Numerical experiments on the drift wave-zonal flow paradigm for nonlinear saturation

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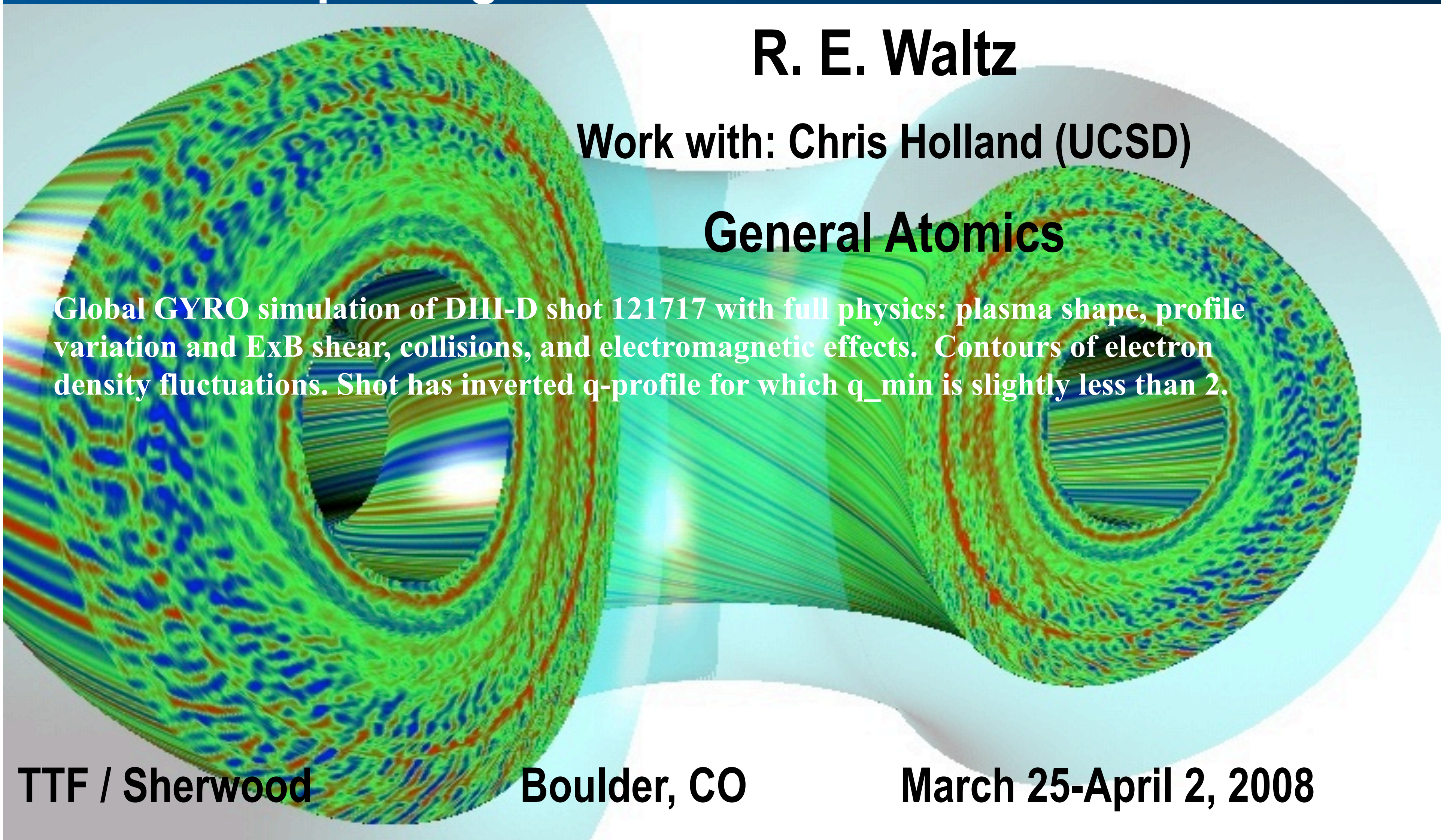
Work with: Chris Holland (UCSD)

General Atomics



Global GYRO simulation of DIII-D shot 121717 with full physics: plasma shape, profile variation and ExB shear, collisions, and electromagnetic effects. Contours of electron density fluctuations. Shot has inverted q-profile for which q_min is slightly less than 2.

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Abstract

The ITG-adiabatic electron (-ae) gyro-landau-fluid simulations of the early 90's [1,2] established that the ExB shear from toroidaly symmetric ($n = 0$) "radial modes" provide the dominant nonlinear saturation mechanism for drift wave turbulence. This is loosely referred to as the "drift wave-zonal flow paradigm" for nonlinear saturation [3]. Actually the radial modes (labeled by a radial wave number $k_x \neq 0$) have several components: a residual or zero frequency "zonal flow" part and an oscillatory "geodesic acoustic mode" (GAM) part. The ExB residual flow is nearly in balance with the ion pressure diamagnetic flow [4], hence radial modes have little net fluid flow. The time average residual flow shears result in equilibrium "profile corrugations" near low order rational surfaces [5]. The zonal flows are weakly damped only by ion-ion collisions (which we ignore) and the GAM's are strongly Landau damped only at low to moderate q. At high-q the Hinton-Rosenbluth residual flow vanish and only the GAM's remain. Curiously none of the rich physics of radial modes has been used in nonlinear saturation models which refer only to the linear growth rates of the [$n > 0,\ k_x = 0$] transport producing modes. What is the difference between the residual zonal flow saturation in the low-q (core) and GAM saturation in the high-q (edge)? Do the mechanisms and "paradigm" apply equally well to TEM and ETG turbulence?

To explore these and other questions, we have done "numerical experiments" with GYRO by modifying components of the nonlinear coupling convolution and modifying the linear physics of the radial modes *while keeping the linear physics of the finite-n modes unchanged*. In the latter we modify the "q" in the radial modes to trade off the zonal flows versus the GAMs, modify the "1/R" curvature in the radial modes to vary the GAM frequency, as well as the turn off the radial mode Landau damping. We find: (1) the *nonlinear coupling triads* $[n_1 \neq 0,],\ [n_2 = 0],\ [n = n_1]$ account for nearly all of the nonlinear saturation; (2) the ExB shear $(\delta\phi)$ components of the radial modes nonlinearly stabilize while the diamagnetic (δf) components nonlinearly destabilize; (3) transport increases as the zonal flow residuals and GAM damping decrease; (4) transport decreases as the GAM frequency decreases; and (5) the transport is largely unchanged without GAM Landau damping. From contour plots of the time-average nonlinear transfer function [$T(\vec{k}) = -2\gamma_k E(\vec{k})$ with $\sum_k T(\vec{k}) = 0$], we determine if (6) the radial modes provide a small net sink of turbulent energy from GAM Landau damping. Finally contrary to previous work, we find all these mechanisms and "the paradigm" are universal: Conclusions (1-5) hold equally well for ITG-ae, ITG/TEM, and purely TEM transport; and (1-2) appear to hold for ETG transport.

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Outline

- The basic drift wave-zonal flow paradigm
  __definition of components:
  zonal flow (ExB vs diamagnetic) residuals vs GAMs

- Zonal flow (core) vs GAM (edge) nonlinear saturation

- What is the dominant nonlinear saturation mechanism and is it the same for ITG, TEM, and ETG ?
  __ How universal is the paradigm?

- Spectrum of entropy generation rate:
  "transport" generation, nonlinear transfer, and dissipation.

Basic drift wave-zonal flow paradigm: definitions

- 90's ITG-ae Gyro-Landau-Fluid simulations [Waltz et al 1994, Hammett et al 1994] showed that ExB shear from toroidally symmetric (n=0) "radial modes" dominate nonlinear saturation (without saturation hardly exists)….loosely referred to as the drift-wave zonal flow paradigm [Diamond et al 2005 review]

- "Zonal flow" refers to the zero frequency (ω =0) or residual component after the oscillatory ($\omega \sim V_{thi}/R$) "geodesic acoustic mode" damps away by Landau damping (Damping from ion-ion collisions is very weak and ignored here.)

- Zonal "flow" normally denotes the ExB flow ($\delta\phi$−part) which is nearly ion force balanced by a diamagnetic flow (δf−part) [Dimits 2001] : $\delta V_{ExB} + \delta V_{*i} \sim \delta V_{\perp} \sim 0$
  _ Zonal flows actually have little mass "flow"

- The fluctuations in the components of zonal flows "linear" flux surface & time average away except near low order rational surfaces (3/2, 2/1, 5/2, 3/1…) where they result result in the observed gradient "profile corrugations" [Waltz et al 2006]

Zonal flow (core) vs GAM (edge) nonlinear saturation

- The Hinton-Rosenbluth ZF residual potential (low k_x) rapidly decreases with increasing q as does the GAM damping: $\delta\phi(t)/\delta\phi(0) = (1 - A_R)\cos(\omega_G t)\exp(-\gamma_G t) + A_R$

$$A_R = 1/[1 + 1.6 q_0{}^2/\varepsilon^{1/2}] \qquad \gamma_G = \omega_G \exp(-q_0^2) \qquad \omega_G = (7/4 + T_e/T_i) v_{thi}/R_0$$

- We consider the GA-std ITG-ae case: $q = 2 \;\; R/a = 3 \;\; s = 1 \;\; a/L_T = 3 \;\; a/L_n = 1 \;\; T_i/T_e = 1$

which fixes the driving rates of the n>0 modes, but we trade off ZF for GAM's by increasing q_0 (core to edge) and slow the GAM oscillations by increasing R_0

q_0	R_0/a	χ_i/χ_{gB}
1	3	2.73 ± 0.04
2 (std)	3	3.61 ± 0.09
4	3	4.21 ± 0.20
6	3*	4.98 ± 0.33
12	3	6.93 ± 1.05

⇑

Although GAM's are less damped, transport increases as ZF residual decreases

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12	3	6.93±1.05
3	6	2.38±0.05
6	6*	1.27±0.16

⇑ ⇓ Although GAM's are less damped, transport increases as ZF residual decreases

⟸ Comparing cases * at fixed high-q, the ExB shearing from GAM's can be much more effective when slowed down…increasing

⟸ $\Delta\theta_0^{max} = \gamma_E^{max}/[\sqrt{2} s \omega_G]$ swing to good curv.

ExB shear from n=0 accounts for nearly all nonlinear saturation for ITG and TEM low-k turbulence

- Keeping linear physics (or n=0 and n > 0) fixed we modify the nonlinear coupling

	ITG-ae
	χ_i/χ_{gB}
all n coupled	3.61 ± 0.09
n=0 only	3.52 ± 0.03
decoup n=0 $\delta\phi$	159 ± 70
decoup n=0 δf *	0.003
decoup n=0 $\delta\phi \& \delta f$	$large$

GA-std:

$q=2 \;\; R/a=3 \;\; s=1 \;\; a/L_T=3 \;\; a/L_n=1 \;\; T_i/T_e=1$

- The n=0 diamagnetic flow component δf * seem to be destabilizing

ExB shear from n=0 accounts for nearly all nonlinear saturation for ITG and TEM low-k turbulence

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	ITG-ae	ITG/TEM		TEM*	
	χ_i/χ_{gB}	χ_i/χ_{gB}	χ_e/χ_{gB}	χ_i/χ_{gB}	χ_e/χ_{gB}
all n coupled	3.61 ± 0.09	10.66 ± 0.20	3.00 ± 0.03	28.78 ± 0.57	33.81 ± 0.67
n=0 only	3.52 ± 0.03	12.06 ± 0.02	3.30 ± 0.04	25.22 ± 0.82	29.48 ± 0.93
decoup n=0 $\delta\phi$	159 ± 70	956 ± 6	198 ± 5	359 ± 7	475 ± 12
decoup n=0 δf *	0.003	0.004	0.04	0.20 ± 0.07	$0.17 \pm .06$
decoup n=0 $\delta\phi \,\&\, \delta f$	large	> 300	> 80	> 300	> 300

GA-std:

$q = 2 \;\; R/a = 3 \;\; s = 1 \;\; a/L_T = 3 \;\; a/L_n = 1 \;\; T_i/T_e = 1$

***GA-TEM2:**

$a/L_T = 3 \Rightarrow 1 \;\; a/L_n = 1 \Rightarrow 3$

- The n=0 diamagnetic flow component δf * seem to be destabilizing

The paradigm appears to be universal: it even seems to apply to high-k ETG

- ETG-ai saturation is often unreliable [Nevins 2007]. Here we employ the λ−model which increases the n=0 zonal flow inertia from ITG-ae ($\lambda=0$) to ETG-ai ($\lambda=1$):

$$\partial[\lambda + k_x^2 \rho^2]\delta\phi_{kx}^0 / \partial t = NL - pump$$

GA-std case	$\lambda=0$ $(ITG-ae)$	$\lambda=0.2$	$\lambda=0.4$	$\lambda=0.6$	$\lambda=1.0$ $(ETG-ai)$
	χ_i/χ_{gB}	χ/χ_{gB}	χ/χ_{gB}	χ/χ_{gB}	χ_e/χ_{gB}
all n coupled	3.61 ± 0.09	3.27 ± 0.13	5.29 ± 0.83	24.1 ± 0.88	$\infty \pm \infty/2$
n=0 only	3.52 ± 0.03	3.36 ± 0.05	6.48 ± 0.63	17.67 ± 1.5	

- At the same NL-pumping rate ETG scale zonal flow are weak because of their larger inertia. Less ExB shearing of n > 0 transport modes makes larger at the same linear driving rate (which in turn makes the pumping larger).

- From coupled ITG/TEM-ETG simulations (i.e. ETG-ki) [Waltz 2007] it appears that actually the ion scale zonal flows are largely responsible for ETG saturation.

Spectrum of the entropy generation rate: "transport" generation, nonlinear transfer, and dissipation

- Flux surface (and phase space) average of entropy conservation equation:

$$\partial[S+W]/\partial t = \sum_{\vec{k}} T_{\vec{k}} + \sum_{\vec{k}}[\sum_s n_0^s \hat{T}_0^s \chi_{\vec{k}}^s / \chi_{gB}](a/L_T^s)^2 + \sum_{\vec{k}} D_{\vec{k}}^{dissip} = 0 (steady\ \ state)$$

$$S = \sum_{\vec{k}} S_k = \left\langle \sum_s \int dv^3 \hat{T}_0^s F_0^s |\delta\hat{f}_{\vec{k}}^s|^2 /2 \right\rangle \qquad W = \left\langle \int dv^3 F_0 \sum_{\vec{k}} [\delta\hat{\phi}_{\vec{k}}^* G_k \{\delta\hat{\phi}_{\vec{k}}\}]/2 \right\rangle \qquad G_k \sim k_\perp^2 \rho_s^2$$

$S >> W$ is the total incremental entropy [Candy 2005], some times called the conserved "turbulent energy"

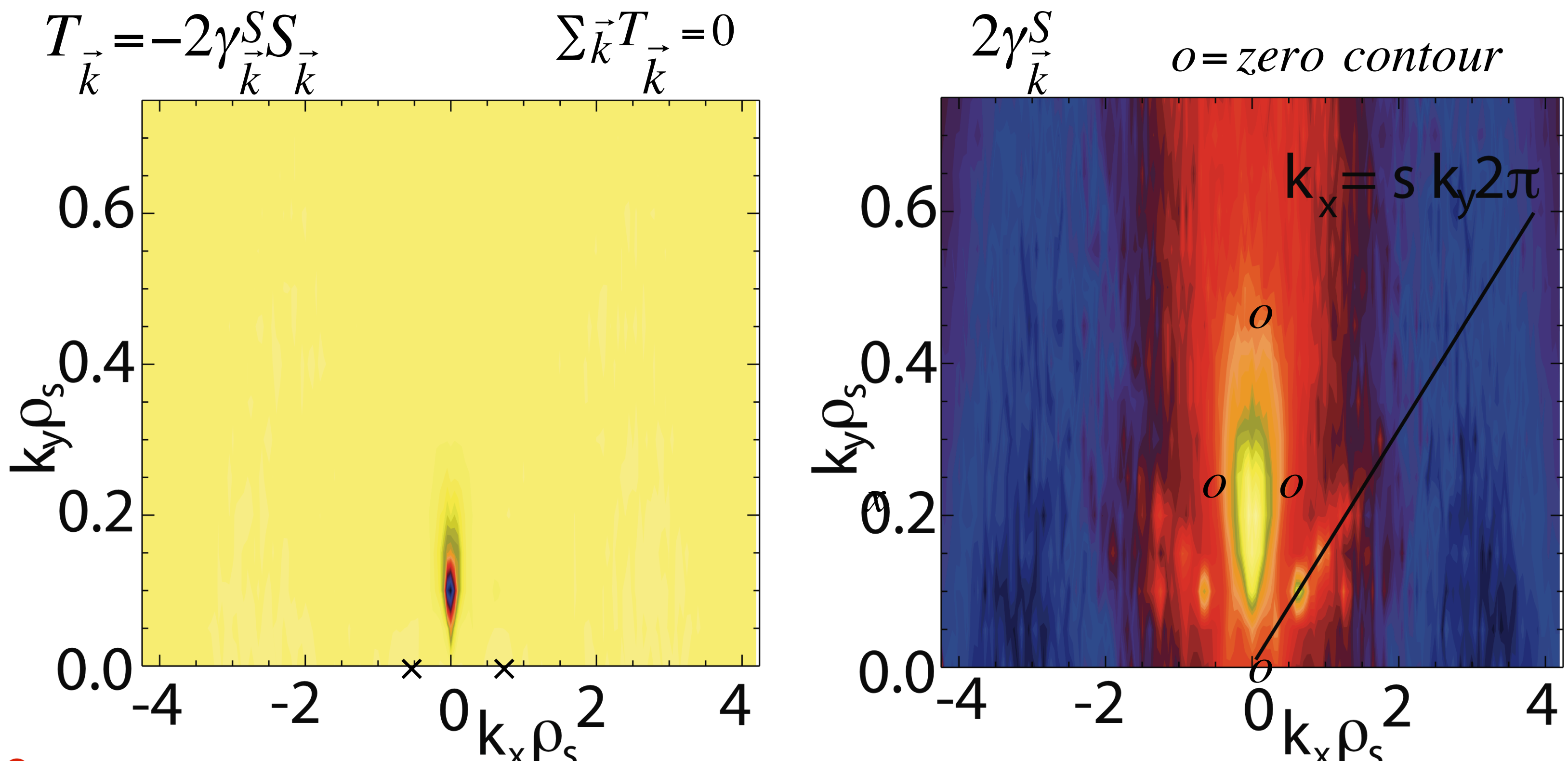
$T_k = \left\langle \sum_s \int dv^3 T_0^s F_0^s \mathrm{Re}[\delta\hat{f}_{\vec{k}}^{s*} [\delta\hat{\vec{v}}_E \cdot \hat{\vec{\nabla}} \delta\hat{f}^s]_k \right\rangle$ is the nonlinear transfer of entropy & $\sum_{\vec{k}} T_{\vec{k}} = 0$

$$-T_{\vec{k}} = \sum_s n_0^s \hat{T}_0^s [\chi_{\vec{k}}^s / \chi_{gB}](a/L_T^s)^2 + D_{\vec{k}}^{dissip} \equiv 2\gamma_{\vec{k}}^S S_{\vec{k}}$$

- $2\gamma_k^S$ is the net rate of entropy generation (transport - dissip.) at each $\vec{k} = [k_x, k_y] \Rightarrow [p, n]$

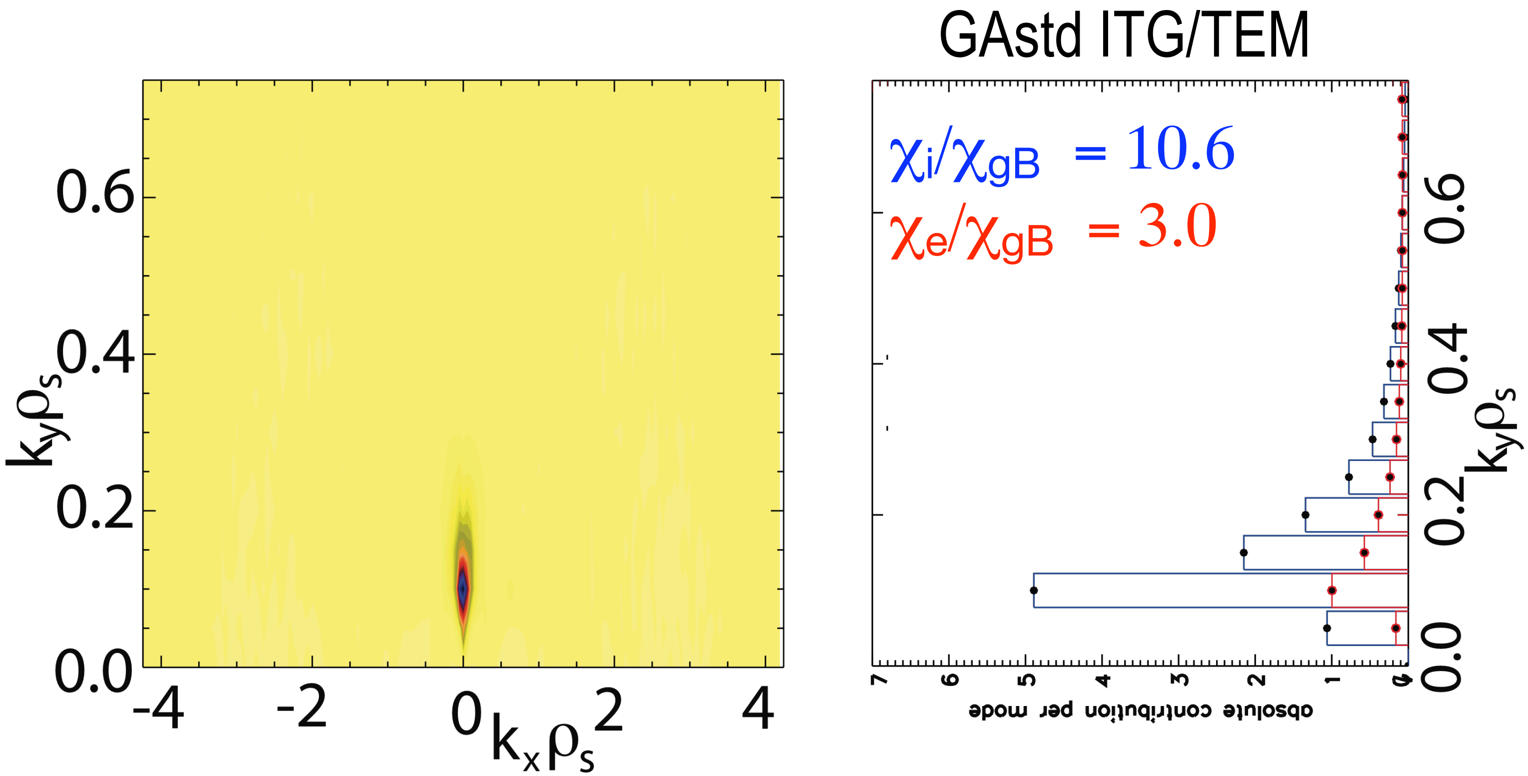
- How much entropy does the n=0 (k_y=0) GAM damping dissipate ?

Spectrum of the entropy generation rate (cont'd)



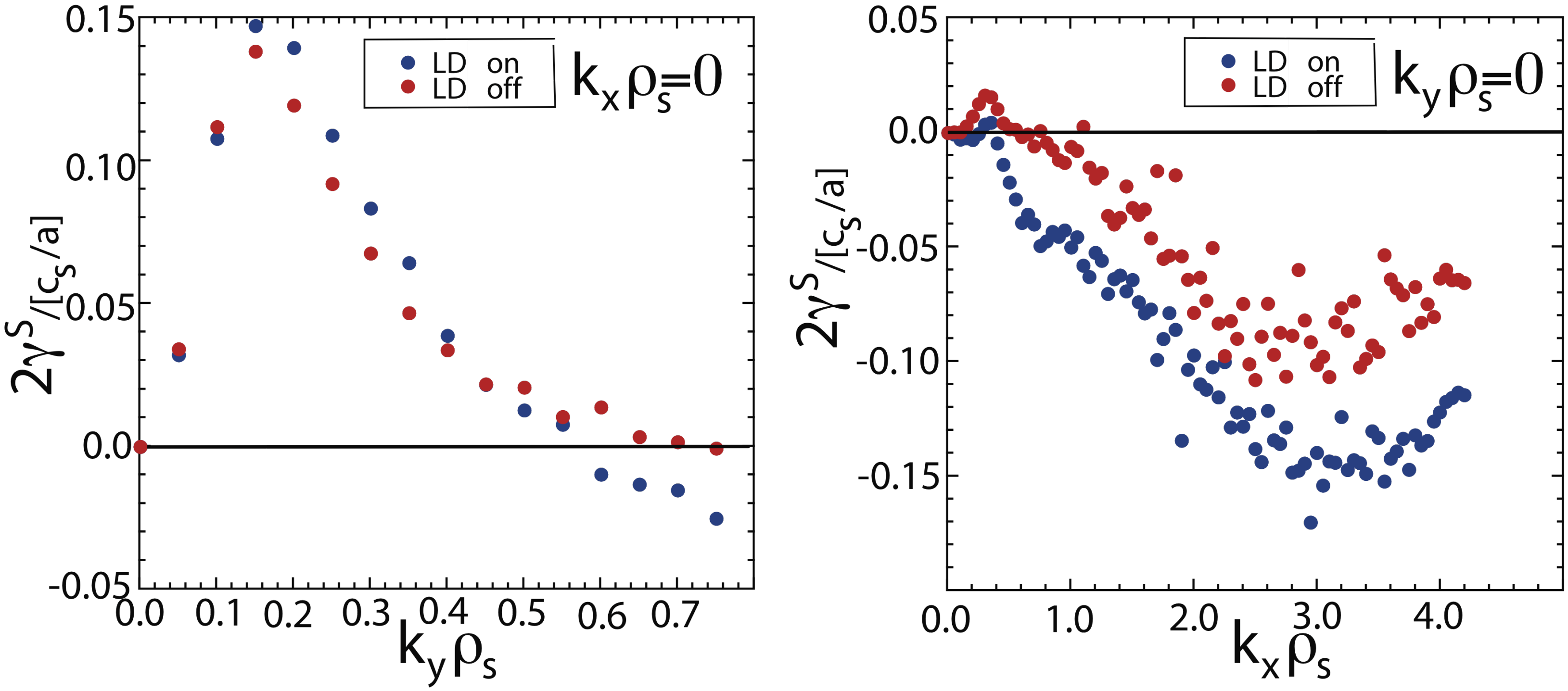
- **The largest negative $T_{\vec k}$ value (transfer from) is at $[\hat k_y,\hat k_x]=[0.1,0.0]$ where the maximum transport occurs and is about ~115x the largest positive value (transfer to) at (x) $[\hat k_y,\hat k_x]=[0.0,\pm 0.65]$ where $2\gamma^S_{\vec k}=-0.037[c_s/a]$ & $2\gamma^S_{\vec k}|_{max}=0.149[c_s/a]$ at $[\hat k_y,\hat k_x]=[0.17,0.0]$**
- **Hence GAM (Landau) damping contributes to the sink, but the sink is spread equally over all n.** **[Note: $2\gamma^{max}=0.45[c_s/a]$ at $[\hat k_y,\hat k_x]=[0.3,0.0]$]**

Spectrum of the entropy generation rate (cont'd)



- **Peak entropy production coincides with with maximum "drain" nonlinear transfer.**

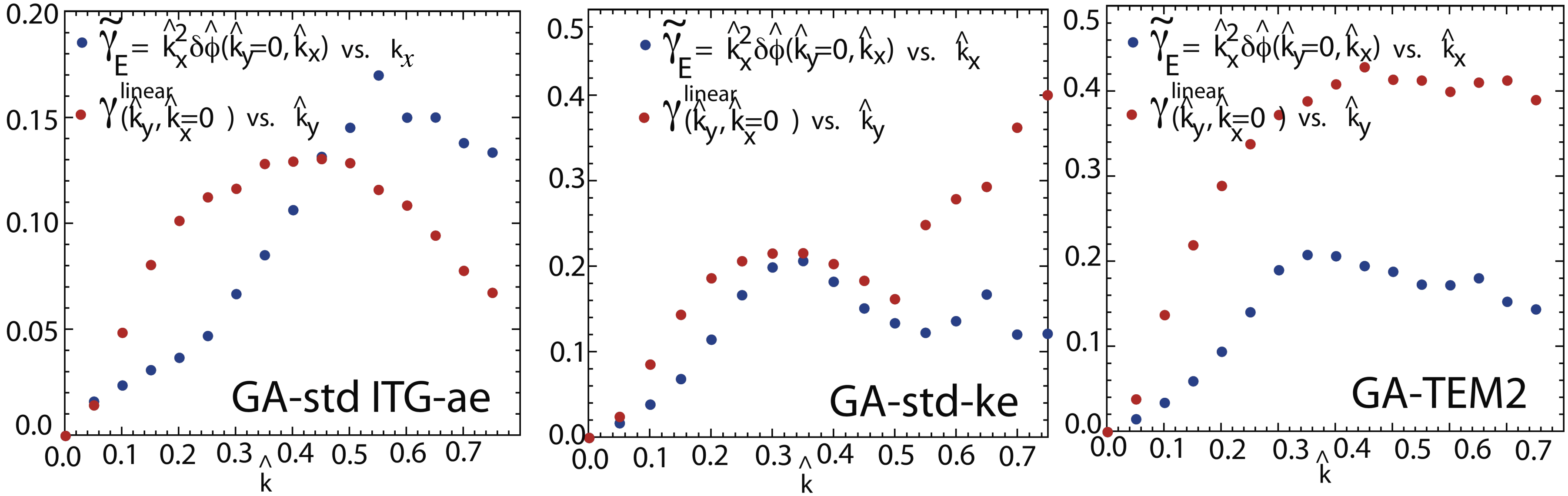
Spectrum of the entropy generation rate: generation along k_y and dissipation along k_x



LD	χ_i/χ_{gB}	χ_e/χ_{gB}	D_e/χ_{gB}
on	11.82	3.27	-1.99
off	13.17	3.55	-1.80

- **Landau damping adds to entropy dissip. and has small stabilizing effect on transport**

Saturation rule for zonal flows



$$\tilde{\gamma}_E^{rms\ time\ ave}(\hat{k}_y=0, \hat{k}_x=\hat{k}) \sim (1.0 \pm 0.5)\ \gamma^{linear}(\hat{k}_y=\hat{k}, \hat{k}_x=0)$$

$\Rightarrow$

$$|\delta\phi|^{rms\ time\ ave}(\hat{k}_y=0, \hat{k}_x=\hat{k}) \sim (1 \pm 0.5)\ \gamma^{linear}(\hat{k}_y=\hat{k}, \hat{k}_x=0)/\hat{k}_x^2$$

Very similar to finite-n saturation rule used in transport models like TGLF:

$$|\delta\phi|^{rms\ time\ ave}(\hat{k}_y, \hat{k}_x=0) \propto \gamma^{linear}(\hat{k}_y, \hat{k}_x=0)/\hat{k}_y^2$$

Which makes no account of ZF-GAM physics !!!!

Summary:

- The n=0 zonal flows in the low-q core are more effective than GAMs at the high-q edge in nonlinearly stabilizing the high-n transport (at the same driving rate).

- The dominant nonlinearly coupling is n> 0 <–> n=0.
  - remaining n1>0 <–>n2>0 coupling is very small.
  - n=0 ExB flow shearing of n>0 is stabilizing
  - n=0 diamagnetic flow shearing of n>0 is destabilizing.

- The "zonal flow-drift wave nonlinear saturation paradigm is universal:
  applies to ITG-ae, ITG/TEM, TEM, ETG-ai

- Landau damping of GAMs contributes to the entropy dissipation sink for the "transport" entropy generated source, but the "sink rate" is spread equally over all n

- An zonal flow-GAM ExB shear saturation rule is provide by

$$\tilde{\gamma}_E^{rms\ time\ ave}(\hat{k}_y=0,\ \hat{k}_x=\hat{k}) \sim (1.0\pm0.5)\ \gamma^{linear}(\hat{k}_y=\hat{k},\ \hat{k}_x=0)$$

Five year synopsis of GYRO physics results

GYRO[Candy 2003a] publications demonstrating:

[2002]

* Bohm to gyroBohm transition at decreasing rho-star in global gyrokinetic ITG- adiabatic electron simulations [Waltz 2002].

[2003]

* Bohm scaling in physically realistic+ gyrokinetic simulations of DIII-D L-mode rho-star pair matching transport within error bars on ion temperature gradients[Candy 2003b]

+(ITG ions and finite-beta and collisional electron physics with real shaped geometry, profile ExB shear, experimental profiles, etc)

[2004]

* small turbulent dynamo in tokamak current-voltage relation[Hinton 2004]
* local gyrobohm flux simulations to be vanishing rho-star limit of global simulations [Candy 2004].

•transport is smooth across minimum-q surface [Candy 2004b]

[2005]

* global gyrokinetic transport solutions, i.e. predicted temperature and density profiles from balance of transport and source flows [Waltz 2005a].
* electron temperature gradient drives plasma flow pinches and recovered the D-V description of experimental Helium transport studies[Estrada-Mila 2005].
* weak beta scaling of transport up to about half the MHD beta limit [Candy 2005]
* turbulence draining from unstable radii and spreading to stable radii providing a heuristic model of non-local transport [Waltz 2005b,Waltz 2005c].

Synopsis (cont'd)

[2006]

* connection between velocity space resolution, entropy saturation and conservation, and numerical dissipation [Candy 2006a].

* perfectly projected experimental profiles in rho-star gyroBohm-like DIII-D H-modes to Bohm-scaled local diffusivity while simulation of actual profiles showed gyroBohm scaling and match transport within error bars. Perfectly project Bohm-like DIII-D L-mode simulations remained Bohm [Waltz 2006a]

* profile corrugations at low-order rational surfaces observed in DIII-D minimum q=2 discharges providing an ExB shear layer to initiate a transport barrier [Waltz 2006b].

* that including so-called parallel nonlinearity has no effect on simulated energy transport at rho_stars less than one percent [Candy 200b]

* density peaking from plasma pinch in DIII-D L-mode simulations with actual collisionality[Estada-Mila 2006b].

* first simulation of fusion hot alpha transport from ITG/TEM micro-turbulence found to be small with ITER parameters [Estada-Mila 2006b].

Synopsis (cont'd)

[2007]

* ETG simulations with kinetic ion cures unphysically large saturation levels in controversial and conventional ETG simulations with adiabatic ions [Candy 2006c, Candy 2007]

* high-Rynolds number coupled ITG/TEM-ETG simulations (at close to physical ion to electron mass ratio) show low-k ITG/TEM and high-k ETG transport decoupled when both strongly driven but ITG/TEM can drive ETG transport in ETG stable plasmas; high-k spectrum tends to be isotropic [Waltz 2007a]

* 400+ web parameter scan database of flux tube simulations used to fit nonlinear saturation rule with ExB shear stabilization in TGLF [Staebler 2005, Staebler 2007] theory based transport code model [Kinsey 2005, Kinsey 2006, Kinsey 2007]

* angular momentum pinch from ExB shear and pinch from "coriolis" force important for understanding experiments with intrinsic toroidal rotation; turbulent shift from neoclassical poloidal rotation is small [Waltz 2007b]

* radially integrated turbulent ohmic heating from parallel and drift currents is actually close to an electron-ion energy exchange and small compared to energy transport flow [Waltz 2008]

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