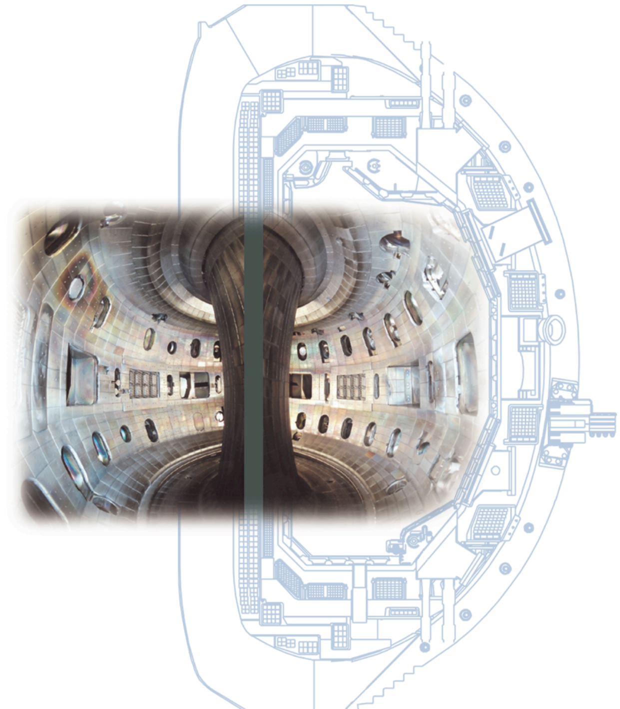
Poloidal Velocity of Impurity Ions in Neoclassical Theory

by
S.K.Wong, V.S.Chan, W.M. Solomon



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Background

- Recent measurement of C^{+6} poloidal velocity disagrees with NCLASS in magnitude and direction
- Toroidal velocity of C^{+6} is close to its thermal velocity, violating the assumption of NCLASS
- Theoretical uncertainty can arise from different calculations in neoclassical theory

Goals

- Calculate analytically poloidal velocities in a two-ion species plasma with large toroidal flows in the banana regime, with boundary layer correction
- Implement the moment approach of Hirshman-Sigmar for a two-ion species plasma
- Compare analytic theory, Hinton-Moore's numerical work, and the moment approach
- Compare with measurements in DIII-D

Large Flow Equilibrium

- **Circular, large aspect ratio flux surfaces** $(\delta = r/R << 1)$ $B_\theta/B << 1$

- **To leading order in gyro-radius/scale length expansion, ions share common toroidal (parallel) flow, rigid on each flux surface, related to radial electric field** $U = E_r/B_\theta$

- **Temperatures are flux functions. Centrifugal force leads to poloidal variations of electric potential and densities:**

$$\frac{e\tilde{\phi}_1}{T_e} = \delta X_e \cos\theta \qquad \frac{\tilde{n}_a}{n_a} = \delta X_a \cos\theta \qquad X_e = \frac{\sum_a Z_a m_a n_a U^2}{n_e T + \sum_a Z_a^2 n_a T_e} \qquad X_a = \frac{m_a U^2}{T} - \frac{Z_a T_e}{T} X_e$$

- **Perpendicular flow is sum of ExB and diamagnetic flows:**

$$u_{a\perp} = -\frac{E_r}{B_\zeta} + \frac{T}{Z_a e B_\zeta} \frac{p_a'}{p_a}$$

Linearized Drift Kinetic Equation

- First order parallel flow is obtained from LDKE
- Written in frame rotating with a flux surface

- Guiding center drifts: ExB, gradB/curvature, inertial forces

- Total energy: kinetic, electrostatic, and centrifugal

- Two classes of particles:

Slow: $\xi = v_{\|} / v \sim \sqrt{\delta}$

Freely circulating: $\xi \sim 1$

Localization Approximation

- Introduce shifted distribution function where $u_\parallel$ is common to all ions $$f_{a1} = \frac{m_a v_\parallel u_\parallel}{T} f_{a0} + \frac{Z_a e \phi_1}{T} f_{a0} + f'_{a1}$$
- Assume:
  freely circulating ions: $f'_{a1} \sim \sqrt{\delta}$ slow ions: $\frac{\partial f'_{a1}}{\partial \xi} \sim 1 \qquad \frac{\partial^2 f'_{a1}}{\partial \xi^2} \sim \frac{1}{\sqrt{\delta}}$
- LDKE for slow ions is simplified:
  G.C. drifts: ExB, grad B, centrifugal force
  Conservation of total energy ε_a leads to $\xi^2 = 1 - \mu B_0 / \varepsilon_a + \delta_a \cos\theta$

  where $\delta_a = \delta\left(1 + \frac{2TX_a}{m_a v^2}\right)$ is effective trapping factor
- Fokker-Planck operator can be replaced by $\frac{\nu_a}{2}\frac{\partial^2 f'_{a1}}{\partial \xi^2}$ LDKE is

$$v_\parallel \frac{\Theta}{r}\left(\frac{\partial f'_{a1}}{\partial \theta}\right)_{\varepsilon_a,\mu} - \frac{\nu_a}{2}\left(\frac{\partial^2 f'_{a1}}{\partial \xi^2}\right)_v = \delta_a \sin\theta \frac{m_a v^2}{2T}\left[u_\parallel \Theta + \frac{T}{Z_a e B_\xi}\left(\frac{n'_a}{n_a} + \overline{\frac{m_a v^2}{2T} - \frac{3}{2}}\frac{T'}{T}\right)\right] f_{a0}$$

Banana Regime

- **Introduce scaled variables** $\xi_* = \xi/\sqrt{\delta_a}$ **and scaled distribution function** g_a

$$f'_a = \sqrt{\delta_a} g_a \frac{m_a v}{T}\left[u_\parallel + \mathrm{sgn}(B_\xi)\frac{T}{Z_a e B_\theta}\left(\frac{n'_a}{n_a} + \overline{\frac{m_a v^2}{2T} - \frac{3}{2}}\frac{T'}{T}\right)\right] f_{a0}$$

It follows: $\xi_* \frac{\partial g_a}{\partial \theta} - \frac{\sin\theta}{2}\frac{\partial g_a}{\partial \xi_*} - \frac{\nu_{a*}}{2}\frac{\partial^2 g_a}{\partial \xi_*^2} = \frac{\sin\theta}{2}$ **where** $\nu_{a*} = \nu_a (\Theta v/r)^{-1}\delta_a^{-3/2}$

- **Solution in banana regime has asymptotic behavior**

$$g_a \cong \mp 0.98\left(1 - 0.92\sqrt{|\nu_{a*}|}\right) \qquad \xi_* \to \pm\infty$$

- **Particle flux can be calculated by flux-friction relation**

$$Z_a e B_\theta \Gamma_a = \mathrm{sgn}(B_\xi)\int \frac{d\theta}{2\pi}\int d^3 v m_a v_\parallel C_a(f_1)$$

- **Integral is dominated by slow ions and can be evaluated using the asymptotic behavior**

Ambipolarity and Common Parallel Flow

- **The main ion flux is (similarly for impurity ions)**

$$-Z_i e B_\theta \Gamma_i = 2.94\sqrt{\delta}\,\frac{m_i^{3/4} m_I^{1/2} n_i}{\tau_{iI}}\left\{k_{i0}\left[\mathrm{sgn}(B_\xi)u_\parallel + \frac{T}{Z_a e B_\theta}\left(\frac{n_a'}{n_a} - \frac{3}{2}\frac{T'}{T}\right)\right] + k_{i1}\frac{T}{Z_a e B_\theta}\frac{T'}{T}\right\}$$

$$k_{in} = \frac{1}{\alpha\sqrt{\rho}}\int_0^\infty dx\,\frac{e^{-x^2}x^{2n+1}\left[G(x)+\alpha G'(\rho x)\right]\left(1+X_i/x^2\right)^{1/2}}{1+0.89\sqrt{\bar{\nu}_{*i}}\left\{\left[G(x)+\alpha G'(\rho x)\right]\Big/\left[x^4\left(1+X_i/x^2\right)^{3/2}\right]\right\}^{1/2}}$$

$$G'(x) = (1-0.5/x^2)\left(2/\sqrt{\pi}\right)\int_0^x e^{-y^2}dy + e^{-x^2}/\sqrt{\pi}\,x \qquad \rho = \sqrt{m_I/m_i} \qquad \alpha = Z_I^2 n_I / Z_i^2 n_i$$

- **The common flow $u_\parallel$ is obtained from ambipolarity, with the result**

$$Z_i\Gamma_i + Z_I\Gamma_I = 0$$

$$u_\parallel = -\frac{T}{eB_\theta}\frac{\mathrm{sgn}(B_\xi)}{k_{i0}+k_{I0}}\left[\frac{k_{i0}}{Z_i}\left(\frac{n_i'}{n_i}-\frac{3}{2}\frac{T'}{T}\right)+\frac{k_{I0}}{Z_I}\left(\frac{n_I'}{n_I}-\frac{3}{2}\frac{T'}{T}\right)+\left(\frac{k_{i1}}{Z_i}+\frac{k_{I1}}{Z_I}\right)\frac{T'}{T}\right]$$

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Impurity Ion Poloidal Velocity

- **Combining parallel and perpendicular flows yields poloidal velocity of impurity ions:**

$$u_{I\theta} = \frac{T}{eB_\xi}\left[c_1\left(\frac{1}{Z_i}\frac{n_i'}{n_i} - \frac{1}{Z_I}\frac{n_I'}{n_I}\right) + \left(\frac{c_2}{Z_i} + \frac{c_3}{Z_I}\right)\frac{T'}{T}\right]$$

$$c_1 = -k_{i0}/(k_{i0} + k_{I0}) \qquad c_2 = -(k_{i1} - 3k_{i0}/2)/(k_{i0} + k_{I0}) \qquad c_3 = 1 - (k_{I1} - 3k_{I0}/2)/(k_{i0} + k_{I0})$$

- **Hinton-Moore's results (weak toroidal rotation) take the same form except k_{ai} are replaced by**

$$k_{i0}^{HM} = \frac{1}{2\alpha\sqrt{\rho}}\frac{F_{11}(1) + \alpha F_{11}(\rho)}{1 + 1.26\bar{\nu}_{i*}\left[F_{11}(1) + \alpha F_{11}(\rho)\right]} \qquad k_{i1}^{HM} = \frac{1}{2\alpha\sqrt{\rho}}\frac{F_{12}(1) + \alpha F_{12}(\rho)}{1 + 0.37\bar{\nu}_{i*}\left[F_{12}(1) + \alpha F_{12}(\rho)\right]}$$

$$F_{11}(\rho) = \frac{\sqrt{1+\rho^2}}{\rho} - \frac{1}{\rho^2}\ln\left(\rho + \sqrt{1+\rho^2}\right) \qquad F_{12}(\rho) = \frac{\rho}{\sqrt{1+\rho^2}}$$

The Moment Approach

- **The basic equations** $\left\langle \vec{B}\cdot\nabla\overleftrightarrow{\Pi}_a \right\rangle = \left\langle BF_{a0} \right\rangle \quad \left\langle \vec{B}\cdot\nabla\overleftrightarrow{Q}_a \right\rangle = \left\langle BF_{a1} \right\rangle$

  are the same as flux-friction relations when δ **is small:**

$$Z_a eB_\theta \Gamma_a = \mathrm{sgn}(B_\xi)\int \frac{d\theta}{2\pi}\int d^3v m_a v_\parallel C_a(f_1) \qquad -Z_a eB_\theta Q_a/T = \mathrm{sgn}(B_\xi)\int \frac{d\theta}{2\pi}\int d^3v m_a v_\parallel \left(m_a v^2/2T - 5/2\right) C_a(f_1)$$

- **LDKE implies relations between poloidal and parallel flows:**

$$u_{a\theta} = u_{a\parallel}\Theta - \frac{E_r}{B_\xi} + \frac{T}{Z_a eB_\xi}\frac{p_a'}{p_a} \qquad \frac{2q_{a\theta}}{5p_a} = \frac{2q_{a\parallel}}{5p_a}\Theta + \frac{T}{Z_a eB_\xi}\frac{T'}{T}$$

- **First assumption: friction integrals can be calculated using**

$$f_{a1} = \left(m_a v_\parallel /T\right)\left[u_{a\parallel} + \left(m_a v^2/2T - 5/2\right)\left(2q_{a\parallel}/5p_a\right)\right] f_{a0}$$

- **Second assumption: radial fluxes are related to poloidal flows:**

$$-Z_i eB_\theta \Gamma_i = 2f_t \frac{m_i^{3/4} m_I^{1/2} n_i}{\tau_{iI}\Theta}\left[k_{i0}^{HS} u_{i\theta} + \left(k_{i1}^{HS} - \frac{5}{2}k_{i0}^{HS}\right)\frac{2q_{i\theta}}{5p_i}\right]$$

$$-Z_i eB_\theta Q_i/T = 2f_t \frac{m_i^{3/4} m_I^{1/2} n_i}{\tau_{iI}\Theta}\left[\left(k_{i1}^{HS} - \frac{5}{2}k_{i0}^{HS}\right)u_{i\theta} + \left(k_{i2}^{HS} - 5k_{i1}^{HS} + \frac{25}{4}k_{i0}^{HS}\right)\frac{2q_{i\theta}}{5p_i}\right]$$

c_1 c_2 c_3

The Moment Approach (Continued)

- **The "viscosity coefficients" k[HS] are obtained by examining the manner of solution of the LDKE**

$$k_{in}^{HS} = \frac{1}{\sqrt{\alpha}\,\rho}\int_0^\infty dx \frac{e^{-x^2} x^{2n+1}\left[G'(x) + \alpha G'(\rho x)\right]}{D_1 D_2}$$

$$D_1 = 1 + 2.33\overline{\nu}_{i*}\frac{G'(x) + \alpha G'(\rho x)}{x^4} \qquad D_2 = 1 + 1.85\frac{qR}{\sqrt{T/m_i}\,\tau_{ii}}\,\frac{H(1,x) + \alpha H(\rho, x)}{x^4}$$

$$H(\rho,x) = \left[3 + \frac{2}{\rho^2} - \frac{3}{2(\rho x)^2}\right]\frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2} dy + \left[\frac{3}{\rho x} - 4\left(1 + \frac{1}{\rho^2}\right)\rho x\right]\frac{e^{-\rho^2 x^2}}{\sqrt{\pi}}$$

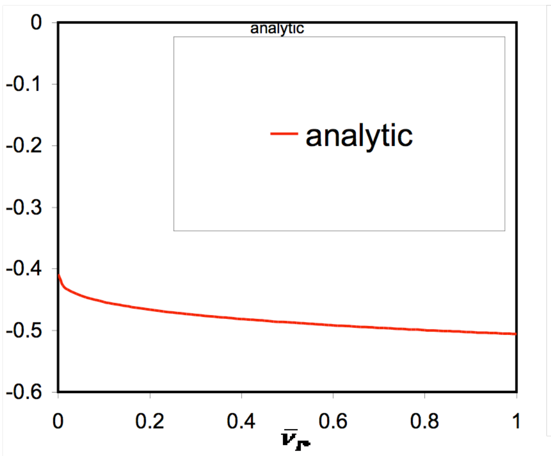
- **Solve four linear equations for** $u_{i\parallel}$, $2q_{i\parallel}/5p_i$, $u_{I\parallel}$, $2q_{I\parallel}/5p_I$

  from which c_1 c_2 c_3 **can be found**

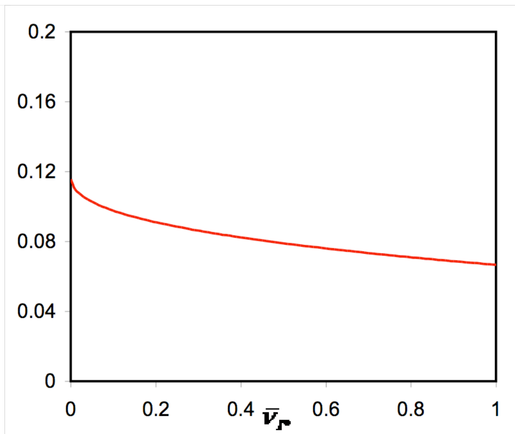
Comparing Calculations (Weak Rotation)

- **Parameters chosen:** $Z_I^2 n_I / Z_i^2 n_i = 1 \quad m_I / m_i = 6 \quad \bar{\nu}_{i*} = 0$
- **Significant differences exist**
- **Moment approach depends on** δ

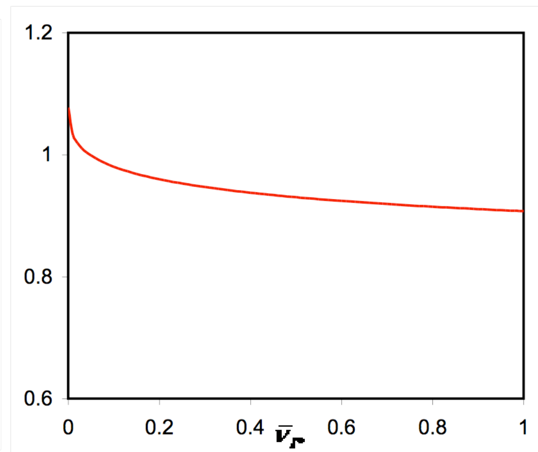
C_1



C_2



C_3



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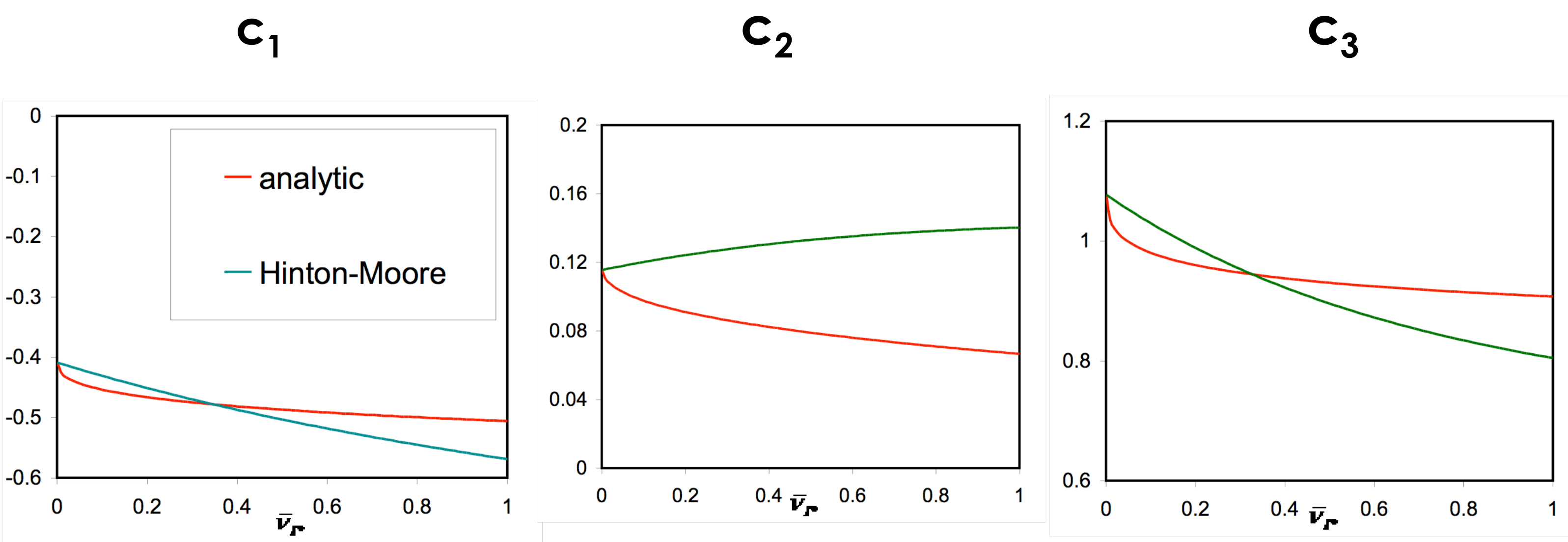

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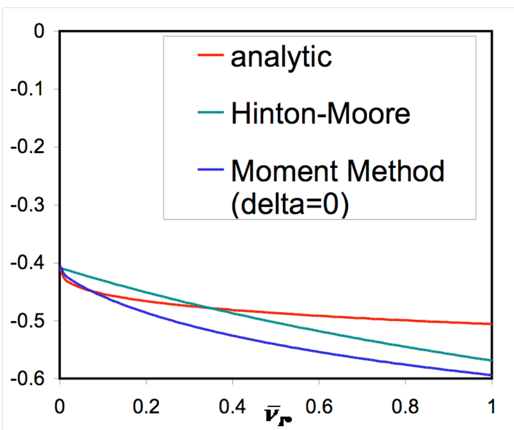
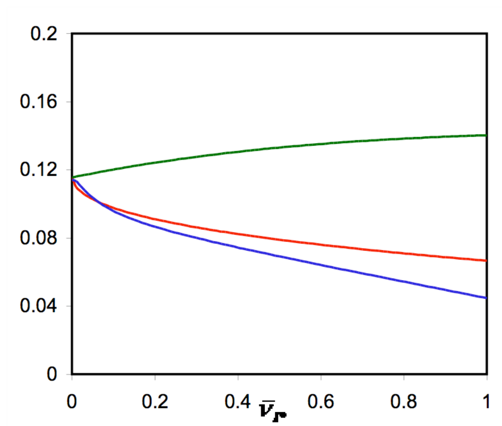
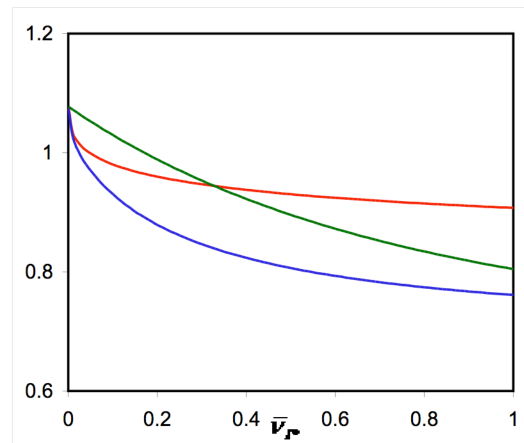


c_2



c_3



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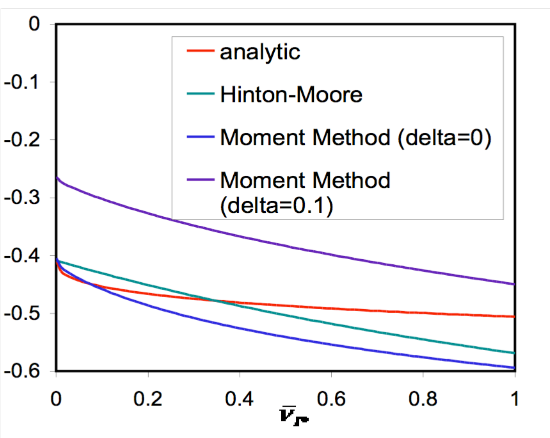
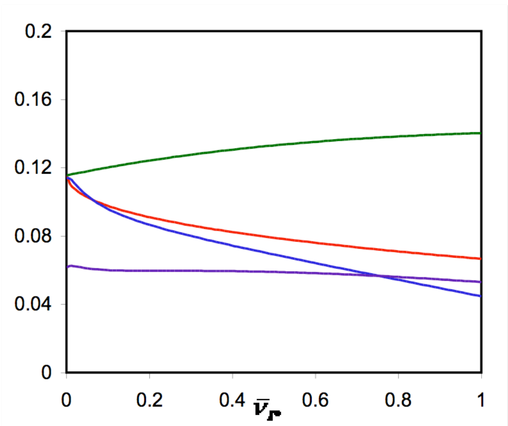
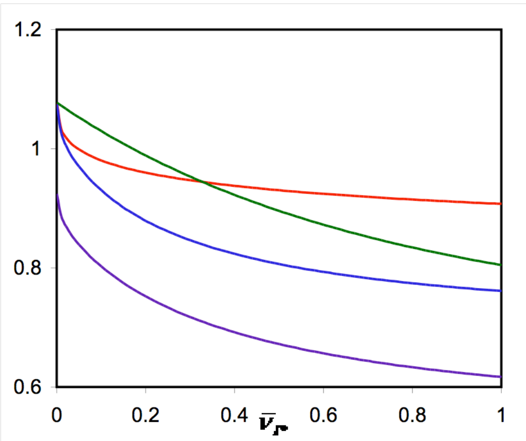


c_2



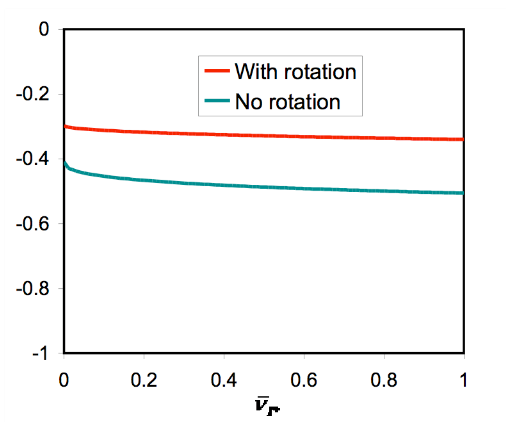
c_3



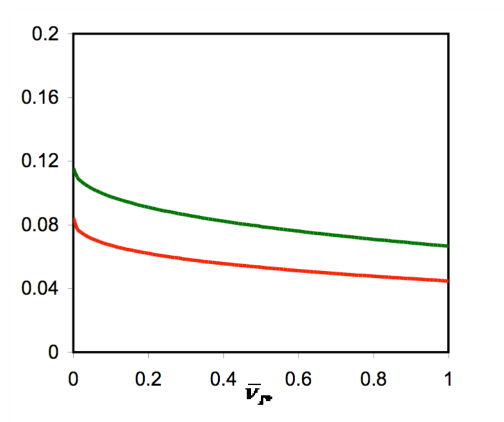
Effect of Toroidal Rotation

*Increase in coefficients drives poloidal velocity in the direction of ion diamagnetic flow

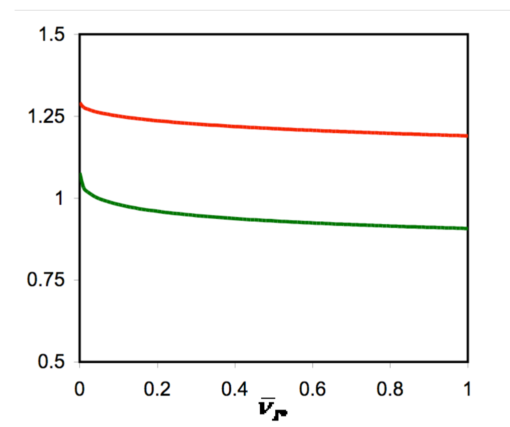
c_1



c_2



c_3

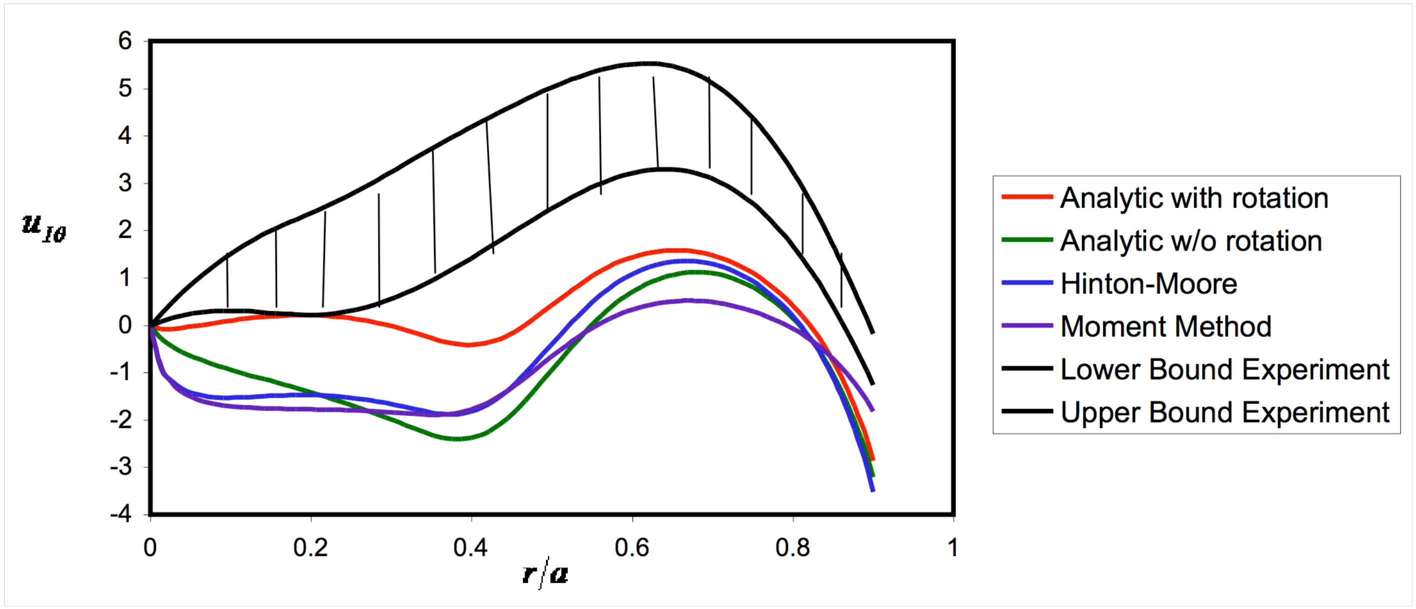


GENERAL ATOMICS

Comparison with DIII-D Experiment

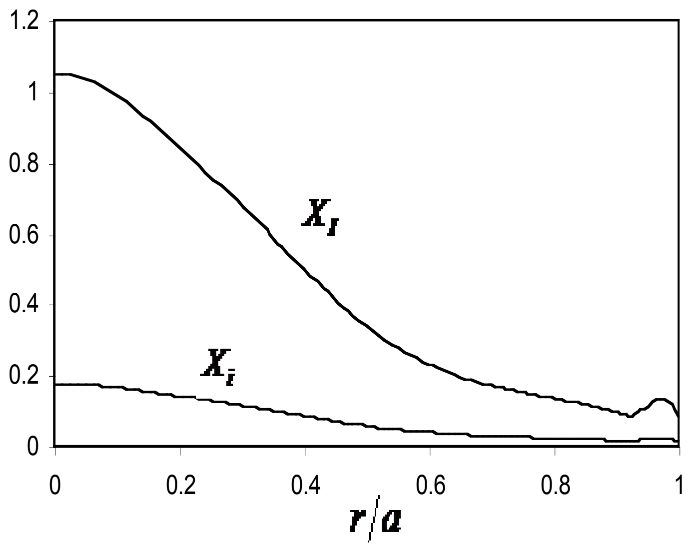
- **Taking toroidal rotation into account alleviates discrepancy with experiment**

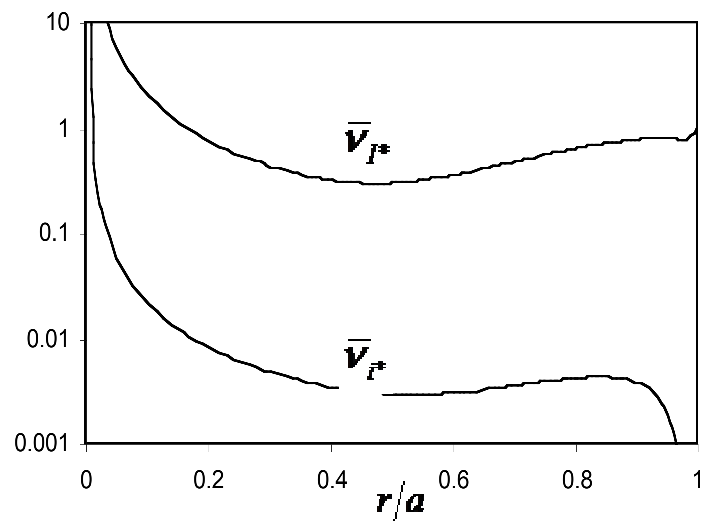
C^{+6} Poloidal Velocity Profile



Rotation Parameter and Collisionality

- **Toroidal rotation is important in central region**
- **Analytic theory is marginal at best with regard to collisionality**





GENERAL ATOMICS

If Steady State with no ion Sources is Assumed

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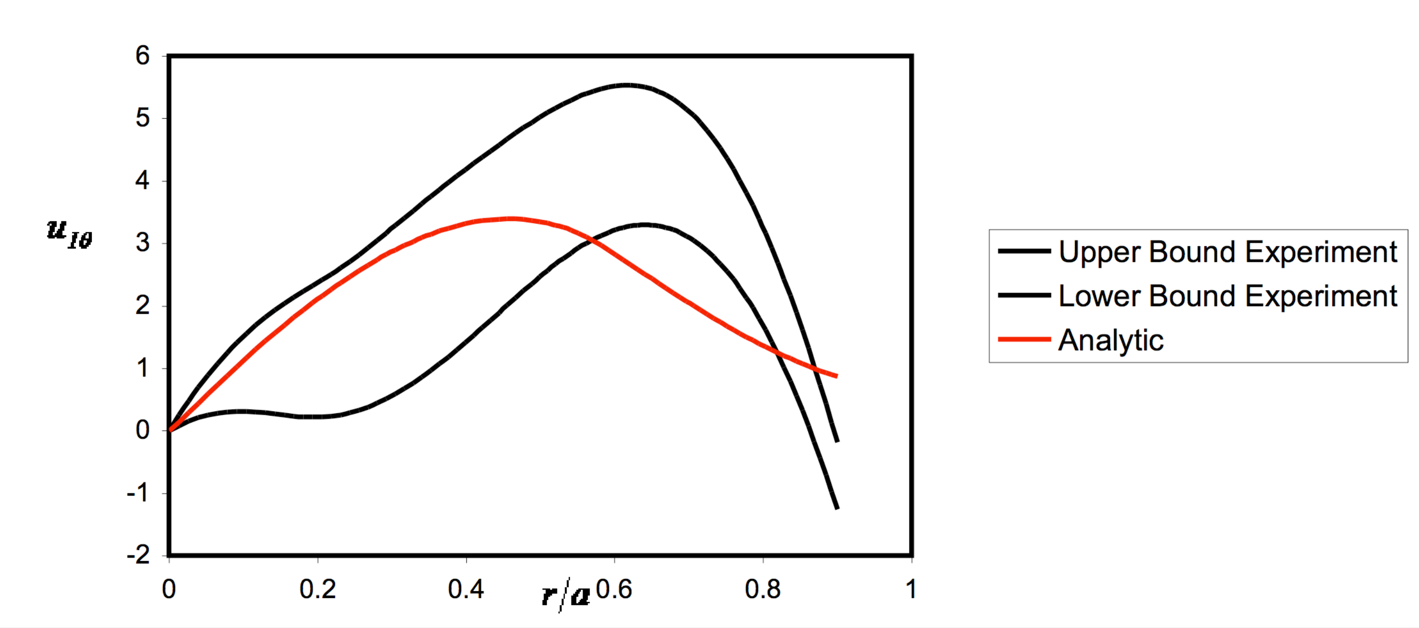
- then $\Gamma_I = \Gamma_i = 0$
- which leads to $u_{I\theta} = \left(\frac{5}{2} - \frac{k_{I1}}{k_{I0}}\right)\frac{1}{Z_I e B_\zeta}\frac{\partial T}{\partial r}$
- All calculations predict reasonable agreements with experiment



Conclusions

- **Taking toroidal rotation into account alleviates discrepancy with experiments**
- **Significant differences exist among different neoclassical calculations**
- **More accurate numerical work needed for experimentally relevant collisionality and magnetic geometry**