US TTF workshop
Boulder, CO, March 25-28, 2008

On the Determination of Plasma Rotation from Neoclassical Viscosity in Toroidal Plasmas

S.Nishimura, D.R.Mikkelsen [a], D.A.Spong [b], H.Sugama,
S.P.Hirshman[b], L.P.Ku [a], H.E.Mynick [a], M.C.Zarnstorff [a]

nishimura.shin@lhd.nifs.ac.jp

National Institute for Fusion Science, 322-6 Oroshi-cho, Toki 509-5292, Japan
[a] *Princeton Plasma Physics Laboratory, Princeton, New Jersey 08543-0451, USA*
[b] *Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6169, USA*

abstract

The importance of rotational stabilization of resistive wall modes (RWM) has prompted more attention to momentum transport studies, particularly to momentum dissipation due to neoclassical toroidal viscosity arising from non-axisymmetric error fields. Some effects of MHD-induced error fields were discussed in Refs.[1-3]. An additional important aspect of trapped particle dynamics has recently been stressed: a positive correlation between neoclassical and anomalous transport through generation of zonal flows in non-symmetric toroidal plasmas [4,5].

In a recent formulation for the neoclassical viscosities in general toroidal plasmas [6], it was shown that three mono-energetic viscosity coefficients [M^*, N^*, L^*] are required to describe the full neoclassical properties of the plasmas. This differs from axisymmetric neoclassical transport codes such as NCLASS, and the additional complications require systematic investigation. For this purpose, these coefficients in two low aspect ratio stellarators (NCSX and QPS) have recently been investigated [7] with various numerical [6] and analytical [8] methods. Even though the basic framework of the unified theory [6] covers both tokamaks and stellarators, some further studies are still required to unify the analytical approximation techniques for the viscosity. Beginning with the recent transport analysis described in [7], the analytical theory for the $1/\nu$ regime is extended to include both the MHD-activity-induced error fields [1-3] and the ripple fields in stellarators.

[1] E.Lazzaro, R.J.Buttery, T.C.Hender, et al., Phys.Plasmas 9, 3906 (2002)

[2] W.Zhu, S.A.Sabbagh. R.E.Bell, et al., Phys.Rev.Lett. **96**, 225002 (2006)

[3] K.C.Shaing, Phys.Plasmas **10**, 1443 (2003), **14**, 049903 (2007)

[4] H.Sugama and T.-H.Watanabe, Phys.Rev.Lett.**94**,115001 (2005)

[5] H.E.Mynick and A.H.Boozer, Phys.Plasmas **14**, 072507(2007)

[6] H.Sugama and S.Nishimura, Phys.Plasmas **9**, 4637 (2002); to be published in Phys.Plasmas (2008)

[7] S.Nishimura, D.R.Mikkelsen, D.A.Spong, et al., to be publidhed in Plasma Fusion Res.

[8] S.Nishimura, H.Sugama, and Y.Nakamura, Fusion Sci.Technol. **51**,61 (2007)

Outline (1)

1. On Basic Framework

(1) Comparison of two methods to clarify the validity of the 13M approximation for the flux surface averaged part

$$\langle \mathbf{B}\bullet\nabla\bullet\boldsymbol{\pi}_a\rangle - \langle n_a\rangle e_a \langle BE_{//}\rangle = \langle BF_{//a1}\rangle,\ \langle \mathbf{B}\bullet\nabla\bullet\boldsymbol{\theta}_a\rangle = \langle BF_{//a2}\rangle$$

H.Sugama and S.Nishimura, to be published in Phys.Plasmas (2008), NIFS report NIFS-885

(2) The poloidally and toroidally varying part determining the impurity transport

$$\mathbf{b}\bullet\nabla p_{a1}^{\mathrm{PS}} = F_{\parallel a1}^{\mathrm{PS}},\ \mathbf{b}\bullet\nabla \theta_{a1}^{\mathrm{PS}} = F_{\parallel a2}^{\mathrm{PS}}$$

2. Neoclassical viscosity coefficients in NCSX and QPS

S.Nishimura, D.R.Mikkelsen, D.A.Spong, et al., to be published in Plasma Fusion Research (http://www.jspf.or.jp/PFR/)

The boundary layer in **v**-space causing coupling effects between the bounce averaged motion of ripple-trapped particles and the non-averaged motion of untrapped particles (collisional entrapping/detrapping)

$\rightarrow 1/\nu^{1/2}$ diffusion, BS currents, rotations

Outline (2)

3. An extension of the 1/ν regime theory to include $|m-qn| \approx 1$ modes in B-field spectra. [$B = \Sigma\, B_{mn} \cos(m\theta - n\zeta)$, q: safety factor]

For future applications to the plasmas with MHD-activity-induced error fields.

(1) Physics in the vicinity of islands in helical/stellarator devices

(2) Rotational stabilization of RWM in tokamaks

for e.g., a recent "NTV" experiment in NSTX

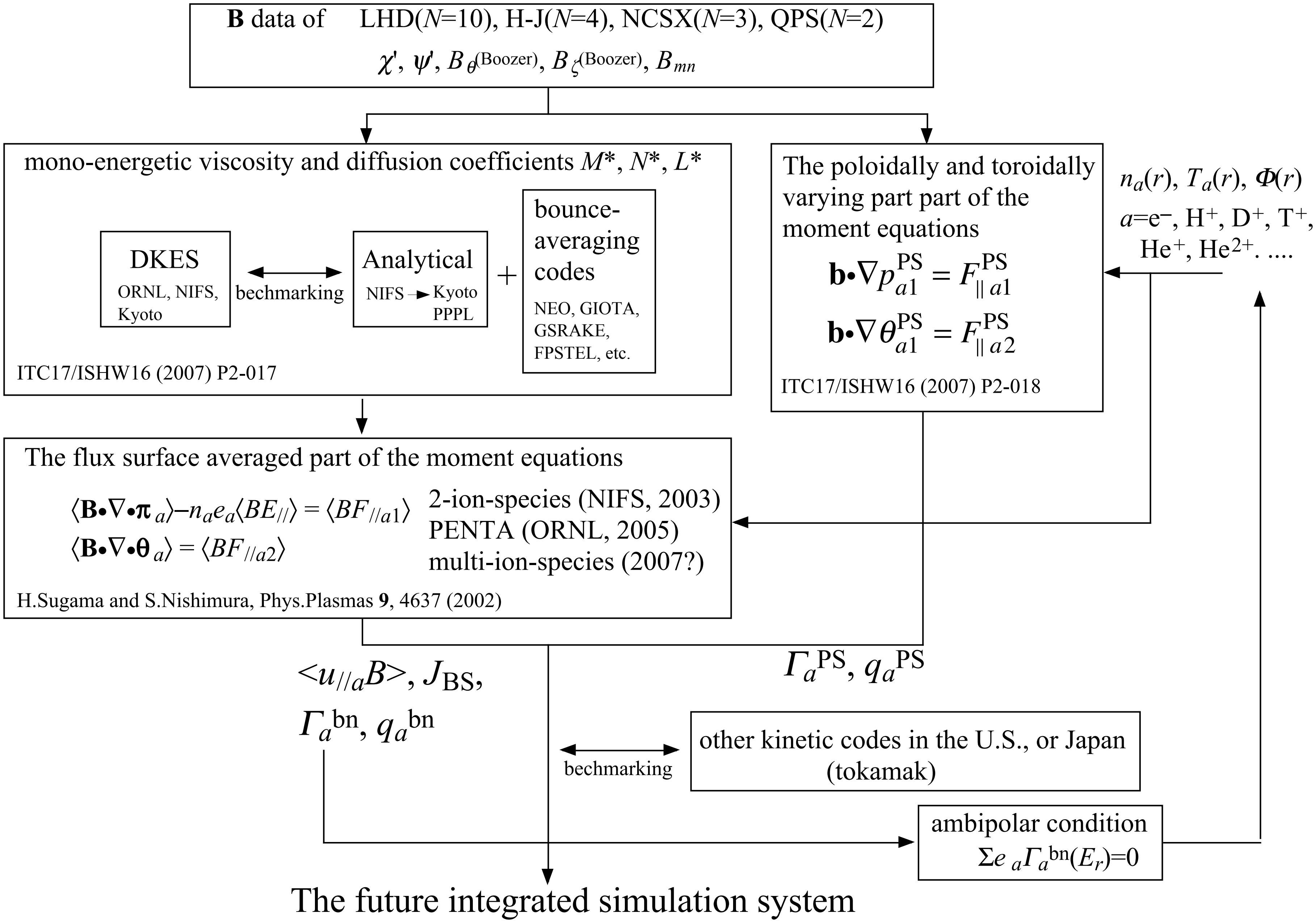
$$\frac{\partial}{\partial t}\left\langle n_{\mathrm{i}} m_{\mathrm{i}} \mathbf{B}_{\mathrm{T}} \bullet \mathbf{V}_{\mathrm{i}} \right\rangle = -\sum_{a} \left\langle \mathbf{B}_{\mathrm{T}} \bullet \nabla \bullet \boldsymbol{\pi}_{a} \right\rangle + S_{\mathrm{M}}$$

(3) Zonal flow in helical/stellarator devices

(Sugama-Watanabe, 2005, Mynick-Boozer, 2007)

4. Summary

A roadmap toward the full neoclassical fluxes



A comparison of two moment-equation methods with generalizations by extending to 21M, 29M approximations

There are two analogous methods known:

H.Sugama & S.Nishimura, Phys. Plasmas 9 (2002)

M.Taguchi, Phys. Fluids B 4(1992)

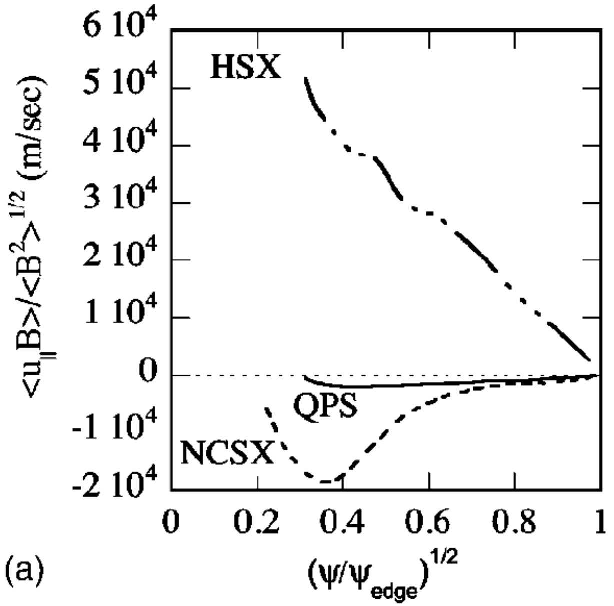
Both papers showed how to take account of collisional momentum conservation (of the Landau collision term) in multi-species plasmas in obtaining the transport coefficients from outputs of commonly used numerical codes such as the DKES, in which the pitch-angle-scattering collision model is used.

However, it is still important to address the theoretical relation between the methods as well as their accuracies from the viewpoint of practical applications.

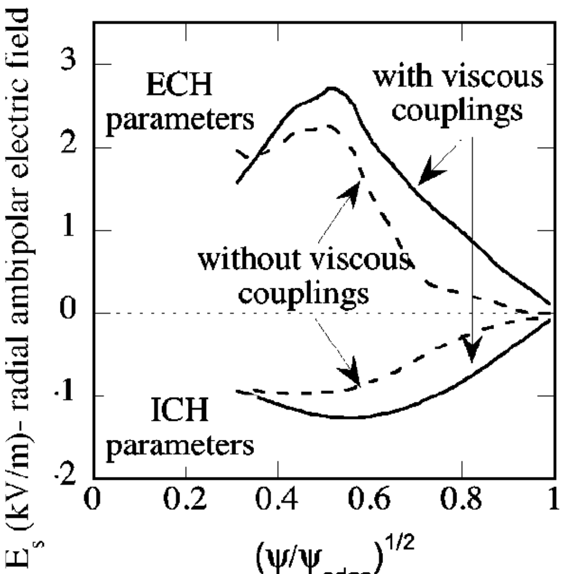
As a result of these generalization and comparison, we will show here the validity of the 13M approximation in Sugama-Nishimura method in 2002, and that it is more suitable for quasi-symmetric systems and tokamaks.

Applications of Sugama-Nishimura method

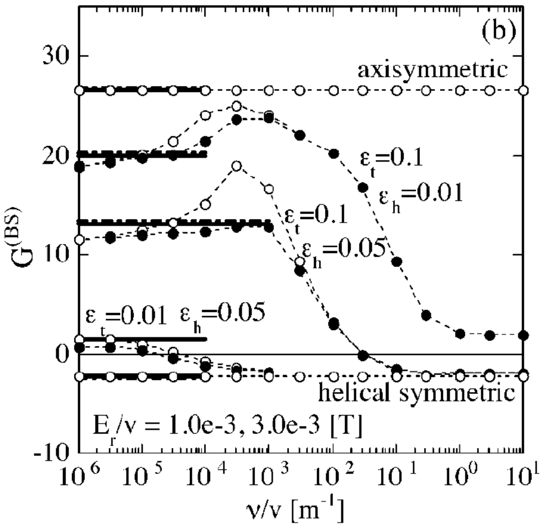
Spontaneous plasma flows on the flux surfaces [Spong, PoP(2005)]



Radial electric fields in QPS [Spong, PoP(2005)]



Geometrical factors for BS currents [Nishimura, Sugama, *et al.* FST(2004)]



These flows and electric fields are determined by the radial gradients of pressures and temperatures.

Common basis of the two methods (Sugama-Nishimura, Taguchi)

Drift Kinetic Equation (DKE)

$$V f_{a1} - C_a(f_{a1}) = \frac{1}{T_a} f_{aM} \left(-\sigma_1^+ \left[X_{a1} + X_{a2} \left(x_a^2 - \frac{5}{2} \right) \right] + e_a \frac{B}{\langle B^2 \rangle^{1/2}} v \xi X_E \right)$$

Collision term

$$C_a(f_{a1}) = \sum_b [C_{ab}(f_{a1}^{(l=1)}, f_{bM}) + C_{ab}(f_{aM}, f_{b1}^{(l=1)})] + \nu_D^a \mathcal{L}(f_{a1} - f_{a1}^{(l=1)})$$

Legendre-Laguerre expansion of the distribution function

Substitute

$$F(\mathbf{x}, v, \xi) = \sum_{l=0}^{\infty} F^{(l)}(\mathbf{x}, v, \xi) \qquad F^{(l)}(\mathbf{x}, v, \xi) = P_l(\xi) \frac{2l+1}{2} \int_{-1}^{1} d\eta P_l(\eta) F(\mathbf{x}, v, \eta)$$

$\int d^3v$ moments with weighting functions such as $v^l L_j^{(l+1/2)}$

$$f_{a1}^{(l=1)} = \frac{2}{v_{Ta}} \xi x_a f_{aM} \left[u_{\|a} + \frac{2}{5} \frac{q_{\|a}}{p_a} \left(x_a^2 - \frac{5}{2} \right) + \cdots \right]$$

$$= \frac{2}{v_{Ta}} \xi x_a f_{aM} \sum_{j=0}^{\infty} u_{\|aj} L_j^{(3/2)}(x_a^2),$$

Relations of $u_{\|aj}$, Γ_{aj} ($j = 0, 1, \ldots, j_{\max}$), X_{a1}, X_{a2}, X_E

Relations of $u_{\|aj}$, Γ_{aj} ($j = 0, 1, \ldots, j_{\max}$), X_{a1}, X_{a2}, X_E

Sugama-Nishimura

Moment Equations :

$$\sum_{k=0}^{j_{\max}} M^a_{j+1,k+1} \langle B u_{\|ak} \rangle / \langle B^2 \rangle + N^a_{j+1,1} X_{a1} - N^a_{j+1,2} X_{a2} = \sum_b \sum_{k=0}^{j_{\max}} l^{ab}_{j+1,k+1} \langle B u_{\|bk} \rangle + \delta_{j0} n_a e_a \langle B^2 \rangle^{1/2} X_E$$

Radial Transport Fluxes :

$$\Gamma^{\mathrm{bn}}_{aj} = \sum_{k=0}^{j_{\max}} N^a_{j+1,k+1} \langle B u_{\|ak} \rangle / \langle B^2 \rangle + L^a_{j+1,1} X_{a1} - L^a_{j+1,2} X_{a2}$$

Taguchi

Moment Equations :

$$\sum_{k=0}^{j_{\max}} A^a_{j+1,k+1} \langle B u_{\|ak} \rangle / \langle B^2 \rangle + B^a_{j+1,1} X_{a1} - B^a_{j+1,2} X_{a2} = \frac{1}{n_a} \sum_{m=0}^{j_{\max}} Z^a_{j+1,m+1} c_m \sum_b \sum_{k=0}^{j_{\max}} l^{ab}_{m+1,k+1} \langle B u_{\|bk} \rangle / \langle B^2 \rangle + Z^a_{j+1,1} e_a X_E / \langle B^2 \rangle^{1/2}$$

Radial Transport Fluxes :

$$\Gamma^{\mathrm{bn}}_{aj} = \sum_{k=0}^{j_{\max}} \mathcal{N}^a_{j+1,k+1} \langle B u_{\|ak} \rangle / \langle B^2 \rangle + \frac{1}{n_a T_a} \sum_{m=0}^{j_{\max}} B^a_{j+1,m+1} \times c_m \sum_b \sum_{k=0}^{j_{\max}} l^{ab}_{m+1,k+1} \langle B u_{\|bk} \rangle / \langle B^2 \rangle + \mathcal{L}^a_{j+1,1} X_{a1} - \mathcal{L}^a_{j+1,2} X_{a2}$$

<u>Neoclassical transport matrix</u> :

It expresses $u_{\|aj}$, Γ_{aj} ($j = 0, 1, \ldots, j_{\max}$) as liner combinations of X_{a1}, X_{a2}, X_E.

$j_{\max}$: maximum Laguerre order

When solving the moment equations,

(1) By using numerical solutions of the approximated DKE (by the DKES), we obtain "coefficients" in the moment equations. In traditional moment equation approach (Hirshman-Sigmar, 1981, Shaing-Callen, 1983), this kind of coefficients is called as "viscosity coefficients".

(2) The Sugama-Nishimura method and the Taguchi's method use different weighting functions.
$\rightarrow$ They result in different moment equations.

$j_{max} \rightarrow \infty$: Both methods are equivalent
finite j_{max} : They give different results. (Which is more correct ?)

For arbitrary $j_{max} \geq 1$, Sugama-Nishimura method gives :
(1) The intrinsic ambipolar condition in the symmetric limits
(2) Transport coefficients satisfying the Onsager symmetry

(But Taguchi's method breaks these conditions in cases with finite j_{max}.)

Comparison of the two methods in applications for axisymmetric limit $\partial B/\partial\zeta=0$ (tokamaks)

The asymptotic banana regime viscosity coefficients can exactly be obtained by analytical solutions.

The neoclassical transport of ion
in a small mass ratio approximation.

poloidal particle and heat flows

$$\begin{bmatrix} u_\theta \\ \frac{2}{5p_i} q_\theta \end{bmatrix} = -\frac{cIX_2}{e\chi'\langle B^2\rangle}\begin{bmatrix} C_{0\theta} \\ C_{1\theta} \end{bmatrix}$$

radial heat diffusion

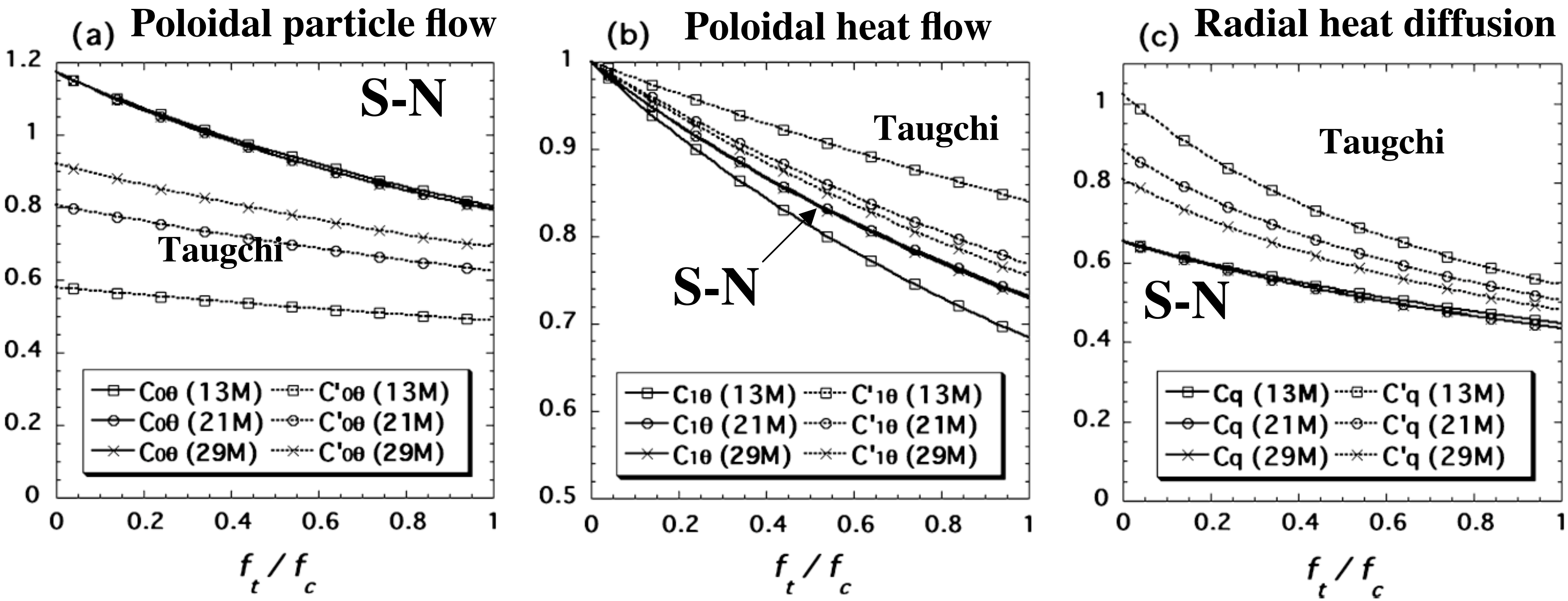
$$q^{\mathrm{b}} \equiv -T_i\Gamma_1^{\mathrm{b}} = C_q \frac{f_t}{f_c}\frac{n_i m_i T_i c^2 I^2}{e^2(\chi')^2\langle B^2\rangle\tau_{ii}} X_2$$

$C_{0\theta}$, $C_{1\theta}$, C_q : numerical factor to be determined by the parallel force balance.

In this axisymmetric limit (tokamaks)
→ Sugama-Nishimura method coincides with Hirshman-Sigmar formulae

Poloidal flows and the radial heat diffusion in the banana regime

Dependence on j_{max} **= 1 (13M), 2 (21M), 3 (29M)**



f_t **(=** $1-f_c$ **) : the fraction of trapped particles**

In high aspcet ratio limits :

f_t **= 1.46** $\varepsilon^{1/2}$

Sugama-Nishimura (13M) $\langle \mathrm{q}_i \cdot \nabla r \rangle \simeq 0.477 \dfrac{n_i \rho_{Ti}^2 q^2}{\epsilon^{3/2} \tau_{ii}} \dfrac{\partial T_i}{\partial r}$

Rosenbluth *et al.* (variational) **0.48**

Taughchi (13M) **0.748**

The radial particle diffusion in the banana regime

In Sugama-Nishimura method:

The j=0 moment coincides with the usual parallel force balance equation.

For arbitrary $j_{\max}$, the intrinsic ambipolarity $\sum_a e_a \Gamma_a = 0$ is retained.

In the axisymmetric limit the ion particle diffusion due to ion-ion collisions should be $\Gamma_{\mathrm{i}}^{\mathrm{b}} = 0$.

(Note that this small mass ratio approximation neglecting ion-electron collisions is only that for a test of the theories.)

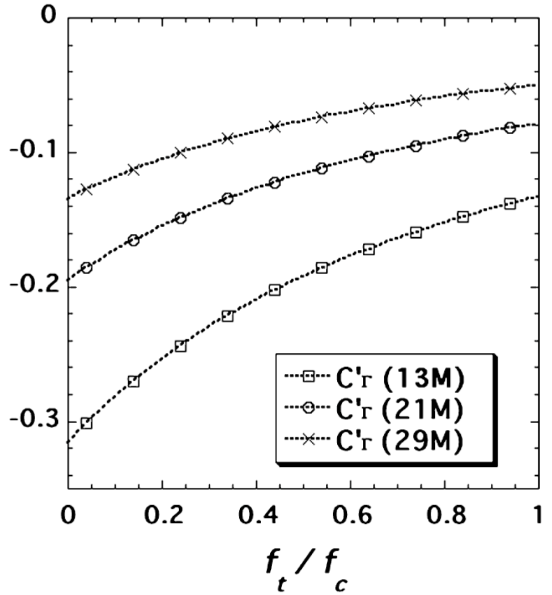
Taguchi's formulas given for stellarators:

The intrinsic ambipolarity in the symmetric systems cannot be satisfied by finite $j_{\max}$ values.

$\rightarrow$ Incorrect finite ion diffusion is given in the small-mass ratio limit.

$$\Gamma^{\mathrm{b}} = C'_{\Gamma} \frac{f_t}{f_c} \frac{n_i m_i T_i c^2 I^2}{e^2 (\chi')^2 \langle B^2 \rangle \tau_{ii}} X_2$$

Incorrect ion diffusion $\mathbf{C'}_{\Gamma}$



Summary of the comparison

Two methods proposed for the neoclassical transport in helical/stellarator devices (Sugama-Nishimura and Taguchi) are derived from common identical basic equation with the momentum conserving collision operator. They can be written for an arbitrary truncation number ($j_{\max}$) of the Laguerre expansion, even though the original papers described only the case of retaining the first two terms in the expansion.

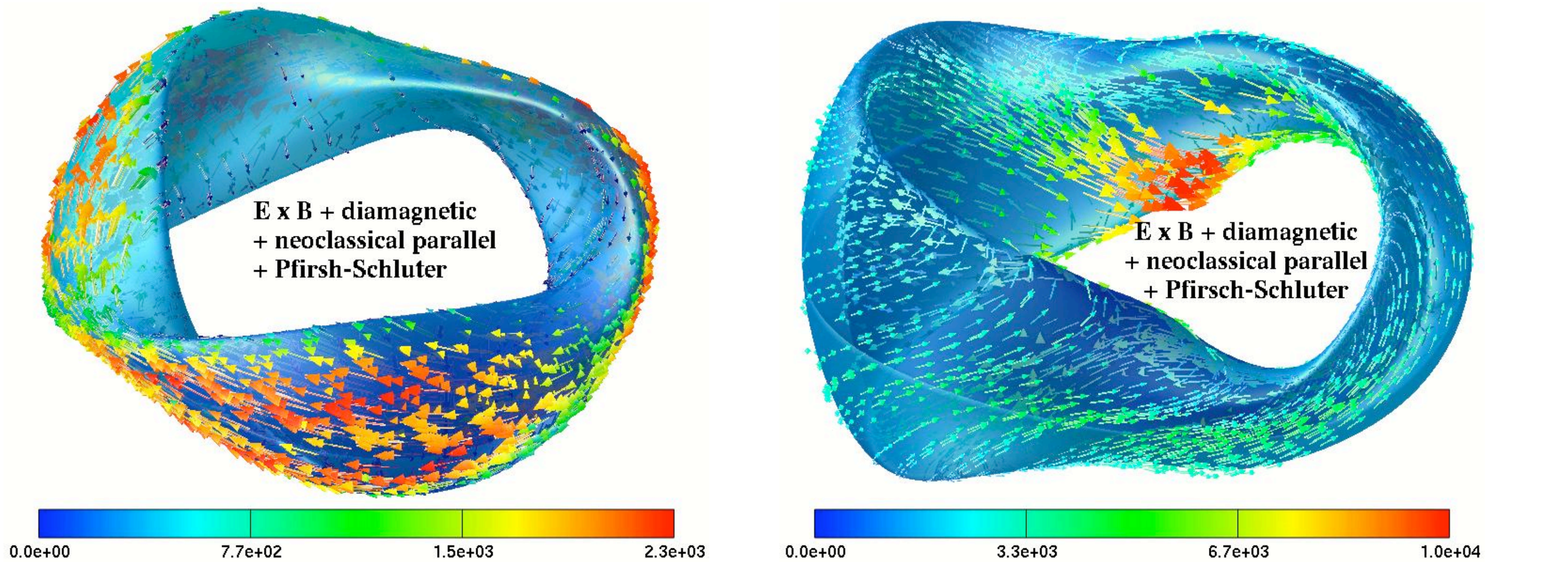
Sugama-Nishimura method and Taguchi's should lead to the same results in the limit of $j_{\max}\to\infty$. However, different results are given from these methods for the finite value of $j_{\max}$.

Sugama-Nishimura method with arbitrary truncation numbers of $j_{\max}\geq 1$ gives the intrinsically ambipolar particle fluxes in symmetric limits, and transport coefficients with the Onsager symmetry.

(→suitable for tokamaks and quasi-symmetric helical systems)

Ref: Sugama & Nishimura, to be published in Phys. Plasmas (2008).

Local structure of the flow pattern before the flux surface averaging has a winding determined by $\nabla\cdot(n_a \mathbf{u}_{//a}) = -\nabla\cdot(n_a \mathbf{u}_{\perp a})$



Even though it is well known that the radial diffusions are dominated by the turbulent transport, plasma flows along the flux surfaces will be determined by the neoclassical processes. The momentum balance including friction forces for the flows determines impurity accumulation and/or shielding.

In contrast to toroidally rotating tokamaks, however, this winding structure will not be simply determined by the incompressible condition $\nabla\cdot\mathbf{u}_a=0$, $\nabla\cdot\mathbf{q}_a=0$.

l=0,1 and j=0,1,2 Legendre-Laguerre moments of DKE

$$\langle p_a\rangle\begin{bmatrix}1&0&0\\0&\frac{5}{2}&0\\0&0&\frac{35}{8}\end{bmatrix}\nabla\bullet\begin{bmatrix}\frac{n_a\mathbf{u}_{\|a}}{\langle n_a\rangle}\\\frac{2\mathbf{q}_{\|a}}{5\langle p_a\rangle}\\\frac{n_a\mathbf{u}_{\|a2}}{\langle n_a\rangle}\end{bmatrix}+\langle n_a\rangle\begin{bmatrix}1&0&0\\-1&\frac{3}{2}&0\\1&-\frac{3}{2}&\frac{15}{8}\end{bmatrix}\left[cE_s\frac{\nabla s\times\mathbf{B}}{\langle B^2\rangle}+u_{\|}^{(\mathrm{rigid})}\mathbf{b}\right]\bullet\nabla\begin{bmatrix}\frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=0)}\\T_{a1}^{(j=1)}\\\frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=2)}\end{bmatrix}$$

$$-\sum_b\begin{bmatrix}0&0&0\\0&e_{11}^{ab}&0\\0&-e_{11}^{ab}&e_{22}^{ab}\end{bmatrix}\begin{bmatrix}\frac{\langle T_b\rangle}{\langle n_b\rangle}n_{b1}^{(j=0)}\\T_{b1}^{(j=1)}\\\frac{\langle T_b\rangle}{\langle n_b\rangle}n_{b1}^{(j=2)}\end{bmatrix}=\frac{c}{e_a}\nabla s\times\mathbf{B}\bullet\nabla\frac{1}{B^2}\begin{bmatrix}\langle T_a\rangle\left(\frac{\partial\langle p_a\rangle}{\partial s}+e_a\langle n_a\rangle\frac{\partial\langle\Phi\rangle}{\partial s}\right)\\\frac{5}{2}\langle p_a\rangle\frac{\partial\langle T_a\rangle}{\partial s}\\0\end{bmatrix}$$

Particle and energy conservations

$$\langle n_a\rangle\begin{bmatrix}1&1&0\\0&\frac{5}{2}&\frac{5}{2}\\0&0&\frac{35}{8}\end{bmatrix}\mathbf{b}\bullet\nabla\begin{bmatrix}\frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=0)}\\T_{a1}^{(j=1)}\\\frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=2)}\end{bmatrix}=\begin{bmatrix}F_{\|a1}\\F_{\|a2}\\F_{\|a3}\end{bmatrix}=\sum_b\begin{bmatrix}l_{11}^{ab}&-l_{12}^{ab}&l_{13}^{ab}\\-l_{21}^{ab}&l_{22}^{ab}&-l_{23}^{ab}\\l_{31}^{ab}&-l_{32}^{ab}&l_{33}^{ab}\end{bmatrix}\begin{bmatrix}n_bu_{\|b}/\langle n_b\rangle\\\frac{2q_{\|b}}{5\langle p_b\rangle}\\n_bu_{\|b2}/\langle n_b\rangle\end{bmatrix}$$

Parallel force balances

It can be solved by a Fourier expansion method for general toroidal configurations.

Numerical examples for the poloidally and toroidally varying part of the moment equations

P-S diffusion coefficients

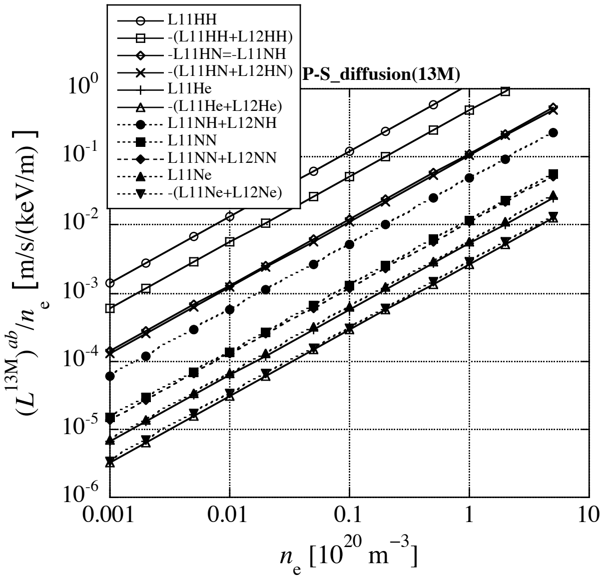
$$\begin{bmatrix} \Gamma_a^{\mathrm{PS}} \\ q_a^{\mathrm{PS}}/T_a \end{bmatrix} = -\frac{c}{e_a}\begin{bmatrix} \left\langle \widetilde{U} F_{\parallel a1} \right\rangle \\ \left\langle \widetilde{U} F_{\parallel a2} \right\rangle \end{bmatrix} \equiv \sum_b \begin{bmatrix} (L^{\mathrm{PS}})_{11}^{ab} & (L^{\mathrm{PS}})_{12}^{ab} \\ (L^{\mathrm{PS}})_{21}^{ab} & (L^{\mathrm{PS}})_{22}^{ab} \end{bmatrix} \begin{bmatrix} X_{b1} \\ X_{b2} \end{bmatrix}$$

$$(L^{\mathrm{PS}})_{ji}^{ba} = (L^{\mathrm{PS}})_{ij}^{ab}$$

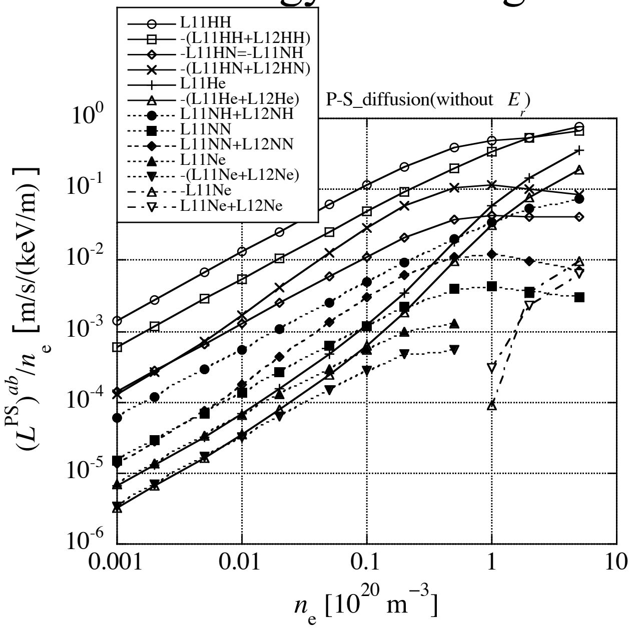
$B=B_0[1-\varepsilon_{\mathrm{t}}\cos\theta_{\mathrm{B}}+\varepsilon_{\mathrm{h}}\cos(L\theta_{\mathrm{B}}-N\zeta_{\mathrm{B}})]$, $(L,N)=(2,10)$,
$B_0=1\mathrm{T}$, $\chi'=0.15\mathrm{T\bullet m}$, $\psi'=0.4\mathrm{T\bullet m}$, $B_\theta=0$, $B_\zeta=4\mathrm{T\bullet m}$.

2 ion-species plasma $H^{+}+Ne^{10+}$ with a ion density ratio corresponding to $Z_{\mathrm{eff}}=5.74$, $T_{\mathrm{e}}=T_{\mathrm{i}}=1\mathrm{keV}$.

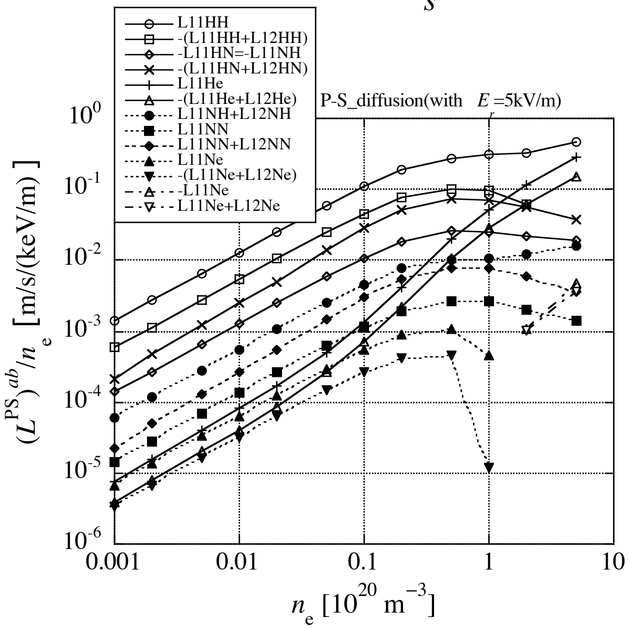
13M approximation



with energy scattering effects



with E_s=5kV/m



M, N, L matrix and flux surface averaged parts of the parallel momentum balance determining $\langle n_a u_{//a} B\rangle$, $\langle q_{//a} B\rangle$ as the integration constants of $\nabla \bullet (n_a u_{//a})$, $\nabla \bullet q_{//a}$

(H.Sugama and S.Nishimura, Phys.Plasmas 9, 4637(2002))

$$\begin{bmatrix} \langle \mathbf{B}\bullet\nabla\bullet\boldsymbol{\pi}_a\rangle \\ \langle \mathbf{B}\bullet\nabla\bullet\boldsymbol{\theta}_a\rangle \\ \Gamma_a^{\rm bn} \\ q_a^{\rm bn}/\langle T_a\rangle \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \dfrac{1}{\langle n_a\rangle}\langle n_a u_{\|a} B\rangle/\langle B^2\rangle \\ \dfrac{2}{5\langle p_a\rangle}\langle q_{\|a} B\rangle/\langle B^2\rangle \\ -\dfrac{1}{\langle n_a\rangle}\dfrac{\partial\langle p_a\rangle}{\partial s} - e_a\dfrac{\partial\langle \Phi\rangle}{\partial s} \\ -\dfrac{\partial\langle T_a\rangle}{\partial s} \end{bmatrix}$$

Given by an approximated DKE (numerically and/or analytically) (with energy integrations)

In symmetric cases $L_{aj} \propto N_{aj} \propto M_{aj}$

combined with the friction-flow relation

$$\sum_b \left(\frac{\delta_{ab}}{\langle B^2\rangle} \begin{bmatrix} M_{a1} & M_{a2} \\ M_{a2} & M_{a3} \end{bmatrix} - \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{21}^{ab} & -l_{22}^{ab} \end{bmatrix} \right) \begin{bmatrix} \dfrac{1}{\langle n_b\rangle}\langle n_b u_{\|b} B\rangle \\ \dfrac{2}{5\langle p_b\rangle}\langle q_{\|b} B\rangle \end{bmatrix}$$

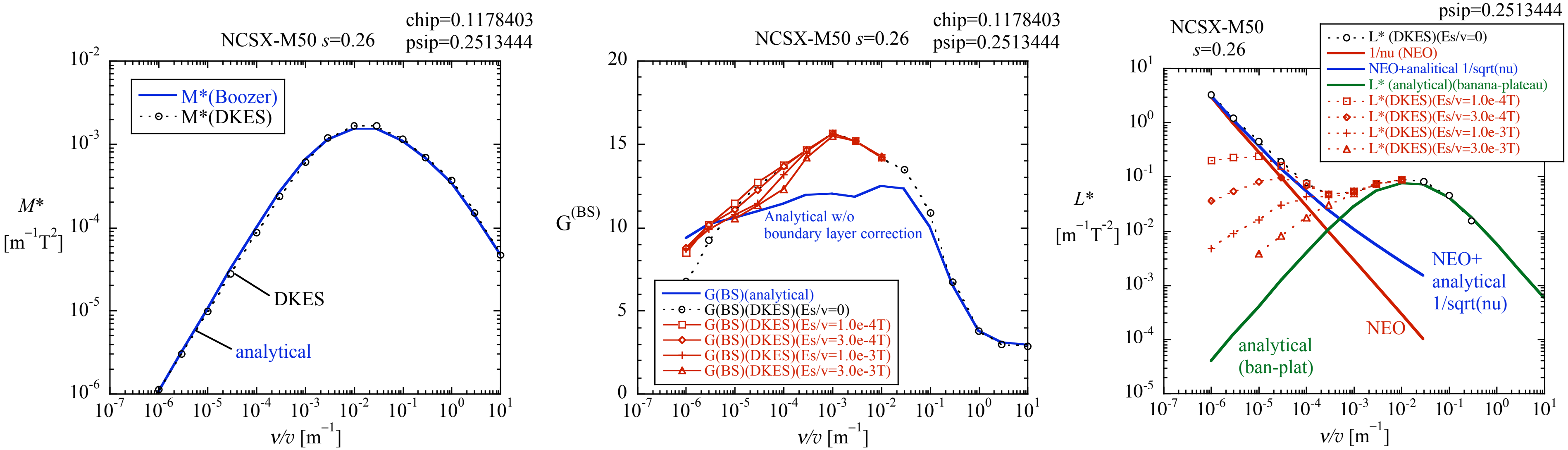
$$= -\begin{bmatrix} N_{a1} & N_{a2} \\ N_{a2} & N_{a3} \end{bmatrix} \begin{bmatrix} -\dfrac{1}{\langle n_a\rangle}\dfrac{\partial\langle p_a\rangle}{\partial s} - e_a\dfrac{\partial\langle \Phi\rangle}{\partial s} \\ -\dfrac{\partial\langle T_a\rangle}{\partial s} \end{bmatrix} + \begin{bmatrix} n_a e_a\langle B E_\|\rangle \\ 0 \end{bmatrix}$$

$a, b = e$, D^+, T^+, He^+, He^{2+}, Li^+, Li^{2+}, Li^{3+}, …

A non-diagonal coupling between particle species is introduced in this step.

A Benchmarking Example in NCSX

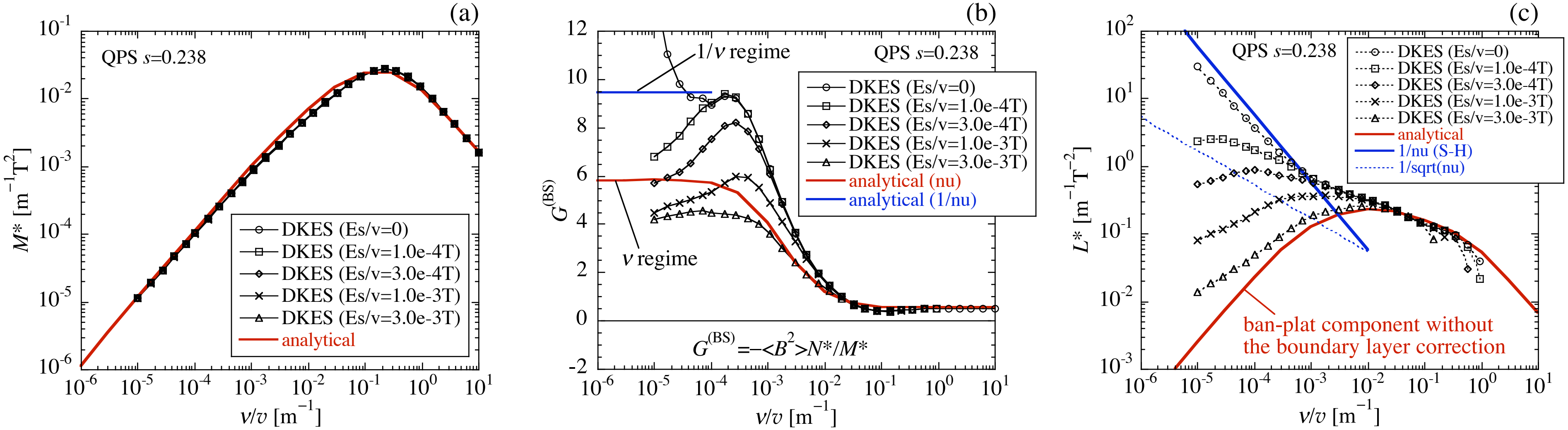
$$G^{(BS)} \equiv -\langle B^2 \rangle N^*/M^*$$



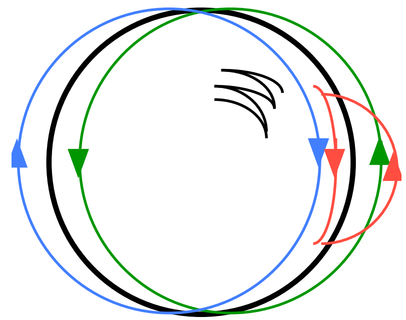
(1) The ripple-trapped/untrapped boundary layer, the distribution function is $\mathbf{B}\bullet\nabla G_{Xa} \neq 0$. For this kind of distribution function components requiring treatments in the 3-D phase space (poloidal angle θ, toroidal angle ζ, pitch angle ξ,), the approximated analytical solutions must be used as effectively as possible. For the $1/\nu^{1/2}$ diffusion, we have to consider procedures to use the analytical solutions and the numerical solutions for the bounce- or ripple-averaged parts $\mathbf{B}\bullet\nabla_{(\mu=\mathrm{const})} G_{Xa}{}^{(\mathrm{avg})}=0$ as complimentary methods.

(2) The N^* given by the DKES transiently becomes larger at $\nu/v \sim 10^{-3}\mathrm{m}^{-1}$ compared with the analytical formula. It is peculiar to the quasi-axisymmetric configurations where the $1/\nu^{1/2}$ component becomes comparable or dominates over the $1/\nu$ component in the radial diffusion.

A benchmarking example in QPS



$$B/B_0 = 1 + \varepsilon_\mathrm{T}(\theta) + \varepsilon_\mathrm{H}(\theta)\cos\{L\theta - N\zeta + \gamma(\theta)\}$$



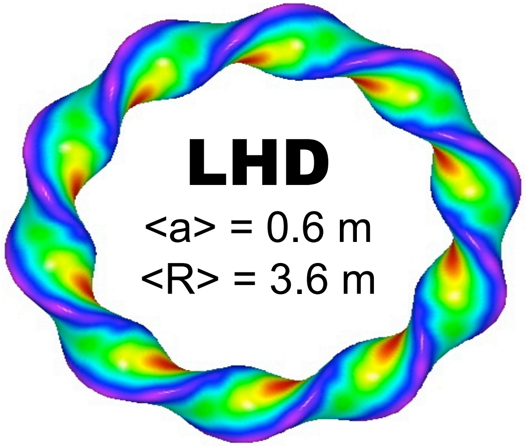
$$N^*_{(\mathrm{boundary})} = -\frac{12}{\pi^3}\frac{\nu_\mathrm{D}^a}{v}\frac{\left\langle B^2\right\rangle}{\chi'\psi' f_\mathrm{c}}\frac{V'}{4\pi^2}\times$$

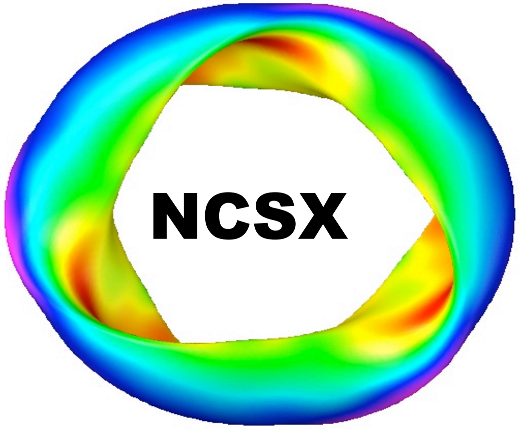
$$\int_0^\pi \mathrm{d}\theta\,(2\delta_\mathrm{eff})^{1/2}(\pi - 2\sin^{-1}\alpha^*)\theta\left(\frac{\partial\varepsilon_\mathrm{T}}{\partial\theta} - \frac{2}{3}\sqrt{1-\alpha^{*2}}\frac{\partial\varepsilon_\mathrm{H}}{\partial\theta}\right)$$

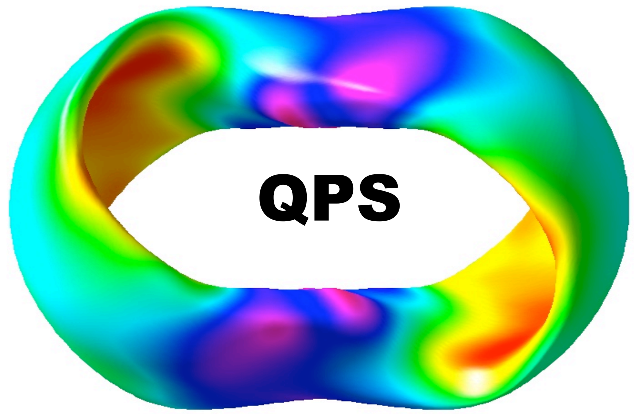
$$L^*_{(-1/2)} = 2.92\frac{2}{\pi^2}\left(\frac{v}{\nu_\mathrm{D}^a}\right)^{1/2}\frac{2^{3/4}}{(\psi')^2}\left(\frac{V'}{4\pi^2}\right)^{1/2}\times$$

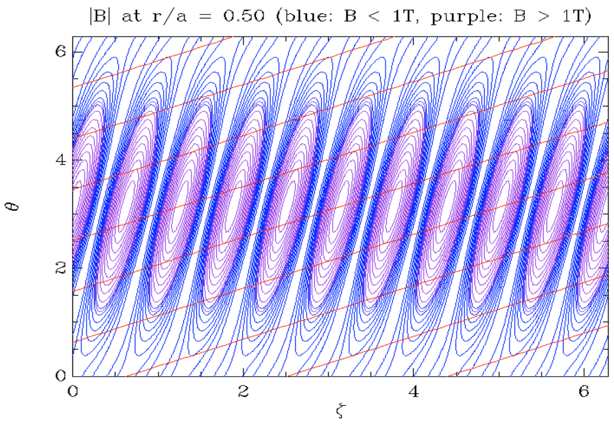
$$\int_{-\pi}^{\pi}\frac{\mathrm{d}\theta}{2\pi}\,\delta_\mathrm{eff}^{3/4}\left(B_0\frac{\pi - 2\sin^{-1}\alpha^*}{N\psi' - L\chi' - \chi'\partial\gamma/\partial\theta}\right)^{1/2}\left\{\left(\frac{\partial\varepsilon_\mathrm{T}}{\partial\theta}\right)^2 - \sqrt{1-\alpha^{*\,2}}\frac{\partial\varepsilon_\mathrm{T}}{\partial\theta}\frac{\partial\varepsilon_\mathrm{H}}{\partial\theta} + \frac{2}{9}(1-\alpha^{*\,2})\left(\frac{\partial\varepsilon_\mathrm{H}}{\partial\theta}\right)^2\right\}$$

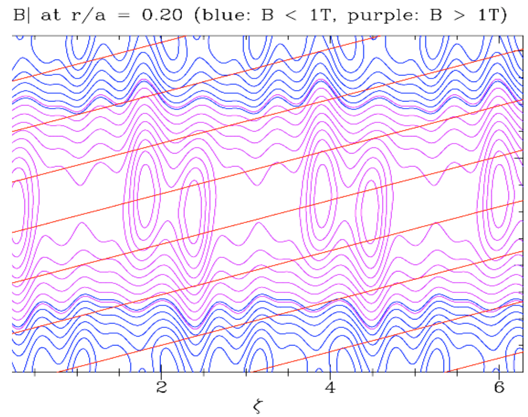
By D.A.Spong, in 15th ISW 2005

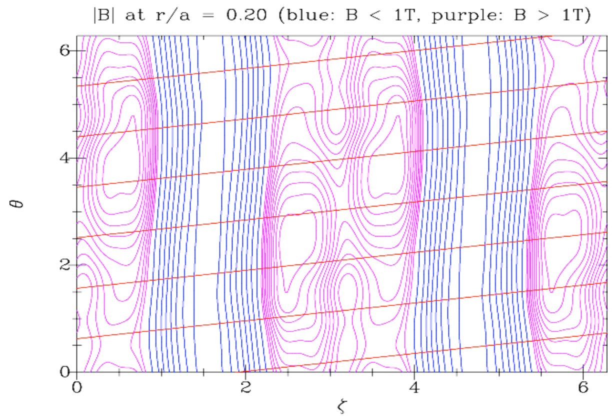




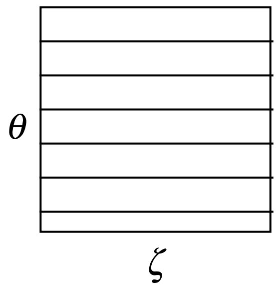




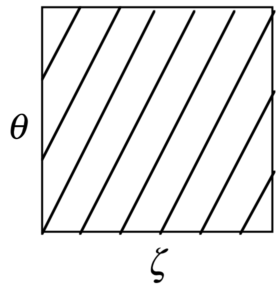




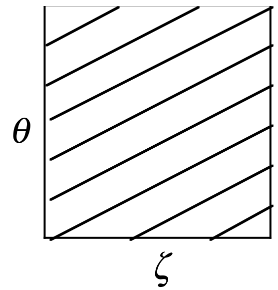
$B=$



$+$



$+$



$$1+\varepsilon_{\mathrm{T}}(\theta) \quad \varepsilon_{\mathrm{H}}(\theta)\cos[L\theta - N\zeta + \gamma(\theta)] \quad \sum_{n=1}^{\infty}\begin{bmatrix} A_n(\theta)\cos(n\zeta_0) \\ +B_n(\theta)\sin(n\zeta_0) \end{bmatrix} \quad \zeta_0 = q\theta - \zeta$$

Effects of these $|m-nq|\approx 1$ modes

(1) In both of tokamaks and stellarators:
The bounce center of toroidally trapped particles drift across the flux surfaces.

The theory for tokamaks by K.C.Shaing, et al.,
PRL **87**, 245003 (2001), PoP **9**, 3470 (2002), PoP **9**, 4633 (2002),
PoP **10**, 1443 (2003), PoP **10**, 4728 (2003), PoP **11**, 625 (2004), PoP **11**, 5525 (2004),
PoP **12**, 072523 (2005), PoP **13**, 022501 (2006), PoP **14**, 024501 (2007)

His theory had recently been tested in NSTX experiments.
W.Zhu, S.A.Sabbagh, R.E.Bell, et al., PRL **96**, 22002 (2006)

The modes change

(2) For ripple trapped particles
in stellarators

$$\frac{\partial G_{Xa}^{(1/\nu)}}{\partial \mu} = \frac{cB_0}{e_a v_D^a \psi'}\left[\left(\frac{\partial \varepsilon_T}{\partial \theta} + \frac{1}{3}\sqrt{1-\alpha^{*2}}\frac{\partial \varepsilon_H}{\partial \theta}\right) - \frac{2}{3}\frac{\kappa^2 E(\kappa)}{E(\kappa)-(1-\kappa^2)K(\kappa)}\sqrt{1-\alpha^{*2}}\frac{\partial \varepsilon_H}{\partial \theta}\right]$$

For existing stellarator codes, the "full torus" calculation including this B-structure means:

(1) For variational methods (DKES): It is substantially an increase of toroidal Fourier mode range for B and the distribution function.

LHD: ×10, W7X: ×5, HSX: ×4, NCSX: ×3, QPS: ×2

(2) For field line integral methods (NEO):
It will require the trace of the field line for the infinite length.

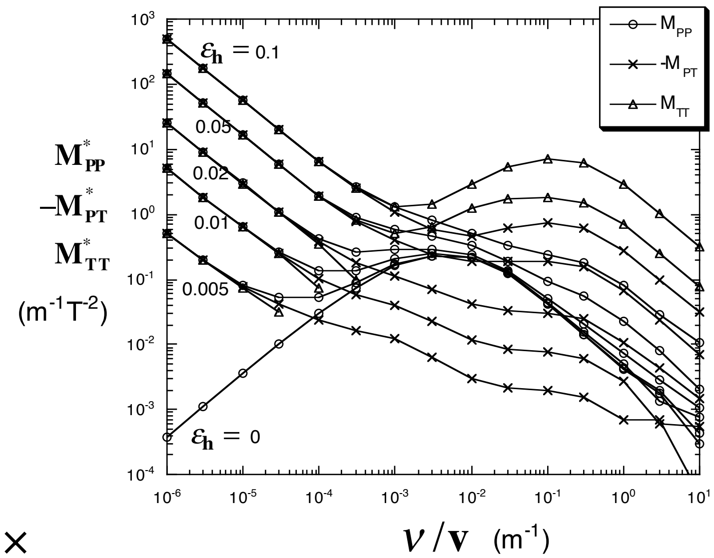
The role in the rotations and calculating method

An equivalence of the $[M_a, N_a, L_a]$ matrices with the poloidal and toroidal viscosities in toroidal momentum balance analysis in the tokamak experiments.

$$\begin{bmatrix} \langle \mathbf{B}_{\mathrm{P}} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle \\ \langle \mathbf{B}_{\mathrm{P}} \cdot \nabla \cdot \boldsymbol{\theta}_a \rangle \\ \langle \mathbf{B}_{\mathrm{T}} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle \\ \langle \mathbf{B}_{\mathrm{T}} \cdot \nabla \cdot \boldsymbol{\theta}_a \rangle \end{bmatrix} = \begin{bmatrix} M_{a1\mathrm{PP}} & M_{a2\mathrm{PP}} & M_{a1\mathrm{PT}} & M_{a2\mathrm{PT}} \\ M_{a2\mathrm{PP}} & M_{a3\mathrm{PP}} & M_{a2\mathrm{PT}} & M_{a3\mathrm{PT}} \\ M_{a1\mathrm{PT}} & M_{a2\mathrm{PT}} & M_{a1\mathrm{TT}} & M_{a2\mathrm{TT}} \\ M_{a2\mathrm{PT}} & M_{a3\mathrm{PT}} & M_{a2\mathrm{TT}} & M_{a3\mathrm{TT}} \end{bmatrix} \begin{bmatrix} \dfrac{1}{\langle n_a \rangle} n_a u_a^{\theta} / \chi' \\ \dfrac{2}{5\langle p_a \rangle} q_a^{\theta} / \chi' \\ \dfrac{1}{\langle n_a \rangle} n_a u_a^{\zeta} / \psi' \\ \dfrac{2}{5\langle p_a \rangle} q_a^{\zeta} / \psi' \end{bmatrix}$$

$$\begin{bmatrix} M_{aj\mathrm{PP}} & M_{aj\mathrm{PT}} \\ M_{aj\mathrm{PT}} & M_{aj\mathrm{TT}} \end{bmatrix} = \frac{4\pi^2}{V'} \begin{bmatrix} \chi' B_\theta^{(\mathrm{Boozer})} / \langle B^2 \rangle & -\dfrac{e_a}{c} \psi' \chi' \\ \psi' B_\zeta^{(\mathrm{Boozer})} / \langle B^2 \rangle & \dfrac{e_a}{c} \psi' \chi' \end{bmatrix} \begin{bmatrix} M_{aj} & N_{aj} \\ N_{aj} & L_{aj} \end{bmatrix} \times$$

$$\begin{bmatrix} \chi' B_\theta^{(\mathrm{Boozer})} / \langle B^2 \rangle & \psi' B_\zeta^{(\mathrm{Boozer})} / \langle B^2 \rangle \\ -\dfrac{e_a}{c} \psi' \chi' & \dfrac{e_a}{c} \psi' \chi' \end{bmatrix}$$



H.Sugama and S.Nishimura, Phys.Plasmas 9, 4637(2002)

Formulas for components due to non-bounce averaged motions

$$W(\lambda)=\sum_{\substack{(m,n)\\ \neq(0,0)}}\frac{Rm+Sn}{\chi' m-\psi' n}\left[\begin{array}{l}-2\dfrac{\partial\alpha_{mn}^{(\mathrm{Boozer})}}{\partial\lambda}\left\langle\dfrac{|v_{/\!/}|}{v}\cos(m\theta_{\mathrm{B}}-n\zeta_{\mathrm{B}})\right\rangle\\ +f_{\mathrm{c}}^{-1}\dfrac{\langle B^2\rangle}{B_{\mathrm{M}}^2}\cos(m\theta_{\mathrm{M}}^{(\mathrm{Boozer})}-n\zeta_{\mathrm{M}}^{(\mathrm{Boozer})})\left\{\dfrac{3}{2}\alpha_{mn}^{(\mathrm{Boozer})}(\lambda=1)+d_{mn}^{(\mathrm{Boozer})}\right\}\end{array}\right]$$

$$+\frac{\chi'\psi'}{\langle B^2\rangle}\frac{4\pi^2}{V'}\sum_{\substack{(m,n)\\ \neq(0,0)}}\frac{B_\zeta m+B_\theta n}{\chi' m-\psi' n}\left[\begin{array}{l}-2\dfrac{\partial\beta_{mn}}{\partial\lambda}\left\langle\dfrac{|v_{/\!/}|}{v}\cos(m\theta_{\mathrm{B}}-n\zeta_{\mathrm{B}})\right\rangle\\ +f_{\mathrm{c}}^{-1}\dfrac{\langle B^2\rangle}{B_{\mathrm{M}}^2}\cos(m\theta_{\mathrm{M}}^{(\mathrm{Boozer})}-n\zeta_{\mathrm{M}}^{(\mathrm{Boozer})})\left\{\dfrac{3}{2}\beta_{mn}(\lambda=1)+e_{mn}\right\}\end{array}\right]$$

$$N^{*\,(\mathrm{asym})}=-\frac{1}{2\chi'\psi'\langle B^2\rangle^{1/2}}\sum_{m,n}\frac{\{\chi'(1-H_2)m+\psi'(1+H_2)n\}(\chi' m-\psi' n)\{B_{mn}^{(\mathrm{Boozer})}\}^2}{\left\{\left(\dfrac{8}{\pi}|\chi' m-\psi' n|\right)^{3/2}+\left(5\dfrac{\nu_{\mathrm{T}}^{\mathrm{a}}(K)}{v}\langle B^2\rangle^{1/2}\dfrac{V'}{4\pi^2}\right)^{3/2}\right\}^{2/3}}$$

$$+\frac{1}{\langle B^2\rangle^{1/2}}\sum_{m,n}\frac{\left(\dfrac{\partial G}{\partial\zeta_{\mathrm{B}}}\dfrac{\partial B}{\partial\theta_{\mathrm{B}}}-\dfrac{\partial G}{\partial\theta_{\mathrm{B}}}\dfrac{\partial B}{\partial\zeta_{\mathrm{B}}}\right)_{mn}(\chi' m-\psi' n)B_{mn}^{(\mathrm{Boozer})}}{\left\{\left(\dfrac{8}{\pi}|\chi' m-\psi' n|\right)^{3/2}+\left(5\dfrac{\nu_{\mathrm{T}}^{\mathrm{a}}(K)}{v}\langle B^2\rangle^{1/2}\dfrac{V'}{4\pi^2}\right)^{3/2}\right\}^{2/3}}$$

$$\frac{1}{T_{\mathrm{a}}}(\sigma_{X\mathrm{a}}^{(\mathrm{asym})},\,G_{X\mathrm{a}}^{(\mathrm{asym})})\Big/\left[\left(\frac{c}{e_{\mathrm{a}}}\right)^2 m_{\mathrm{a}}v_{T\mathrm{a}}K^{3/2}\right]$$

$$=\frac{1}{(2\chi'\psi')^2\langle B^2\rangle^{1/2}}\frac{V'}{4\pi^2}\sum_{mn}\frac{\{\chi'(1-H_2)m+\psi'(1+H_2)n\}^2\{B_{mn}^{(\mathrm{Boozer})}\}^2}{\left\{\left(\dfrac{8}{\pi}|\chi' m-\psi' n|\right)^{3/2}+\left(5\dfrac{\nu_{\mathrm{T}}^{\mathrm{a}}(K)}{v}\langle B^2\rangle^{1/2}\dfrac{V'}{4\pi^2}\right)^{3/2}\right\}^{2/3}}$$

$$-\frac{1}{\chi'\psi'\langle B^2\rangle^{1/2}}\frac{V'}{4\pi^2}\sum_{mn}\frac{\left(\dfrac{\partial G}{\partial\zeta_{\mathrm{B}}}\dfrac{\partial B}{\partial\theta_{\mathrm{B}}}-\dfrac{\partial G}{\partial\theta_{\mathrm{B}}}\dfrac{\partial B}{\partial\zeta_{\mathrm{B}}}\right)_{mn}\{\chi'(1-H_2)m+\psi'(1+H_2)n\}B_{mn}^{(\mathrm{Boozer})}}{\left\{\left(\dfrac{8}{\pi}|\chi' m-\psi' n|\right)^{3/2}+\left(5\dfrac{\nu_{\mathrm{T}}^{\mathrm{a}}(K)}{v}\langle B^2\rangle^{1/2}\dfrac{V'}{4\pi^2}\right)^{3/2}\right\}^{2/3}}$$

$$+\frac{1}{\langle B^2\rangle^{1/2}}\frac{V'}{4\pi^2}\sum_{mn}\frac{\left(\dfrac{\partial G}{\partial\zeta_{\mathrm{B}}}\dfrac{\partial B}{\partial\theta_{\mathrm{B}}}-\dfrac{\partial G}{\partial\theta_{\mathrm{B}}}\dfrac{\partial B}{\partial\zeta_{\mathrm{B}}}\right)_{mn}^2}{\left\{\left(\dfrac{8}{\pi}|\chi' m-\psi' n|\right)^{3/2}+\left(5\dfrac{\nu_{\mathrm{T}}^{\mathrm{a}}(K)}{v}\langle B^2\rangle^{1/2}\dfrac{V'}{4\pi^2}\right)^{3/2}\right\}^{2/3}}$$

S.Nishimura, H.Sugama, and Y.Nakamura, Fusion Sci.Technol.**51**, 61 (2007)

These are applicable for arbitrary B_{mn} spectra even when including the MHD-activity-induced error field with $|m-qn|\approx 1$. (Note that $m-qn=0$ of $1/B^2$ in the Boozer and of B^2 in the Hamada coordinates are forbidden.)

In contrast to them, the formulas relating to the bounce averaged motions ($L^*_{(-1)}$, $L^*_{(-1/2)}$, $N^*_{(\mathrm{boundary})}$) assuming $Nq-L>>1$ must be extended to include the $|m-qn|\approx 1$ modes.

The analytical method for stellarators

Bounce averaged DKE for the toroidally trapped particles (1/ν)

$$-\nu_{\mathrm{D}}^{a} m_a \frac{\partial}{\partial \mu}\mu\left(\oint \frac{\mathrm{d}l}{B} v_{//}\right)\frac{\partial G_{Xa}}{\partial \mu} = \frac{m_a c}{e_a \chi'}\frac{\partial}{\partial \zeta_0}\oint v_{//}\mathrm{d}l$$

By integrating it,

$$\frac{\partial G_{Xa}}{\partial \mu} = -\frac{c}{e_a \chi' \nu_{\mathrm{D}}^{a}} \frac{\oint \frac{\left|v_{//0}\right|}{\left(B_{\mathrm{axisymmetric}}\right)^2}\frac{\partial B}{\partial \zeta_0}\mathrm{d}\theta_{\mathrm{B}}}{\oint \frac{\left|v_{//0}\right|}{\left(B_{\mathrm{axisymmetric}}\right)^2}\mathrm{d}\theta_{\mathrm{B}}}$$

$$\left|v_{//0}\right| \equiv v\left(1-\frac{\mu B_{\mathrm{axisymmetric}}}{w}\right)^{1/2}$$

$$B_{\mathrm{axisymmetric}} \equiv B_0\left\{1+e_{\mathrm{T}}(\theta_{\mathrm{B}})\right\}$$

The integral period length for $\oint \mathrm{d}\theta_{\mathrm{B}}$ **is determined by** $B/B_0=1+\varepsilon_{\mathrm{T}}(\theta_{\mathrm{B}})+\varepsilon_{\mathrm{H}}(\theta_{\mathrm{B}})$

(Since the contributions of $|m-qn|>>1$ modes become small in this integral, modes of $|m-qn|>Nq-L>>1$ can be omitted.)

$$L^{*}_{(\mathrm{MHD})} \propto \frac{1}{\nu_{\mathrm{D}}^{a}}\int_{\mathrm{trapped}}\mathrm{d}\mu \; \frac{\sum_{n=1}^{\infty} n^2\left\{\oint \frac{\left|v_{//0}\right|}{\left(B_{\mathrm{axisymmetric}}\right)^2} A_n(\theta_{\mathrm{B}})\,\mathrm{d}\theta_{\mathrm{B}}\right\}^2}{\oint \frac{\left|v_{//0}\right|}{\left(B_{\mathrm{axisymmetric}}\right)^2}\mathrm{d}\theta_{\mathrm{B}}}$$

Though this integral can only be obtained numerically, this estimation is still easier than applications of existing methods for stellarators (DKES, NEO).

Summary

- The basic framework proposed by us is most favorable for studies of tokamaks with the MHD-activity-induced error fields and quasi-symmetric helical systems. It determines all of neoclassical quantities consistently in arbitrary collisioanlity regimes in general toroidal configurations.

- Not only existing numerical tools for stellarators can obtain the required viscosity coefficients, but also simple analytical approximations for the DKE can be used for this purpose.

  Tests of these analytical formulas are being carried out in various helical/stellarator configurations.

- For the test of an extension to include the MHD-activity-induced error fields in the analytical formula for the $1/\nu$ regime of stellarators, low aspect stellarator configurations with few toroidal periods seem to be favorable as the first step, in viewpoint of the toroidal Fourier mode range of the DKES. This extension will be useful for studies of physics in the vicinity of islands in helical/stellarator devices.