# Resonance Coherency, Transport Events and Spreading of CTEM Turbulence

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Caveat Emptor: Work in Progress

# Outline

## A.) Prelude: General Issues in "Turbulence Spreading" and Non-Locality

- questions re: turbulence propagation
- diffusion vs front propagation (c.f. X. Garbet, et. al.; PoP 2007)
- symptoms of spreading in stationary state

## B.) Spreading and Resonance in CTEM

- why CTEM - strong resonance coherency
GTC sims $\leftrightarrow$ granulation (life beyond Z.F.)
- how describe propagation/spreading in kinetic models?
- granulation emission enhances spreading
- Future Plans

A.) Issues - Models of Nonlocality

Gyro-Bohm breaking
- fast pulse propagation
excitation in stable regions
⋮

} "non-local" character to/in turbulence

- models
  - gradient competition → avalanching SOC, critical gradient....
  - intensity transport → turbulence spreading (c.f. a la $k$-$\epsilon$)

⇒ Many Questions: (Partial Listing)

- front propagation vs. nonlinear diffusion ?
- self-generated noise effects – {Z. Wang, et al. this meeting}
- how identify/quantify spreading in ≈ stationary states (re: modelling) ? (C. Holland)
- systematics, fully kinetic formulation ? resonant avalanching
  → CTEM simulations (Lin; EPS 07),
  → EPM avalanching (Zonca, et al.; '05)

→ Fronts vs. Diffusion? { X. Garbet, et. al. PoP '07

- model combining { marginality / intensity transport

(extends Gurcan, P.D. et. al. '05; Naulin, et. al. '06)

$$\partial_t \mathcal{E} = \underbrace{\gamma_0(\partial_x T - \kappa_c)}_{\text{(growth)}} \underbrace{H(\partial_x T - \kappa_c)}_{\text{(threshold)}} \mathcal{E}$$

$$- \underbrace{\gamma_{NL} \mathcal{E}^2}_{\text{(nonlinear saturation)}} + \underbrace{D_0 \partial_x (\mathcal{E} \partial_x \mathcal{E})}_{\text{(diffusive scattering of intensity by NL)}}$$

$$\partial_t T = \underbrace{D_0 \partial_x (\mathcal{E} \partial_x T)}_{\text{turbulent transport}} + \underbrace{D_c \partial_x^2 T}_{\text{collisional transport}} + \underbrace{S}_{\text{heat source}}$$

- bottom line:

- both diffusive and ballistic front dynamics possible, but:

- self-similar "Fisher" front if: (heat flux critical $\Phi_{crit} \sim \frac{k_c^2 \gamma_0 \delta k}{\gamma_{NL}}$) { near margins; finite perturb. scale

- $V_f \sim 100 - 1000$ m/sec. possible → fast propagat.

→ Spreading in Stationary State? (G. Holland)

- Key Question ⇒ Is turbulence at $r = r_0$ a consequence of:
  - local growth ?
  - influx from spreading process ?

→ especially relevant if local $\left\{ \begin{array}{l} \text{fluctuations} \\ \text{flux} \end{array} \right.$ underpredicted at increasing $\rho$.

- Need examine match fluctuation intensity profile.

i.e. generically:

$$\partial_t \mathcal{E} = \gamma \mathcal{E} - \gamma_{NL} \mathcal{E}^2 + \partial_x ( D_0 \mathcal{E} \partial_x \mathcal{E} )$$

for interval $[x_1, x_2]$

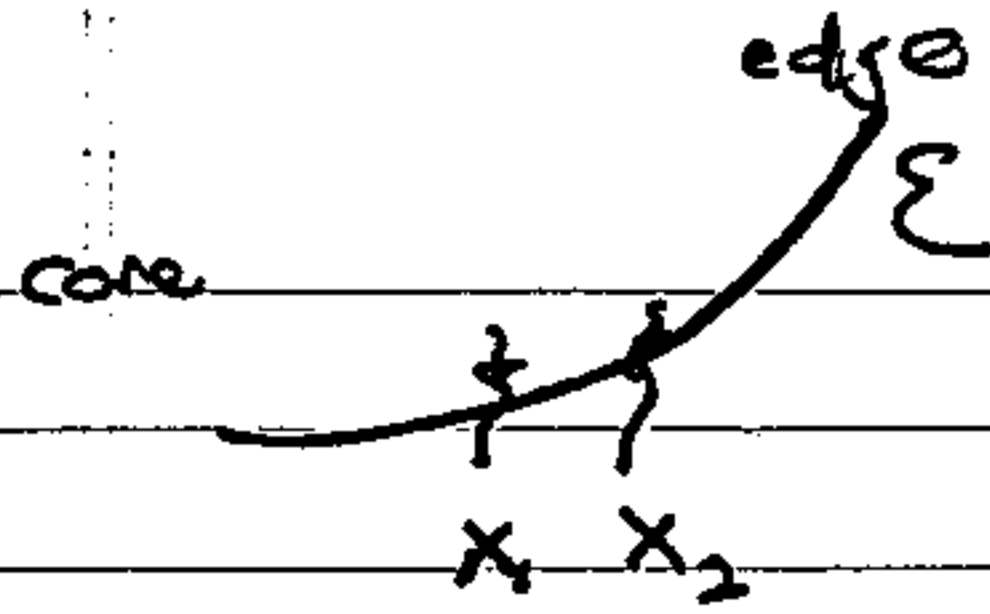
- growth locally → $\int_{x_1}^{x_2} dx \, \gamma(x) \, \mathcal{E}(x)$

influx/outflow → $D_0 \partial_x (\mathcal{E}^2/2) \Big|_{x_1}^{x_2}$

i.e. $\underbrace{\Delta}_{\text{jump}} (\mathcal{E}^2)' > 0$ → influx

$< 0$ → outflow

i.e.




$\left\{ \dfrac{\text{corner}}{\text{is critical}} \right\}$ not slope

⇒ suggests intensity rise at edge will drive transition layer of strong turbulence between:

- edge
- stiff core

(c.f. Parail, JET, Hahm, et al. '05)

⇒ quench of edge turbulence in H-mode can result in turbulence reduction in outer core (more than change in b.c.) → $\Delta(\mathcal{E}^2)$ flips sign.

Suggests:

- fluctuation intensity profile should be routine output of modelling and focus of validation studies ⇒ curvature

* - regions of large, + curvature in $\mathcal{E}$ likely spreading driven ...

- some interest in direct analysis of intensity flux (akin "spatial bi-coherence") from simulation, experiment

B.) Spreading and Resonance - CTEM

→ interesting features/challenges (beyond zonal structures)

- drift resonance resembles 1D:

i.e. $1/(\omega - \overline{\omega}_D \epsilon)$
↳ bounce avg. precession freq.

⇒ wave-particle decorrelation rate

$$1/\tau_{ac} = \left| \frac{d\omega}{dk_\perp} - \frac{\omega}{k_\perp} \right| |\Delta k_\perp|$$

- weak dispersion ⇒

→ coherence time long, even for broad spectrum, "turbulence"
→ strong resonance distortion (i.e. granulation, trapping .....)
→ coherent over extended region ...
→ granulations Cerenkov emit

∴ Spreading, with resonance coherency

→ model must:

- be fundamentally kinetic, ab-initio

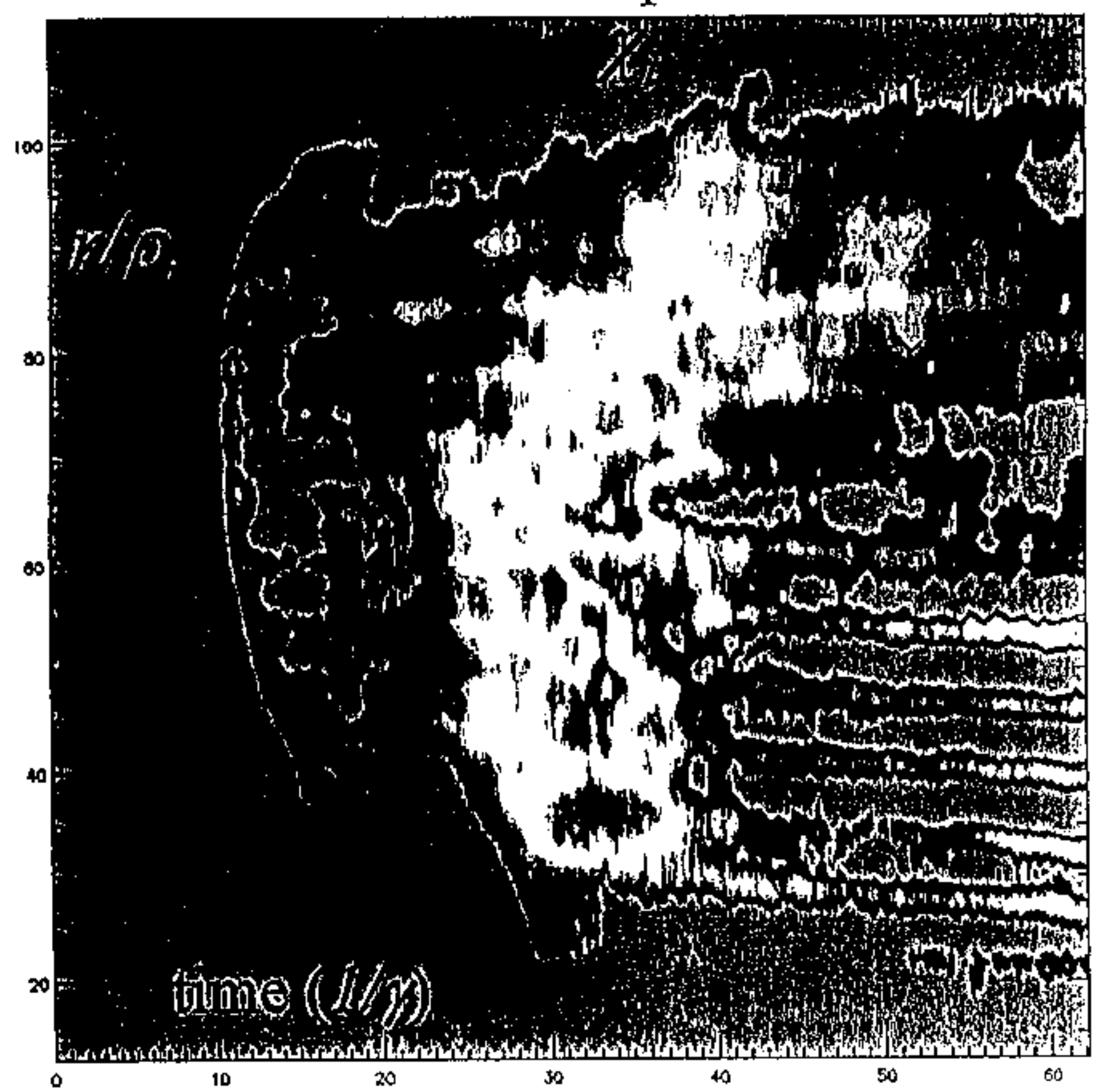
- treat resonance coherency

# CTEM Nonlinear Bursting & Spreading

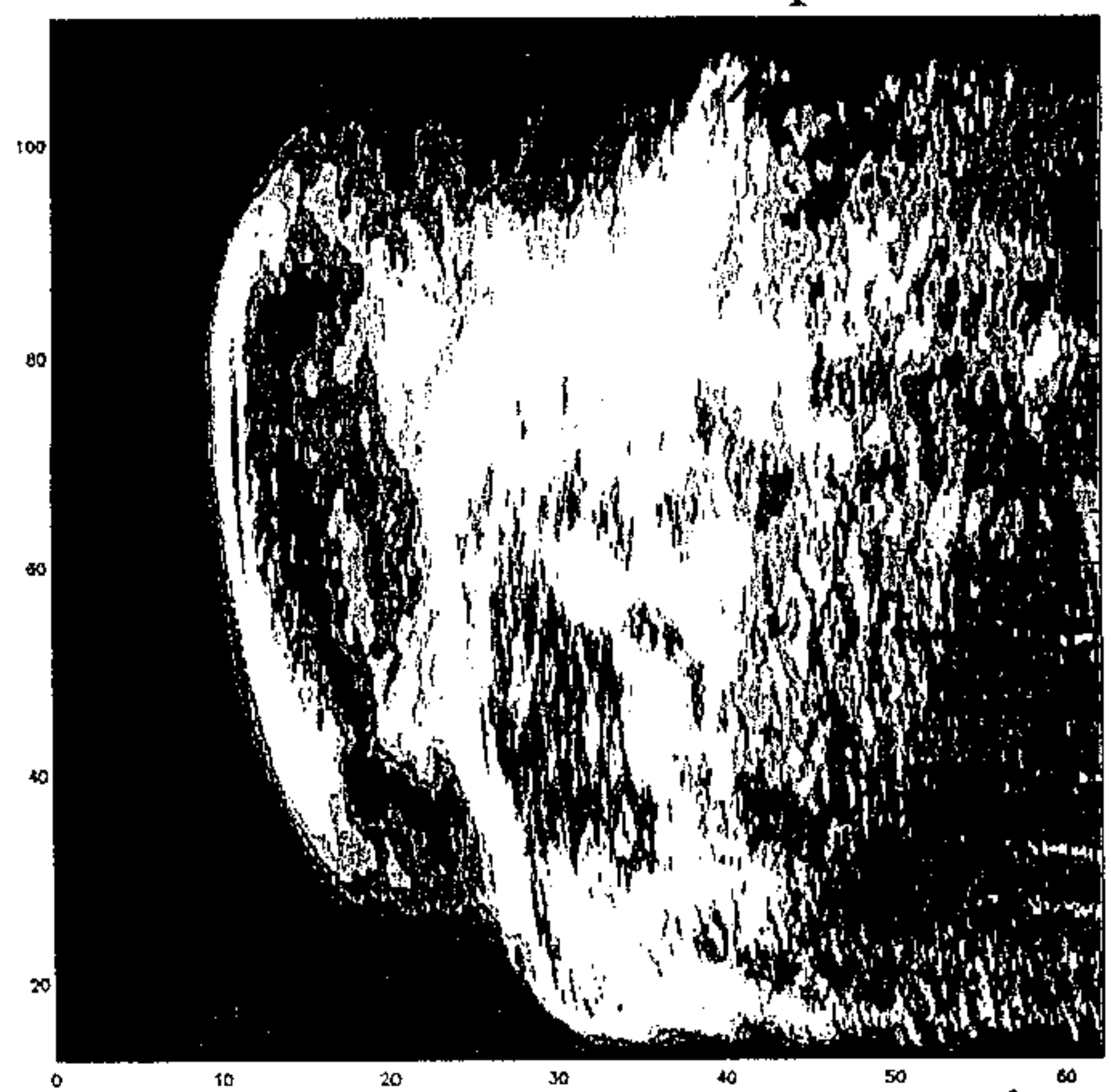
*UCI*

- Burst originates at $r=0.5a$, spreading both inward & outward Lin, et. al.
- Inward spreading ballistic with a speed close to drift velocity (!)
- Outward propagation much faster (?)

Ion transport




Electron transport



GTC. Global and Mesoscales.

→ **Spreading of Vlasov Turbulence**

– consider first 1D – "velocity spreading"
i.e. expansion of resonant region

fundamental quantity : $\langle \delta f(1)\, \delta f(2) \rangle$

phase space density correlation function

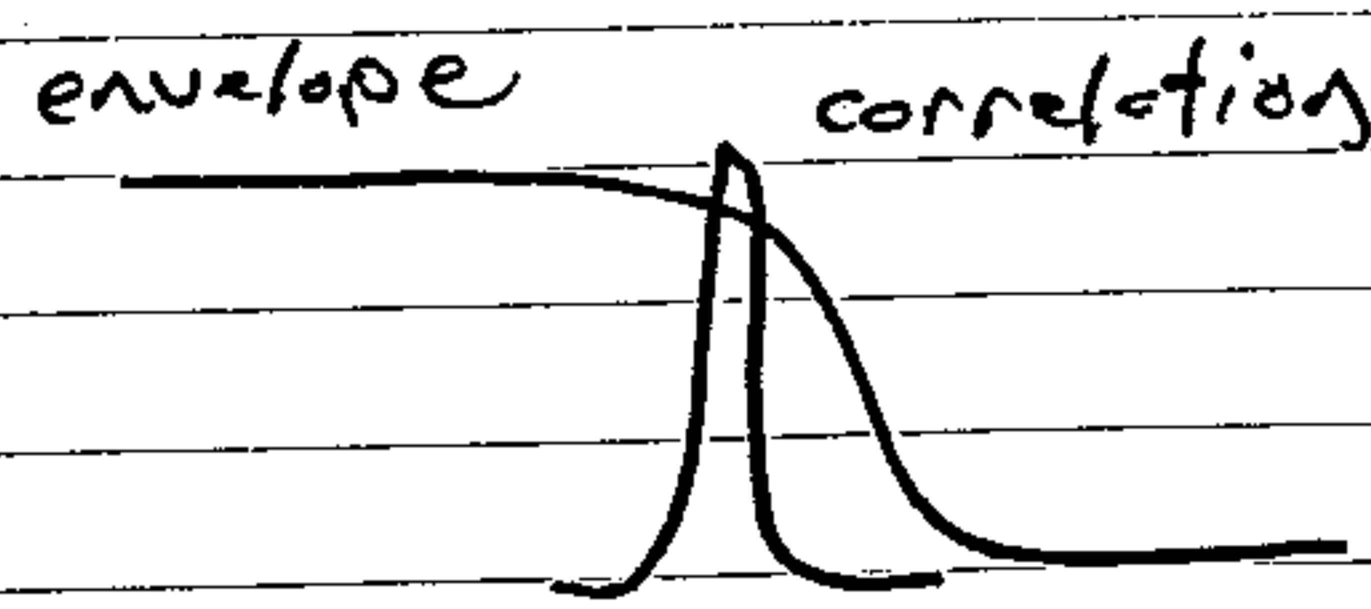
$$\langle \delta f(1)\, \delta f(2) \rangle = C(x_-, v_-;\, x_+, v_+)$$

$x_-, v_-$ → granulation structure (Dupree, et. al.)

– coupled

$x_+, v_+$ → envelope ↔ phase space intensity field structure

i.e.




⇒ need study $\langle \delta f^2 \rangle = I_{ph}$ → entropy intensity
to describe evolution of turbulence field

$$= \lim_{x_-, v_- \to 0} \langle \delta f(1)\, \delta f(2) \rangle$$

## → Describing Intensity Spreading $\left(\frac{q}{M} \equiv 1\right)$

a) $$\frac{\partial \delta f}{\partial t} + v\frac{\partial}{\partial x}\delta f + \tilde{E}\frac{\partial \delta f}{\partial v} = -\tilde{E}\frac{\partial \langle f \rangle}{\partial v}$$

so the quickie is: $\left\{\text{treat } I_{\Pi} \text{ as an advected "stuff"}\right.$

$$\frac{\partial}{\partial t} I_{\Pi} + v\frac{\partial}{\partial x} I_{\Pi} + \left\langle \frac{\tilde{E}}{2}\frac{\partial}{\partial v} I_{\Pi} \right\rangle = -\langle \tilde{E}\delta f \rangle \frac{\partial \langle f \rangle}{\partial v}$$

closure + average ($x \leftrightarrow$ symmetry)

$$\frac{\partial \langle I_{\Pi} \rangle}{\partial t} - \overset{①}{\frac{\partial}{\partial v} D \frac{\partial \langle I_{\Pi} \rangle}{\partial v}} = \overset{②}{-\langle \tilde{E}\delta f \rangle \frac{\partial \langle f \rangle}{\partial v}}$$

where $\langle I_{\Pi} \rangle = \int \frac{dx}{L} \langle \delta f^2 \rangle$

[ $I_{\Pi}$ pinche possible if $\underline{\nabla} \cdot \underline{V}_E \neq 0$

$$\langle \tilde{E}\delta f \rangle = \lim_{2 \to 1} \langle \tilde{E}(2)\, \delta f(1) \rangle$$

$$D = D_{QL} \quad \text{(simplest closure)}$$

basic constituents:

① – intensity diffusion / scattering $O(\tilde{E}^4)$

② – drive i.e. $\frac{\partial \langle f \rangle}{\partial t} = -\frac{\partial}{\partial v}\langle \tilde{E}\delta f \rangle$

b.) **Systematics** → Where is this in 2pt Theory ?

$\rightarrow \frac{d}{dt} f = 0 \quad \Rightarrow \left\{ \frac{d}{dt}_{②} \langle \delta f^2 \rangle = -\frac{\partial}{\partial t} \langle f \rangle^2 \equiv S_{1,2} \right.$

$$\left( \frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial x_1} + v_2 \frac{\partial}{\partial x_2} \right) \langle \delta f(1) \delta f(2) \rangle$$

$$+ \frac{\partial}{\partial v_1} \langle \tilde{E}(1) \delta f(1) \delta f(2) \rangle + \frac{\partial}{\partial v_2} \langle \tilde{E}(2) \delta f(1) \delta f(2) \rangle = S_{1,2}$$

$x_{\pm} = (x_1 \pm x_2)/2, \quad v_{\pm} = (v_1 \pm v_2)/2$

$\Rightarrow$

$$\left( \frac{\partial}{\partial t} + v_+ \frac{\partial}{\partial x_+} + v_- \frac{\partial}{\partial x_-} \right) \langle \delta f(1) \delta f(2) \rangle$$

$$+ \frac{\partial}{\partial v_-} \langle \underbrace{(\tilde{E}(1) - \tilde{E}(2))}_{\text{relative accel.}} \delta f(1) \delta f(2) \rangle + \frac{\partial}{\partial v_+} \langle \underbrace{(\tilde{E}(1) + \tilde{E}(2))}_{\text{envelope evol'n.}} \delta f(1) \delta f(2) \rangle$$

$$= S_{1,2}$$

→ LHS → rapid variation on small scales

→ RHS → slow variation, only

– with closure:

$$\left(\frac{\partial}{\partial t} + T_{rel}^{(-)} + T_{envelope}^{(+)}\right) \langle \delta f(1)\,\delta f(2)\rangle = S_{1,2}$$

$$T_{rel} = v_- \frac{\partial}{\partial x_-} - \frac{\partial}{\partial v_-} D_- \frac{\partial}{\partial v_-}$$ (a la' Dupree)

$$D_- = \sum_{k,\omega} \left(1-\cos(kx_-)\right) |\tilde{E}_{k,\omega}|^2 \pi \delta(\omega - kv)$$

→ rapid, relative decorrelation → correlation

$$1/\tau_c \sim (k^2 D)^{1/3}$$

$$T_{envelope} = v_+ \frac{\partial}{\partial x_+} - \frac{\partial}{\partial v_+} D \frac{\partial}{\partial v_+}$$

$$D = D_{QL}$$

→ slower envelope evolution → spreading
usually ignored re: correlation

Point:
– $x_-, v_- \to 0$ and $\langle\ \rangle_x$ → quickie

– $S_{1,2}$ well behaved in $1 \to 2$ limit

- for intensity $\leftrightarrow \langle \delta f^2 \rangle$

$$\frac{\partial}{\partial t} I_n - \frac{\partial}{\partial v} D \frac{\partial}{\partial v} I_n = S_{1,2}[I]$$

- recovers "quickie" reaction-diffusion structure

- re: $S_{1,2}$ $\rightarrow \left\{ \begin{array}{l} \text{slight variation as} \\ 1 \rightarrow 2, \text{ only} \end{array} \right.$

$$S_{1,2} = -\langle \tilde{E} \delta f \rangle \frac{\partial \langle f \rangle}{\partial v} \quad \leftrightarrow \quad \frac{\partial \langle f \rangle}{\partial t}$$

$$\delta f = \underset{\substack{\text{coherent} \\ \text{response}}}{f^c} + \underset{\text{granulation (phase space vortex)}}{\tilde{f}}$$

$$f^c_{k,\omega} = -\tilde{E}_{k,\omega} R_{k,\omega} \, \partial \langle f \rangle / \partial v$$

$\Rightarrow$

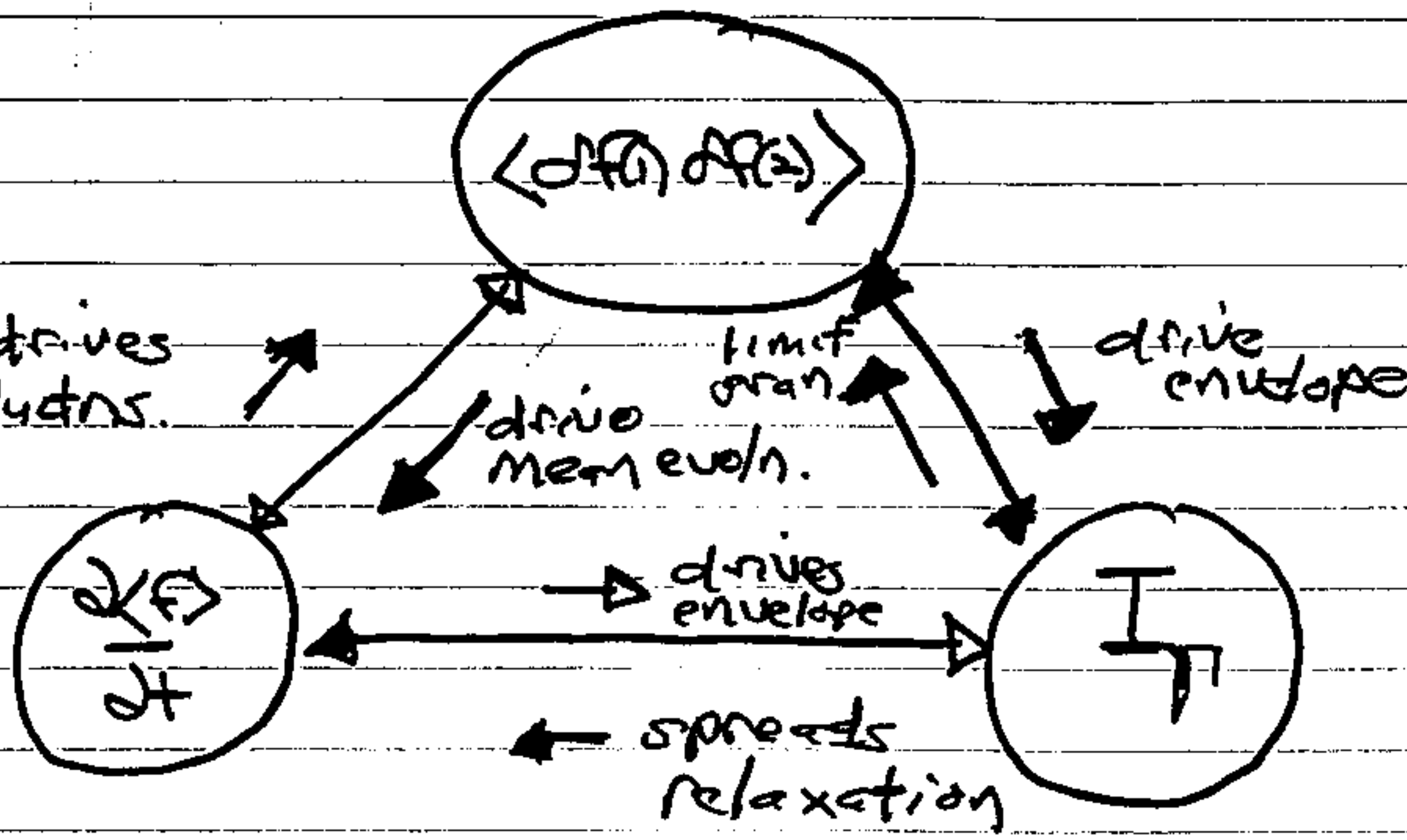
$$S_{1,2} = D_{QL} \left( \frac{\partial \langle f \rangle}{\partial v} \right)^2 - \underbrace{\langle \tilde{E} \tilde{f} \rangle \frac{\partial \langle f \rangle}{\partial v}}_{\text{drag / dynamical friction}}$$

$D_{QL}$: linked to linear drive ....

where: $ik \, \epsilon(k,\omega) \, E_{k,\omega} = \int \tilde{f}_{k,\omega} \, dv$

$\rightarrow$ enforces self-consistency

## → An Observation




For evolving inhomogeneity,

$\langle \delta f(1)\, \delta f(2) \rangle$, $I_\Gamma$, $\frac{\partial \langle f \rangle}{\partial t}$ all strongly coupled ....

"primacy hierarchy"?

→ CTEM Spreading

- straightforward to extend simple Vlasov theory, drawing on P.D., et. al. '82

- $I_\pi \equiv \lim_{1\to 2} \langle \delta g(1)\, \delta g(2) \rangle$

Markovian approximation

⇒ Fisher structure:

$$\underbrace{\frac{\partial}{\partial t} I_\pi - \frac{\partial}{\partial r_t} D \frac{\partial}{\partial r_t} I_\pi}_{} + \bar{d}_T I_\pi = S[I_\pi]$$

$\frac{\partial}{\partial r_t} D \frac{\partial}{\partial r_t}$ → envelope spreading (spatial coupling)

$\bar{d}_T I_\pi$ → local transfer → small scale

$S[I_\pi]$ → drive $\sim \gamma_{eff} I_\pi$

where:

$$\begin{cases} \bar{d}_T = \sum_{k',\omega'} \overline{(k \cdot k' \times \hat{z})}^2 \frac{c^2}{B_0^2} \left| \langle \phi_{k',\omega'} \rangle_{b.a.} \right|^2 \pi \delta(\omega' - \bar{\omega}_D' G) \\ \overline{k \cdot k' \times \hat{z}\, \phi} \to k_\theta k_\theta' \hat{s}\, (2\pi M + \theta_k) \langle \phi_{k,\omega} \rangle_{b.a.} \end{cases}$$

with $\left| \langle \phi_{k',\omega'} \rangle_{b.a.} \right|^2 \sim I_\pi$

$$D = \sum_{k',\omega'} k_\theta'^2 \frac{c^2}{B_0^2} |\hat{\phi}_{k',\omega'}|^2 \pi \delta(\omega' - \bar{\omega}_D' G)$$

with $|\hat{\phi}_{k',\omega'}|^2 \sim I_\pi$

but: $\underline{\gamma_{eff} \neq \gamma_{Lin}}$ !

→ for $\gamma_{eff}$ :

$$\mathcal{S}' = 2\langle f\rangle \sum_{k',\omega'} \left(\omega' - \omega_{*T}'\right) \mathrm{re}R_{k',\omega'}(\epsilon)\, W_{k',\omega'} \frac{\overset{\sim I_{\Gamma}}{\langle \tilde{g}\tilde{n}\rangle_{k'}}\, d^{ion}_{IM}(k',\omega)}{|d(k',\omega)|^2}$$

- reflects/includes drag and diffusion (dynamical friction)
- $\mathcal{S}' \sim \mathrm{Im}\, d^{ion}$ → electron granulations can scatter off ions

- $\gamma$ not tied to linear growth rate
  ⇒ granulation scattering can boost penetration of stable region

With pole approx:

$$\mathcal{S}'(\epsilon) = 2\langle f\rangle \sum_{k'} \overset{(<0)}{(\omega_D'\epsilon - \omega_{*T})} \frac{W_{k'} \overset{\sim I_{\Gamma}}{\langle \tilde{g}\tilde{n}\rangle_{k'}}\, \overset{(<0)}{d^{ion}_{IM}(k',\bar{\omega}_D\epsilon)}}{\left|(k'-k_{res})\frac{\partial d}{\partial k'}\right|^2 + |\mathrm{Im}\, d|^2}$$

$\sim 1/|\mathrm{Im}\, d|$ → { emission into weakly over-saturated modes (stiff) ⟷ strong $\mathcal{S}$

⇒

→ Amplitude Equation for $I_\Gamma$:

$$\boxed{\frac{\partial I_\Gamma}{\partial t} - \frac{\partial}{\partial r} D \frac{\partial}{\partial r} I_\Gamma + \bar{d}_T I_\Gamma = \gamma_{eff} I_\Gamma}$$

where:

$$\gamma_{eff} \cong \left(\omega_{k_{res}} - \omega_{*T}\right) W_k\, d^{ion}(k_{res}, \omega_k) \Big/ |d_{IM}| \left|\frac{\partial d_{real}}{\partial k_r}\right|$$

→ Fisher structure, so

$$V_{front} \cong (\gamma_{eff} D)^{1/2}$$

and since

$$D = D[\langle\phi^2\rangle] = D[I_\Gamma]$$

$$\therefore \quad V_{eff} \sim \gamma_{eff}$$

⇒ expect rapid penetration of stiff region by resonant particle emission (simulations)

⇒ stronger than "simple model" observation, where $\gamma$ fixed and small

⇒ "entropy fluctuation front" can propagate via inter-species drag ⊕ intensity diffusion $\nu_{ion}$. profile not critical.

## Conclusions

→ Work ongoing, but:

* – framework for kinetic theory of turbulence spreading, avalanching established

* – spreading driven locally by dynamical friction ⟷ not tied to linear growth rate

* – resonant granulation emission into quasi-marginal modes ⇒ strongly enhanced spreading velocity

→ { predict fast spreading into stiff regions
 speed underestimated by simple models

⇒ overflow of edge/outside to core ...

## Future Work

– 2 field dynamics $I_{\Gamma}$, $T_e(r)$ ⟷ how is stiffness manifested in kinetic theory

– noise ⟷ front interaction $\{\delta^{\Delta n}$ ¿?

* – sub-critical fronts ($\gamma_{100} < 0$) dynamics ¿?