ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

CRPP

Turbulent transport of beam ions

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Introduction

Energetic ions in fusion plasmas

- Fusion born α particles

  $\rightarrow$ isotropic velocity distribution

  $\rightarrow$ fixed birth energy 3.5 MeV

- Auxiliary heating and current drive

  - Ion Cyclotron Resonance Heating (ICRH)

    $\rightarrow$ pitch angle $\lambda = v_{\parallel}/v \ll 1$

    $\rightarrow$ energies 100keV-1MeV

  - Neutral Beam Injection (NBI)

    $\rightarrow$ pitch angle $\lambda \sim 1$

    $\rightarrow$ typical beam energy 40keV-100keV up to 1MeV in ITER

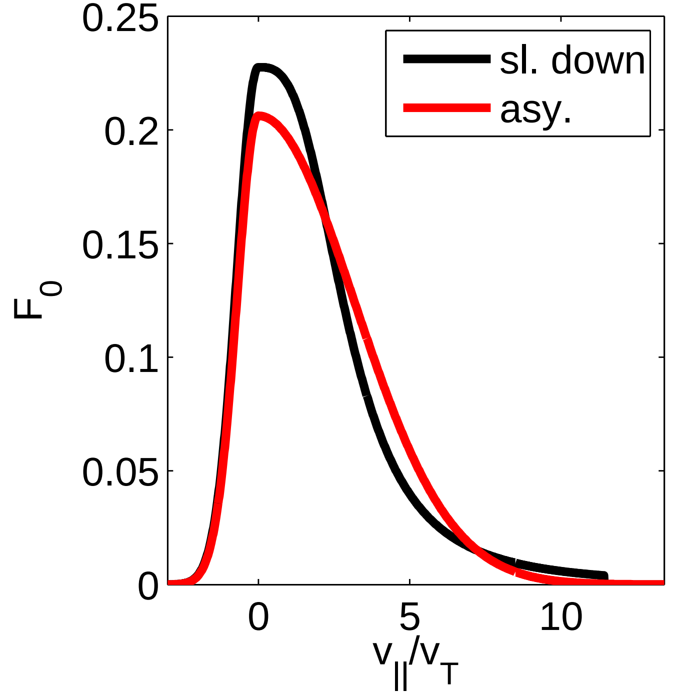
Turbulence driven by ITG modes.

Model

Beam ions from NBI:

- injected tangentially, large parallel velocity
- (only) thermal perpendicular velocity
- co- or counter $\mathbf{B}$
- no trapping

Modelled by an asymmetric, anisotropic Maxwellian with 3 characteristic temperatures $T_-, T_+, T_\perp$



$$F_{0b}(v_\parallel, \mu) = \mathcal{N} e^{-\mu B_0/T_{\perp 0}} \left(e^{-(v_\parallel/v_{T-})^2} (1 - \Theta(v_\parallel)) + e^{-(v_\parallel/v_{T+})^2} \Theta(v_\parallel) \right)$$

$\Rightarrow$ Analytical treatment of the equations possible

Normalizing velocities to thermal velocities, introduces the quantities

$$\tau_\pm = T_{0\pm}/T_0 \qquad \tau_\perp = T_{0\perp}/T_0$$

Model continued

Beam ions are treated

- fully gyrokinetically
- as passive tracers
- with asymmetric $v_\|$ grid

$N_{v_\|,-} = 16 \quad N_{v_\|,+} = 48$

$v_{\|,\mathrm{min}} = -3v_T \quad v_{\|,\mathrm{max}} = 12.33v_T$

$$N_x = 32|96 \qquad N_{ky} = 4|16$$
$$N_z = 16$$
$$N_{v_\|} = 32 \qquad N_\mu = 8$$
$$v_{\|,\mathrm{max}} = 3v_T \qquad \mu\mathrm{max} = 9TB_{\mathrm{ref}}^{-1}$$
$$L_x = 25|100\rho_s \qquad k_{y,\mathrm{min}}\rho_s = 0.15|0.05$$

Simulations done with the GENE code in flux-tube geometry. [Dannert & Jenko, 2005, Jenko *et. al*, 2000] All changes to the gyrokinetic equations due to the equilibrium distribution function has been taken into account. Also all modification to the calculation of the moments.

Standard parameters for the beam ions are

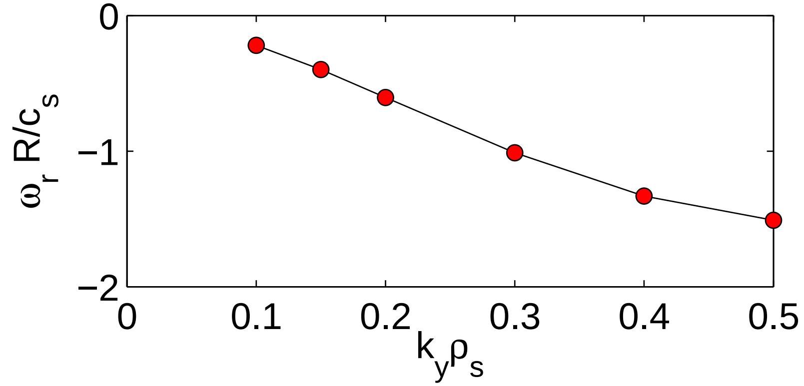
$$R/L_n = 15 \qquad \tau_\perp = \tau_- = 1 \qquad \tau_+ = 40$$

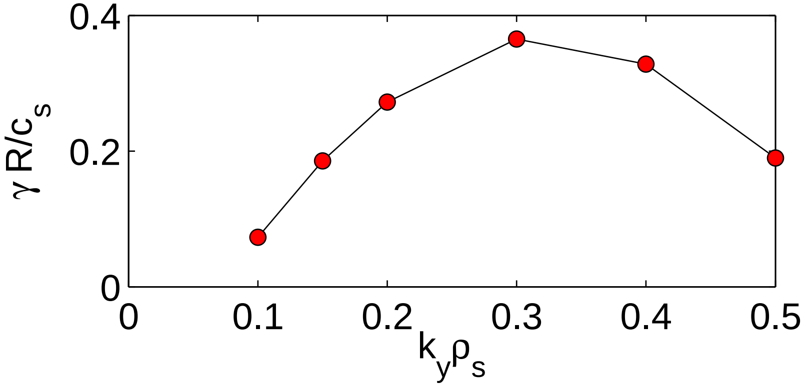
Linear GENE Simulations

First insight

Linear growth rate spectrum for the underlying ITG mode

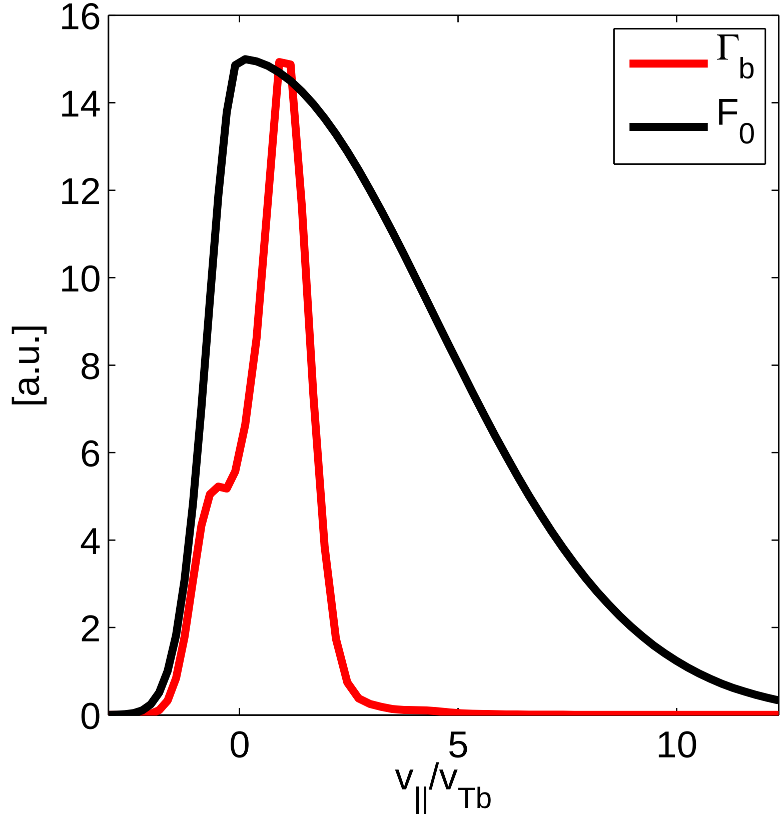
$(R/L_n = 3, R/L_{Te} = 8, R/L_{Ti} = 9, \hat{s} = 0.8, q = 1.4, \beta_e = 0.1\%, T_e = T_i, r/R = 0)$





$\longrightarrow$ drift in the negative y direction (ion diamagnetic direction)

Beam ion particle flux over parallel velocity



$\longrightarrow$ no high $v_\|$ contributions to particle flux

Reduced model for the beam ions

Switching off different terms in Vlasov equation $\Rightarrow$ reduced model:

$$\frac{\partial F_1}{\partial t} + \frac{v_\parallel^2}{\sigma_j} \mathcal{K}_y \frac{\partial F_1}{\partial y} + \omega_{n_j} F_0 \frac{1}{\hat{B}} \frac{\partial \bar{\Phi}_1}{\partial y} = 0$$

$\rightarrow$ advection in y direction with the curvature drift $v_c = v_\parallel^2 \mathcal{K}_y / \sigma_j$

$\rightarrow$ coupling to the background field via the drive term

Fourier expansion leads to a response for F_m due to an external perturbation $\bar{\Phi} = \sum_m \Phi_m \exp\{-i\omega t + i k_m y\}$

$$F_m = \frac{k_m \omega_{n_j} F_0}{\hat{B}\,(\omega' + i\gamma)} \Phi_m$$

with the Doppler shifted frequency $\omega' = \omega_r \left(1 - \frac{v_\parallel^2 \mathcal{K}_y / \sigma_j}{\omega_r / k_m}\right) = \omega_r \left(1 - \frac{v_c}{v_{\rm ph}}\right)$.

$\rightarrow$ the drifting beam ion “sees” a shifted background mode frequency

$\rightarrow$ high velocities lead to high frequencies $\Rightarrow$ no interaction

Beam ion diffusivity

Further calculate the particle flux dependent on $v_{\|}$ as

$$\Gamma_m(x, z, v_{\|}, t) = \omega_{n_j} \gamma^2 D_{\mathsf{turb}}(x, k_m, z, t) \frac{F_{0j}(v_{\|})}{\omega'^2 + \gamma^2}$$

Dividing the flux by F_0 and the gradient, and using $D_{\mathsf{turb}} = |v_{E,m}|^2/\gamma$, we get a $v_{\|}$ dependent diffusivity

$$D(k_m, z, v_{\|}) = \frac{\gamma^2}{\omega'^2 + \gamma^2} D_{\mathsf{turb}}(k_m, z)$$

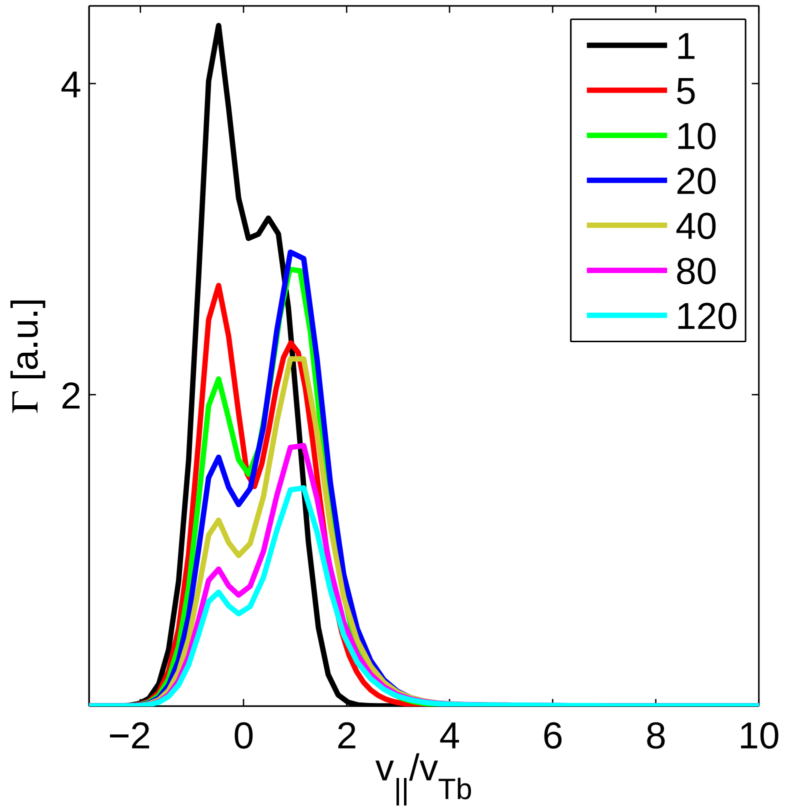
$\longrightarrow$ independent of the beam energy τ_+

$\longrightarrow$ dependent on background mode

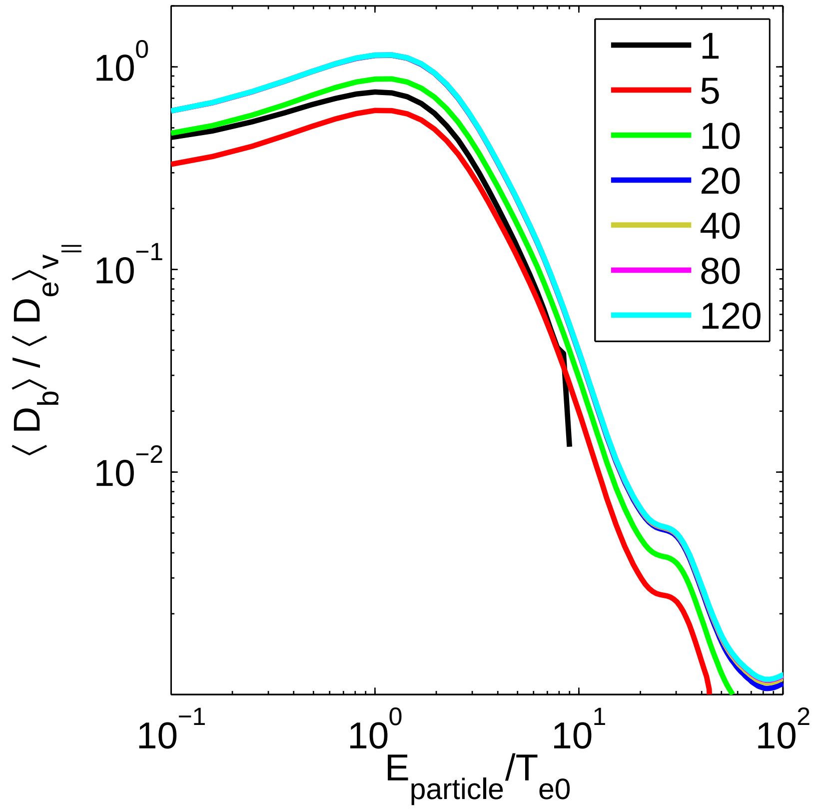
$\longrightarrow$ dependent on magnetic geometry

Graphical representation

Flux over parallel velocity



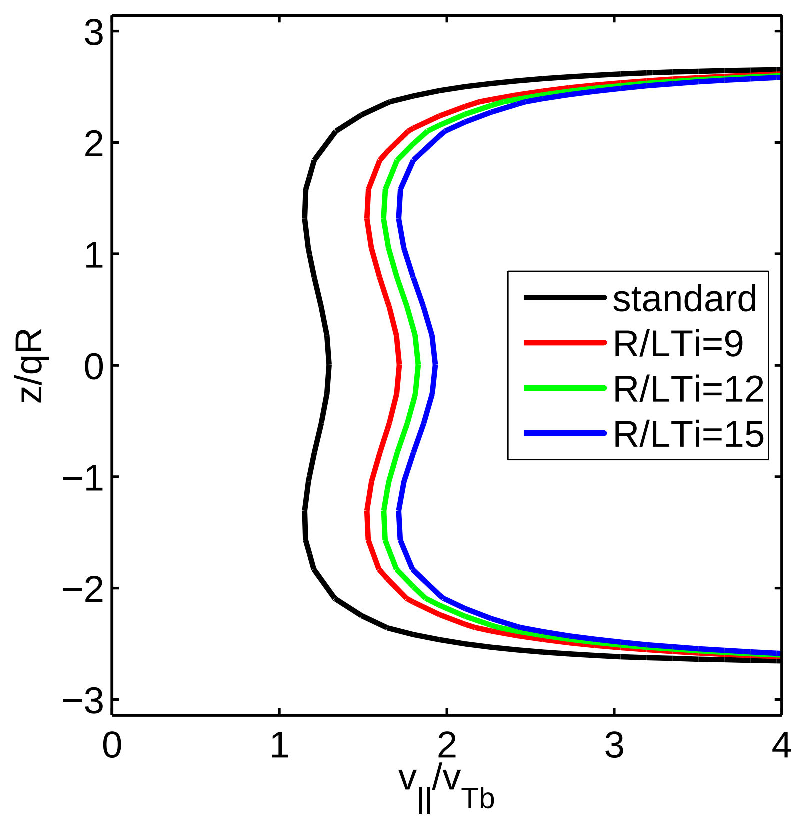
Diffusivity over particle energy

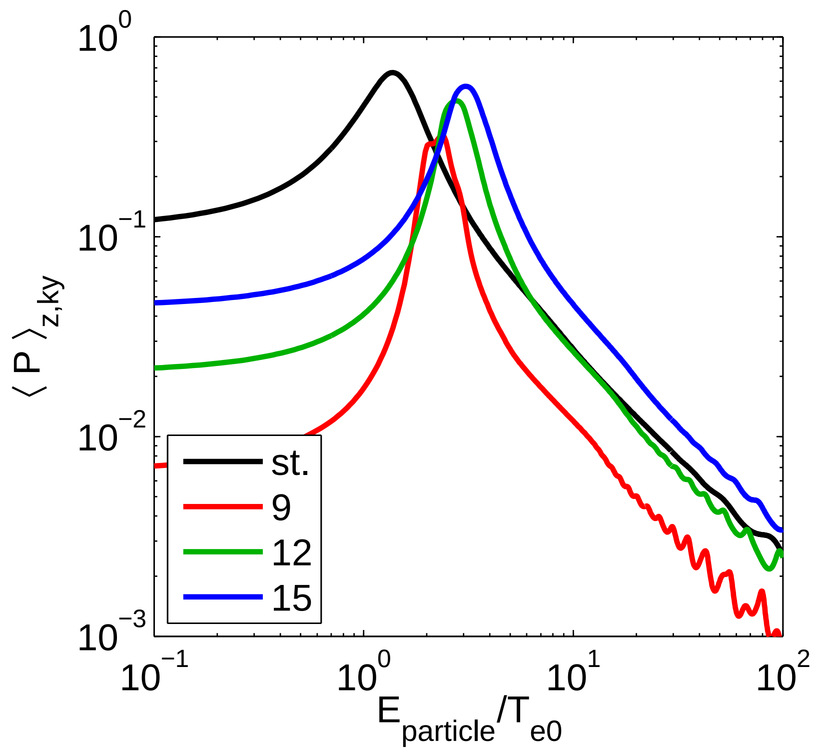


Analysis of the reduced model, changing gradients

Analyze the prefactor $\frac{\gamma^2}{\omega'^2+\gamma^2}$

Maximum at $\omega' = 0 \Longrightarrow v_{\|} = \pm\sqrt{\sigma_j/\mathcal{K}_y \cdot \omega_r/k_m} = \pm\sqrt{\frac{\sigma}{-2(\cos z + \hat{s}z \sin z)} \cdot \frac{\omega_r}{k_m}}$



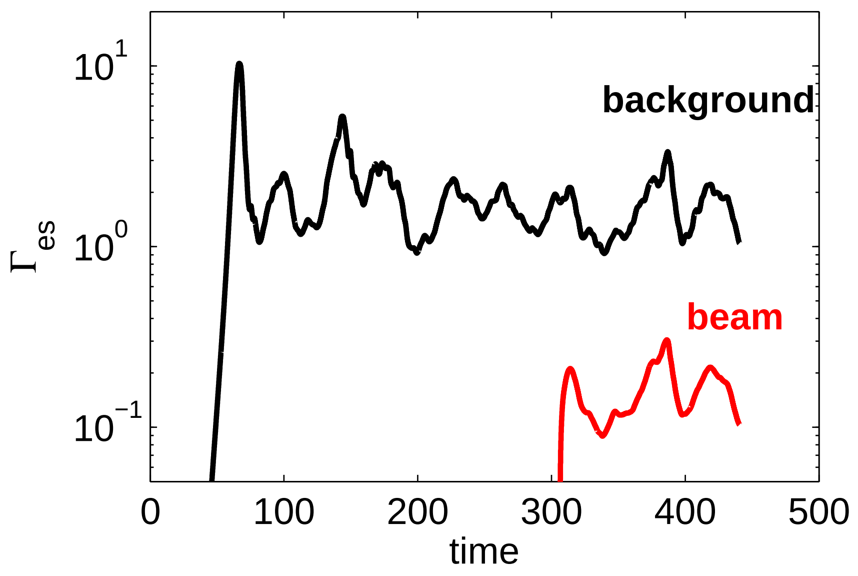


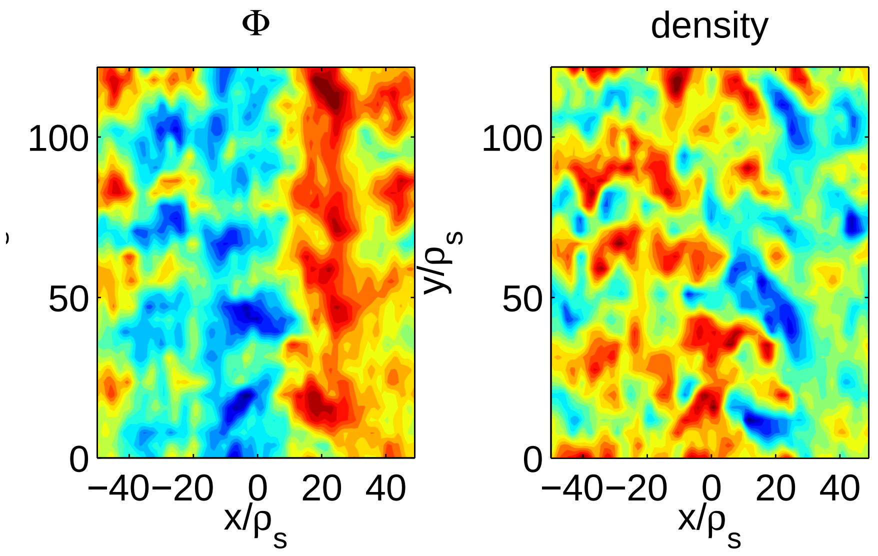
→ higher transport for higher particle energies

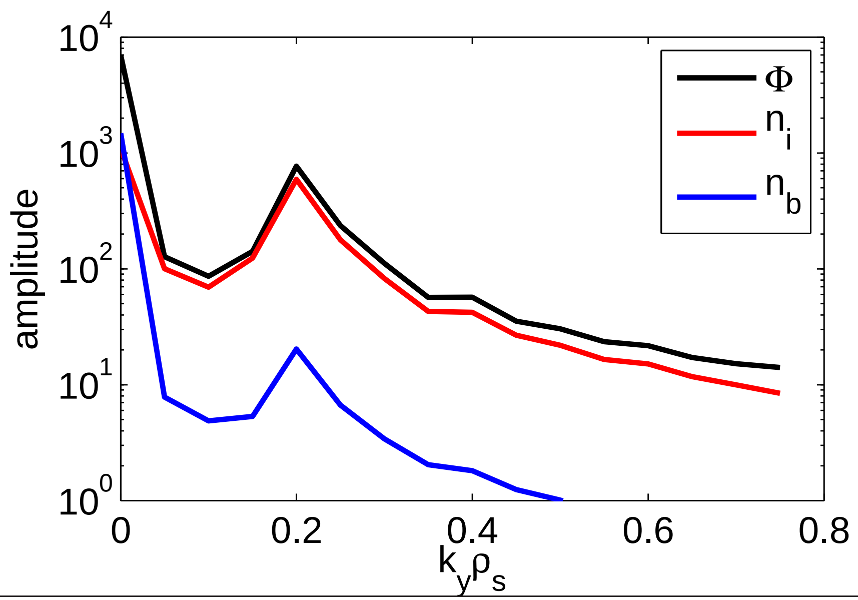
Nonlinear GENE Simulations

Methodology

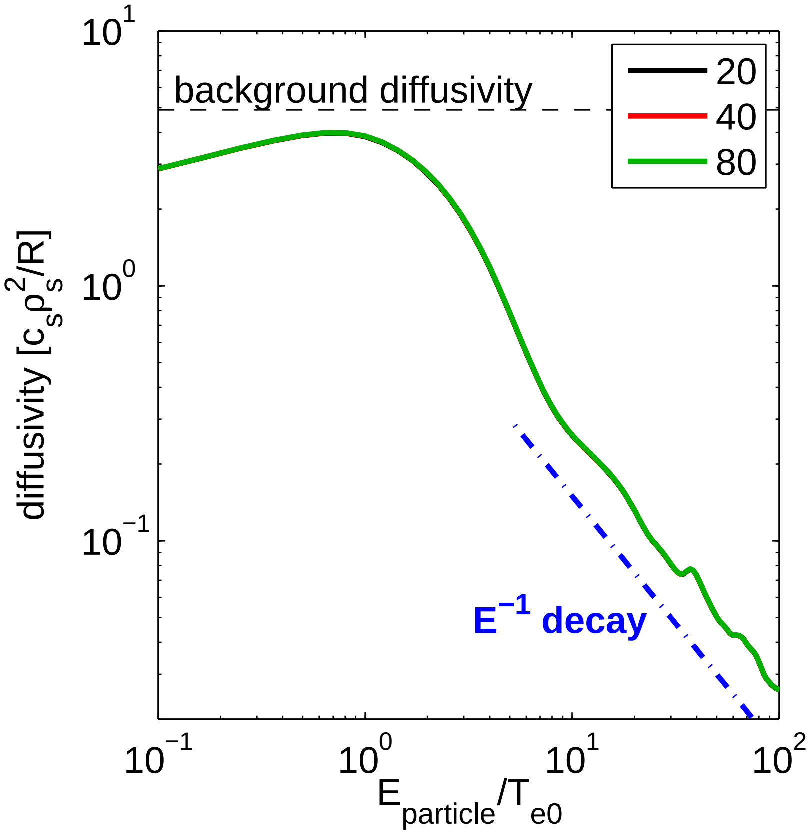
Introduce the fast ions as third species in well saturated turbulence.







Beam ion diffusivity



- shape is similar to the linearly calculated curves
- difference for higher particle energies: nonlinearly we get a $(E_{\text{particle}}/T_{\text{e0}})^{-1}$ decrease for $E_{\text{particle}} \gtrsim 10T_{\text{e0}}$

Summary and Outlook

- Beam ion diffusivity up to turbulent background diffusivity
- Significant particle transport up to $E_{\text{particle}}/E_{\text{thermal}} \sim 10$
- Diffusivity only depends on geometry and the background

- Possible explanation of Asdex Upgrade current drive results
- Electromagnetic simulations
- Active treatment of the beam ions

Appendix

Time scales for turbulence and beam ions

Velocities: $v_{Ti} \sim v_{Tb} < v_{\text{beam}} \ll v_{Te}$

Typical frequency of the ITG turbulence:

diamagnetic frequency $\omega_* \approx (k_y \rho_s) \frac{R}{L_{Ti}} \frac{c_s}{R}$

Particle frequencies

- transit frequency $\omega_{\text{tr}} = v_\| / qR = \frac{v_\|}{c_s} \frac{1}{q} \frac{c_s}{R}$
- curvature drift frequency $\omega_{\text{c}} = \frac{v_\|^2}{\Omega R} k_\perp = \frac{v_\|^2}{c_s^2} (k_\perp \rho_s) \frac{c_s}{R}$

Time scales for turbulence and beam ions

Velocities: $v_{Ti} \sim v_{Tb} < v_{\text{beam}} \ll v_{Te}$

Typical frequency of the ITG turbulence:

diamagnetic frequency $\omega_* \approx (k_y \rho_s) \frac{R}{L_{Ti}} \frac{c_s}{R} \approx (1 - 3) \frac{c_s}{R}$

Particle frequencies

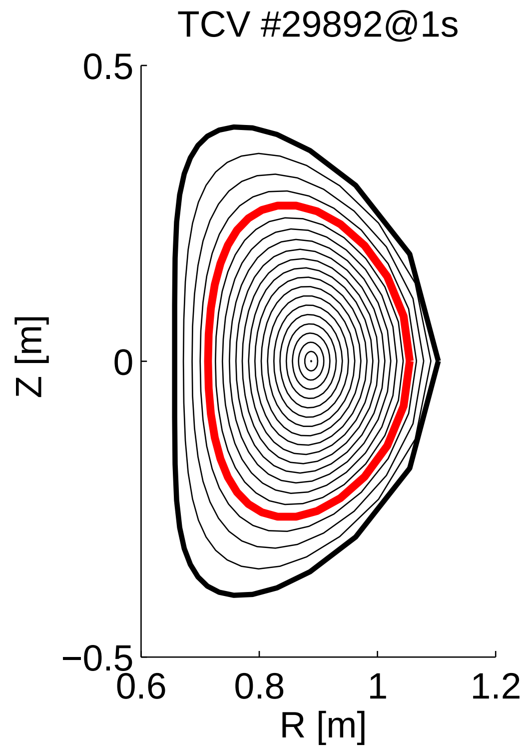
- transit frequency $\omega_{\text{tr}} = v_\parallel / qR = \frac{v_\parallel}{c_s} \frac{1}{q} \frac{c_s}{R} \approx (1 - 40) \frac{c_s}{R}$
- curvature drift frequency $\omega_{\text{c}} = \frac{v_\parallel^2}{\Omega R} k_\perp = \frac{v_\parallel^2}{c_s^2} (k_\perp \rho_s) \frac{c_s}{R} \approx (0.4 \ldots) \frac{c_s}{R}$

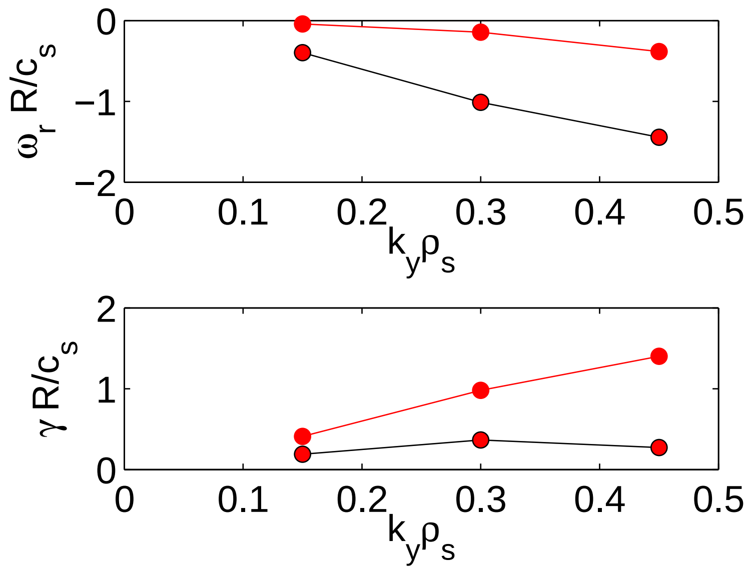
⇒ all frequency ranges are overlapping

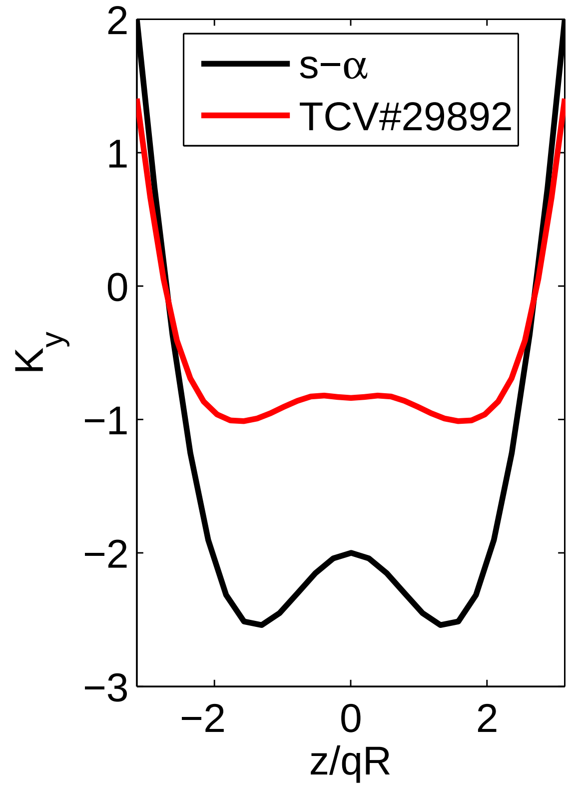
⇒ need simulations to sort them out

Dependence on magnetic geometry

Use discharge #29892@1.0s from TCV, CHEASE equilibrium, and the same gradients.



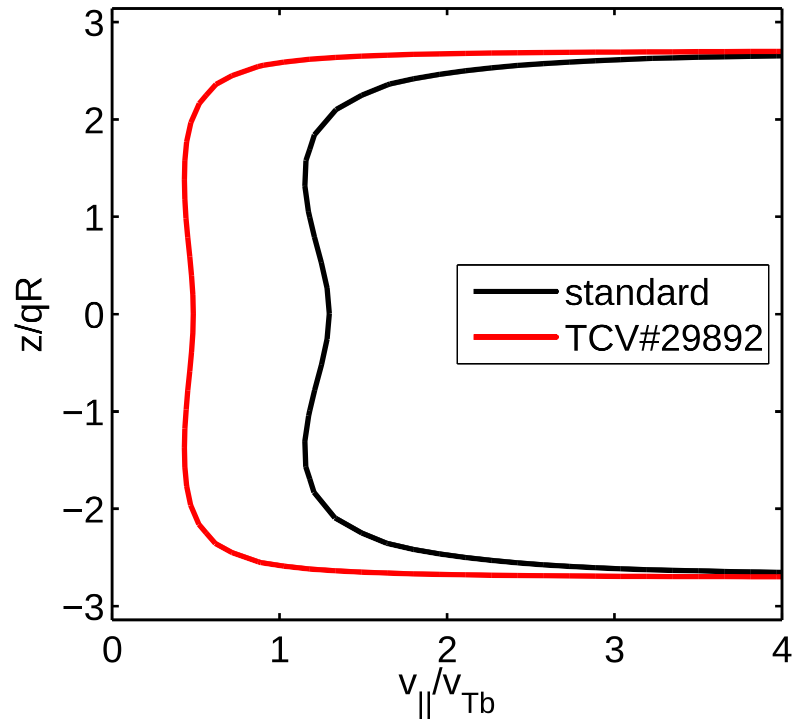


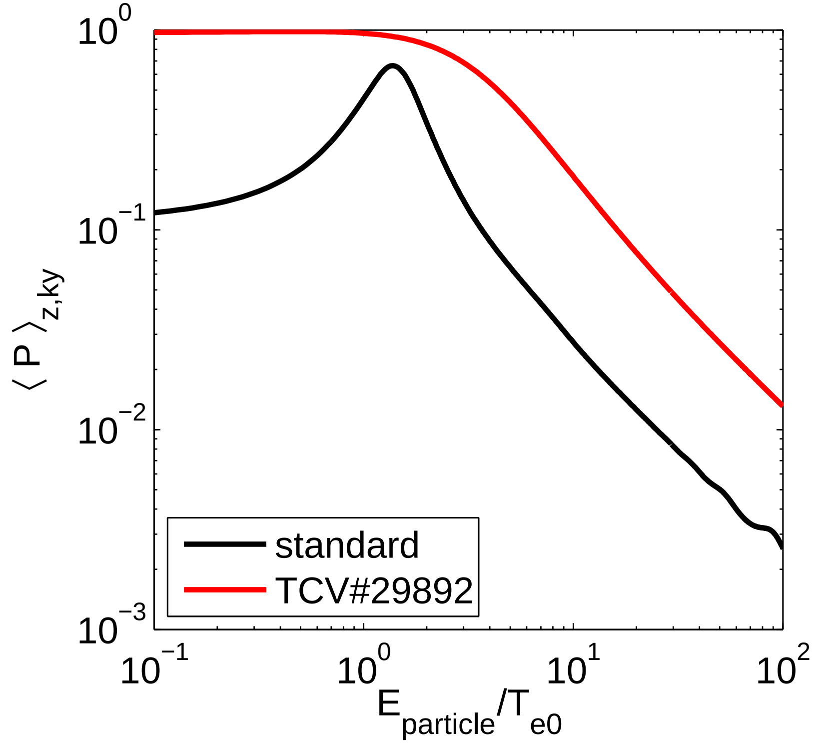


$|\mathcal{K}_y|$ reduced, growth rate increased, real frequency reduced

Dependence on magnetic geometry

Investigating the prefactor $\frac{\gamma^2}{\omega'^2+\gamma^2}$





Full Vlasov Equation

$$\frac{\partial g}{\partial t} + \alpha_j v_\parallel \frac{\partial F_1}{\partial z} + \sigma_j \alpha_j v_\parallel \left(\frac{F_{0-}}{\tau_-} + \frac{F_{0+}}{\tau_+} \right) \frac{\partial \bar{\Phi}_1}{\partial z} + \frac{1}{\widehat{B}} \left(\frac{\partial \chi}{\partial x} \frac{\partial g}{\partial y} - \frac{\partial \chi}{\partial y} \frac{\partial g}{\partial x} \right)$$

$$+ \mathcal{D} \left(\frac{1}{\widehat{B}} \frac{\partial \chi}{\partial y} - \frac{\mu \widehat{B} + 2 v_\parallel^2}{2 \sigma_j} \mathcal{K}_x \right) + \frac{\mu \widehat{B} + 2 v_\parallel^2}{2 \sigma_j} \left(\mathcal{K}_x \frac{\partial g}{\partial x} + \mathcal{K}_y \frac{\partial g}{\partial y} \right)$$

$$+ \left[\frac{F_0 \mu \widehat{B}}{2 \tau_\perp} + v_\parallel^2 \left(\frac{F_{0-}}{\tau_-} + \frac{F_{0+}}{\tau_+} \right) \right] \left(\mathcal{K}_x \frac{\partial \chi}{\partial x} + \mathcal{K}_y \frac{\partial \chi}{\partial y} \right) = 0$$

with $\mathcal{K}_x = -\frac{2L_\perp}{R} \sin z$ and $\mathcal{K}_y = -\frac{2L_\perp}{R_0} (\cos z + \hat{s} z \sin z)$.