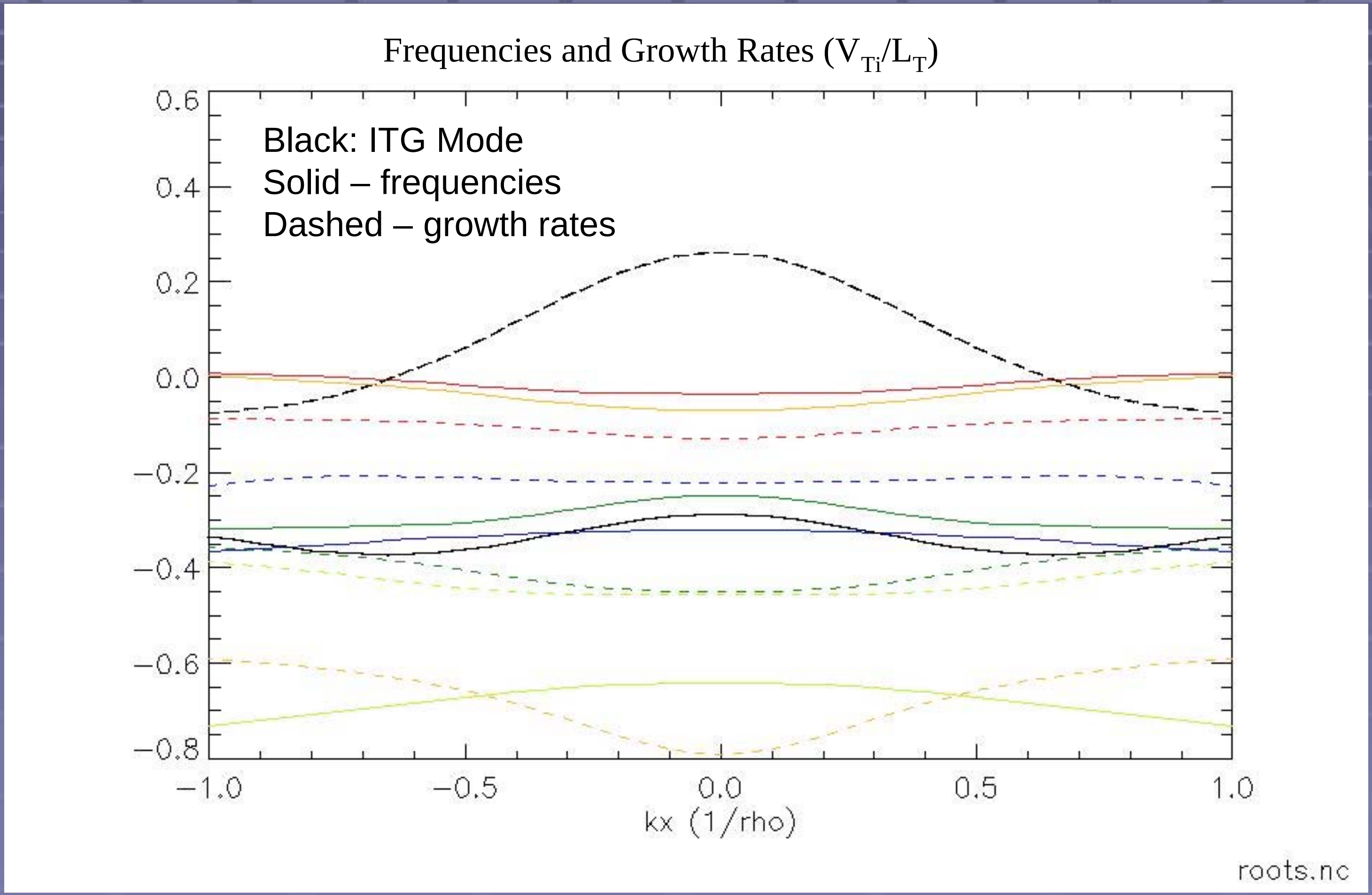




One unstable mode (ITG) and five other weakly damped modes with frequencies similar to ITG.

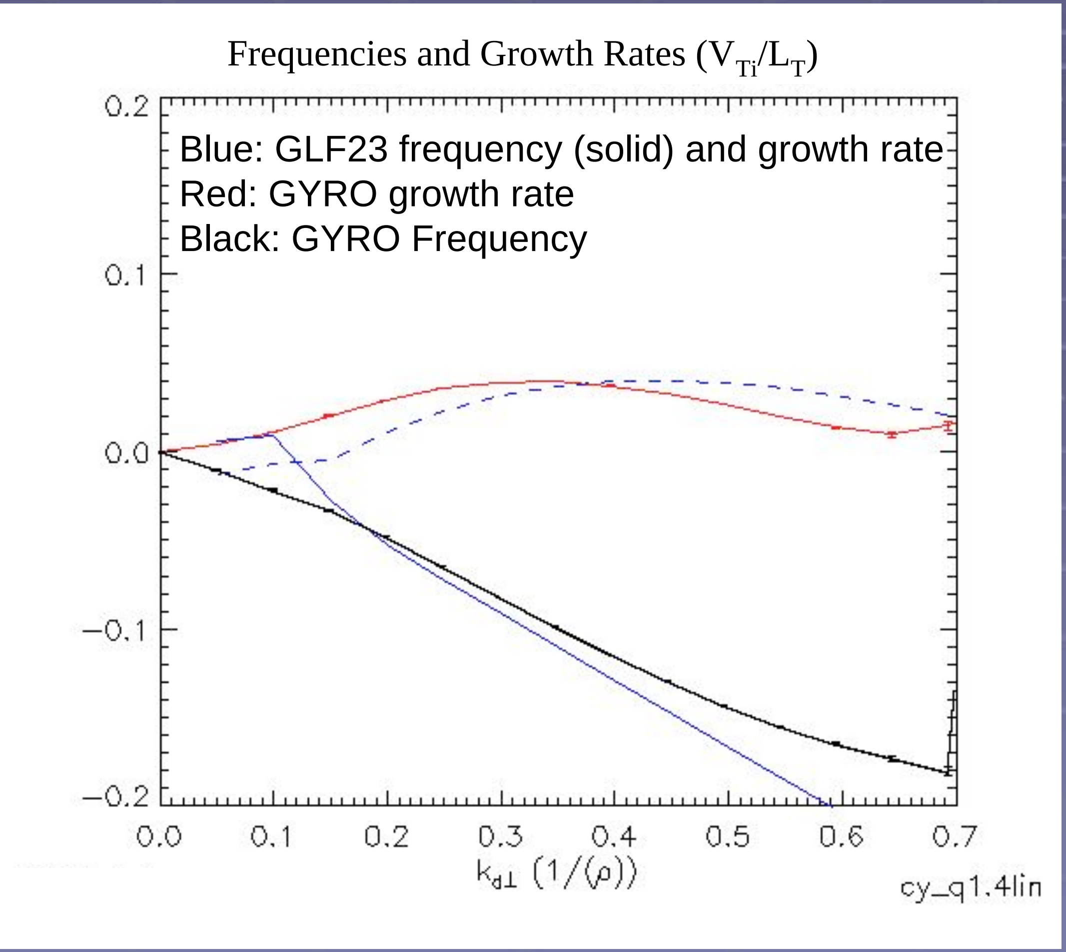
Two modes are very high frequency and strongly damped – probably unimportant in dynamics.

GLF23 Frequencies and GR's



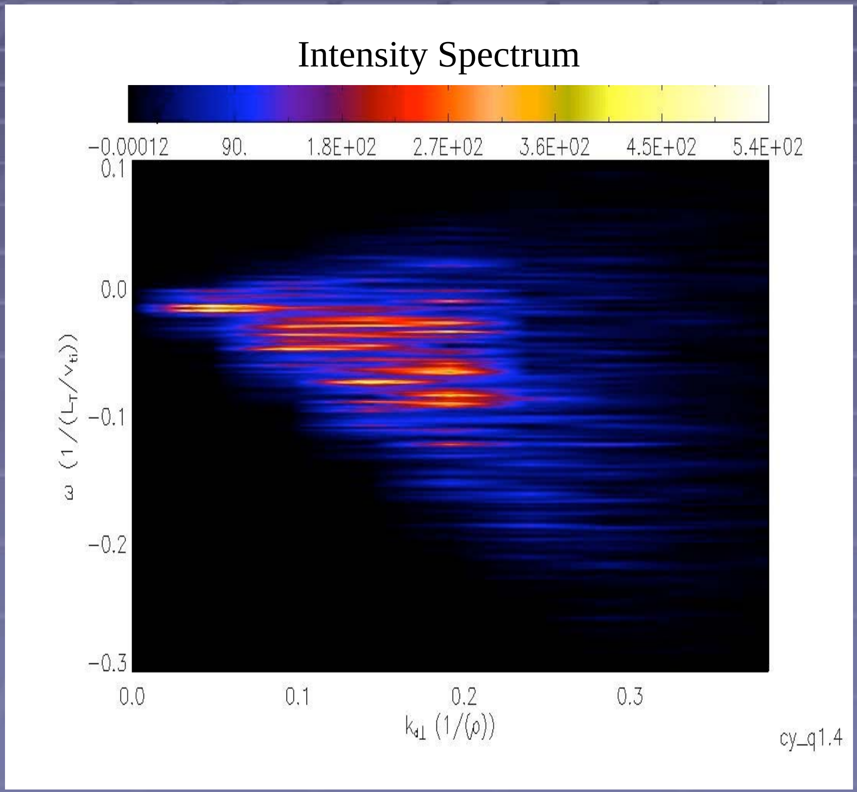
The six modes at ky = .3

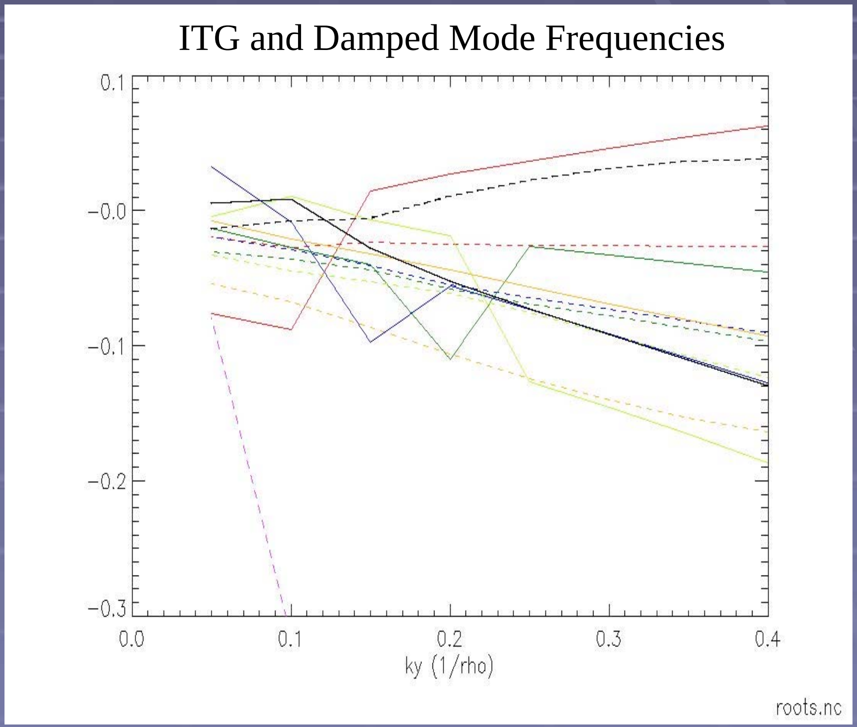
GLF23, GYRO Comparison



Frequencies and growth rate are close but not perfect. (GLF23 input file could be improved) Good enough for qualitative comparisons.

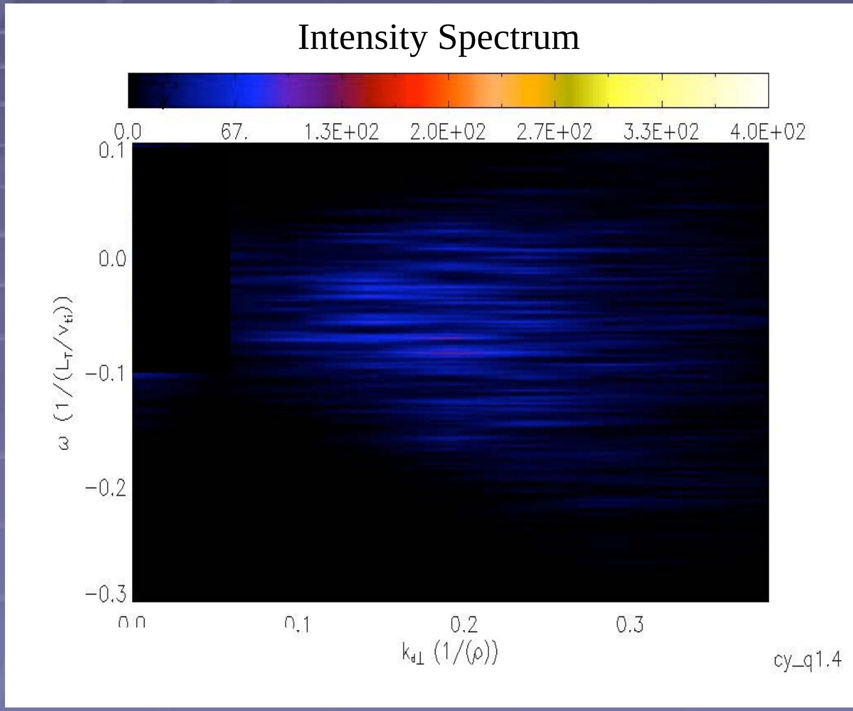
Spectrum and Frequency Comparisons k_x=0

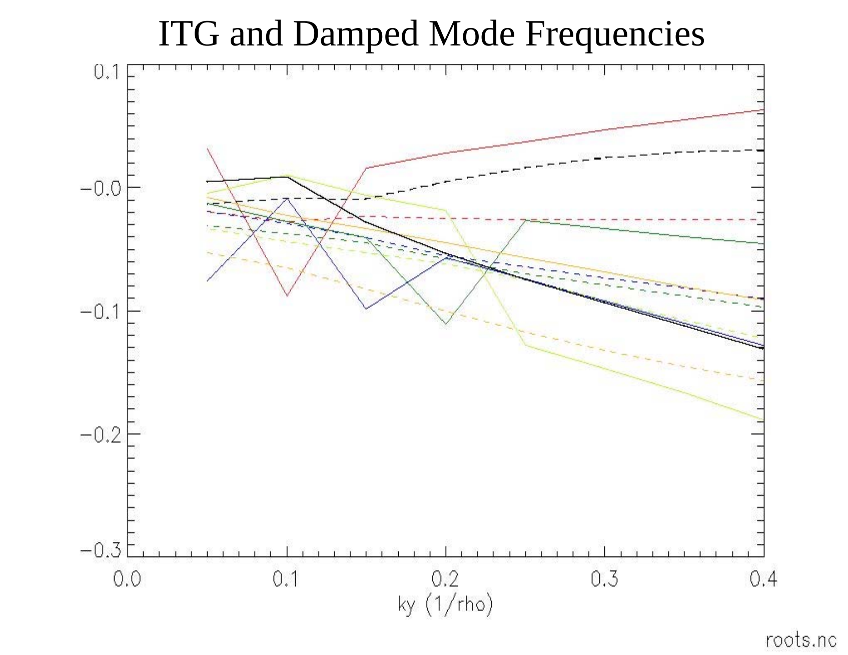




Spread in spectrum matches very closely the spread in mode frequencies. All five damped mode frequencies are within the nonlinear spectrum and are in a sense representative of the spectrum, spreading as k_y increases.

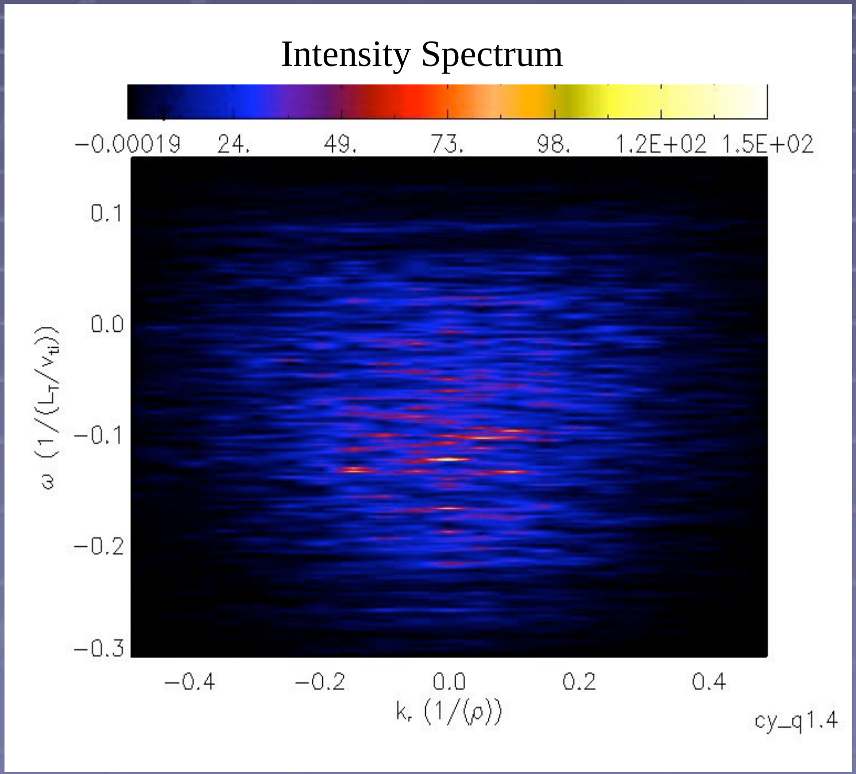
Spectrum and Frequency Comparisons k_x=.2

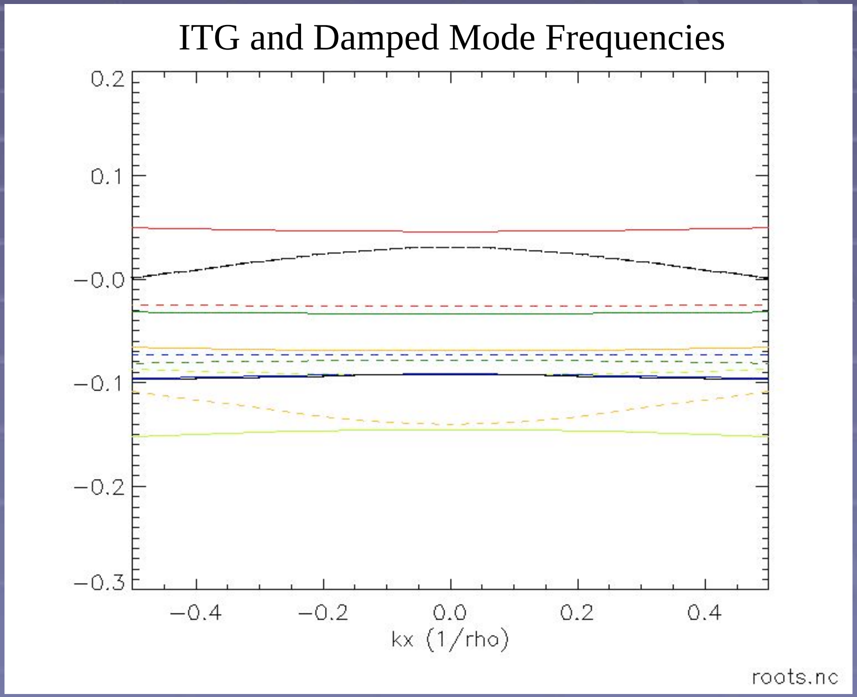




Spread in spectrum matches very closely the spread in mode frequencies.

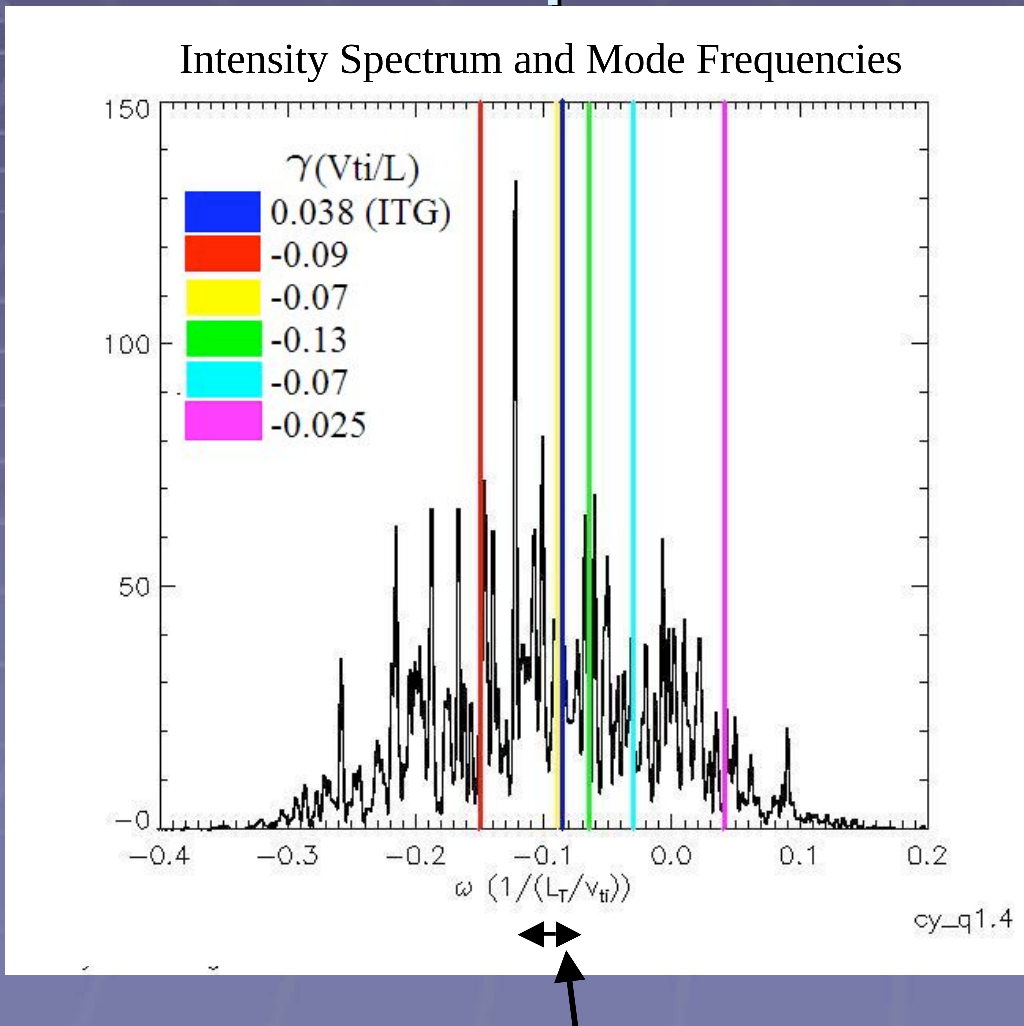
Spectrum and Frequency Comparisons k_y=.3





Spread in spectrum matches very closely the spread in mode frequencies.

Spectrum and Frequency Comparisons k_x=0, k_y=.3



All five damped mode frequencies fall in the spread of the nonlinear spectrum.

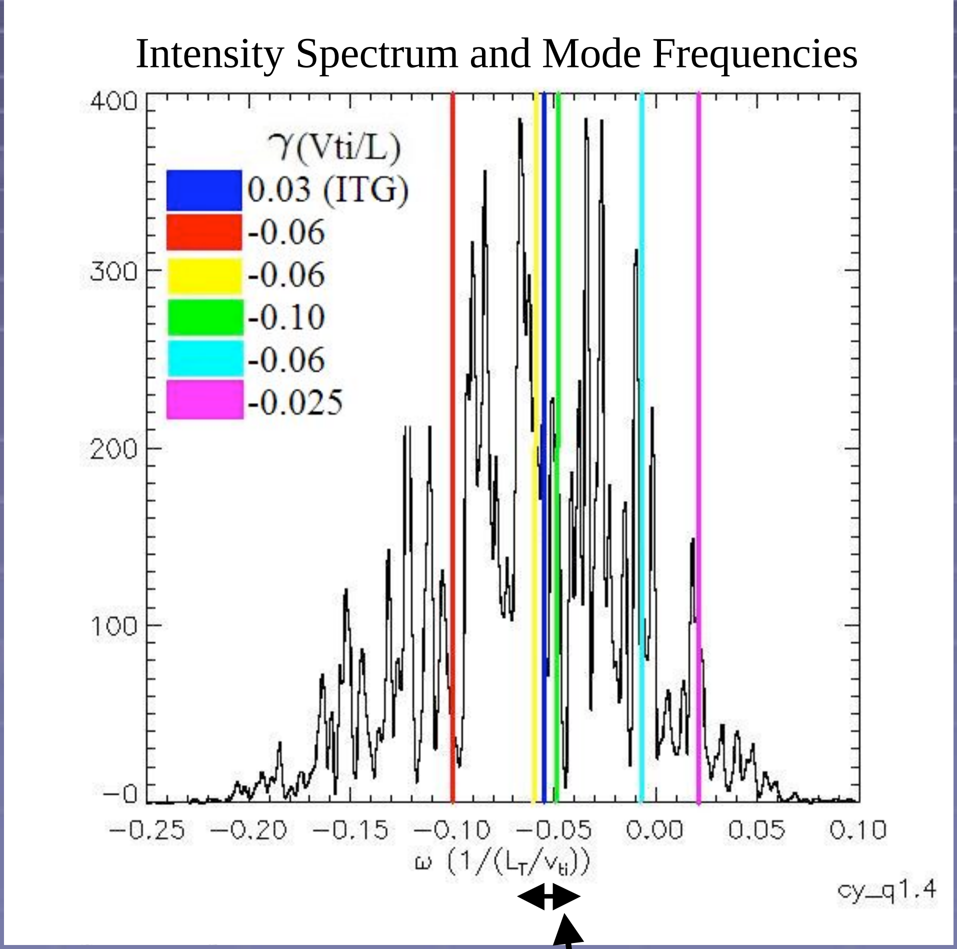
Vertical lines are the mode frequencies. The ITG mode frequency taken from linear GYRO data, the other five frequencies taken from GLF23.

Growth rates of each mode shown on left.

Width of the spectrum is due to frequencies of excited damped eigenmodes.

Expected: $\Delta\omega \sim \gamma \sim 0.04$

Spectrum and Frequency Comparisons k_x=0, k_y=.2

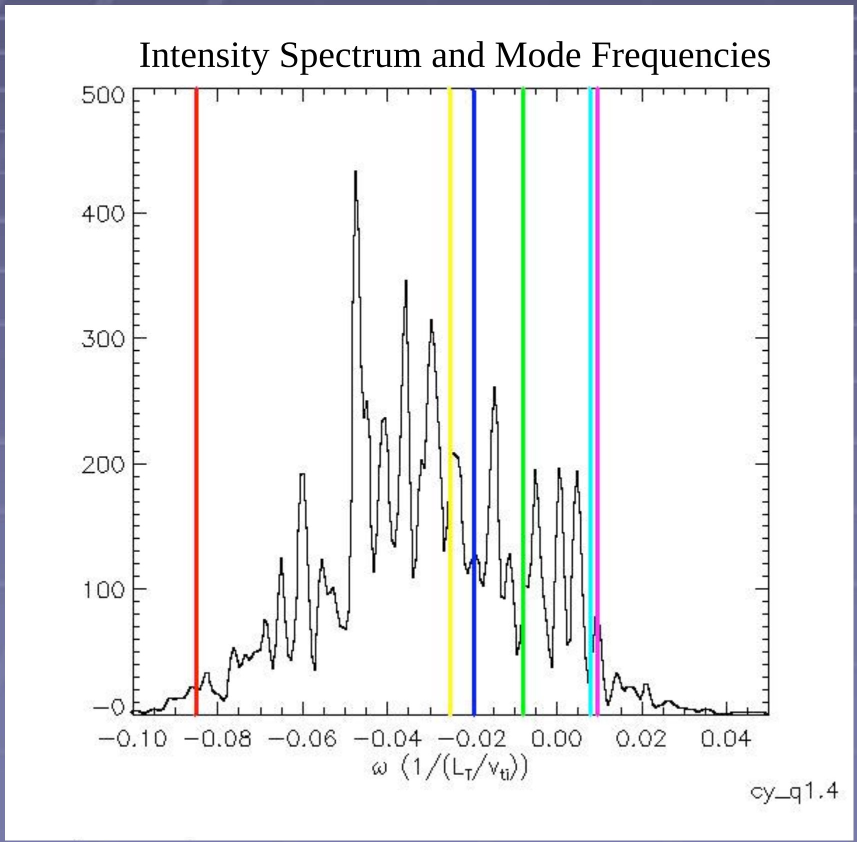


All five damped mode frequencies fall in the spread of the nonlinear spectrum.

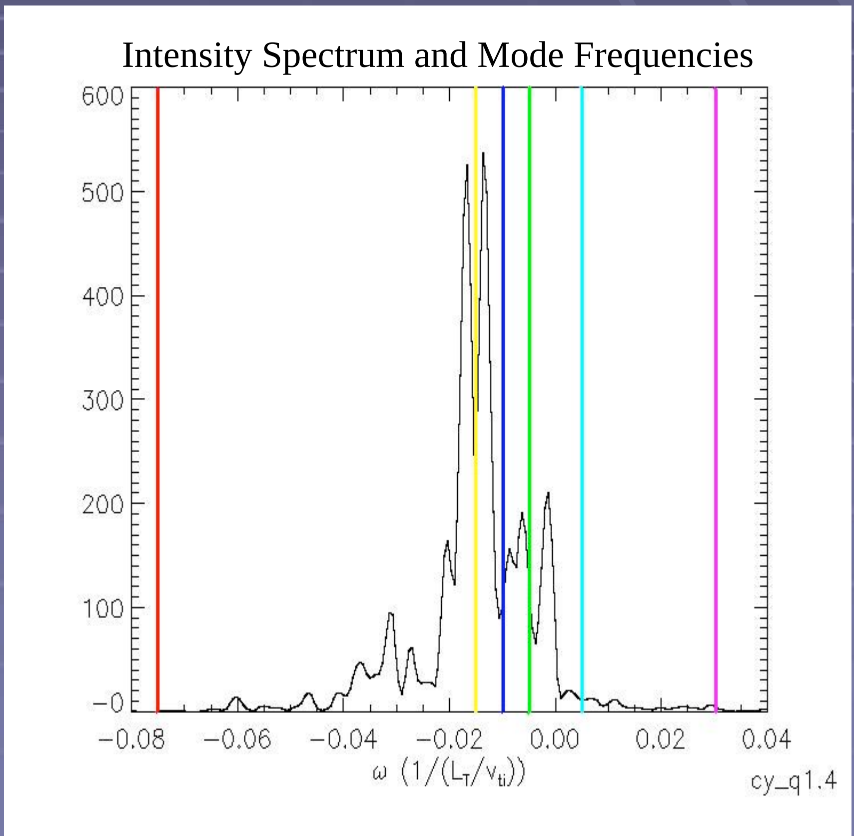
Width of the spectrum is due to frequencies of excited damped eigenmodes.

Expected: $\Delta\omega \sim \gamma \sim 0.03$

Spectrum and Frequency Comparisons k_x=0, low k_y



k_x=0, k_y=0.1



k_x=0, k_y=0.05

Conclusions

For GYRO simulation of cyclone ITG:

Difficult to account for saturation without invoking damped eigenmodes.

Deviation from ITG phase angle for phi/pressure.

Damped eigenmode excitation reduces heat flux – quasilinear overestimates true flux.

Damped mode frequencies appear in nonlinear spectrum, accounting for large $\Delta\omega$.

Future Work

Derive gyrokinetic nonlinear growth rate and implement diagnostic.

Examine CTEM for damped eigenmode excitation.

Expand reduced model to 3-D nonlocal to understand damped eigenmodes and relation to parallel mode structure.

Examine damped eigenmodes in gyrofluid models.

Future Work - Nonlinear Growth Rate