Ion particle transport in the tokamak edge pedestal

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A generalized pinch-diffusion relation can be derived from momentum balance and combined with the particle continuity equation to obtain a generalized diffusion theory appropriate for the plasma edge pedestal. The FSA toroidal component of the momentum balance may be written $n_j^0 m_j \nu_{jk}^0 \left(\left(1+\beta_j\right) \upsilon_{\phi j}^0 - \upsilon_{\phi k}^0 \right) = n_j^0 e_j E_\phi^A + e_j B_\theta^0 \Gamma_{rj} + M_{\phi j}^0$, where $M_{\phi j}$ is beam momentum input, and $\beta_j \equiv \dfrac{\nu_{dj}^0 + \nu_{\perp j}^0 + \nu_{anomj}^0 + \nu_{nj}^0 + \nu_{elcxj}^0 + \nu_{ionj}}{\nu_{jk}^0} \equiv \dfrac{\nu_{dj}^*}{\nu_{jk}^0}$ represents the ratio of radial momentum transfer by gyroviscous (ν_{dj}), perpendicular viscous $(\nu_{\perp j})$, anomalous viscous (ν_{anomj}), inertial (ν_{nj}), elastic scattering and charge-exchange (ν_{elcxj}), and ionization (ν_{ionj}) processes to the interspecies collisional momentum transfer (ν_{jk}). Combining the FSA toroidal and radial components of the momentum balance, $E_r^0 = \dfrac{1}{n_j^0 e_j} \dfrac{\partial p_j^0}{\partial r} + \upsilon_{\phi j}^0 B_\theta^0 - \upsilon_{\theta j}^0 B_\phi^0$, leads to a pinch-diffusion relation $\Gamma_{rj} \equiv \left\langle n_j \upsilon_{rj} \right\rangle = n_j D_{jj} \left(L_{nj}^{-1} + L_{Tj}^{-1} \right) - n_j D_{jk} \left(L_{nk}^{-1} + L_{Tk}^{-1} \right) + n_j \upsilon_{pj}$, where the "diffusion coefficients" are given by $D_{jj} \equiv \dfrac{m_j T_j \left(\nu_{dj}^* + \nu_{jk} \right)}{\left(e_j B_\theta \right)^2}$, $D_{jk} \equiv \dfrac{m_j T_k \nu_{jk}}{e_j e_k (B_\theta)^2}$ and the "pinch velocity" is given by $n_j \upsilon_{pj} \equiv -\dfrac{M_{\phi j}}{e_j B_\theta} - \dfrac{n_j E_\phi^A}{B_\theta} + \dfrac{n_j m_j \nu_{dj}^*}{e_j B_\theta} \left(\dfrac{E_r}{B_\theta} \right) + \dfrac{n_j m_j f_p^{-1}}{e_j B_\theta} \left(\left(\nu_{jk} + \nu_{dj}^* \right) \upsilon_{\theta j} - \nu_{jk} \upsilon_{\theta k} \right)$. Combining this pinch-diffusion relation with the continuity equation $\nabla \cdot \Gamma_j \equiv \nabla \cdot n_j \upsilon_j = S_j$ leads to the generalized radial diffusion equation that conserves momentum and particles

$$-\frac{\partial}{\partial r}\left(D_{jj} \frac{\partial n_j}{\partial r} \right) - \frac{\partial}{\partial r}\left(D_{jk} \frac{\partial n_k}{\partial r} \right) - \frac{\partial}{\partial r}\left(D_{jj} \frac{n_j}{T_j} \frac{\partial T_j}{\partial r} \right) - \frac{\partial}{\partial r}\left(D_{jk} \frac{n_j}{T_k} \frac{\partial T_k}{\partial r} \right) + \frac{\partial \left(n_j \upsilon_{pj} \right)}{\partial r} = S_j \qquad (1)$$

The radial electric field and the rotation velocities dominate particle transport via the last term on the LHS, the density gradients of other "k" species and the temperature gradients also drive diffusive transport of species "j", and radial momentum transfer via viscosity and atomic physics affects the self-diffusion coefficients D_{jj}. Only the first term on the LHS is usually retained in edge plasma fluid codes, thereby neglecting much of the physics relevant to transport.