Magnetic Fluctuation Induced Particle Transport and Parallel Ion Velocity Fluctuations on MST

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Motivation

Understanding of density relaxation due to magnetic fluctuations

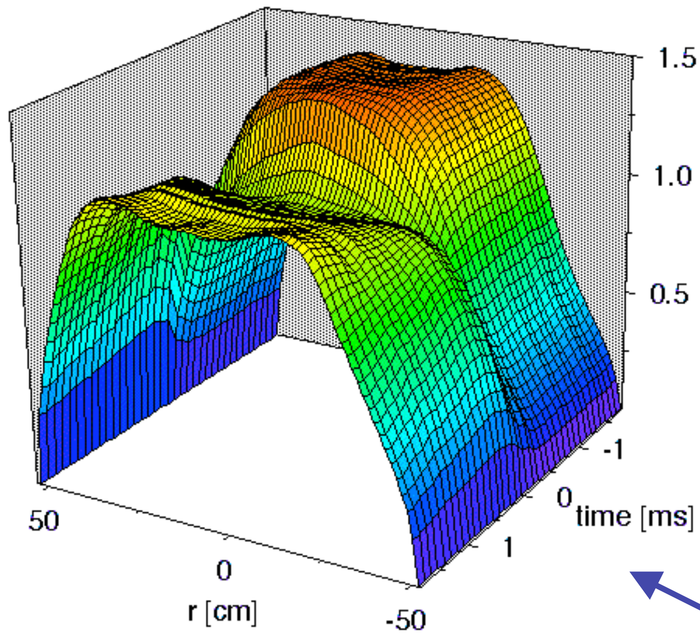
(1) How does magnetic fluctuation cause electron (and ion) particle transport?

(2) What is ion particle diffusivity in MHD approximation?

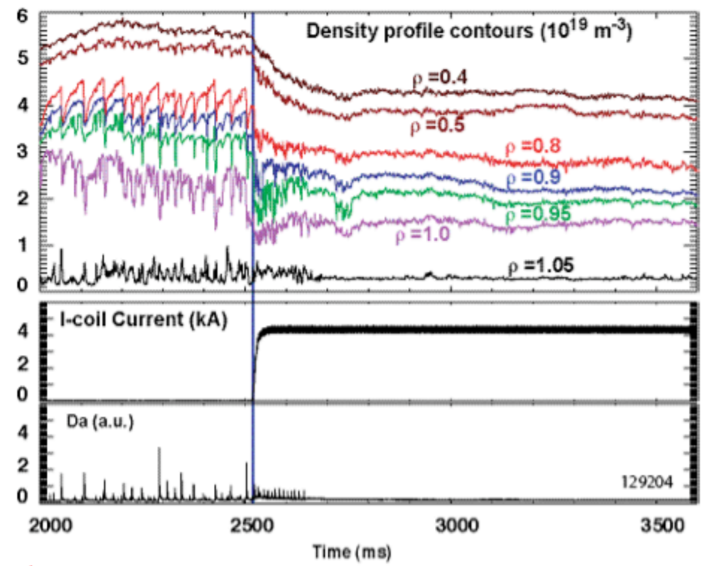
Electron Density Relaxation Due to Stochastic Field

MST

$n_e(r)\ [10^{19} m^{-3}]$



DIII-D



- Another n=3 RMP discharge with ITER-similar shape and low collisionality, but RMP is applied in later time.
- Density reduces even at ρ=0.4

L.Zeng (UCLA) et al DIII-D by reflectometry

?

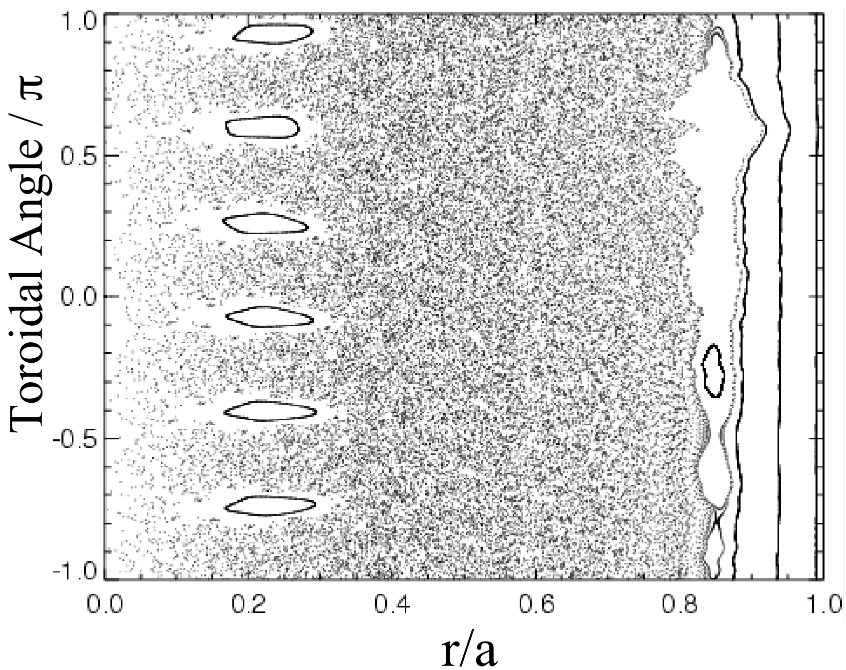
Stochastic Magnetic Field

Driven by Tearing Instabilities in MST

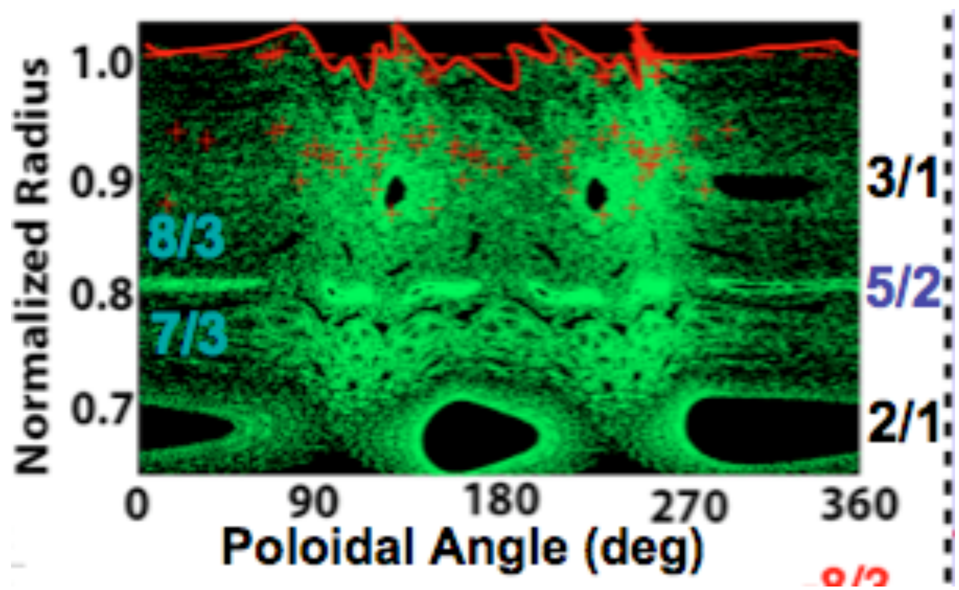
Driven by I-coil externally in DIII-D

Stochastic Magnetic Fields

Puncture Plot of B Field in MST



DIII-D B field with I-Coil



R.Moyer--Error field workshop 2007

Calculation(DEBS) +Exp

Vacuum Field Calculation

Particle Flux due to Stochastic Magnetic Fields

$$\Gamma_r = < \Gamma_{//} \vec{b} \bullet \vec{e}_r >$$

Particle transport arises from particle streaming along stochastic field lines

$$\Gamma_{//} = \overline{\Gamma}_{//} + \tilde{\Gamma}_{//}$$

$$\vec{b} = \vec{B}_0 + \tilde{b}_r$$

Fluctuation-induced particle flux ⟶ $\Gamma_r = \dfrac{< \tilde{\Gamma}_{//} \tilde{b}_r >}{B_0}$

$$\tilde{\Gamma}_{//} = n \tilde{u}_{//} + \tilde{n} u_{//}$$

$$\Gamma_r = u_{//} \frac{< \tilde{n} \tilde{b}_r >}{B_0} + n \frac{< \tilde{u}_{//} \tilde{b}_r >}{B_0}$$

Outline

Experimental Measurement:

(1) Stochastic Field Induced Particle Transport and Electron Density Relaxation on MST;

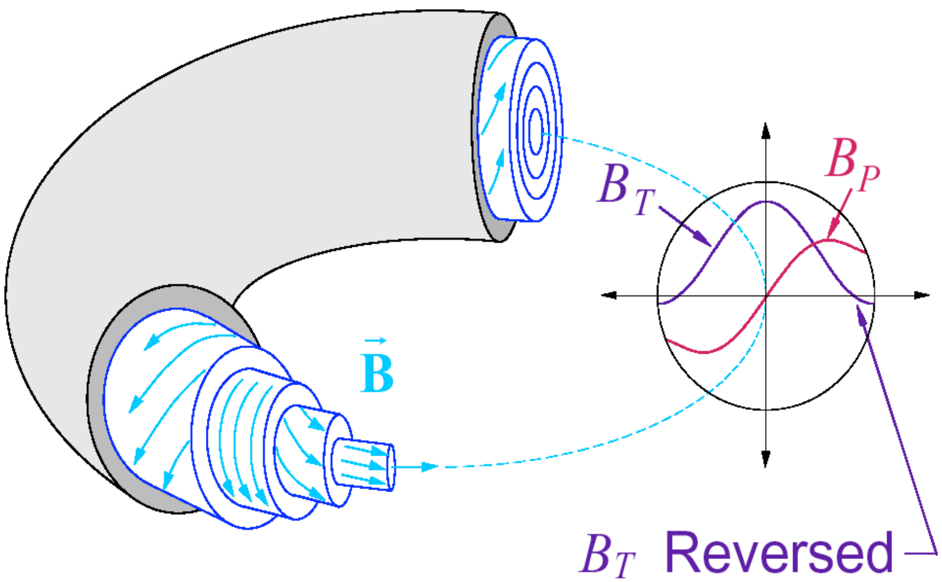
$$\Gamma_r = u_{//} \frac{< \tilde{n}\tilde{b}_r >}{B_0} + n \frac{< \tilde{u}_{//}\tilde{b}_r >}{B_0} \longrightarrow \frac{\partial n_e}{\partial t}$$

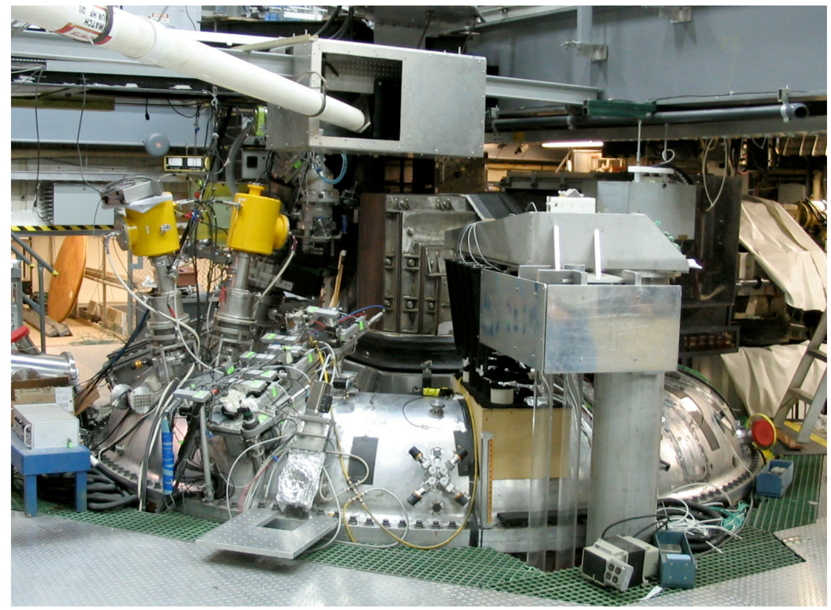
For both electron and ion

(2) Discussion on origin of ion velocity $\tilde{u}_{//}$

Madison Symmetric Torus

MST Reversed-Field Pinch (RFP) is toroidal configuration with relatively weak toroidal magnetic field B_T (i.e., $B_T \sim B_p$)





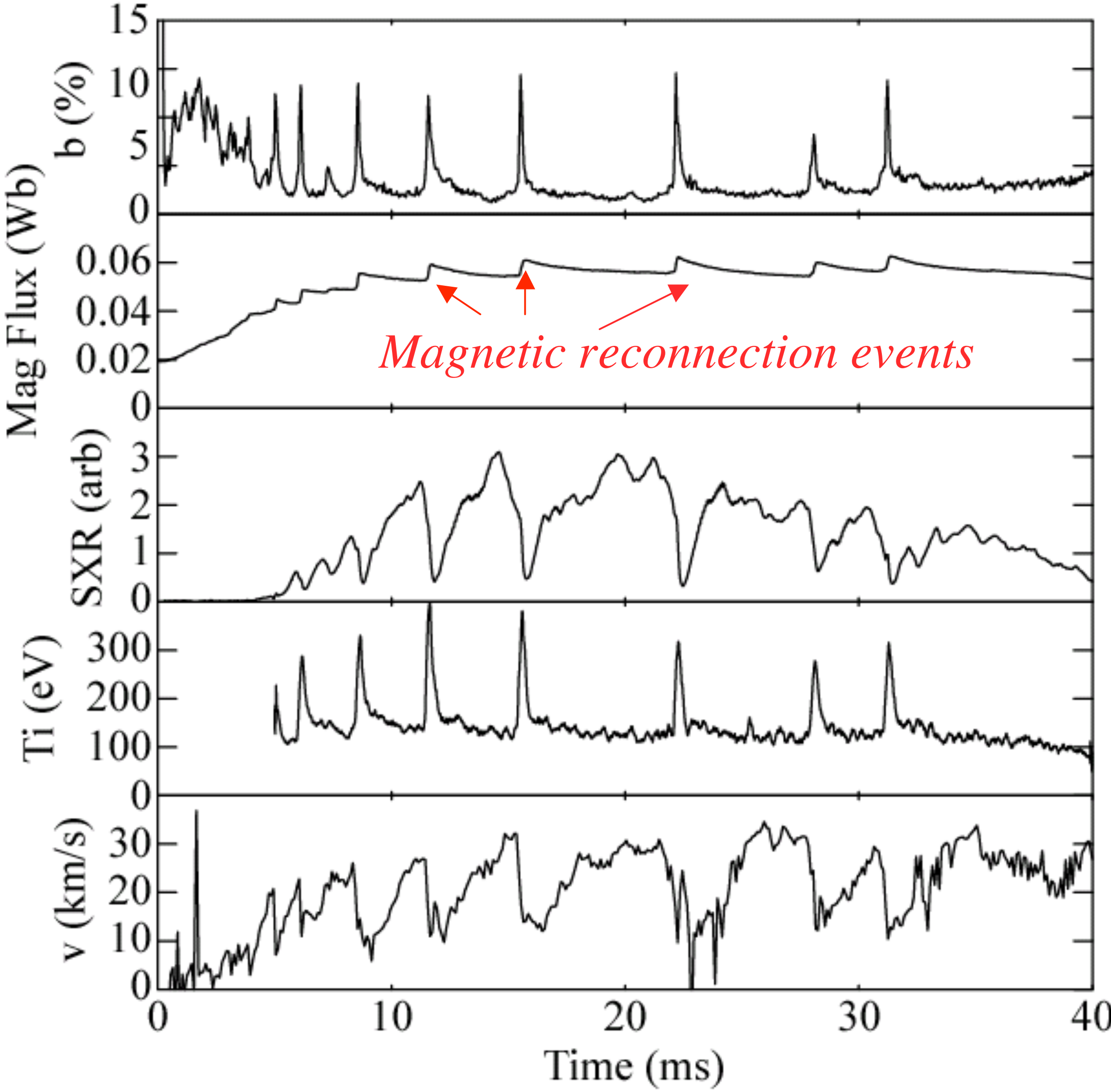
$$q(r) = \frac{r}{R}\frac{B_T}{B_P} < 1$$

R_0 = 1.5 m, a = 0.51 m, I_p < 600 kA

$B_T \sim$ 3-4 kG, $n_e \sim 10^{19}$ m^{-3}, T_{e0} < 1.3 keV

$\tau_E \sim$ 10 ms, $\beta = \langle p \rangle / B^2(a)$=15%

Effects of Fluctuations on Transport in the RFP



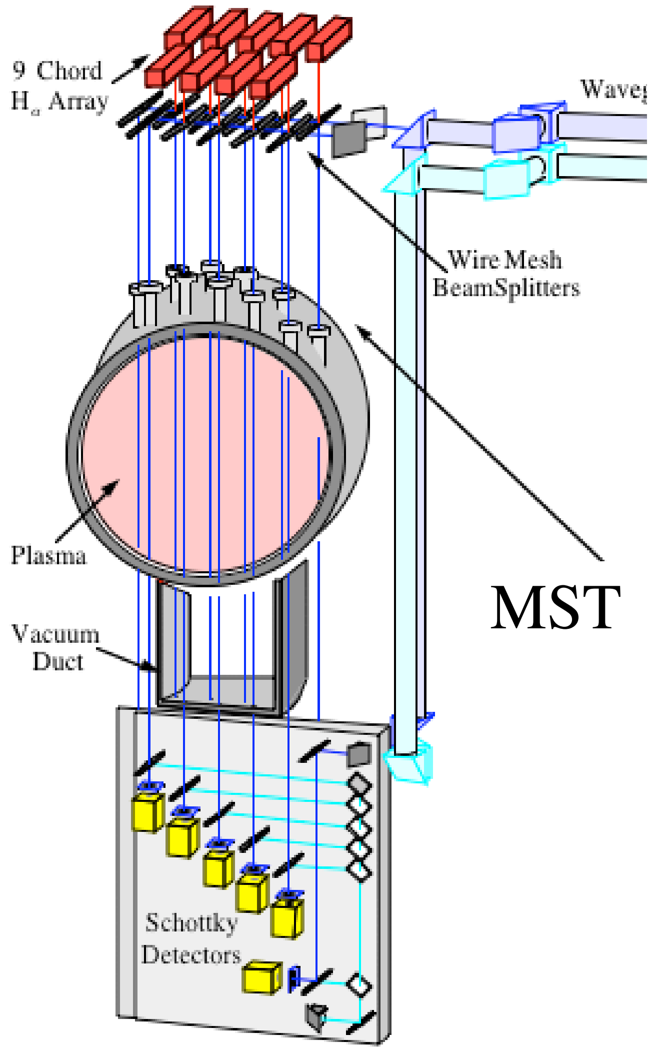
Generation / manipulation of magnetic flux (dynamo)

Particle and energy transport

Ion heating

Momentum transport

FIR Polarimeter-Interferometer System



11 chords, space separation 8 cm

$$\phi \sim \int n dl + \int \tilde{n} dl$$

$$\Phi \sim \int n\vec{B} \bullet d\vec{l} + \int n\tilde{\vec{b}} \bullet d\vec{l} + \int \tilde{n}\vec{B} \bullet d\vec{l}$$

Interferometer $\longrightarrow$ density

Faraday rotation $\longrightarrow$ magnetic field

Measurement of magnetic fluctuation induced transport flux

$$\Gamma_r^e = u_{//,e} \frac{<\tilde{n}_e \tilde{b}_r>}{B} + n \frac{<\tilde{u}_{//,e} \tilde{b}_r>}{B}$$

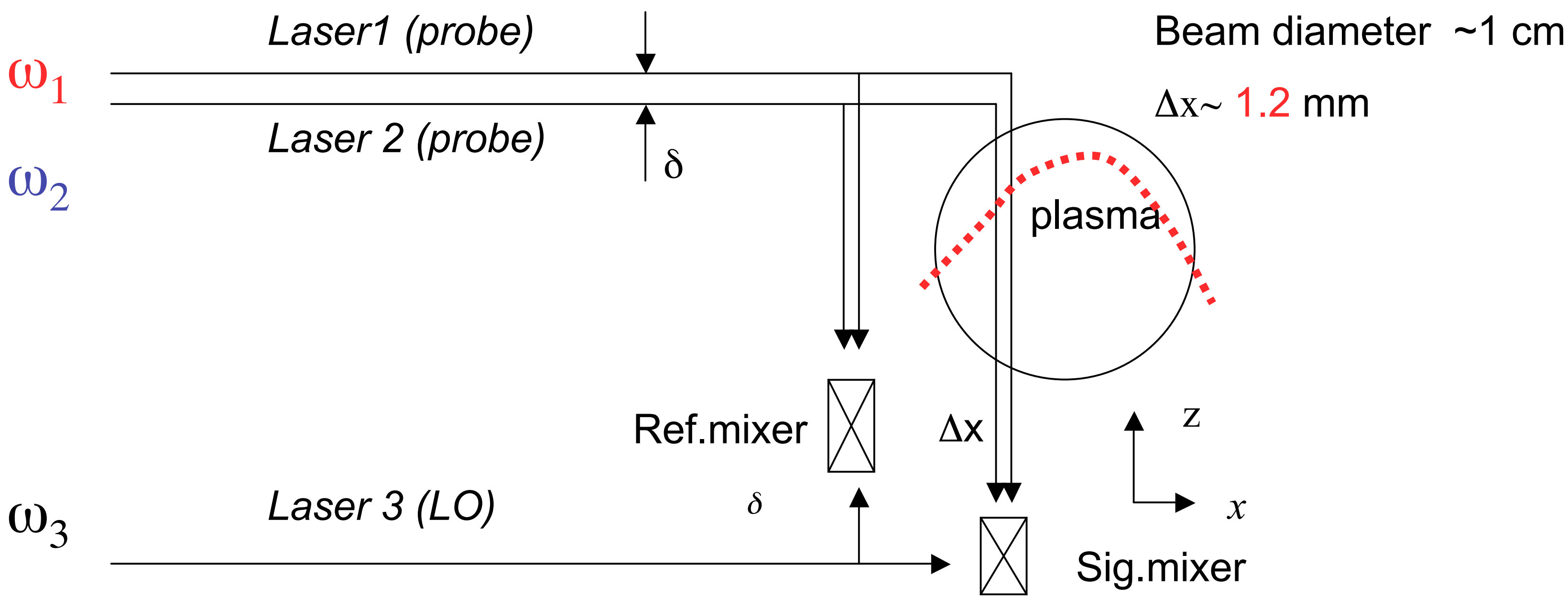
$V_{//,e}$, B Laser Faraday rotation B $\rightarrow$ $J = \nabla \times B$ $\rightarrow$ $V_{//,e} \approx \frac{J_{//}}{n_e e}$

$\tilde{b}_r(r)$ Laser Faraday rotation measures

$\tilde{n}_e(r)$ Laser interferometer ($\nabla \tilde{n}_e$)

$\tilde{u}_i$ Ion Doppler Spectroscopy (IDS)

Density Gradient Measurement by Differential Interferometer



Phase ($\omega_1 - \omega_3$) →

Phase ($\omega_1 - \omega_2$) →

$$n_e(x) = \frac{1}{L}\int_x n_e(r)dz, \qquad r^2 = x^2 + z^2$$

$$\frac{\partial n_e(x)}{\partial x} = \frac{1}{L}\int_x \frac{\partial n_e(r)}{\partial r}\frac{x}{r}dz$$

$\Delta x = 1$ mm

Localization of Density Fluctuations

(1) *For mean density*

$$\frac{\partial \bar{n}_e(x)}{\partial x} = \frac{1}{L}\int_x \frac{\partial n_e(r)}{\partial r}\cos\theta dz$$

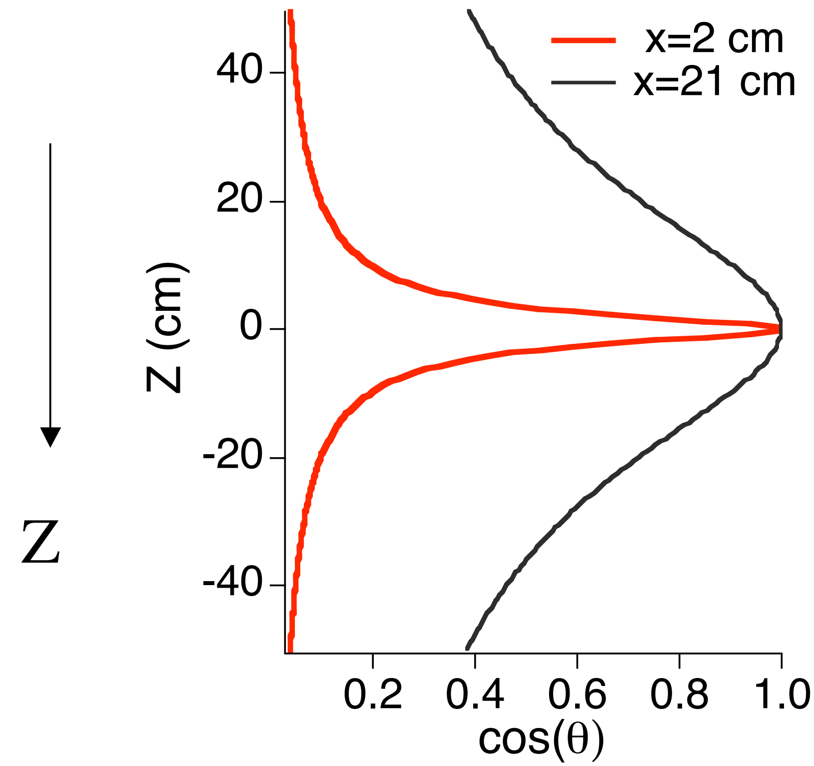
$$\approx \frac{\partial \bar{n}_e(r)}{\partial r} \times (0.3-0.5) \quad \text{for x} \rightarrow 0$$

(2) *For density fluctuations (m=1)*

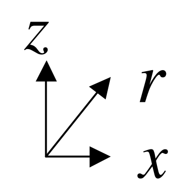
$$\tilde{\phi} = \int \tilde{n}_e(r)\cos\theta dz$$

$$\tilde{\phi}(x) \approx \tilde{n}_e(r)L \times (0.3-0.5)$$

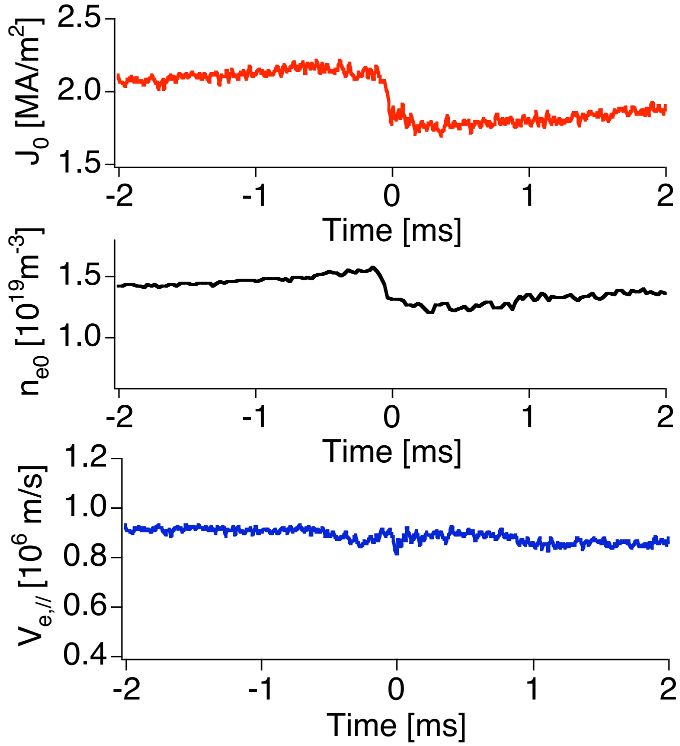
$$x \rightarrow 0$$

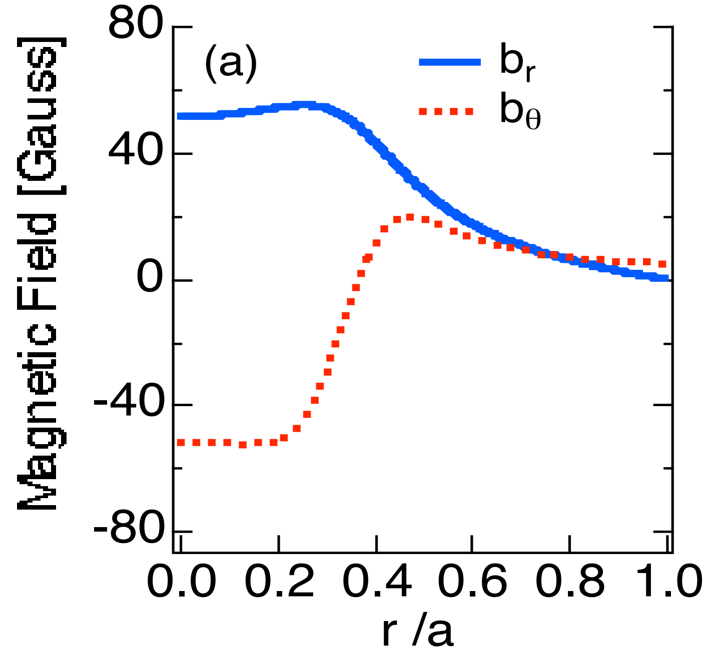


$$\cos\theta = \frac{x}{r}$$



Measurement of transport by Faraday rotation and interferometer





$$V_{//,e} = \frac{J_0}{en_{e0}}$$

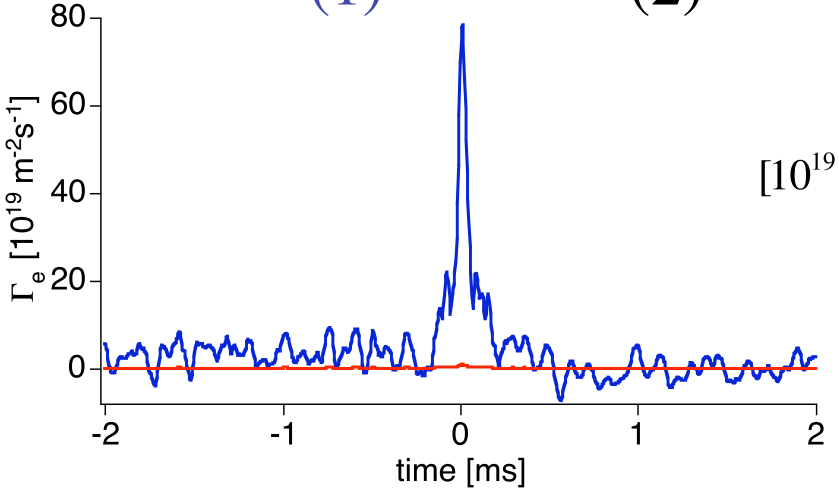
$$\Gamma_r = V_{//,e}\frac{<\tilde{n}_e\tilde{b}_r>}{B}$$

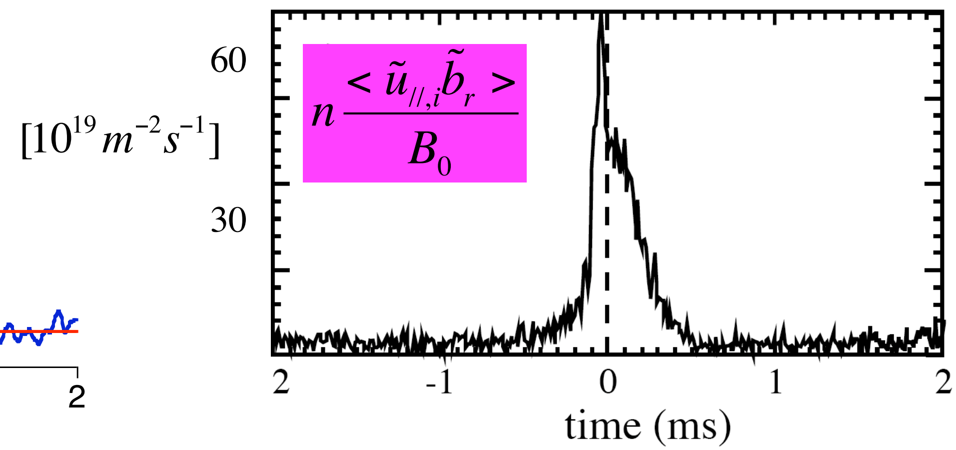
$$\nabla\bullet\Gamma_r = V_{//,e}\nabla\bullet\frac{<\tilde{n}_e\tilde{b}_r>}{B} \approx \frac{2V_{//,e}}{B}(<\tilde{b}_r\frac{\partial}{\partial r}\tilde{n}_e> + <\tilde{n}_e\frac{\partial}{\partial r}\tilde{b}_r>)$$

Electron Particle Flux and Ion Flux at r/a<0.2

$$\Gamma^e = u_{//,e}\frac{<\tilde{n}\,\tilde{b}_r>}{B_0} + n\frac{<\tilde{u}_{//,e}\tilde{b}_r>}{B_0} \approx \Gamma^i = u_{//,i}\frac{<\tilde{n}\,\tilde{b}_r>}{B_0} + n\frac{<\tilde{u}_{//,i}\tilde{b}_r>}{B_0}$$

(1) (2) ? (3) (4)





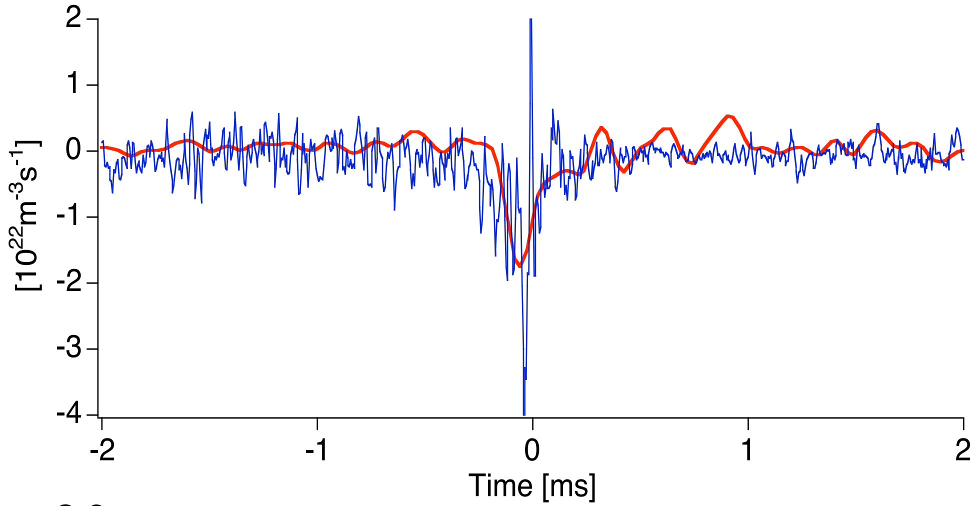
Charge flux measurement →

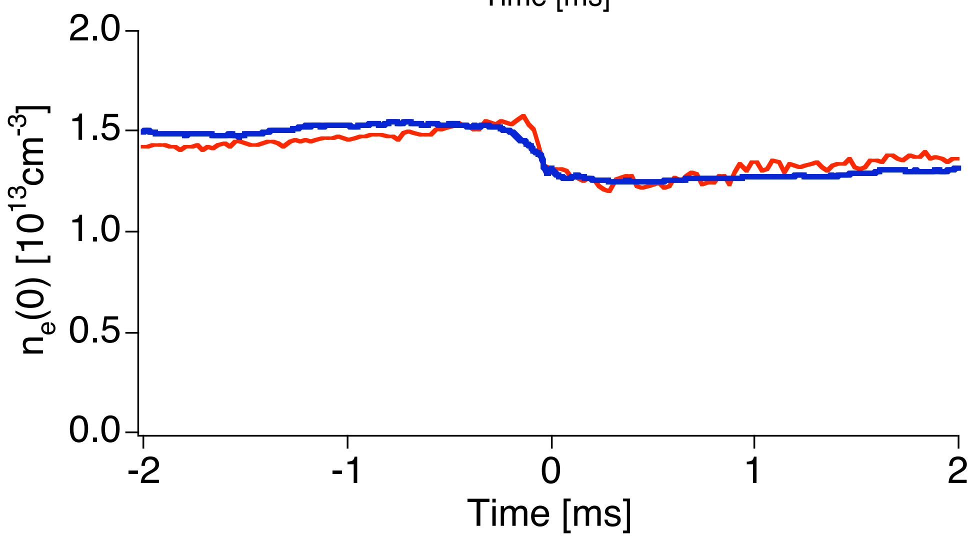
$$\Gamma_q = \Gamma^i - \Gamma^e = \frac{<\tilde{j}_{//}\tilde{b}_r>}{eB} \approx \frac{1}{eB}\frac{R}{nB}(\vec{k}\bullet\vec{B}) < \frac{1}{r}\tilde{b}_r\frac{\partial}{\partial r}r\tilde{b}_\theta > \approx 0$$

(1) Electron convective flux is significant

(2) Ion convective flux is negligible and pinch flux is significant

Density Change is Balanced by Particle Transport





Red: $\frac{\partial \langle n_e \rangle}{\partial t}$

Blue: $-\nabla \bullet \Gamma_r$

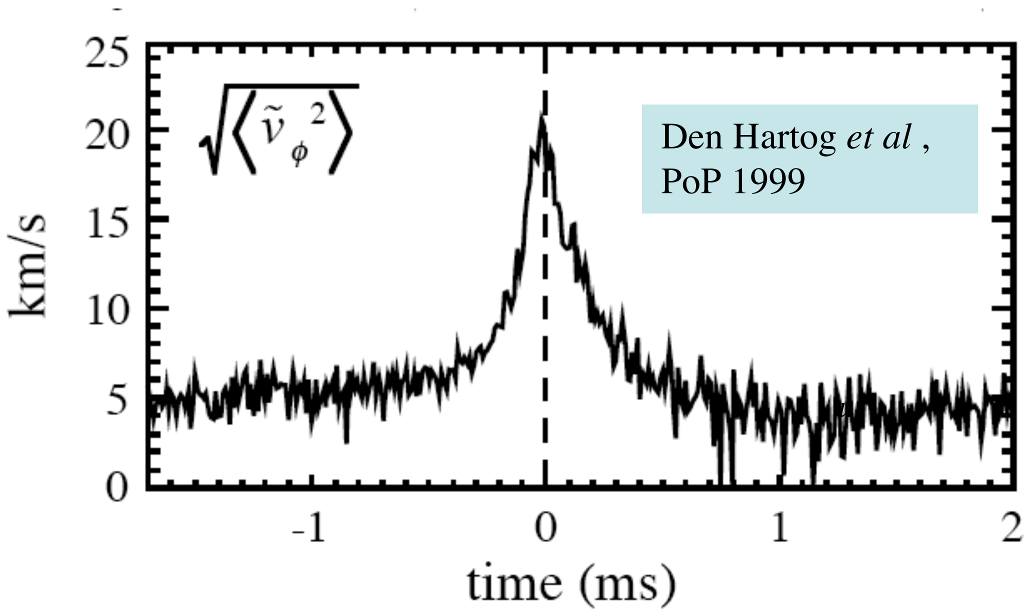
$$\frac{\partial \langle n_e \rangle}{\partial t} + \nabla \bullet \Gamma_r \approx 0$$

Red: $\langle n_e \rangle$

Blue: $-\int \nabla \bullet \Gamma_r \, dt$

Flux includes multiple dominant mode contribution (n=6,7,8,9)

Origin of Parallel Ion Fluctuations

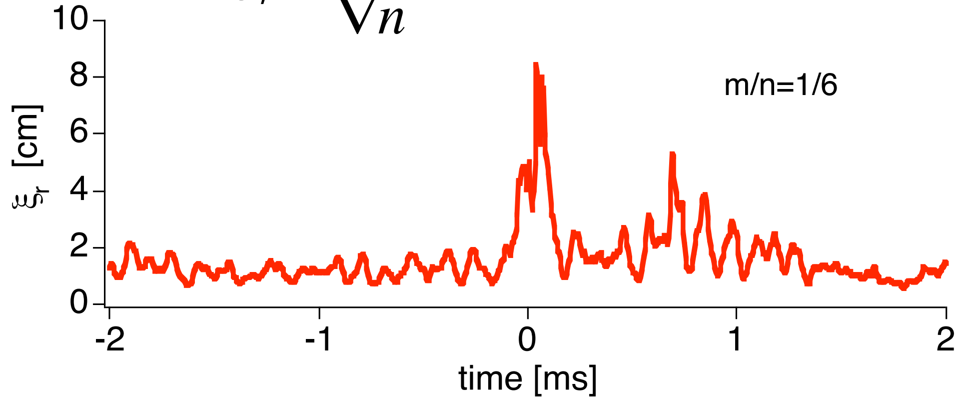


$$\tilde{u}_{//} \sim \tilde{\xi}_r \nabla u_{//}$$

The advection of mean parallel velocity is a potential source of parallel velocity fluctuations

Estimation of parallel velocity fluctuations

$$\xi_r = \frac{\delta n}{\nabla n}$$



Estimated fluctuation amplitude is less than the measured 20km/s

$$\tilde{u}_{//} \sim \xi_r \frac{u_{//}}{a} \sim 3\ [km/s]$$

If $\quad u_{//} \to 0, \quad \tilde{u}_{//} \to 0, \ \Gamma_i = n \dfrac{< \tilde{u}_{//} \tilde{b}_r >}{B_0} \to 0$

Which is not consistent with density relaxation even if plasma has no rotation

Other source of parallel ion velocity fluctuations?

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho(\vec{V} \bullet \nabla)\vec{V} + \mu \nabla^2 \vec{V} = \vec{J} \times \vec{B}$$

$$\rho \frac{\partial \tilde{u}_{//}}{\partial t} + \mu_{\perp} \nabla^2 \tilde{u}_{//} = (\tilde{J} \times B)_{//} + (J \times \tilde{B})_{//}$$

$$J = J_{//} + J_{\perp} \qquad \tilde{B} = \tilde{b}_{//} + \tilde{b}_{\perp} + \tilde{b}_r$$

$$(J \times \tilde{B})_{//} = J_{\perp} \times \tilde{b}_r = -\frac{\nabla p}{B} \tilde{b}_r$$

$$\tilde{f} = -\frac{\nabla p}{B} \tilde{b}_r$$

Fluctuating torque

Parallel Ion velocity fluctuations driven by fluctuating torque

$$\rho \frac{\partial \tilde{u}_{//}}{\partial t} + \mu_{\perp} \nabla^2 \tilde{u}_{//} = -\frac{\nabla p}{B} \tilde{b}_r$$

$$\nabla p = T\nabla n + n\nabla T = T(1 + \frac{L_n}{L_T})\nabla n$$

$$L_n^{-1} = \frac{1}{n}\frac{\partial n}{\partial r}$$

$$L_T^{-1} = \frac{1}{T}\frac{\partial T}{\partial r}$$

Estimate parallel ion velocity at crash phase

$$L_n \sim L_T \ \ , \ \ T = T_i + T_e \approx 700eV, \ \nabla n \approx 1 \times 10^{19} m^{-4}$$

$$\frac{b_r}{B} = 1.0 \times 10^{-2}, \rho = nM = 3.34 \times 10^{-8} kg/m^3, \Delta t = 100 \mu s$$

$$\Delta \tilde{u}_{//} \sim 67 \ [km/s]$$

For m=1,n=6 mode

Ion particle flux in steady state

Estimate flow fluctuations away sawtooth crash

$$\rho \frac{\partial \tilde{u}_{//}}{\partial t} = 0$$

$$\nu_{\perp} \nabla^2 \tilde{u}_{//} = -\frac{\nabla p}{\rho B} \tilde{b}_r$$

$$\nu_{\perp} \sim Z_{eff} \frac{\rho_i^2}{\tau_{ii}}$$

$$\nabla^2 \sim k_r^2,$$

$$\tilde{u}_{//} = -\frac{1}{Z_{eff}} \frac{\tau_{ii}}{(\rho_i k_r)^2} \frac{\nabla p}{\rho B} \tilde{b}_r$$

If $k_r \sim \frac{2\pi}{w}$ $(\rho_i k_r)^2 \sim 1$ $\tilde{u}_{//} \sim 5 \ [km/s]$

Particle Diffusivity due to Stochastic Field

In steady state or away sawtooth crash

$$\rho \frac{\partial \tilde{u}_{//}}{\partial t} = 0$$

$$\nu_{\perp} \nabla^2 \tilde{u}_{//} = -\frac{\nabla p}{\rho B} \tilde{b}_r \qquad \nu_{\perp} \sim \frac{\rho_i^2}{\tau_{ii}} \qquad \nabla^2 \sim k_r^2,$$

$$\tilde{u}_{//} = -\frac{\tau_{ii}}{(\rho_i k_r)^2} \frac{\nabla p}{\rho B} \tilde{b}_r$$

$$\Gamma_i = n \frac{< \tilde{u}_{//} \tilde{b}_r >}{B} = -\frac{\tau_{ii}}{(\rho_i k_r)^2} (1 + \frac{L_n}{L_T}) \left(\frac{\tilde{b}_r}{B} \right)^2 c_s^2 \nabla n = -D_i \nabla n$$

$$D_i = \frac{1}{(\rho_i k_r)^2} (1 + \frac{L_n}{L_T}) \lambda_m \left(\frac{\tilde{b}_r}{B} \right)^2 c_s$$

R-R theory

$$D \sim l_c \left(\frac{\tilde{b}_r}{B} \right)^2 v_i$$

Comparison between Heat and Particle Diffusivity

$$D_i = \frac{1}{(\rho_i k_\perp)^2}(1+\frac{L_n}{L_T})\lambda_{mfp}\left(\frac{\tilde{b}_r}{B}\right)^2 c_s$$

$$\chi_e^{RR} \cong l_c\left(\frac{\tilde{b}_r}{B}\right)^2 v_e$$

Rechester-Rosenbluth QL theory

$$\frac{\chi_e^{RR}}{D_i} = \sqrt{\frac{M}{m_e}}\frac{l_c}{\lambda_{mfp}}(\rho_i k)^2 \frac{1}{1+\frac{L_n}{L_T}}$$

$$\frac{\chi_e^{RR}}{\chi_i^{RR}} = \sqrt{\frac{M}{m_e}} = 43$$

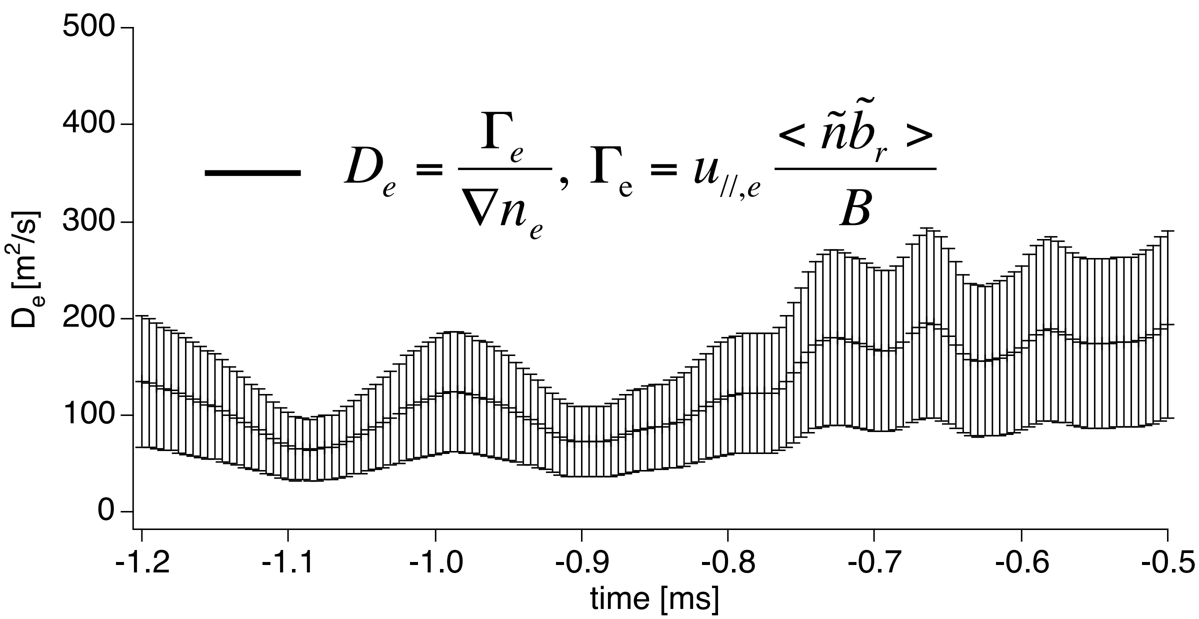
$\chi_e \sim 300\ [m^2/s]$ From power balance in a steady state

$D_e \sim 137\ [m^2/s]$ $\Gamma_e \sim 5.50\times10^{19}[\mathrm{m}^{-2}\mathrm{s}^{-1}]$, $\nabla n \sim 4\times10^{17}[m^{-4}]$, $D_e = \frac{\Gamma_e}{\nabla n_e}$

Electron heat diffusivity is comparable to particle diffusivity $\frac{\chi_e^{mst}}{D_e^{mst}} \sim 2$

For typical MST parameters

$$D_i = \frac{1}{Z_{eff}(\rho_i k_r)^2}(1+\frac{L_n}{L_T})\lambda_{mfp}\left(\frac{\tilde{b}_r}{B}\right)^2 c_s \approx 76.7\ \frac{1}{(\rho_i k_r)^2}\quad [\mathrm{m}^2/s]$$



$$D_i = D_e \quad \longrightarrow \quad \rho_i k_r \sim (0.5-1)$$

A few cm scale velocity fluctuation is needed to explain observed ion particle flux.

Summary

(1) Convective electron particle flux can account for electron density relaxation;

(2) Fluctuating torque can drive parallel ion velocity fluctuations and particle flux

$$-\frac{\nabla p}{B}\tilde{b}_r$$

(3) The ratio of heat transport to particle transport can depend on magnetic fluctuations characteristics, not only mass ratio.