Gyrokinetic δf particle simulation of energetic particles driven modes

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Outline

- Gyrokinetic simulations with kinetic electrons
  - Obtain the TAE frequency predicted by the MHD theory
  - Unstable mode is driven by energetic particles
- Hybrid model with fluid electrons
  - MHD theory
  - Simulation results
  - Future work
- Summary

Gyrokinetic simulations with kinetic electrons

Introduction to GEM

- GEM is an explicit δf particle code that includes kinetic electrons and electromagnetic perturbations (Y. Chen and S. E. Parker, J. Comput. Phys. **189** (2003) 463) , which has been employed to simulate plasma turbulence and transport.

- This code can also handle general toroidal equilibrium, magnetic configuration and arbitrary equilibrium density and temperature profiles (Y. Chen and S. E. Parker, J. Comput. Phys. **220** (2007) 839).

- The gyrokinetic model can accurately describe the slab shear Alfven waves and toroidal kinetic ballooning modes.

Simulation parameters

- Basic parameters:

$$B_0 = 1.91\,\mathrm{T},\ \ T_i = T_e = 2\ \ \mathrm{K}eV,\ 1/L_T = 1/L_n = 0,\ \ R_0 = 1.67\,m,\ \ a = 0.36R_0$$

- Profile:

$$q(r) = 1.3\left(\frac{r}{r_0}\right)^{0.3},\ \ r_0 = a/2,\ \ r_0 q'/q = 0.3$$

- Plasma to magnetic pressure ratio and mass ratio:

$$\beta = 1.0\times 10^{-5},\ \ m_p / m_e = 500$$

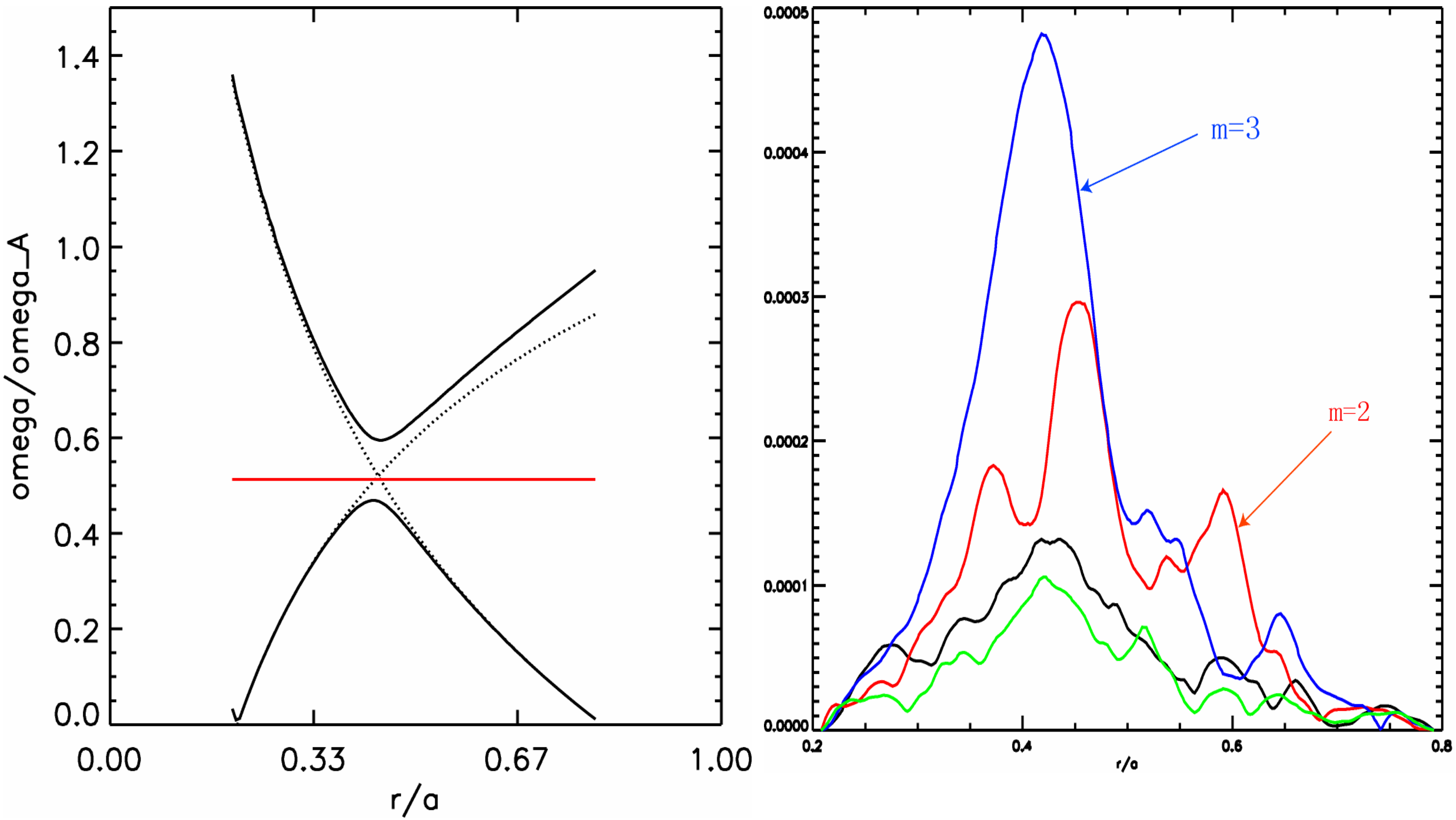
- Simulation domain:

$$[0.2a, 0.8a]$$

- External drive:

Add external n=2 current for 200 steps, then observe the subsequent oscillation and mode structure

TAE frequency eigenmode observed at low β with kinetic electrons



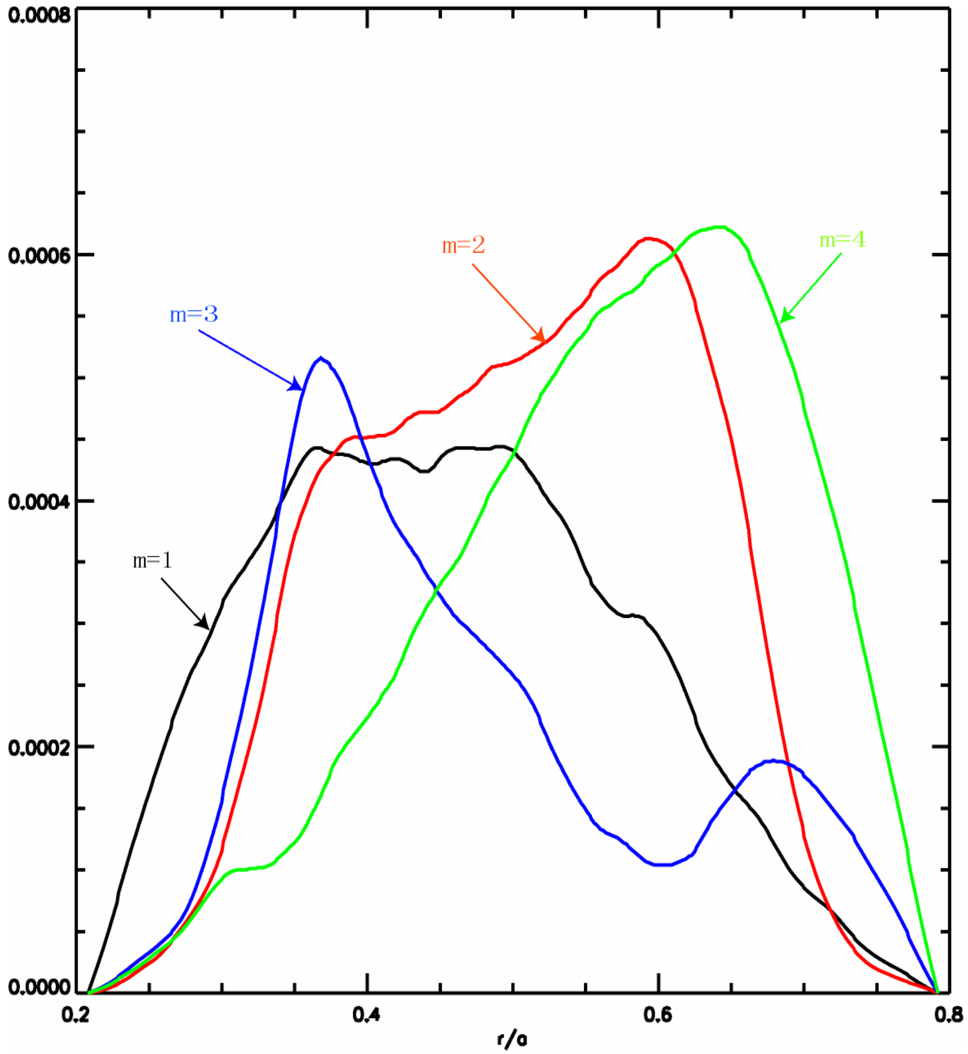
Preliminary results of energetic particle driven instability

- With energetic particles

- Mass ratio:

$$m_p / m_e = 500, \quad m_i / m_p = 2$$

- The mode structure has changed



Hybrid model with fluid electrons

The MHD equations for the shear Alfven wave

- **Quasi-neutrality**

$$-\frac{n_0 m_i}{B(\vec{r})^2}\nabla_\perp^{\ 2}\phi = \delta n_i - \delta n_e$$

- **Continuity equations**

$$\left.\begin{aligned}
&\frac{\partial \delta n_i}{\partial t} + n_0 \hat{b}\cdot\nabla u_{\|i} + \vec{E}\times\hat{b}\cdot\nabla n_0 = 0\\
&\frac{\partial \delta n_e}{\partial t} + n_0 \hat{b}\cdot\nabla u_{\|e} + \vec{E}\times\hat{b}\cdot\nabla n_0 = 0
\end{aligned}\right\}$$

$$\frac{\partial}{\partial t}(\delta n_i - \delta n_e) + n_0 \hat{b}\cdot\nabla(u_{\|i} - u_{\|e}) = 0$$

- **Ampere's law**

$$-\nabla_\perp^{\ 2} A_\| = \mu_0 q n_0 (u_{\|i} - u_{\|e})$$

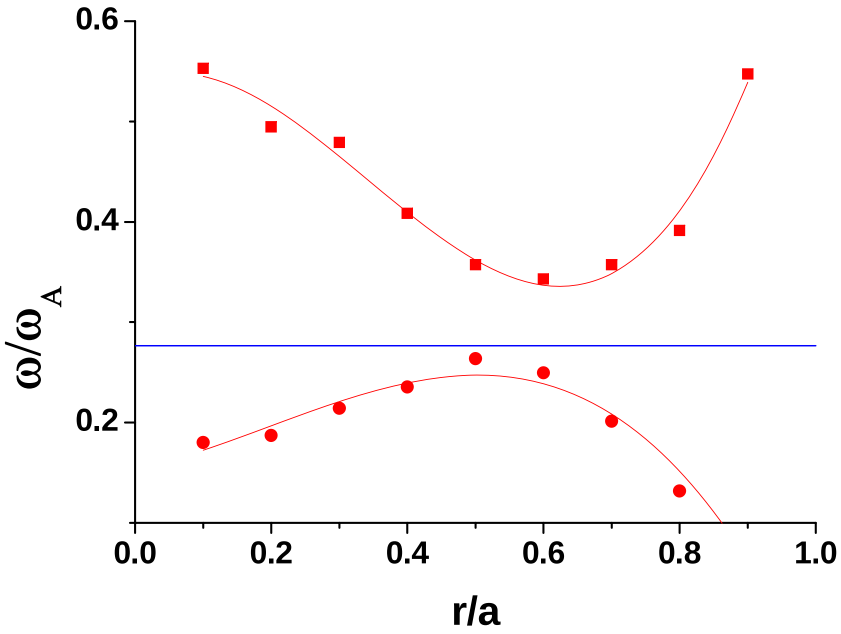
- **Faraday's law**

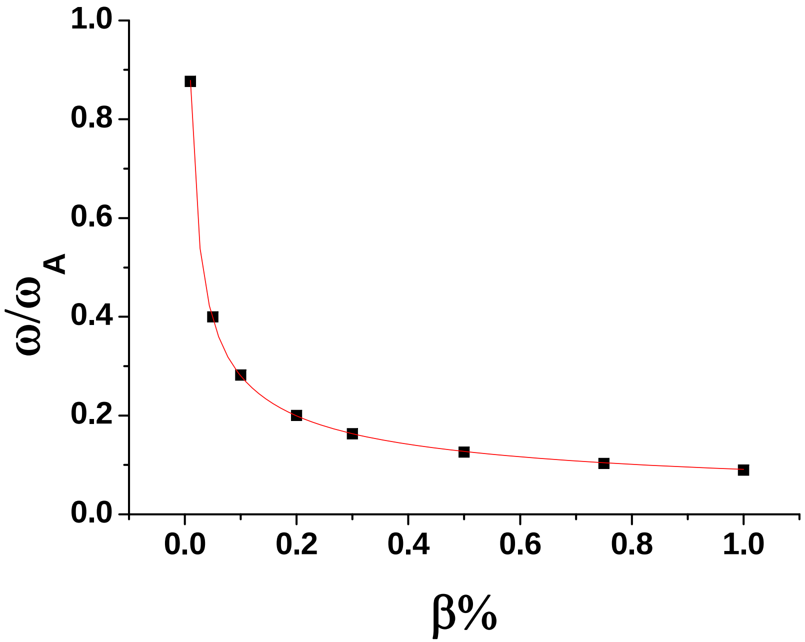
$$\frac{\partial A_\|}{\partial t} + \hat{b}\cdot\nabla_\perp\phi = 0$$

- **MHD TAE equation**

$$\frac{\partial^2}{\partial t^2}\frac{1}{V_A(\vec{r})^2}\nabla_\perp^{\ 2}\phi = \hat{b}\cdot\nabla\nabla_\perp^{\ 2}\hat{b}\cdot\nabla\phi$$

GEM observes the global TAE eigenmode and the continuum

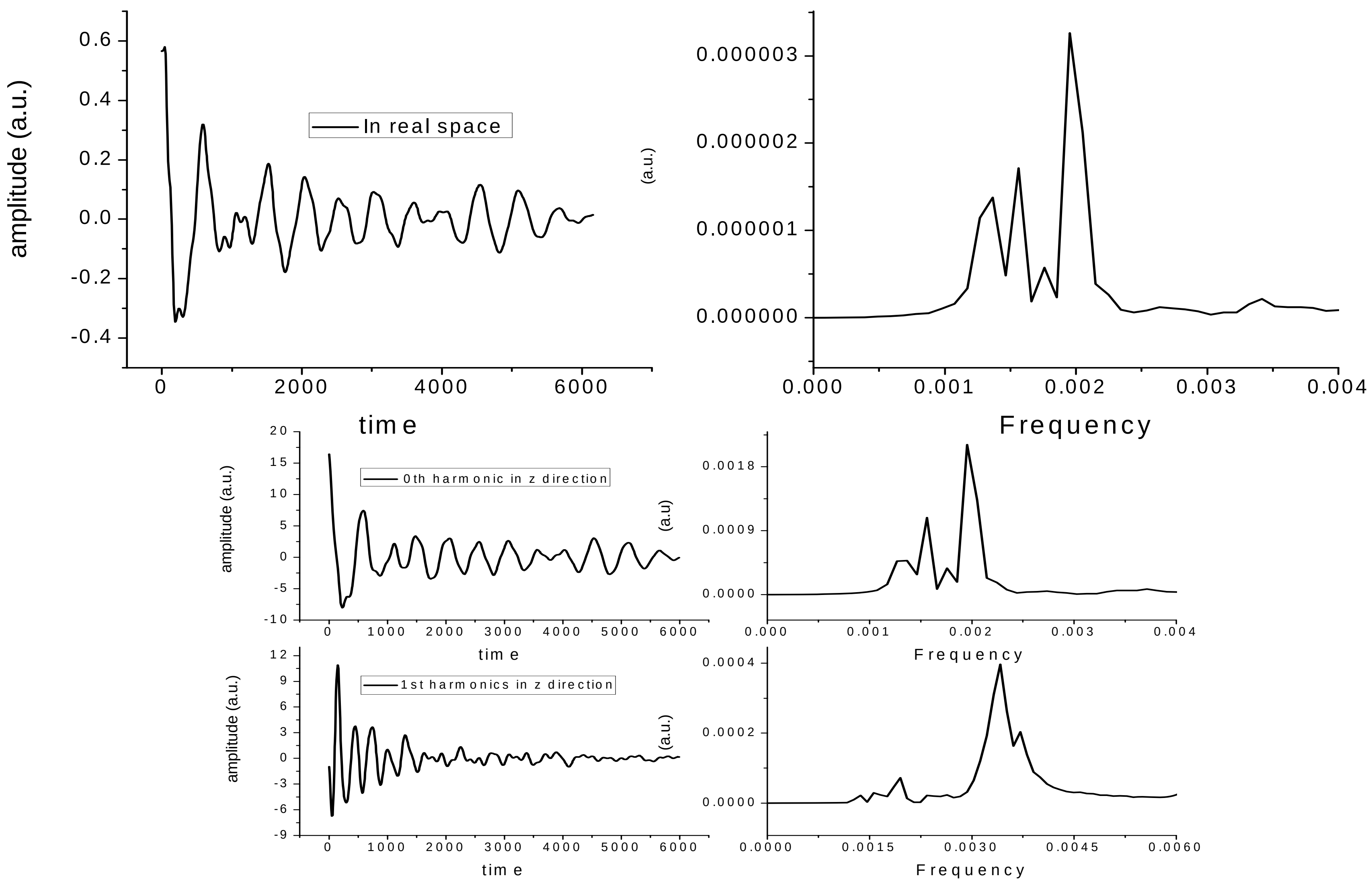




- Field-line-following coordinates are employed in the simulations
- Global geometry is applied
- The gap spectra are observed
- There is a global Alfven wave with the frequency fallen in the gap

- BUT, the frequency of the global Alfven wave does not agree with analytical MHD result (G. Y. Fu and J. W. Van Dam, Phys. Fluids B **1**, 1949 (1989))

Measurement for the mode frequency



Numerical analysis for the eigenmode

TAE eigenmode analysis

- Field-line-following coordinates:

$$x = r - r_0$$

$$y = \frac{r_0}{q_0}\left(\int_0^{\theta} \hat{q}\, d\theta - \zeta \right)$$

$$z = q_0 R_{\Psi 0} \theta$$

- The left hand side operator of the MHD TAE equation:

$$\frac{1}{V_A^2}\frac{\partial^2}{\partial t^2}\nabla_\perp^2 \phi = -\frac{\omega^2}{V_A^2}\left(|\nabla x|^2 \frac{\partial^2}{\partial x^2} + 2\nabla x \cdot \nabla y \frac{\partial^2}{\partial x \partial y} + |\nabla y|^2 \frac{\partial^2}{\partial y^2} + \nabla^2 x \frac{\partial}{\partial x} + \nabla^2 y \frac{\partial}{\partial y} \right)\phi$$

- The right hand side operator of the MHD TAE equation :

$$\hat{b}\cdot\nabla\nabla_\perp{}^2 \hat{b}\cdot\nabla\phi = \hat{b}\cdot(\nabla x \times \nabla y)\frac{\partial}{\partial z}\left(|\nabla x|^2 \frac{\partial^2}{\partial x^2} + 2\nabla x \cdot \nabla y \frac{\partial^2}{\partial x \partial y} + |\nabla y|^2 \frac{\partial^2}{\partial y^2} + \nabla^2 x \frac{\partial}{\partial x} + \nabla^2 y \frac{\partial}{\partial y} \right)$$

$$\hat{b}\cdot(\nabla x \times \nabla y)\frac{\partial}{\partial z}\phi$$

- Keep one toroidal mode ($k_y = \frac{2\pi}{l_y}$)

$$\frac{\partial \phi}{\partial y} = i k_y \phi, \quad \frac{\partial^2 \phi}{\partial y^2} = -k_y^2 \phi$$

- Discretize the operators in radial (r) and field-line (z) directions

Numerical solutions

$$left = -\omega^2 \begin{pmatrix} & & & \vdots & & & \\ \cdots & L_{i*N_z+k,(i-1)*N_z+k} & \cdots & L_{i*N_z+k,i*N_z+k} & \cdots & L_{i*N_z+k,(i+1)*N_z+k} & \cdots \\ & & & \vdots & & & \end{pmatrix} \begin{pmatrix} \phi_{0,0} \\ \vdots \\ \phi_{i,k-1} \\ \phi_{i,k} \\ \phi_{i,k+1} \\ \vdots \\ \phi_{N_r,N_z} \end{pmatrix}$$

$$right = \begin{pmatrix} & & & & & \vdots & & & & & \\ X & R_{i*N_z+k,(i-1)*N_z+k} & X & \cdots & X & R_{i*N_z+k,i*N_z+k} & X & \cdots & X & R_{i*N_z+k,(i+1)*N_z+k} & X \\ & & & & & \vdots & & & & & \end{pmatrix} \begin{pmatrix} \phi_{0,0} \\ \vdots \\ \phi_{i,k-1} \\ \phi_{i,k} \\ \phi_{i,k+1} \\ \vdots \\ \phi_{N_r,N_z} \end{pmatrix}$$

- Matrix size: (Nr*Nz, Nr*Nz), elements are determined by the equilibrium quantities
- Left hand side: 3 nonzero elements per row
- Right hand side: 9 nonzero elements per row

Search for the global eigenmode

- Eigen-functions depend on both r and z
- Global eigen-functions are smooth in both r and z directions
- Continuum eigen-functions are smooth in z direction but singular in r direction
- Numerical eigen-functions caused by discretization are jumpy in both r and z directions

Future work

- Use the numerical eigenmode analysis to benchmark the simulation results
- Try to explain the disagreement between the our simulation results and the analytical MHD results
- Destabilize the global Alfven eigenmode by adding the gyrokinetic ions and energetic particles (Alpha particles) to the simulation

Summary

- **TAE with kinetic electrons at low beta**
  - The TAE eigen-frequency and global mode structure are achieved
  - Limited numbers of grids and particles restrict the simulations being in low beta regime ($\beta \sim 10^{-5}$)
- **Energetic particle driven instability with kinetic electrons**
  - Mode is driven unstable by energetic particles
  - The mode structure is different with the TAE mode
- **TAE simulation with the hybrid model**
  - Global TAE mode is observed
  - The eigen-frequency is different from the MHD theory prediction
- **Benchmark the simulation with the MHD eigenmode analysis**
  - Solve the MHD eigenmode equations according to GEM in field-line-following coordinates
  - The eigen-frequency will be benchmarked by the numerical eigenmode calculation and be driven unstable by the hot Alpha particles