Role of coherent structures for transport in fully-developed turbulence

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12th US-EU TTF, San Diego, 17th April 2007

Outline

1. *Turbulence and wavelets*
   Turbulence and averages
   Continuous and orthogonal wavelets
   Extraction of coherent structures

2. *Role of coherent structures*
   2D Numerical experiments (DNS)
   2D Laboratory experiment (PIV)
   3D Numerical experiments (DNS)

3. *Interpretation and perspectives*
   Interpretation of the turbulent cascade
   Conclusion and perspectives :
   Coherent Vortex Simulation (CVS)

Fully-developed turbulence

Turbulence is a property of flows which involves
a **large number of degrees of freedom** interacting together, *i.e.*,
a **crowd *(turba,ae)* of vortices *(turbo, turbinis)*.**

Fluid hypothesis : observation is made at scales
much larger than the mean free path of particles.

Turbulent flows are solutions of **Navier-Stokes equations** :

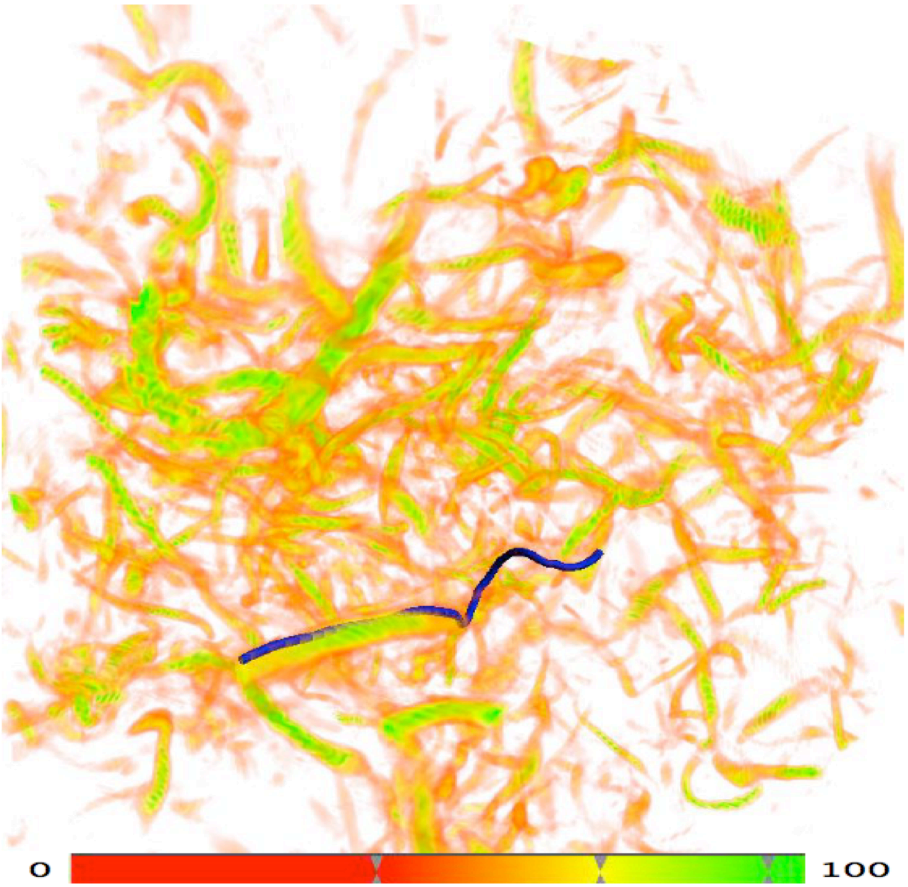
$$\partial_t \vec{\omega} + (\vec{v} \cdot \nabla) \vec{\omega} - \vec{\omega} \cdot \nabla \vec{v} = \nu \nabla^2 \omega + \nabla \times \vec{F}$$

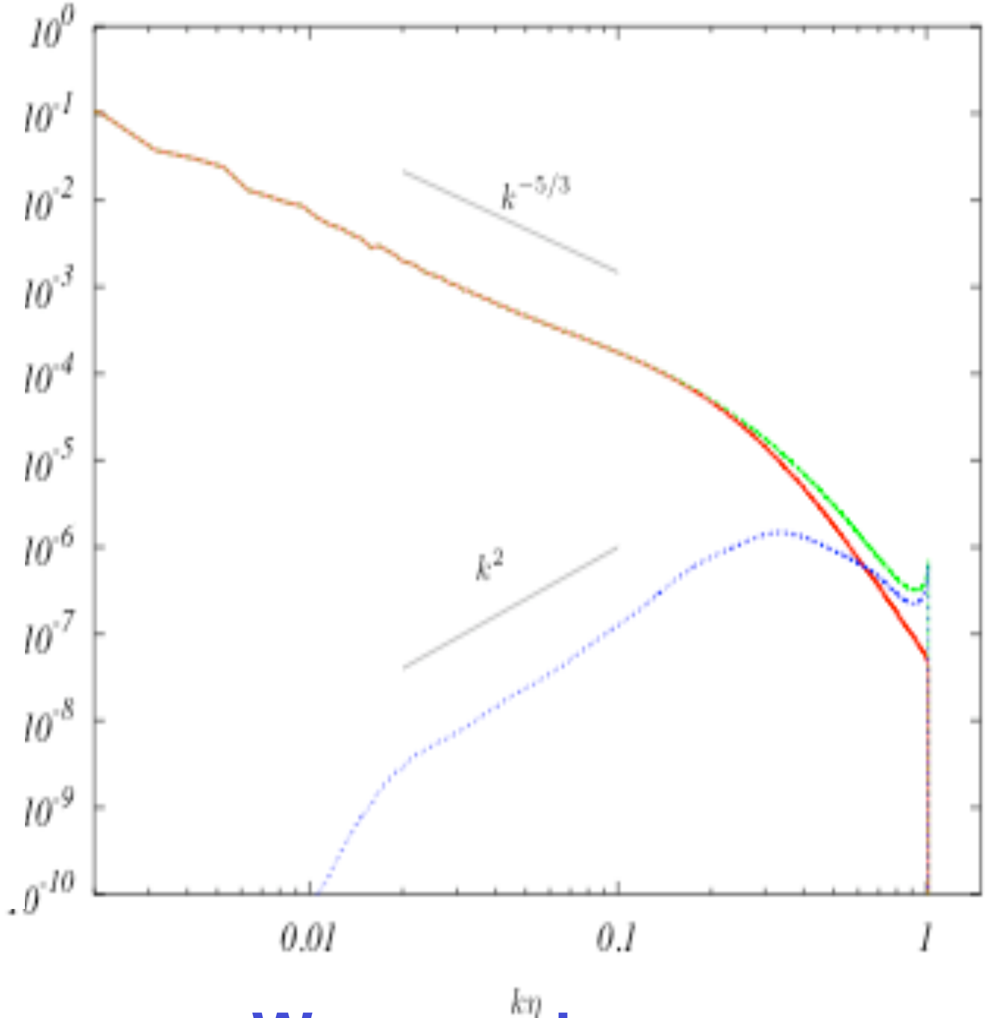
$$\vec{\omega} = \nabla \times \vec{v} \qquad \text{and} \qquad \nabla \cdot \vec{v} = 0$$

ω vorticity, v velocity, F external force, ν viscosity and $\rho=1$ density,
plus initial conditions and boundary conditions

Fully-developed turbulence when Reynolds number
is very large, *i.e.*, when the **nonlinear term strongly
dominates the viscous linear term**.

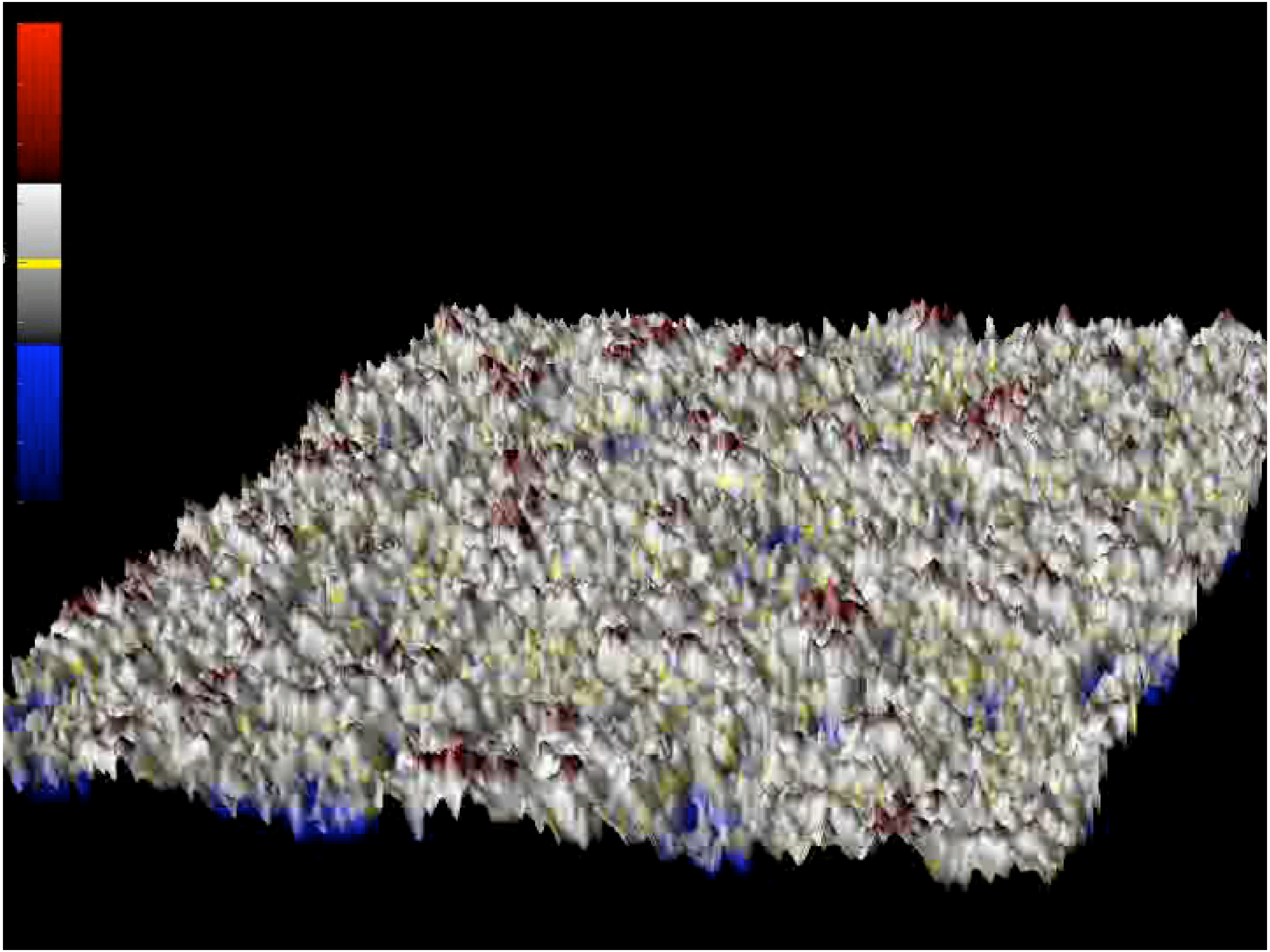
One realization of a fully developed turbulent flow



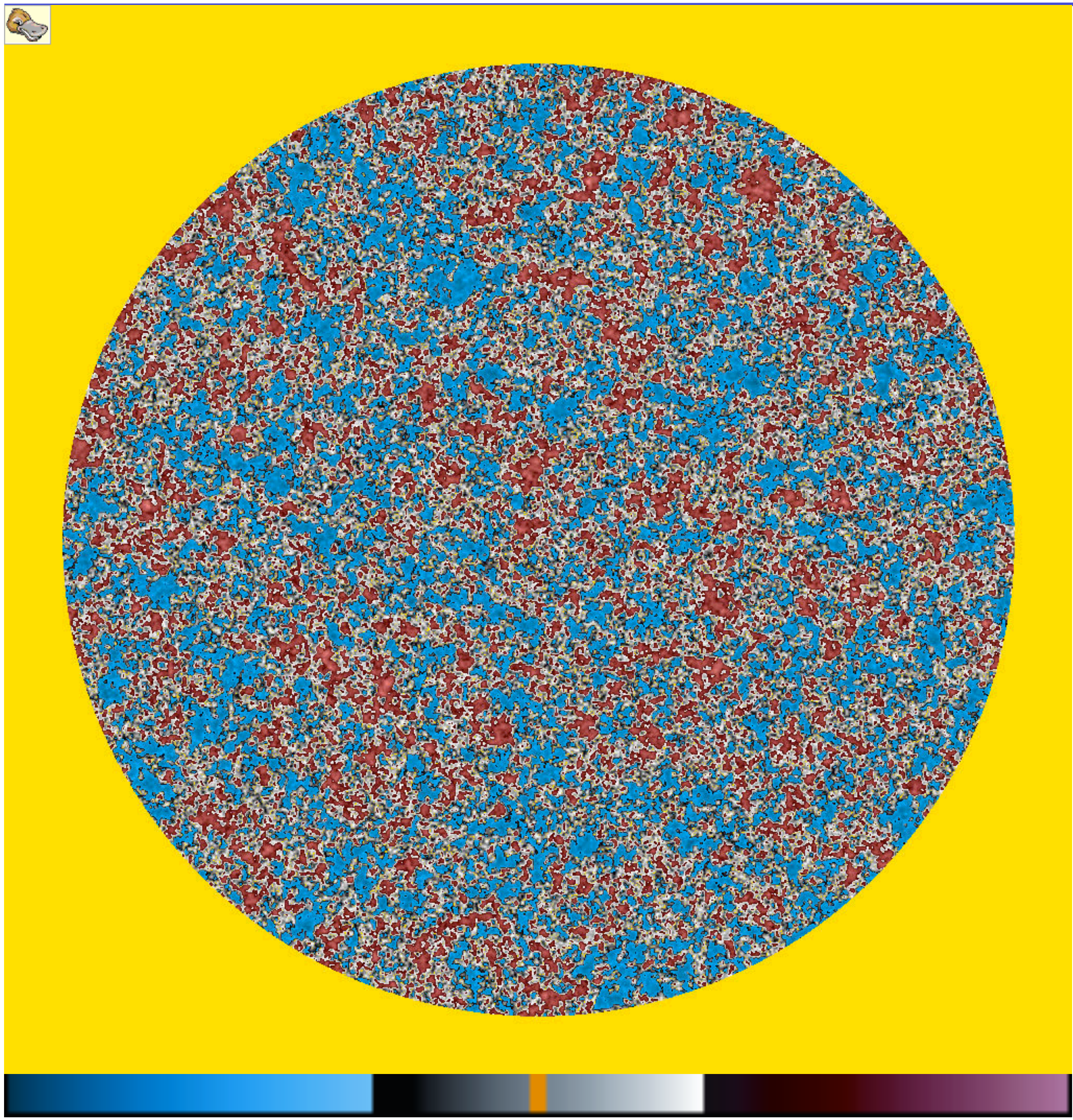


Physical space:
Modulus of vorticity

Wavenumber space:
Energy spectrum



2D turbulent flow in a cylindrical container



DNS
N=1024^2

Random initial conditions

No-slip boundary conditions using volume penalization

Schneider & Farge
Phys. Rev. Lett.,
December 2005

Turbulence practice is the 'art of averaging'

Reynolds averaging (1883) :

Field f = Mean $\overline{f}$ + Fluctuations f'

with $\overline{f'} = 0$ $\overline{\overline{f}} = \overline{f}$

$$\overline{f + g} = \overline{f} + \overline{g} \qquad \overline{\partial f} = \partial \overline{f}$$

but nonlinearity is hard to handle since there is no scale separation :

$$\overline{fg} = \overline{f}\,\overline{g} + \overline{f'g'}$$

Proposition :

$$f' = f'_c + f'_i$$

Fluctuations = coherent fluctuations + incoherent fluctuations

Research program to study turbulence

'In the last decade we have experienced a conceptual shift in our view of turbulence. For flows with strong velocity shear, or other organizing characteristics, many now feel that the spectral description has inhibited fundamental progress. ***The next "El Dorado" lies in the mathematical understanding of coherent structures*** *in weakly dissipative fluids: the formation, evolution and interaction of metastable vortex-like solutions of nonlinear partial differential equations...'*

Norman Zabusky,
Physics Today, 1984

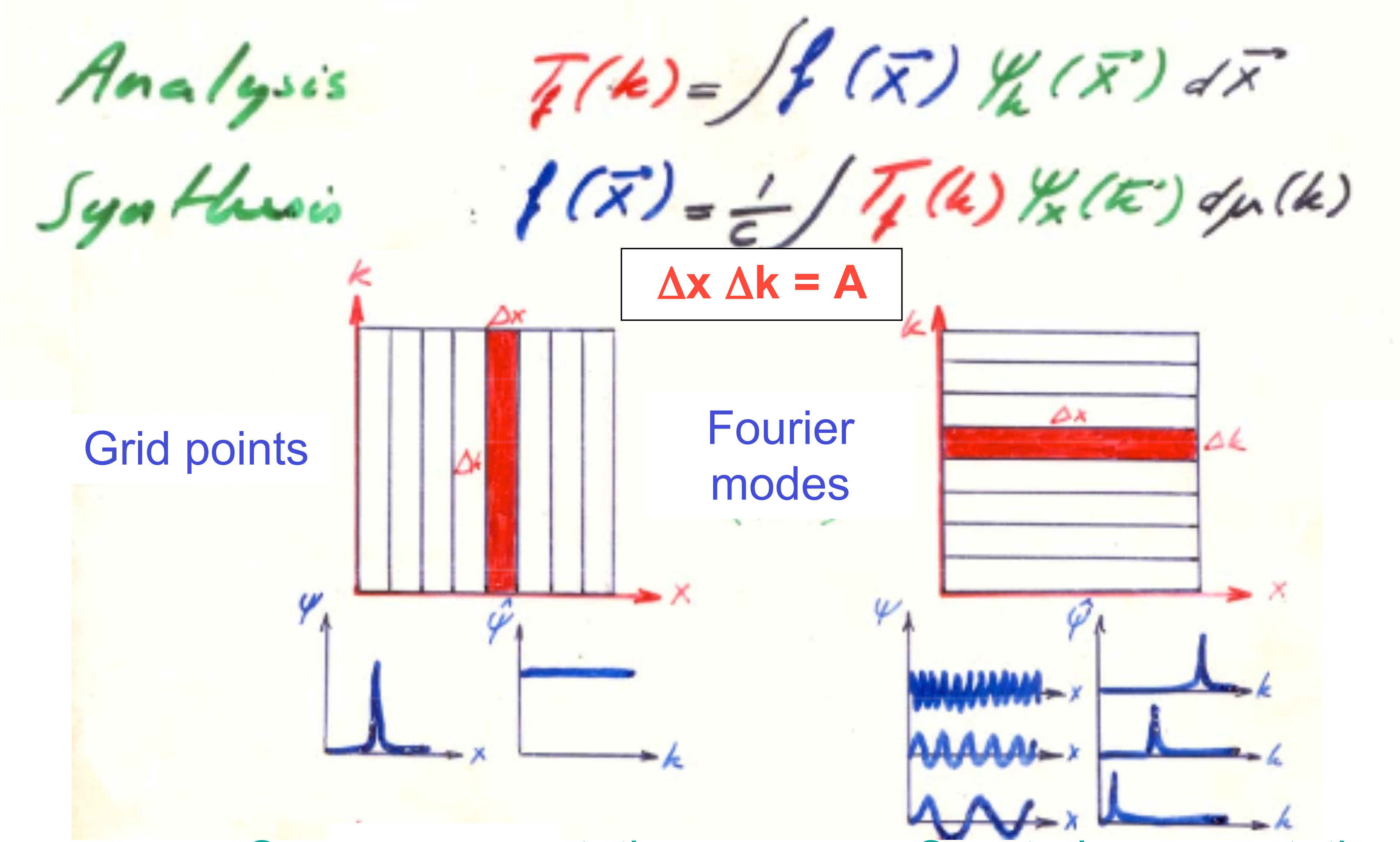
We have proposed to replace
the Fourier representation by the **wavelet representation**
to keep track of **coherent structures** and to replace
classical averages by **nonlinear filtering of wavelet coefficients.**

Farge and Rabreau, 1988,
C. R. Acad. Paris, ***307***

Farge, 1992,
Ann. Rev. Fluid Mech., ***24***

Farge and Schneider, 2006
Encyclopedia Math. Phys., Elsevier

Integral transforms



Δx Δk = A

Grid points

Fourier modes

Space representation

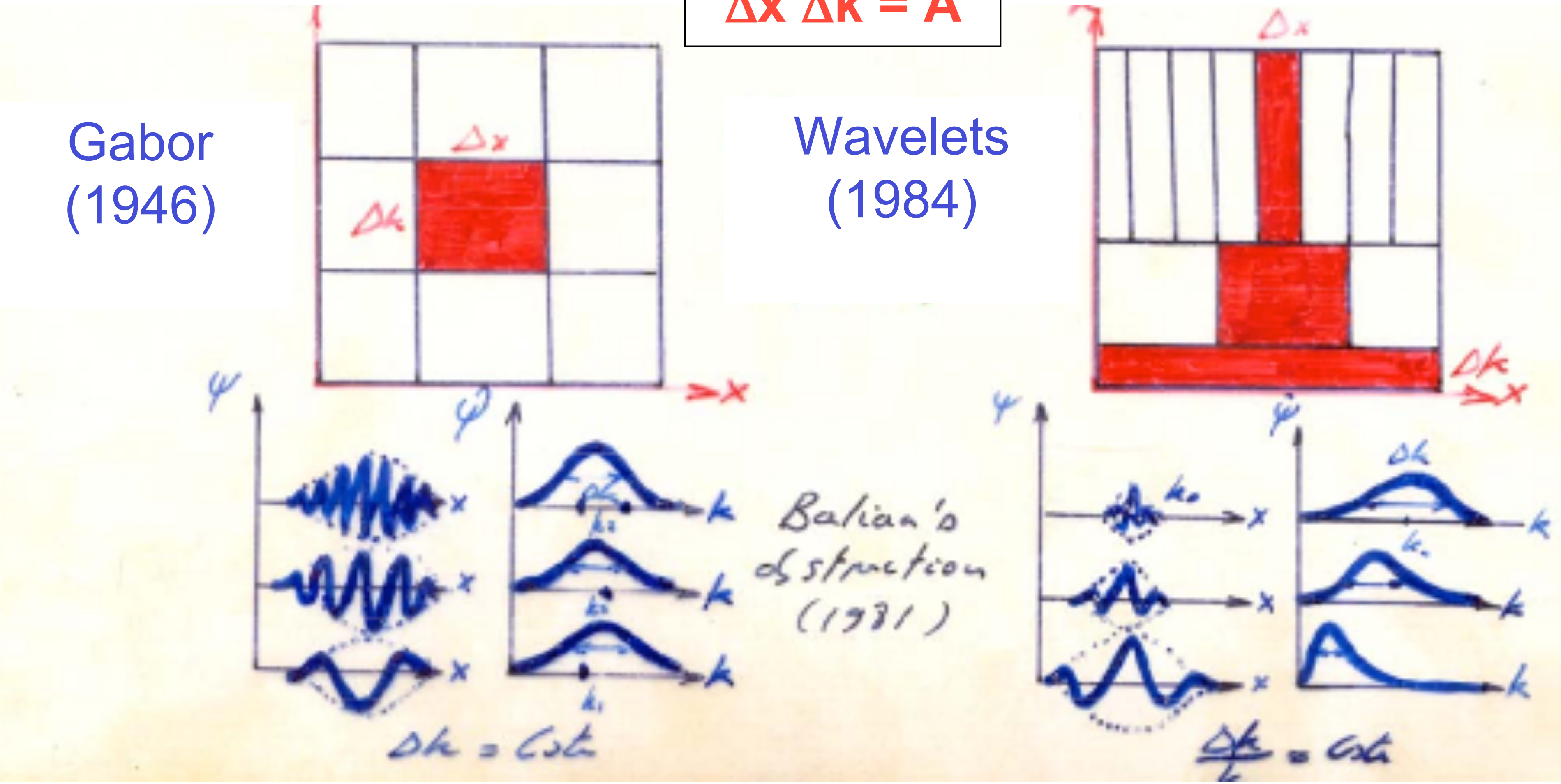
Spectral representation

Other representations

$\Delta x \, \Delta k = A$

Gabor (1946)

Wavelets (1984)



Space-wavenumber representation

Space-scale representation

Wavelet representation

Wavelets are functions **well localized in both physical and spectral space**. The wavelet representation is based on a set of dilated and translated wavelets ψ_{ji} indexed by scale *j* and position *i* .

The space locality of f is kept by each wavelet coefficient $\tilde{f}_{ji}$ since each coefficient is indexed by its scale *j* and its position *i* . Therefore filtering in wavelet basis preserves space locality.

In contrast Fourier coefficients are indexed by wavenumbers and therefore one needs to know the phase of all coefficients to preserve space locality. Therefore filtering in Fourier basis looses space locality.

Special wavelets can generate orthogonal bases. Using them, a signal sampled on *N* points is represented by *N* wavelet coefficients and the computing cost of the fast wavelet transform is proportional to ***N* operations**.

Farge, Ann. Rev. Fluid Mech., 24, 1992

Farge and Schneider, Encyclopedia of Mathematical Physics, 2006

Wavelet transforms

Analyzing functions :

Translates and **dilates**

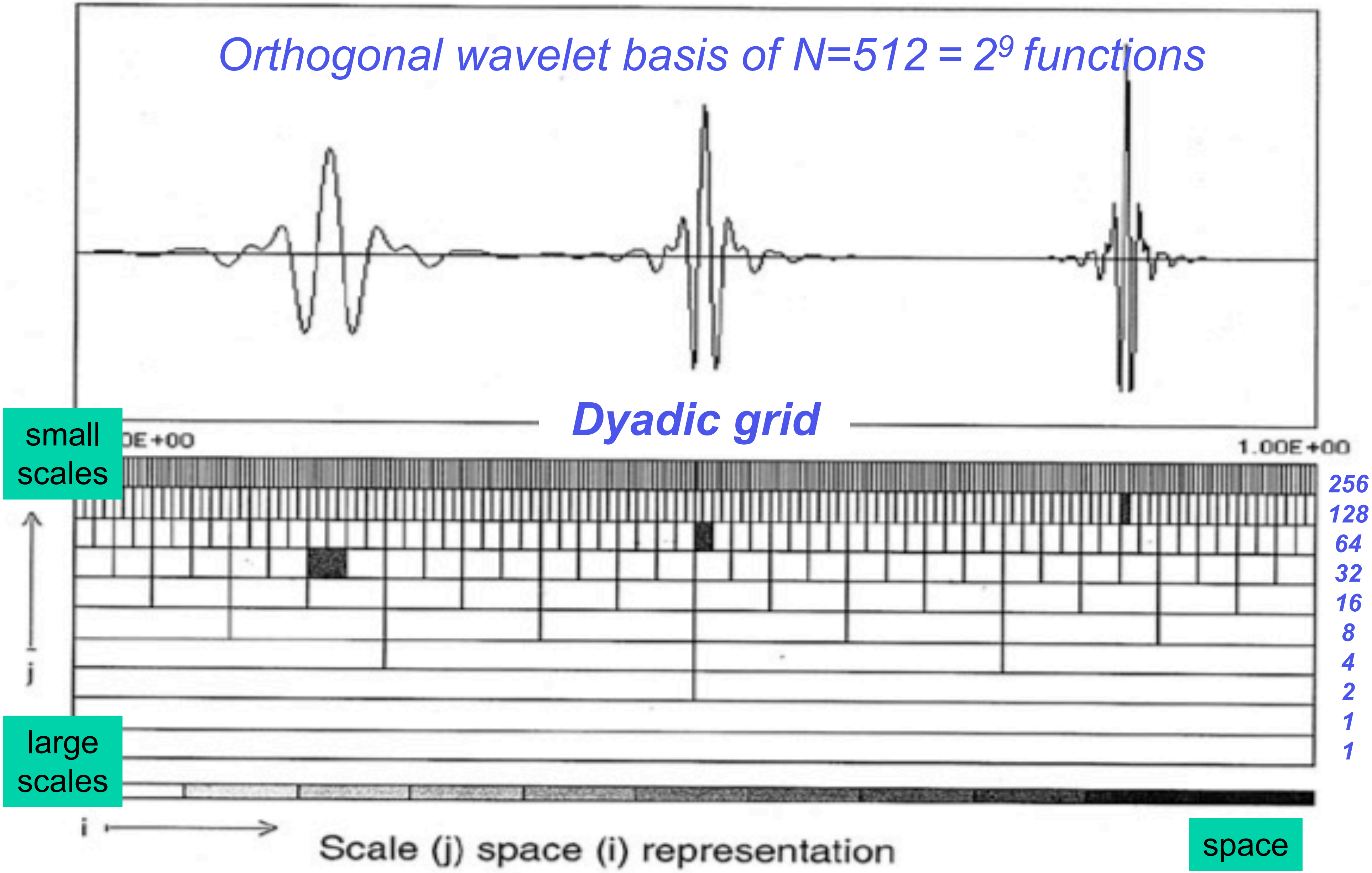
of an oscillating function (zero mean)

Well localized in space and wavenumber

Wavelet coefficients : $\widetilde{f}(l,\vec{x}) = \langle \psi_{l,\vec{x}} \mid f(\vec{x}) \rangle$

Continuous wavelets	**Orthogonal wavelets**
$\psi_{l,\vec{x}} = \frac{1}{l^{n/2}} \psi\left(\frac{\vec{x}' - \vec{x}}{l}\right)$	$\psi_{j,i} = 2^{j/2} \psi(2^j x - i)$
• Translates and dilates can vary continuously • Redundant basis	• Translates and dilates sit on a discrete dyadic grid • Orthogonal basis
• Easily read coefficients • Unfold in both space and scale • Good for analysis	• Filtering and reconstruction • Compression (JPEG 2000) • Good for computation (N oper.)

Multiscale representation



Extraction of coherent structures

Since there is not yet a universal definition of coherent structures observed in turbulent flows (from laboratory and numerical experiments), we adopt an **apophetic method** :
instead of defining what they are, we define what they are not.

We propose the minimal statement:
'Coherent structures are different from noise'

$\Longrightarrow$

Extracting coherent structures becomes a **denoising problem**, not requiring any hypotheses on the coherent structures but only on the noise to be eliminated.

Choosing the simplest hypothesis as a first guess, we suppose we want to eliminate an additive Gaussian white noise.

Farge, Schneider, Kevlahan, Phys. Fluids, 11 (8), 1999

Denoising algorithm

Gaussian **white noise** is by definition **equidistributed** among all modes and the amplitude of the coefficients is given by its r.m.s., whatever the functional basis one considers.

Therefore the coefficients of a noisy signal whose amplitudes are much larger than the r.m.s. of the noise belong to the denoised signal. This procedure corresponds to **nonlinear filtering**.

The advantage of performing such a nonlinear filtering using the wavelet representation is that the **wavelet coefficients** preserve the space locality.

Since we do not know a priori the r.m.s. of the noise, we have proposed an iterative procedure which takes as first guess the r.m.s. of the noisy signal and does not require adjusting any parameter. The more noisy the signal is, the faster the convergence.

Azzalini, Farge, Schneider,
Appl. Comput. Harmonic Analysis, 18 (2), 2005

Wavelet-based denoising

1. Goal:
   Extraction of coherent structures
   from a noise which can then be modelled
   to compute the flow evolution.

2. Apophatic principle:
   - **no hypothesis on the structures**,
   - **only hypothesis on the noise**,
   - simplest hypothesis as our first choice.

3. Hypothesis on the noise:

   $f_n = f + w$

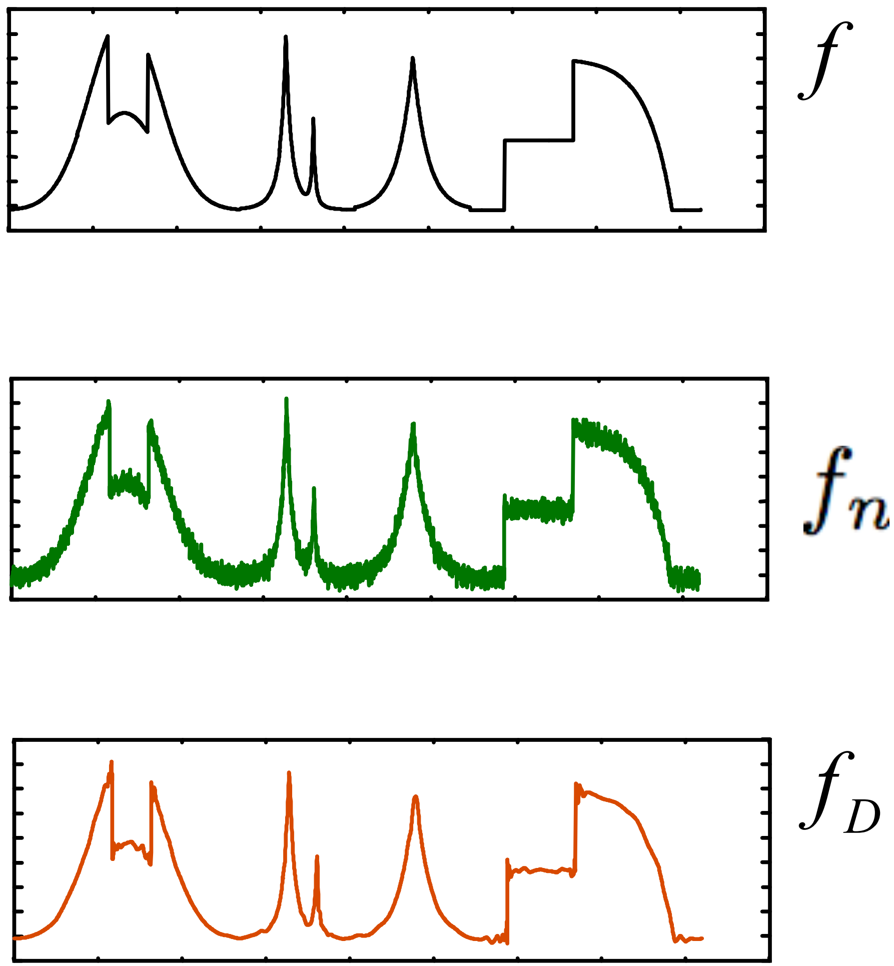
   w : Gaussian white noise,
   σ^2 : variance of the noise,
   N : number of coefficients.

4. Computation of the threshold:

$$\epsilon = \sqrt{2\sigma^2 ln(N)}$$

5. Denoised signal:

$$f_D = \sum_{\lambda : |\tilde{f}_\lambda| < \varepsilon} \tilde{f}_\lambda \psi_\lambda$$



Donoho and Johnstone, Biometrica, 81, 1994

Coherent Vortex Extraction

Farge, Schneider, Kevlahan, Phys. Fluids 11(8), 1999
Farge, Pellegrino, Schneider, Phys. Rev. Lett. 87(5), 2001

- Vorticity $\vec{\omega} = \nabla \times \vec{v}$ at resolution $N = 2^{3J}$
- Wavelet transform $\tilde{\vec{\omega}} = \langle \vec{\omega}, \psi_\lambda \rangle$
- Thresholding: $T = (4/3 Z \ln N)^{1/2}$

$$\tilde{\vec{\omega}}_C = \begin{cases} \tilde{\vec{\omega}} & \text{for} \quad |\tilde{\vec{\omega}}| \geq T, \\ 0 & \text{for} \quad |\tilde{\vec{\omega}}| < T \end{cases} \qquad \tilde{\vec{\omega}}_I = \begin{cases} \tilde{\vec{\omega}} & \text{for} \quad |\tilde{\vec{\omega}}| < T, \\ 0 & \text{for} \quad |\tilde{\vec{\omega}}| \geq T \end{cases}$$

- Inverse wavelet transform to reconstruct $\vec{\omega}_C + \vec{\omega}_I = \vec{\omega}$
- Apply Biot-Savart operator to reconstruct $\vec{v}_C + \vec{v}_I = \vec{v}$ with $\vec{v} = \nabla \times \nabla^{-2} \vec{\omega}$
- Remark: $Z = Z_C + Z_I$ (orth. dec.) and $E \approx E_C + E_I$
- Linear complexity, $\mathcal{O}(\mathcal{N})$

Application to 2D turbulent flows

- Either from **numerical experiment**, computed using Direct Numerical Simulation (DNS) at resolution 512^2

- Or from **laboratory experiment**, measured using Particle Image Velocimetry (PIV) at resolution 128^2

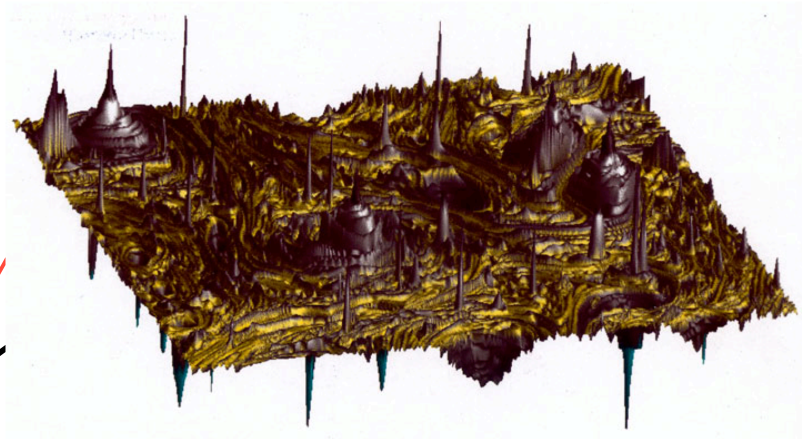
in collaboration with Jori E. Ruppert-Felsot, Erhan Sharon and Harry L. Swinney

Center for Nonlinear Dynamics, University of Texas at Austin

Decomposition of 2D vorticity field
in numerical experiment

DNS
$N=512^2$

0.2 % of coefficients
99.8 % of kinetic energy
93.6 % of enstrophy

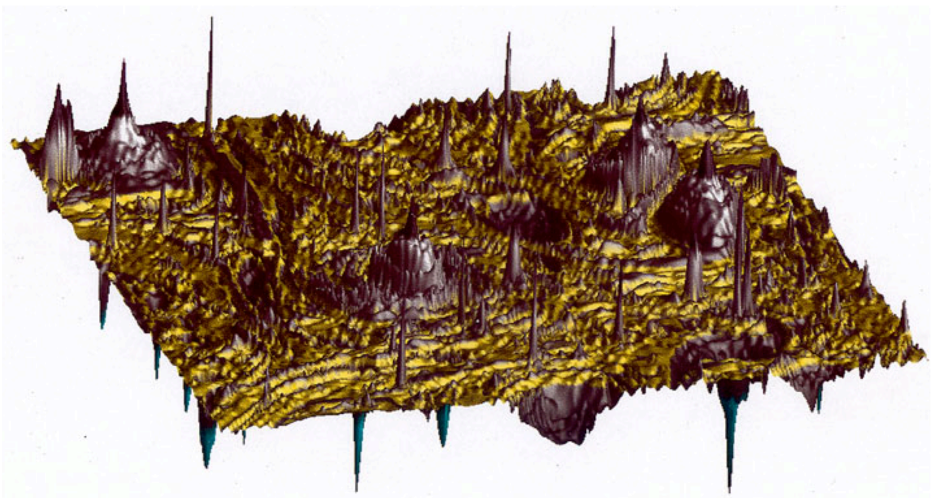


99.8 % of coefficients
0.2 % of kinetic energy
6.4 % of enstrophy

Coherent flow

Total flow

Incoherent flow



+

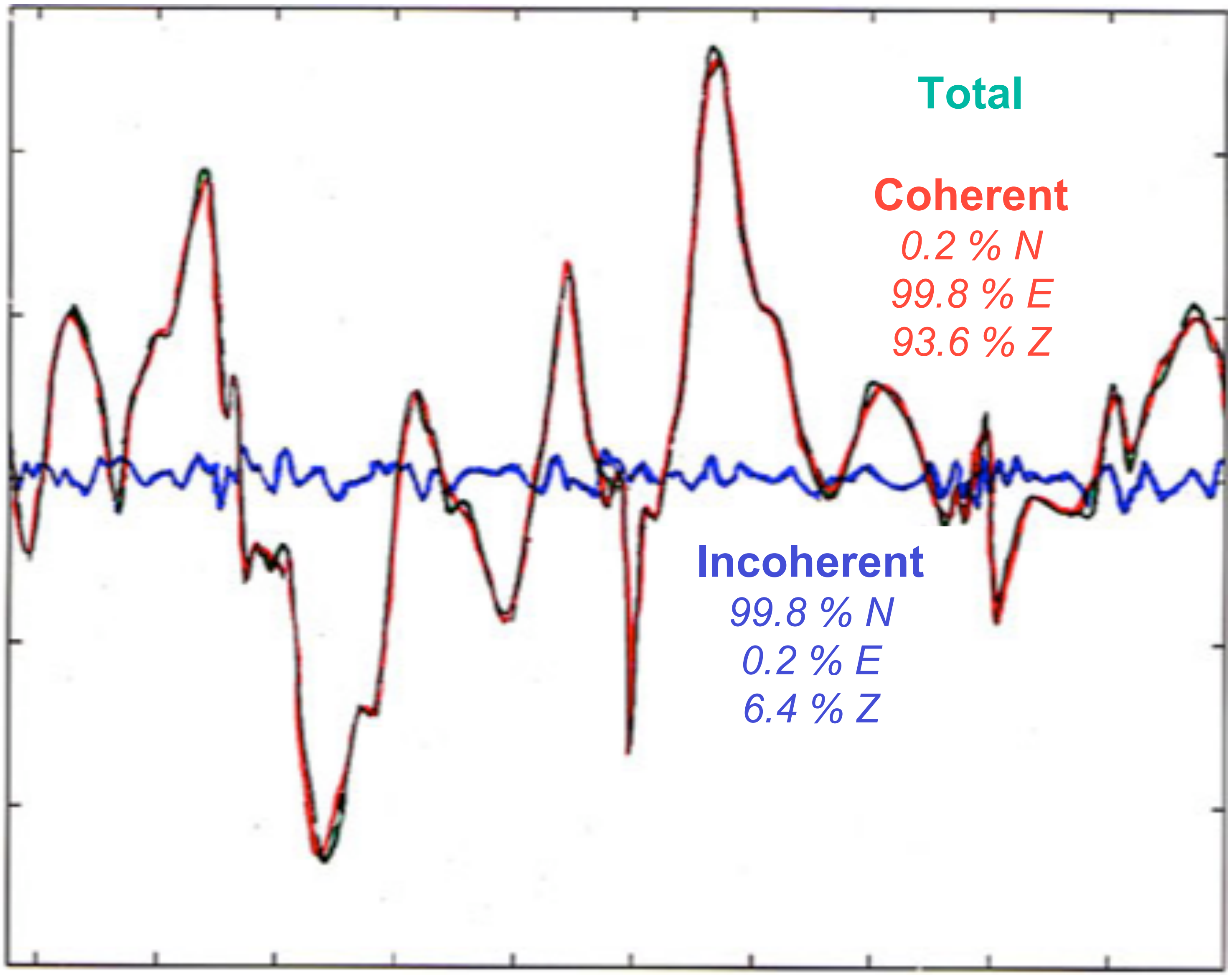
$+\omega_{min}$  $+\omega_{max}$

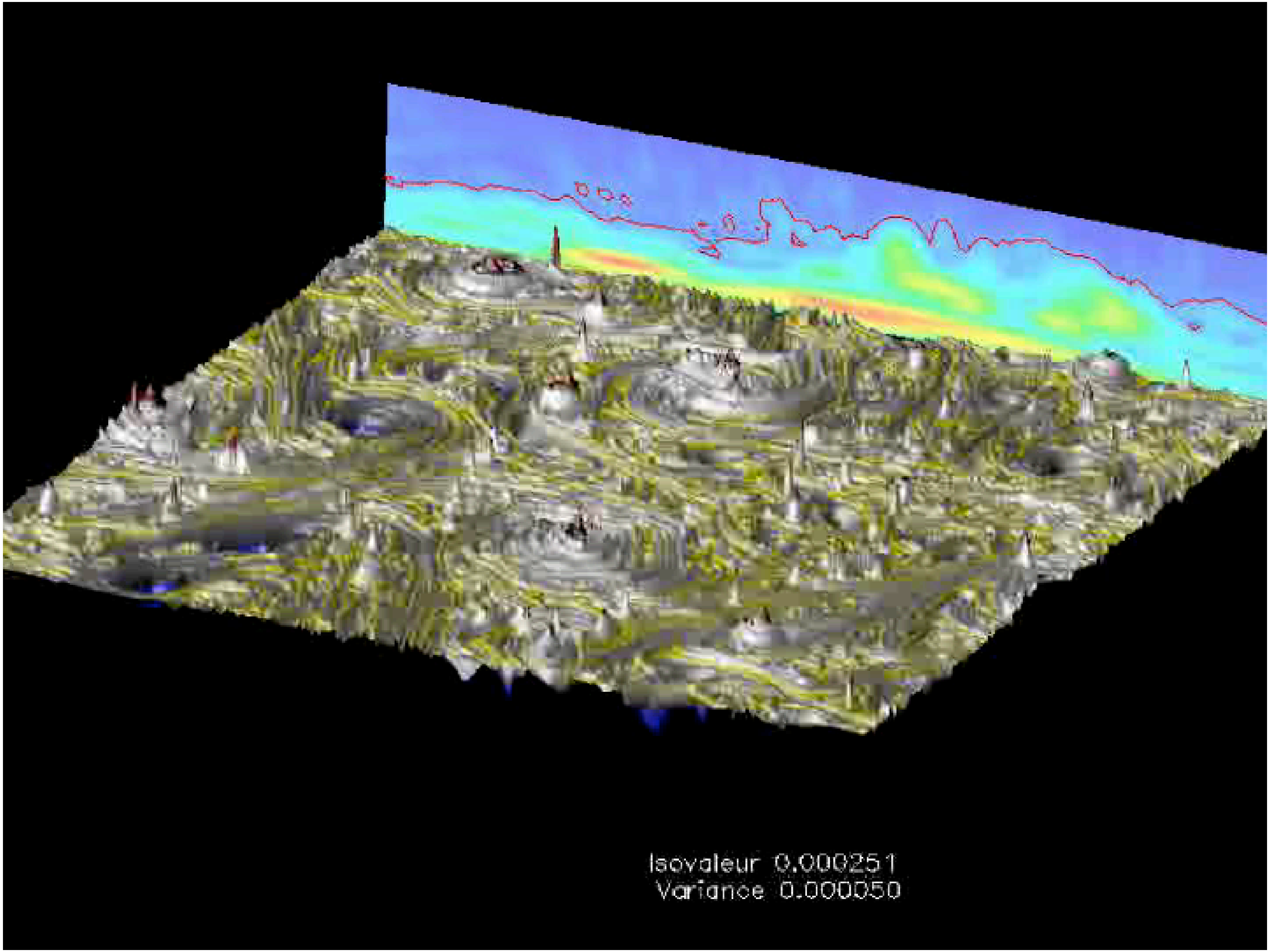
1D cut of the vorticity field
in numerical experiment

DNS
N=512²

$\omega_t = \omega_c + \omega_i$

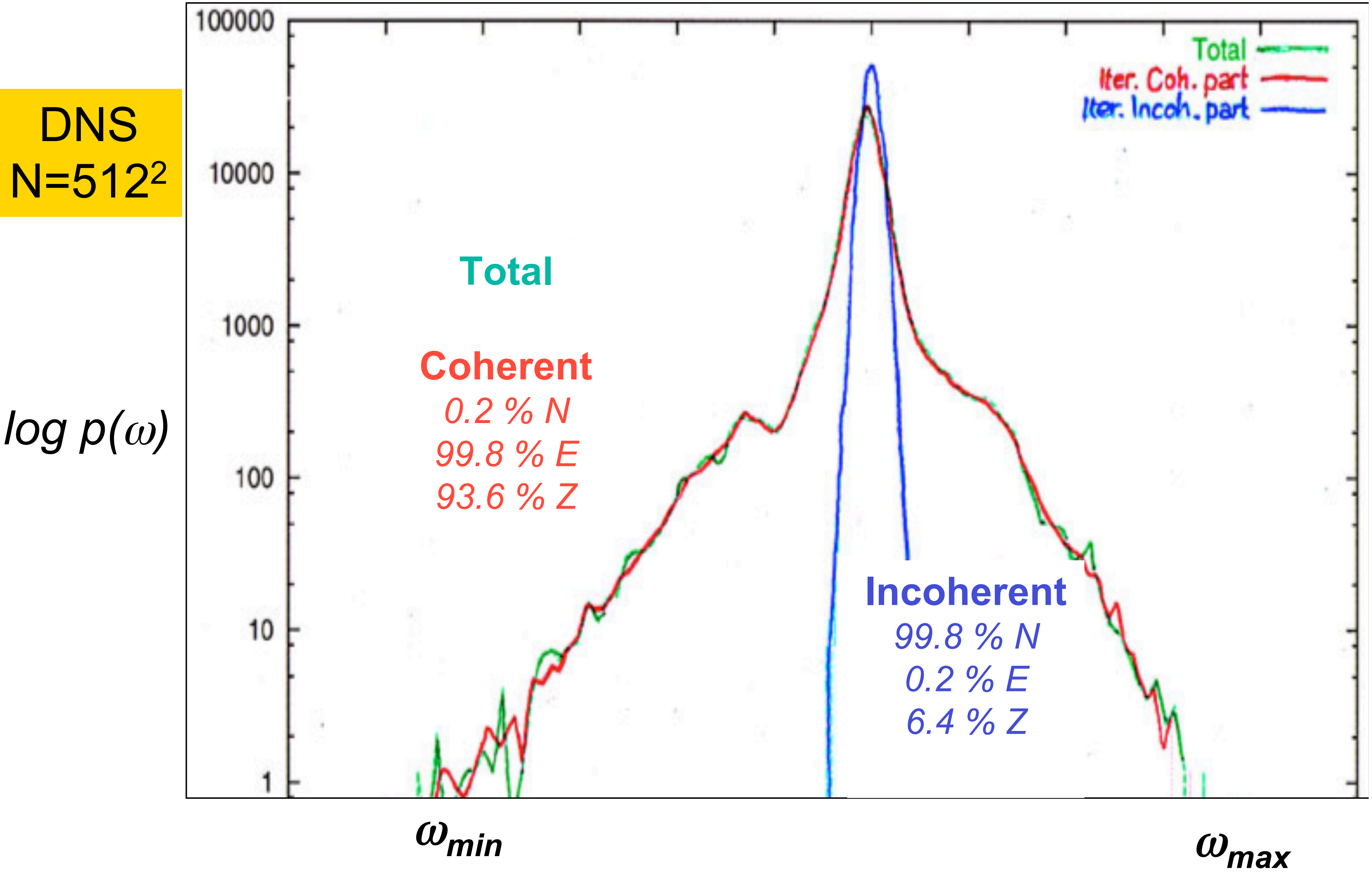
$Z_t = Z_c + Z_i$



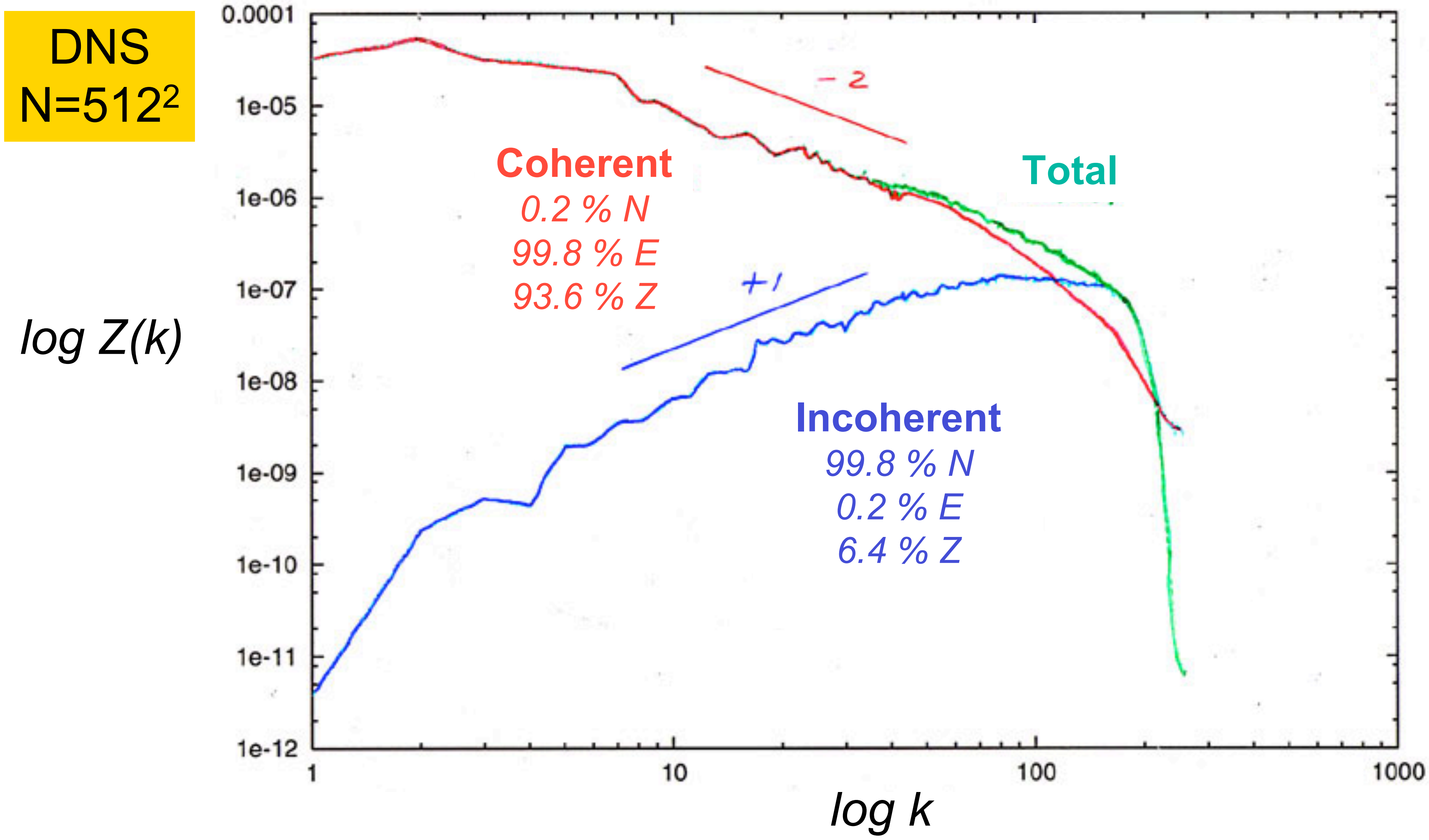

Isovaleur 0.000251
Variance 0.000050

PDF of vorticity
in numerical experiment

DNS
$N=512^2$



Enstrophy spectrum
in numerical experiment

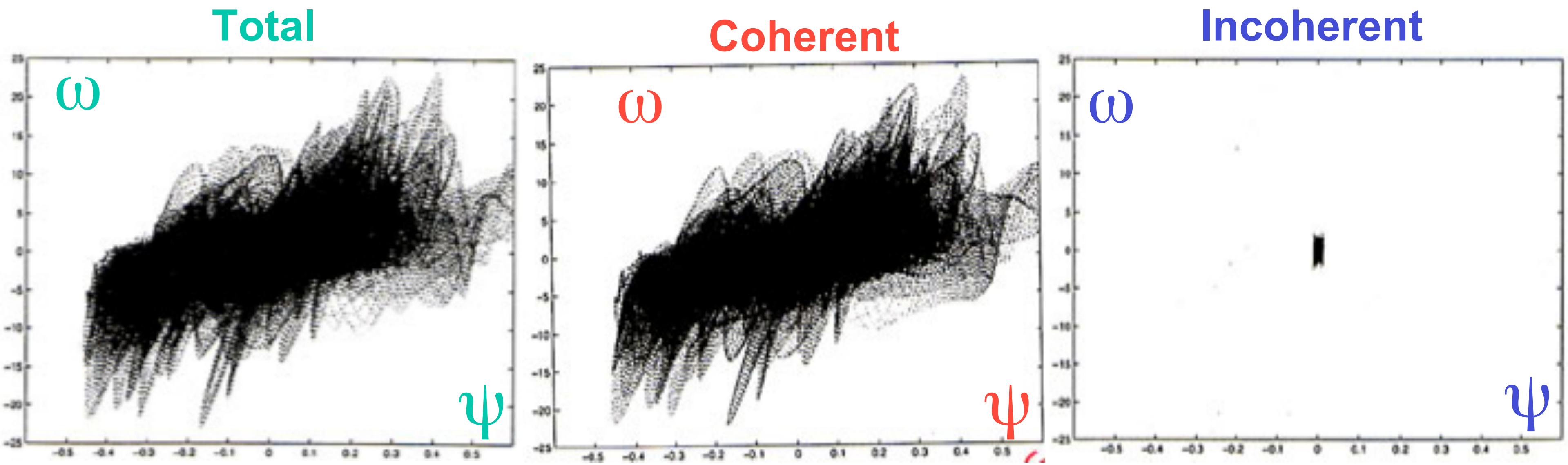


A posteriori proof of coherence
in numerical experiment

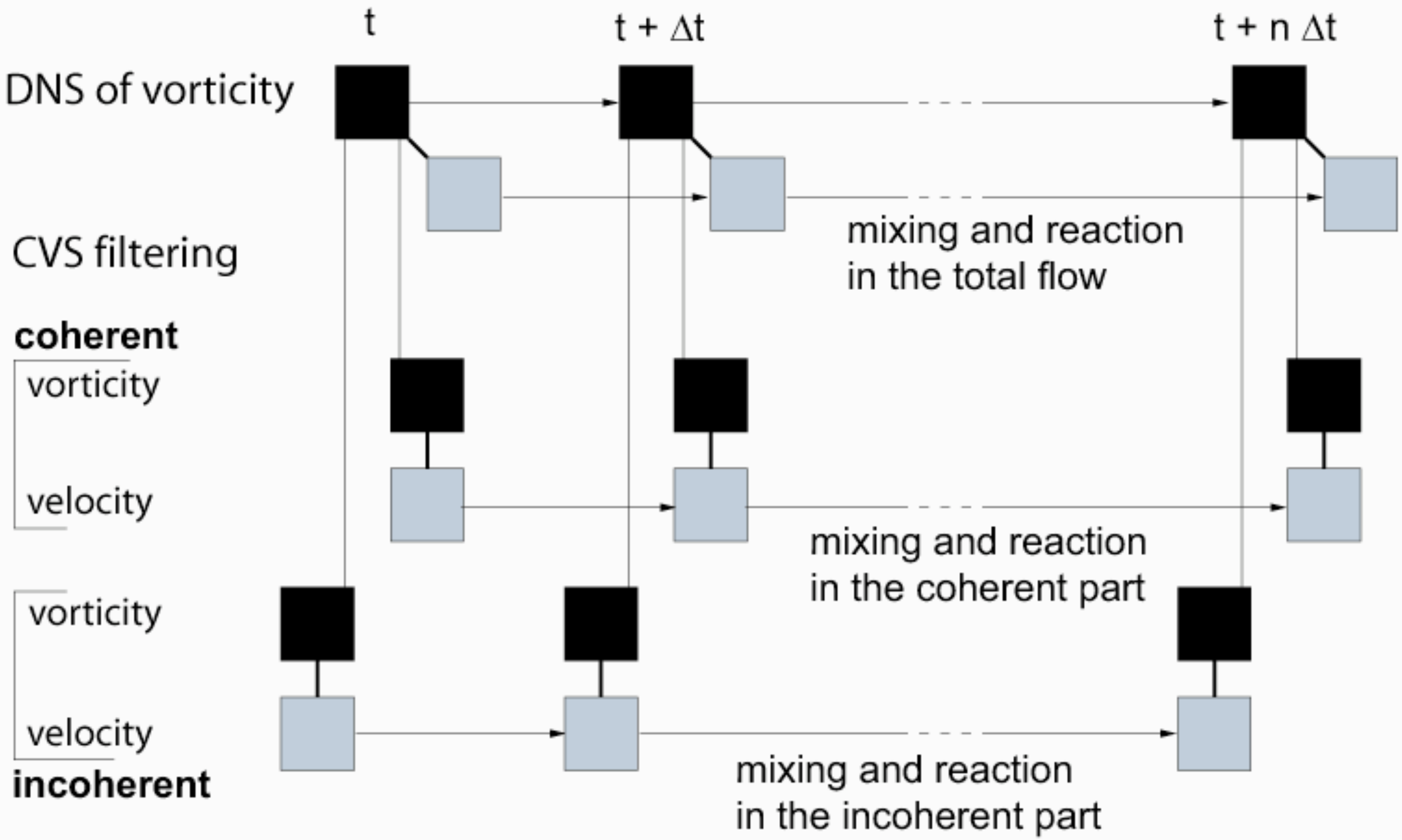
DNS
$N=512^2$

Coherent structures are regions where **nonlinearity is depleted**, thus, for 2D flows: $\omega = \sinh(\psi)$

Arnold, 1965,
Joyce & Montgomery, 1973
Robert & Sommeria, 1991

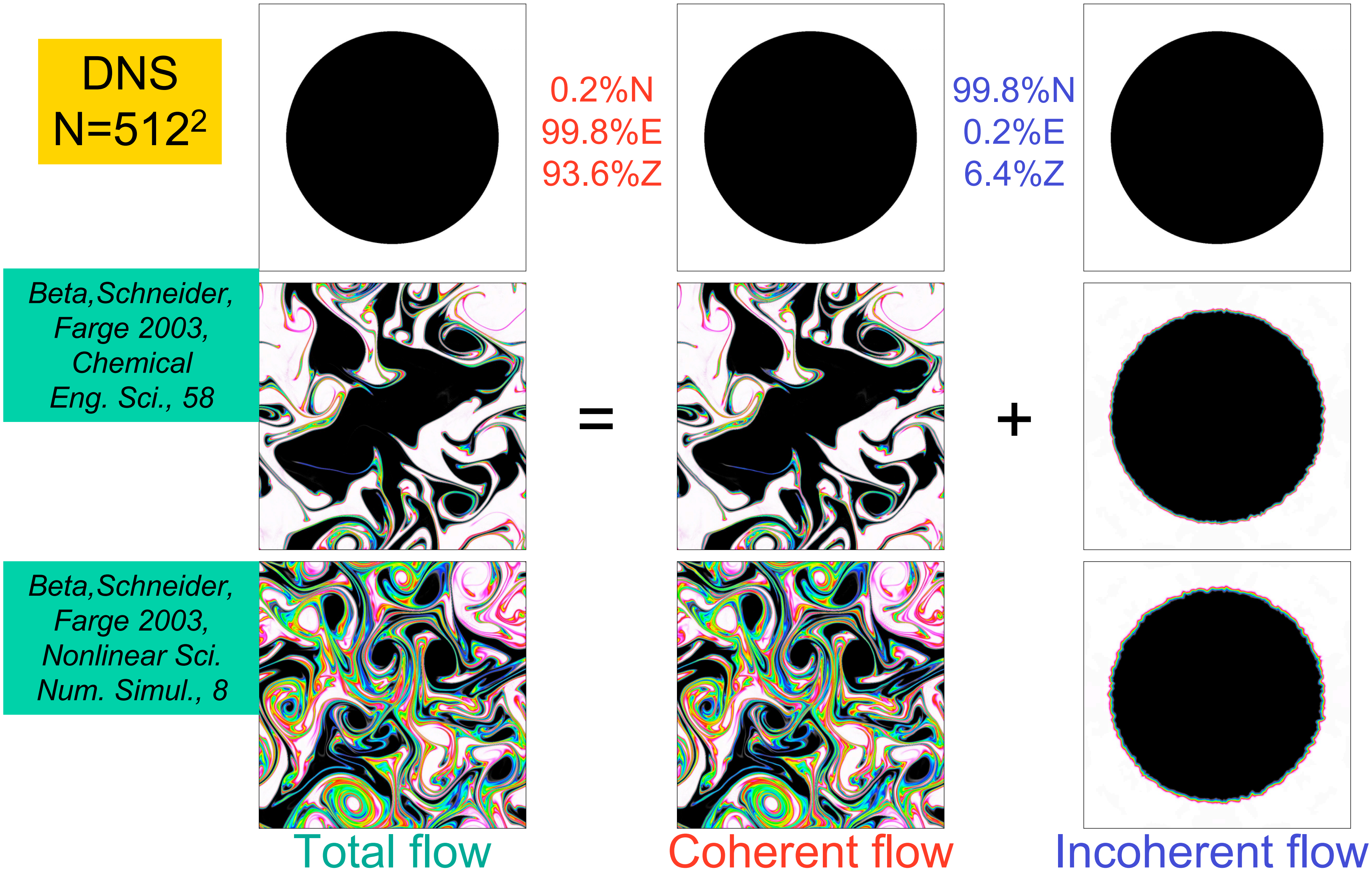


Wavelet filtering of the flow evolution
in numerical experiment



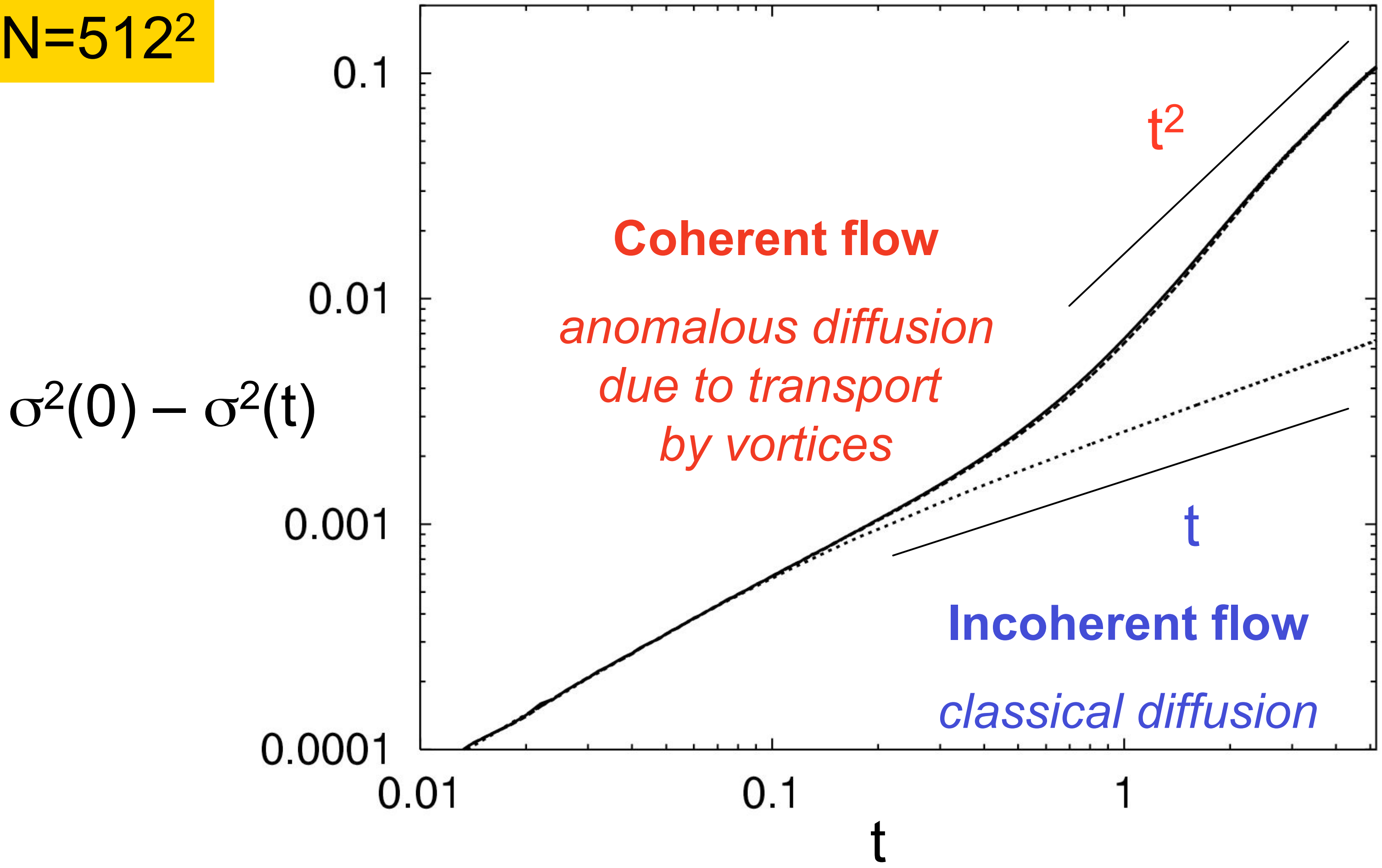
Beta, Schneider, Farge 2003, Nonlinear Sci. Num. Simul., 8

Passive scalar advection
from numerical experiment



Time evolution of the concentration variance
from numerical experiment

DNS
$N=512^2$



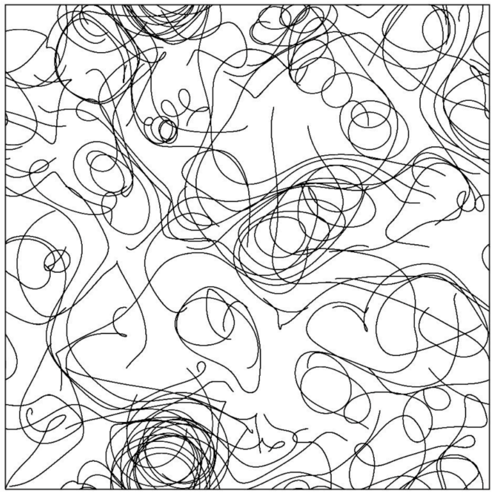
Advection of tracer particles

from numerical experiment

DNS
$N=512^2$

0.2 %	*of coefficients*	*99.8 %*	*of coefficients*
99.8 %	*of kinetic energy*	*0.2 %*	*of kinetic energy*
93.6 %	*of enstrophy*	*6.4 %*	*of enstrophy*

by the total flow **by the coherent flow** **by the incoherent flow**



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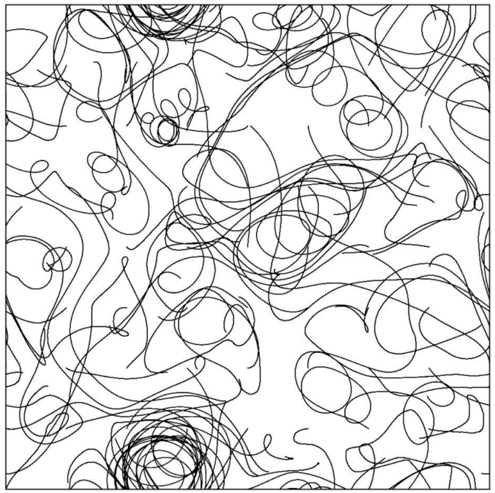

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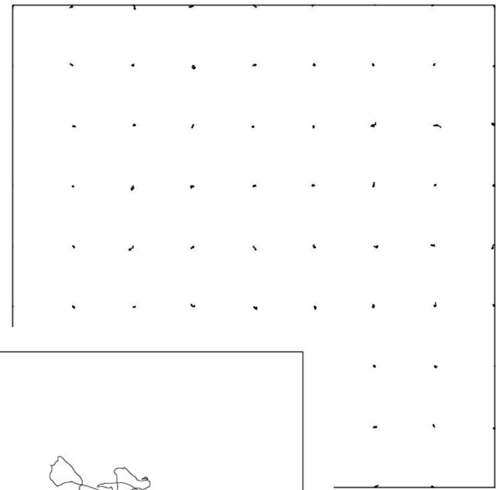
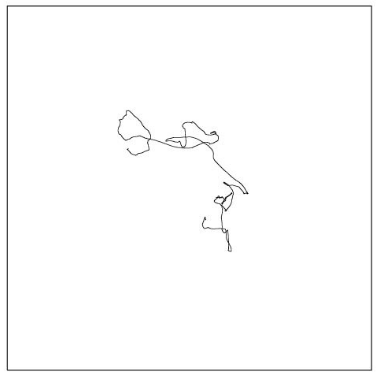


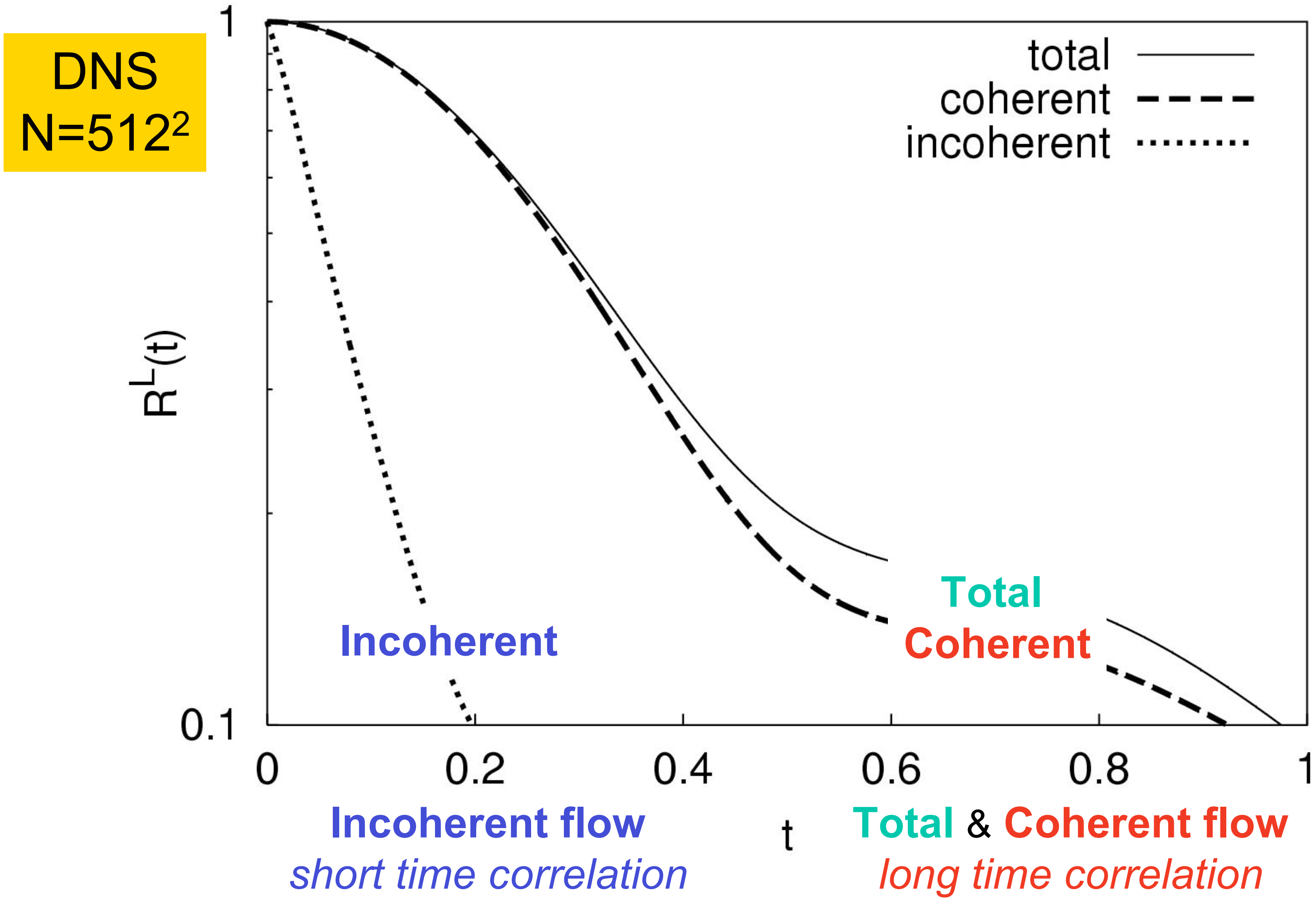


Transport by vortices

Beta,Schneider, Farge 2003,
Nonlinear Sci. Num. Simul., 8

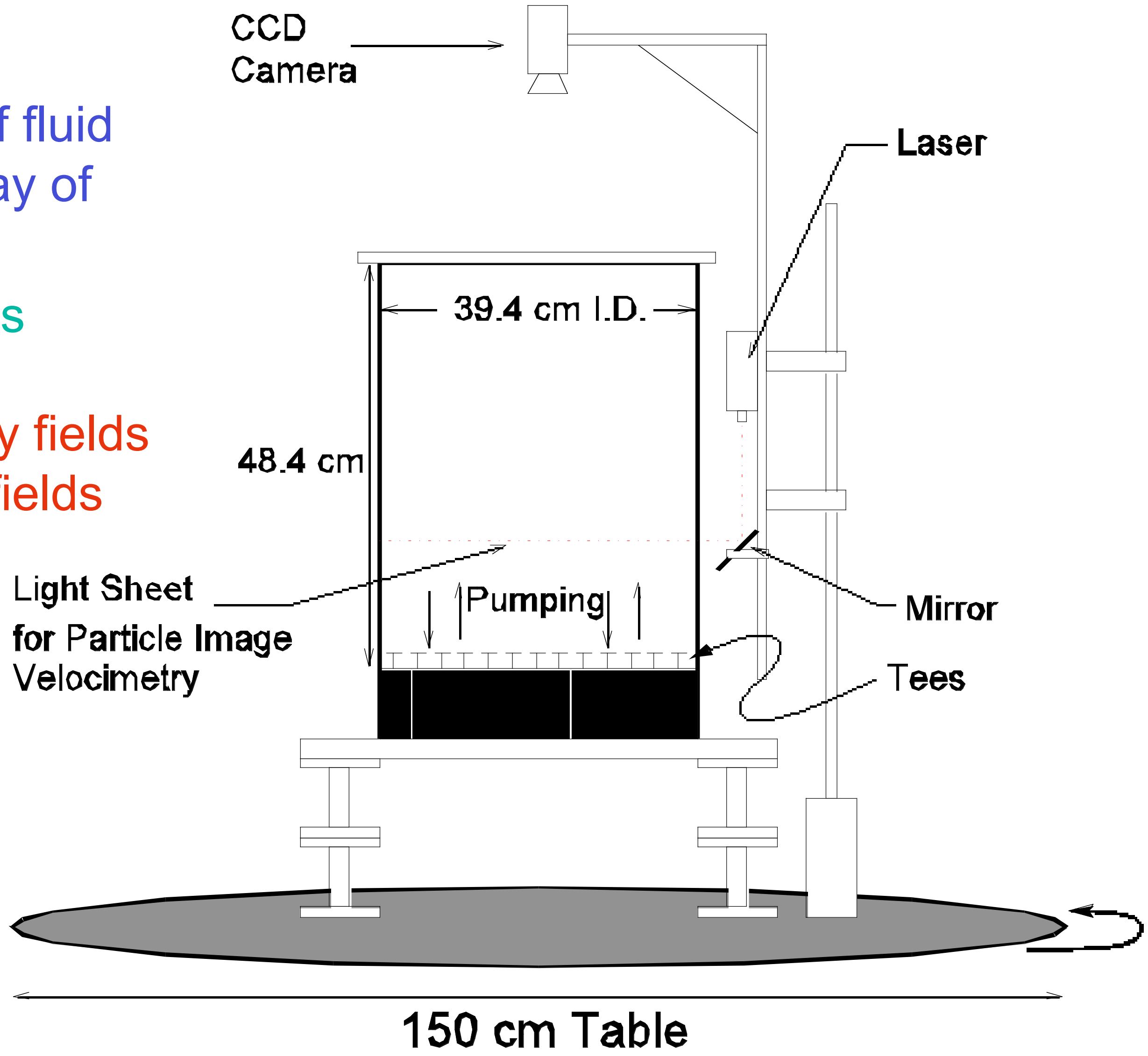
Diffusion by Brownian motion

Time correlation of the Lagrangian velocity
from numerical experiment



Rotating tank experiment

- Rotate up to 1.0 Hz
- Mechanical pumping of fluid through hexagonal array of sources and sinks
- 100 mm seed particules
- PIV to measure velocity fields and calculate vorticity fields



Decomposition of 2D vorticity field

in laboratory experiment

PIV
$N=128^2$

Total vorticity

100% E

100% Z

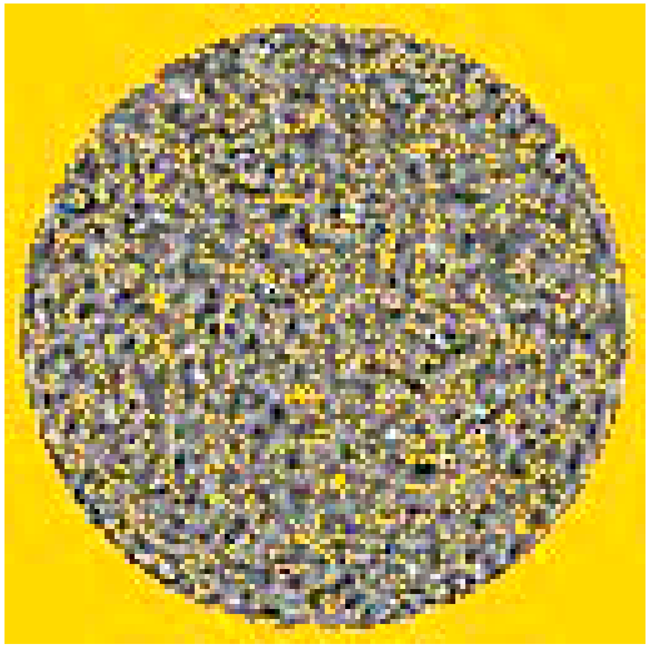
2% N

Coherent vorticity

99% E

80% Z

98% N



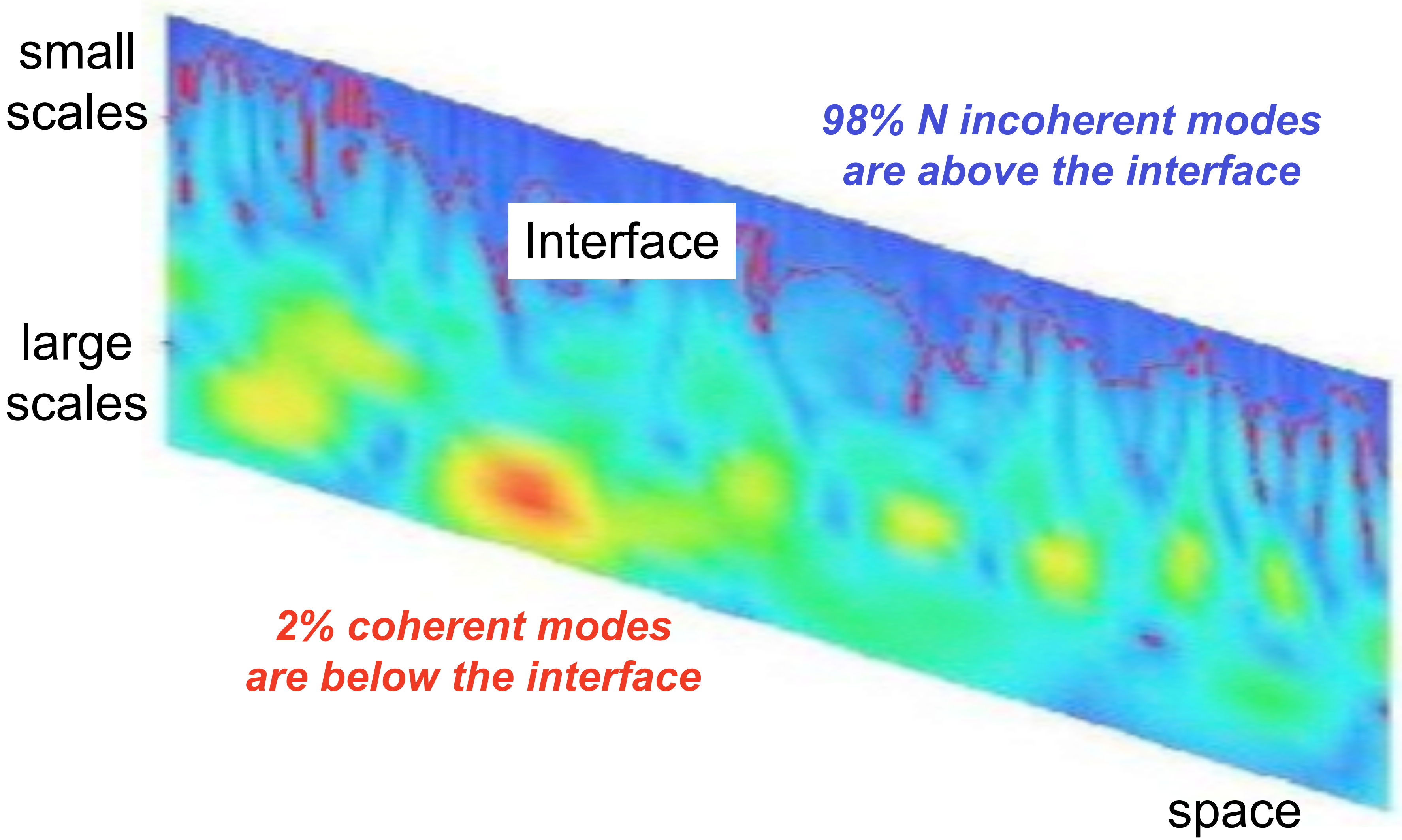
Incoherent vorticity

1% E

20% Z

Enstrophy in wavelet space

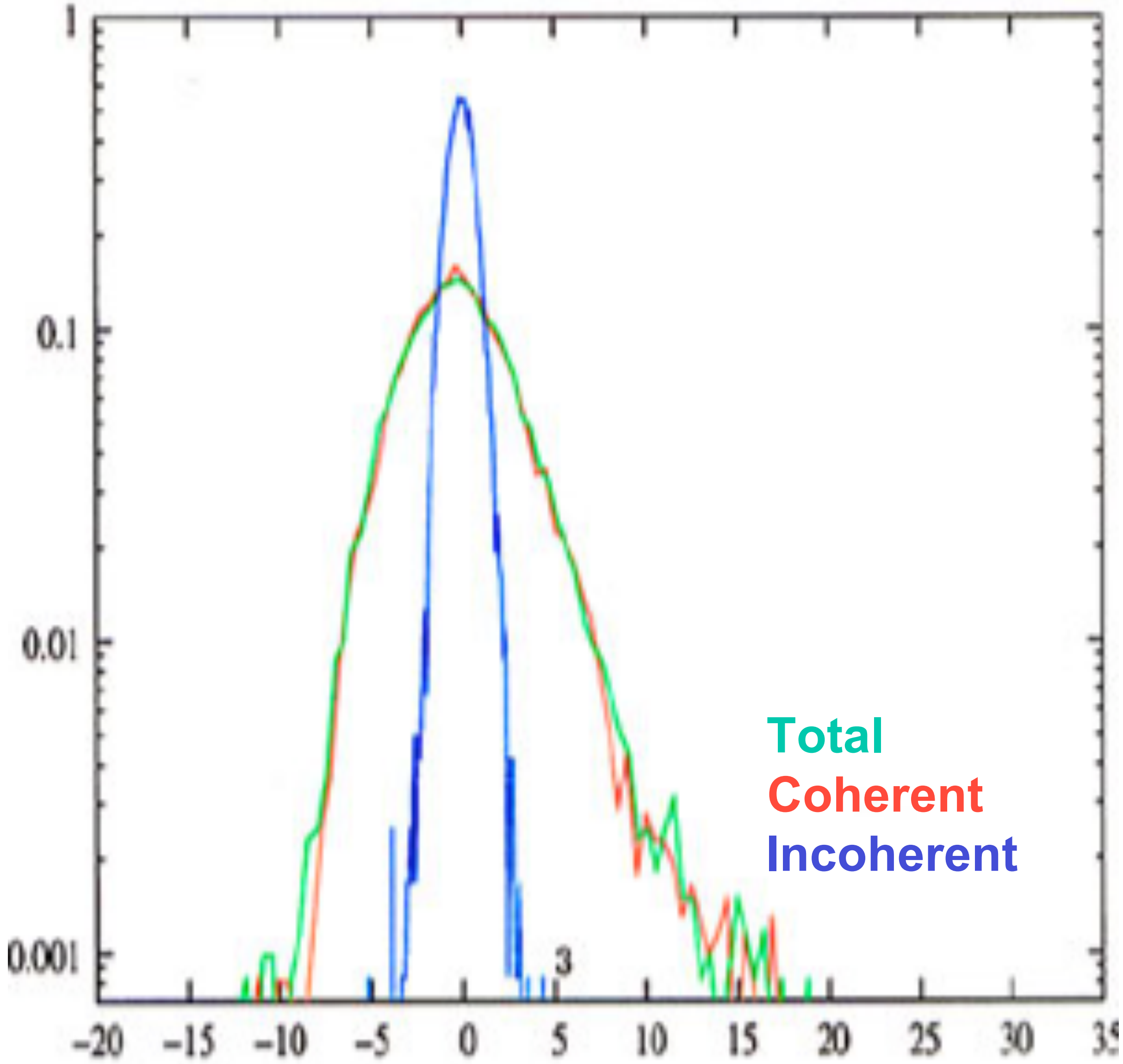
in numerical experiment



PDF of vorticity
in laboratory experiment

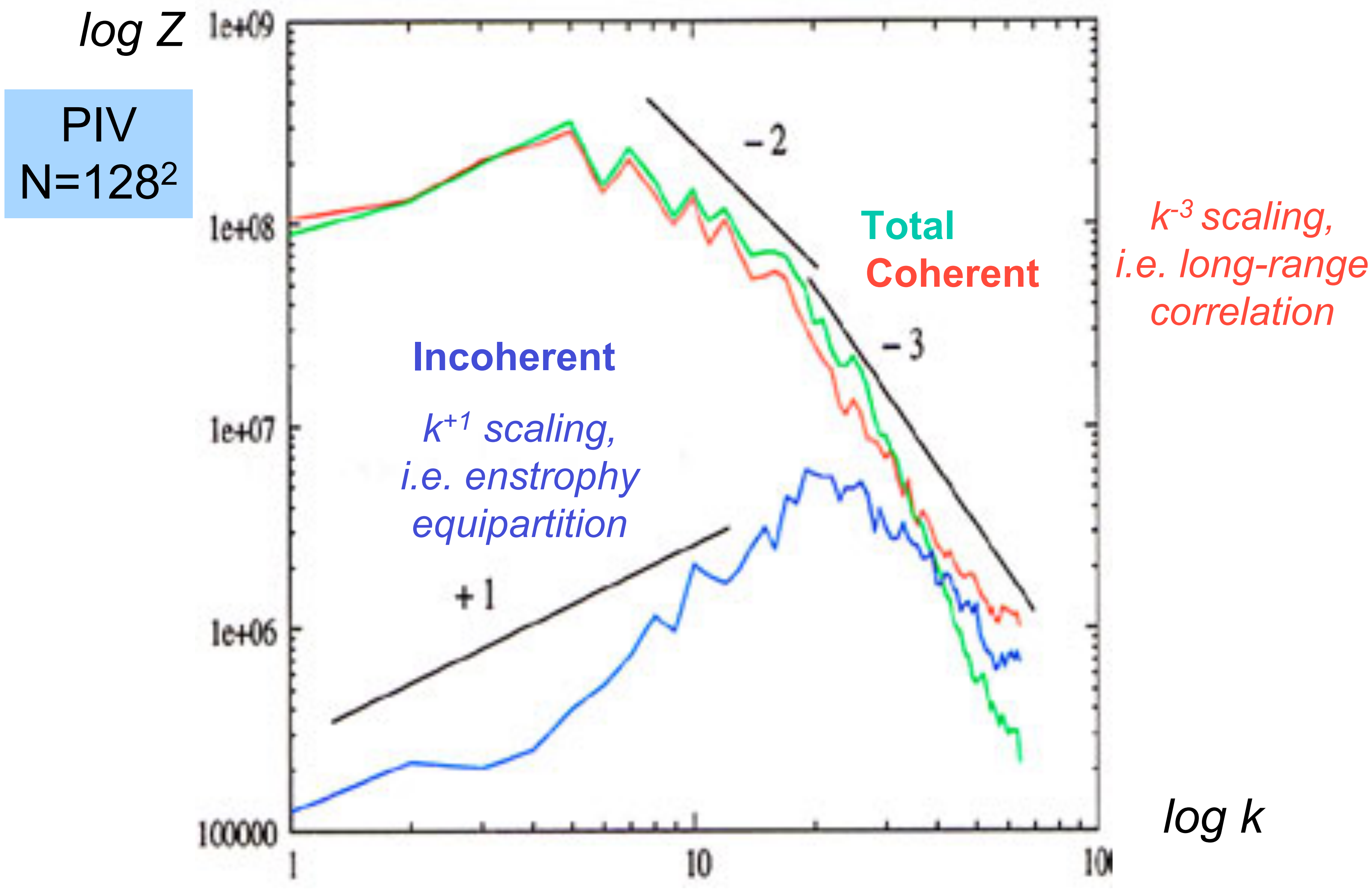
$\log p(\omega)$

PIV
$N=128^2$



$-\omega$

Enstrophy spectrum
in laboratory experiment



A posteriori proof of coherence
in laboratory experiment

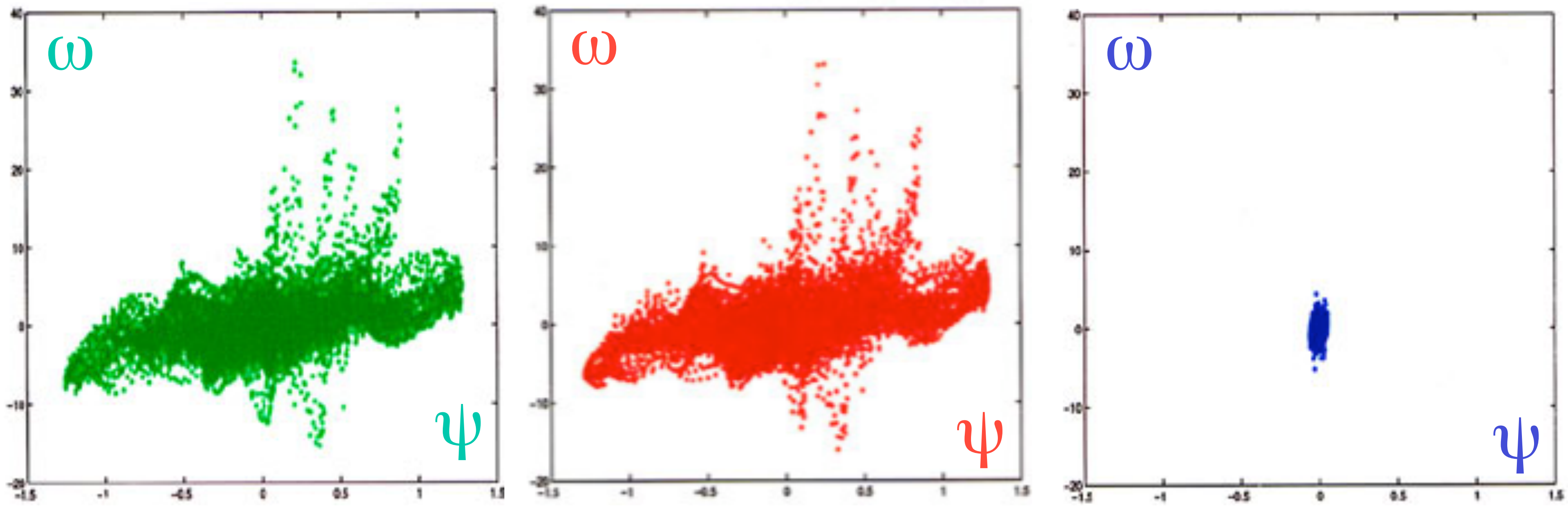
PIV
$N=128^2$

Coherent structures are regions where **nonlinearity is depleted**, thus, for 2D flows:

$$\omega = f(\psi)$$

Arnold, 1965,
Joyce & Montgomery, 1973
Robert & Sommeria, 1991

Total **Coherent** **Incoherent**

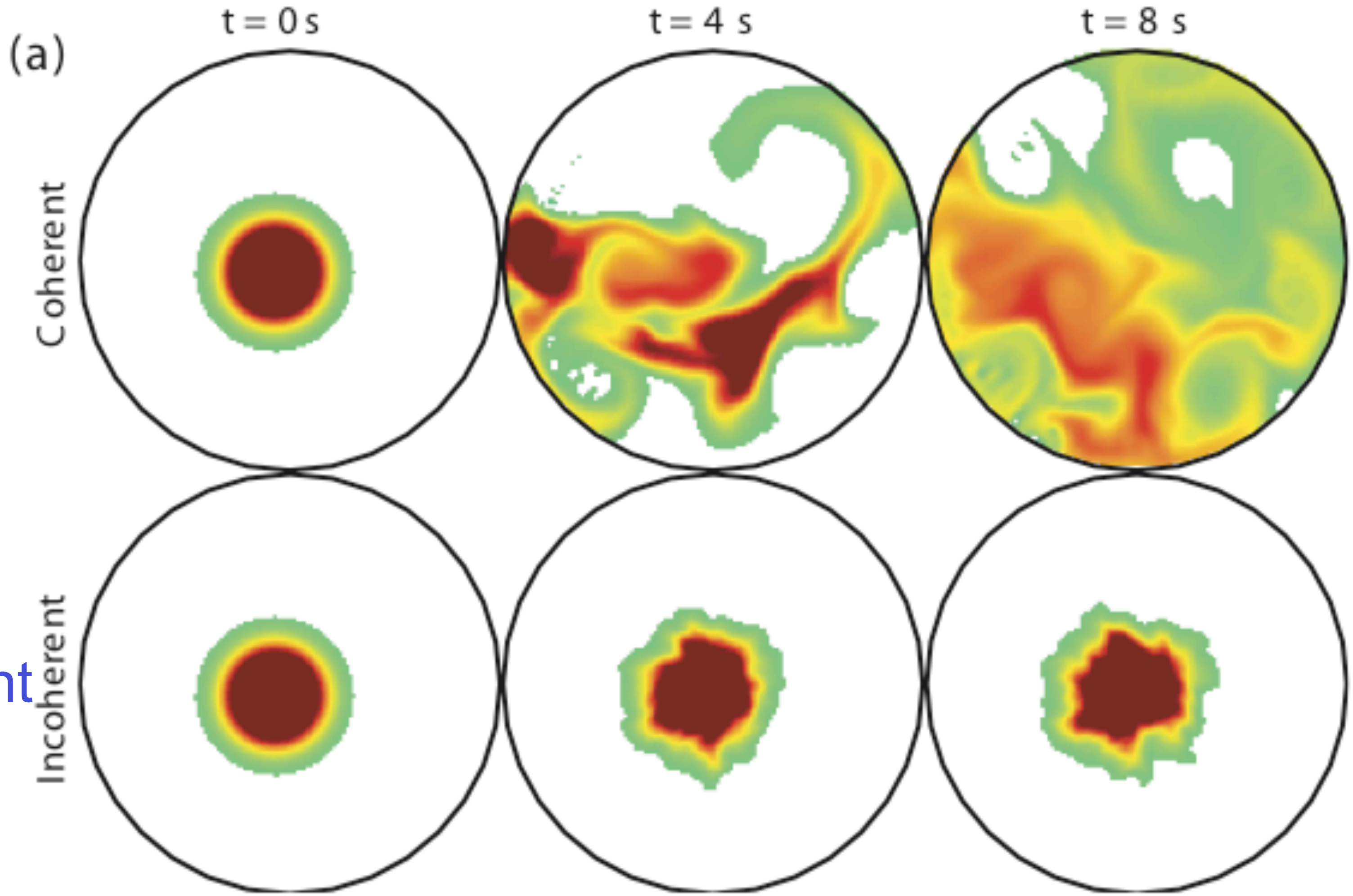


Passive scalar advection
in laboratory experiment

PIV
N=128^2

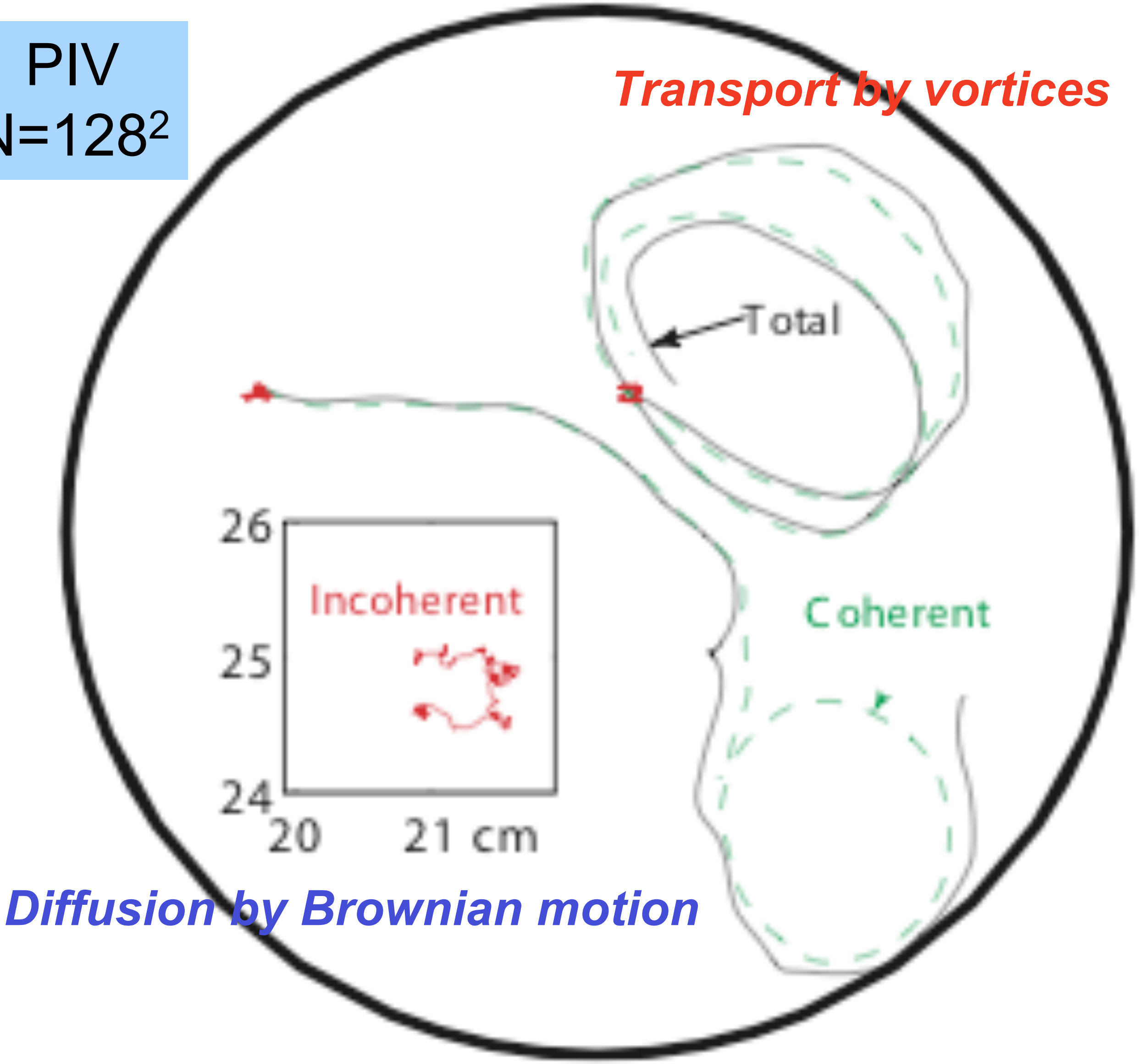
Transport by the coherent vortices :

Diffusion by the incoherent background :



Advection of tracer particles
in laboratory experiment

PIV
N=128^2



Application to 3D turbulent flows

- from **numerical experiment**
  of a turbulent mixing layer,
  computed using DNS at resolution 512 x256 x 128 :

in collaboration with Mike Rogers

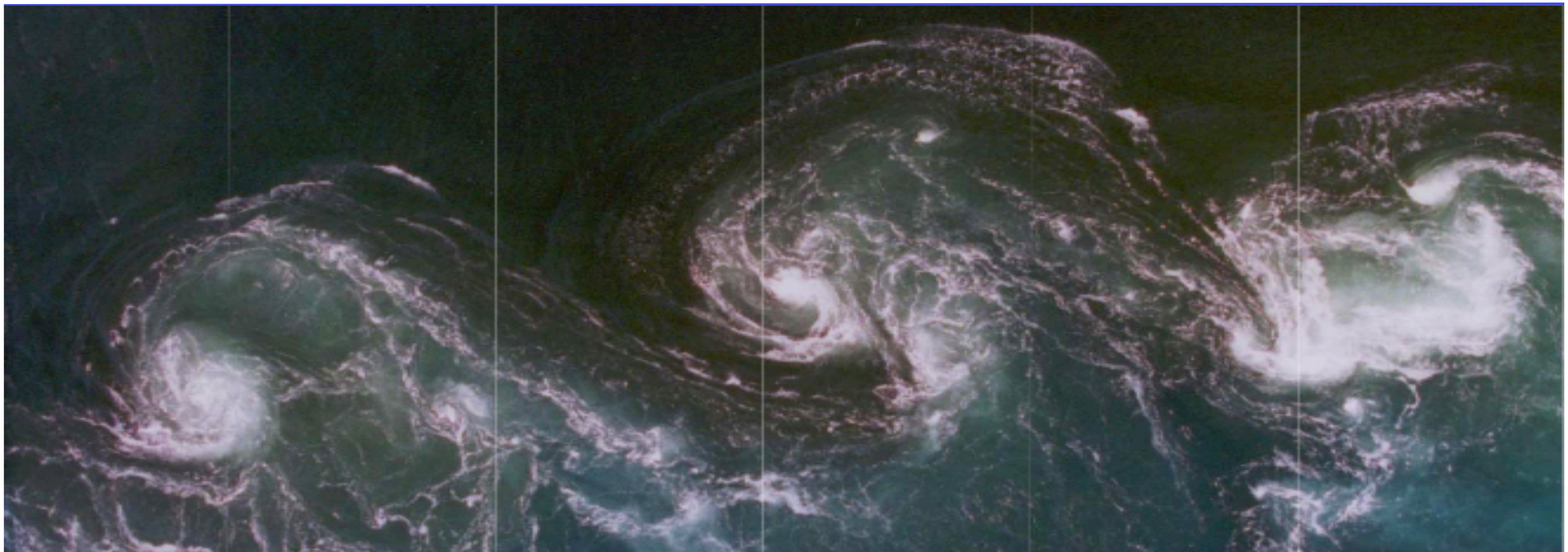
Center for Turbulence Research,
NASA-Ames and Stanford University

- from **numerical experiment**
  of homogeneous isotropic turbulence,
  computed using DNS at resolution 2048^3 :

in collaboration with Yukio Kaneda,
Katsunori Yoshimatsu and Naoya Okamoto

Computer Sciences Department,
Nagoya University

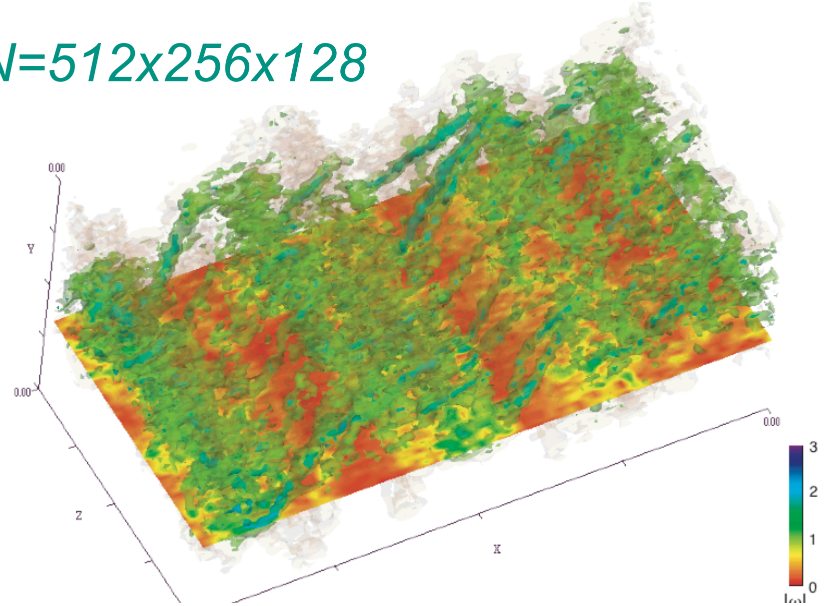
3D turbulent mixing layer

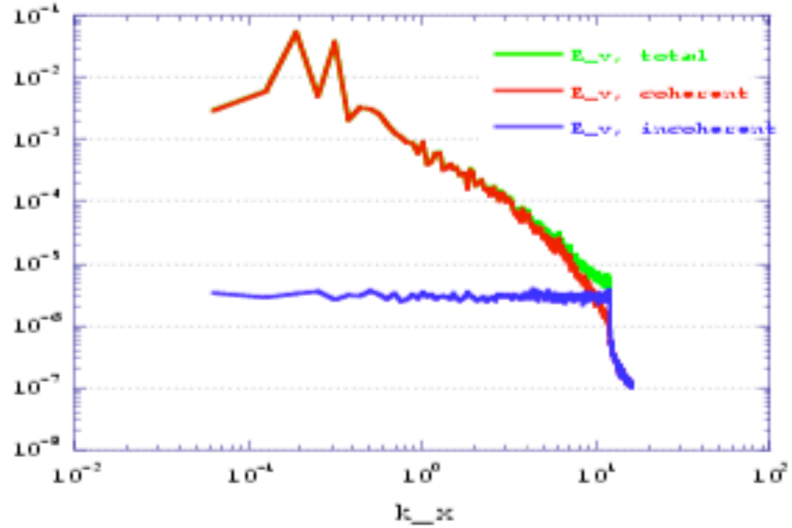


3D turbulent mixing layer

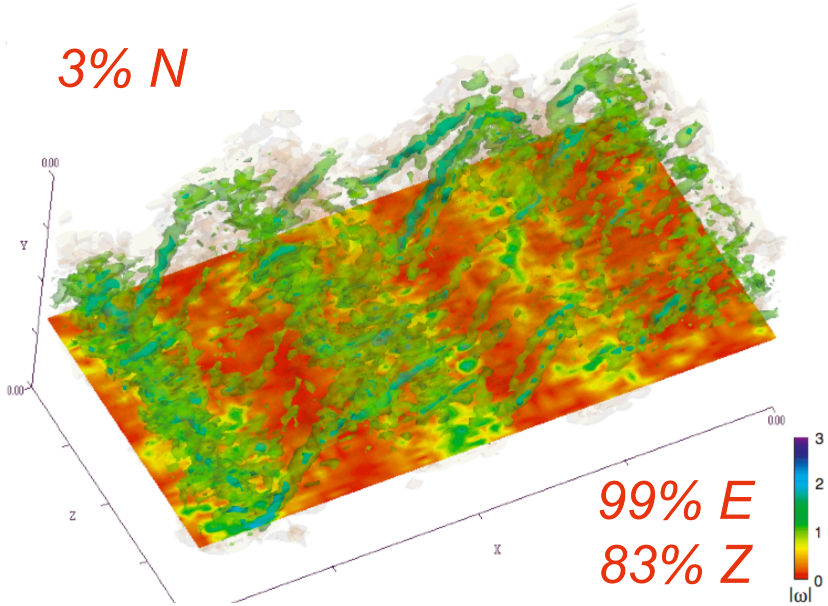
Schneider, Farge, Pellegrino, Rogers, J. Fluid Mech., 534, 2005

N=512x256x128



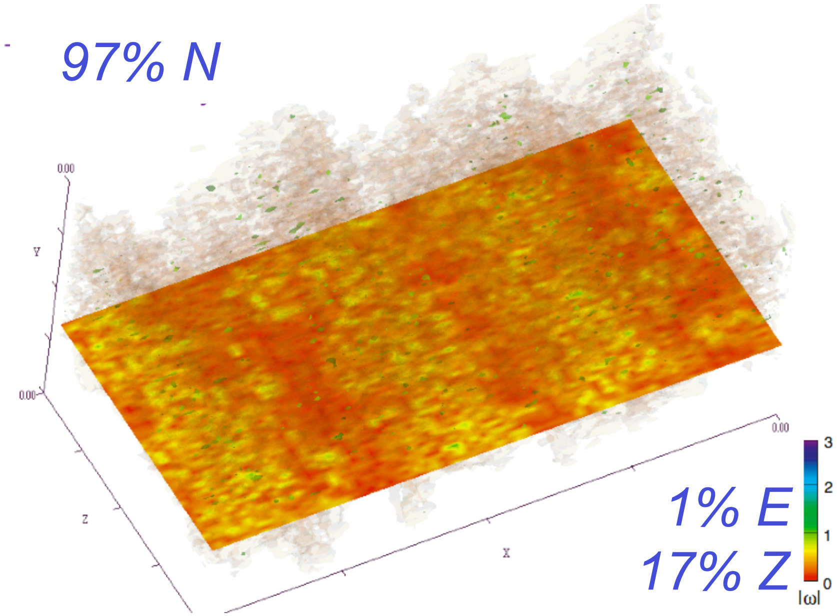


3% N



99% E
83% Z

97% N

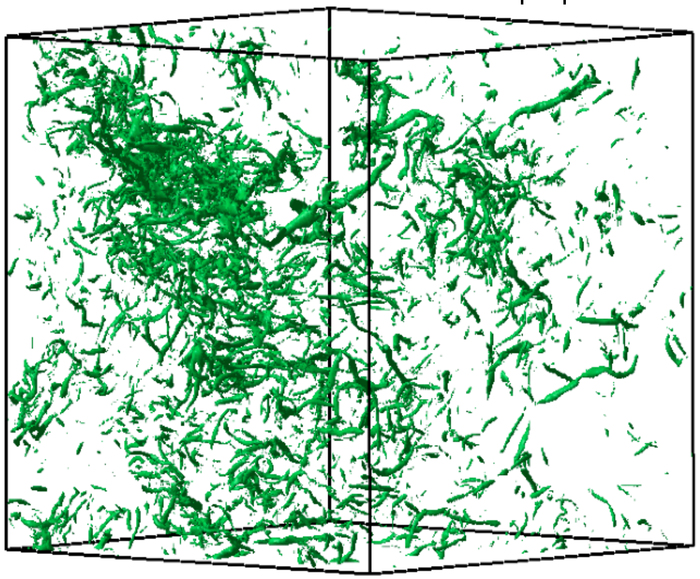


1% E
17% Z

Modulus of vorticity field
in numerical experiment

DNS
$N=2048^3$

$|\omega|=5\sigma$



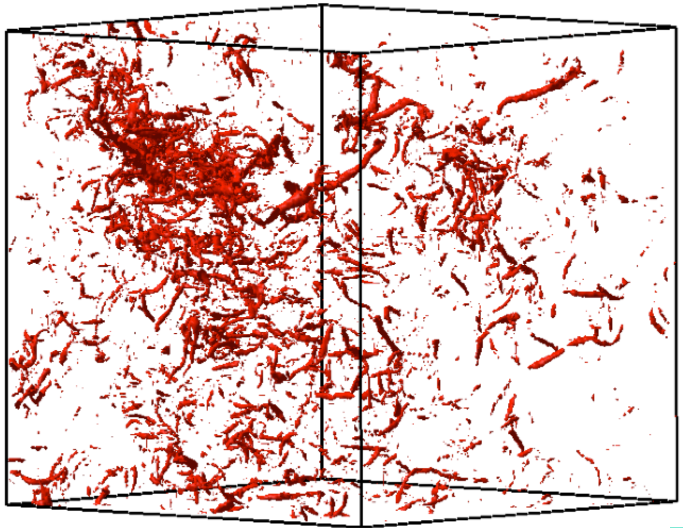
with $\sigma=(2Z)^{1/2}$

Coherent vorticity

2.6 % N coefficients
80% enstrophy
99% energy

Incoherent vorticity

97.4 % N coefficients
20 % enstrophy
1% energy

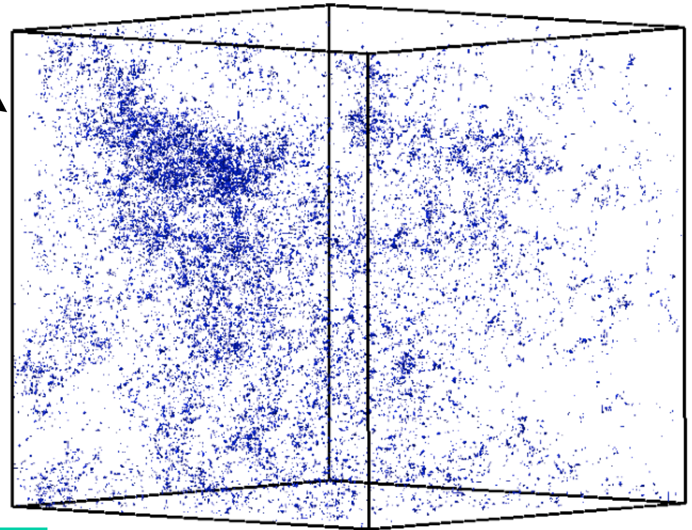


$|\omega|=5\sigma$

Total vorticity

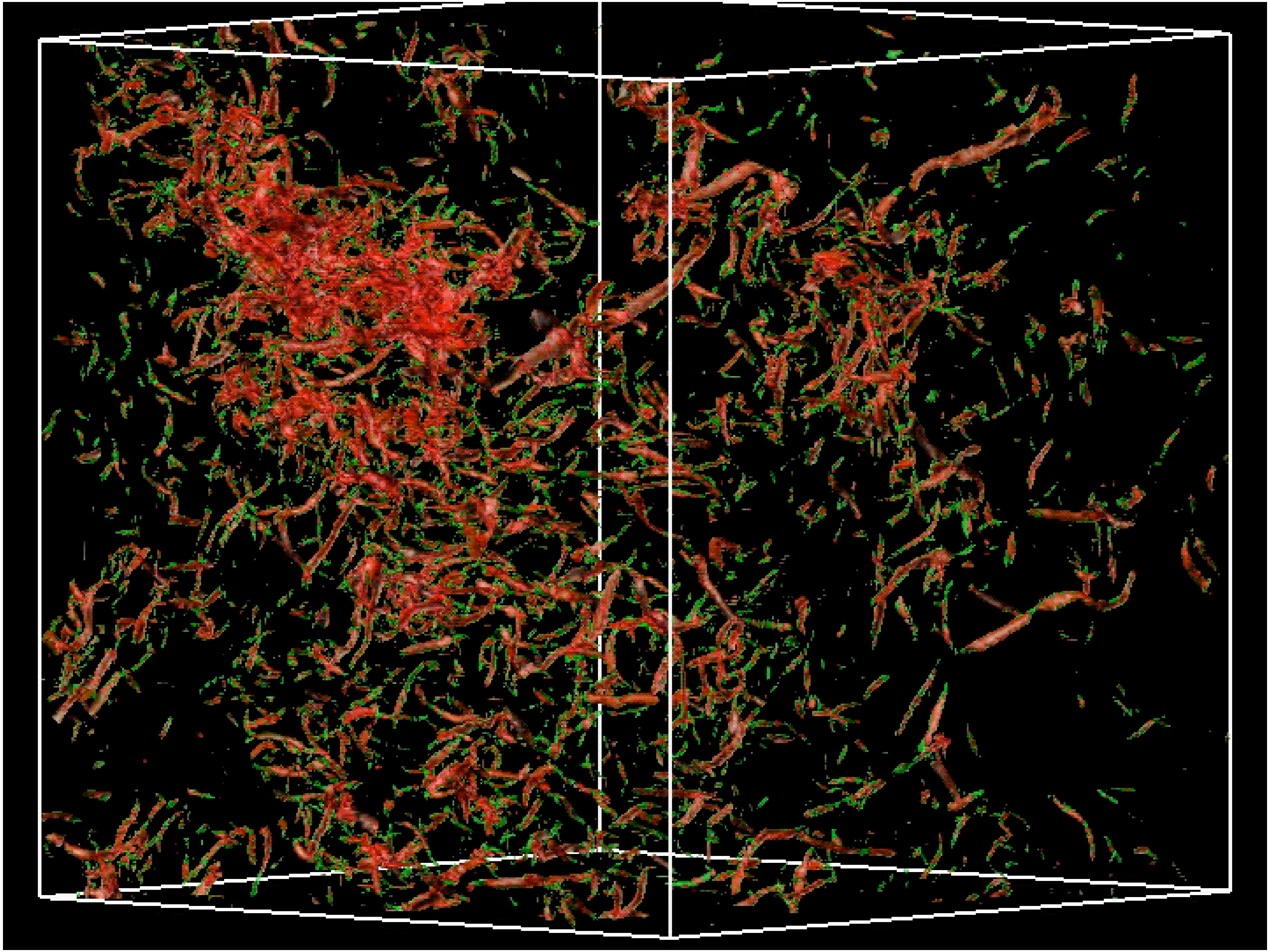
$R_\lambda=732$
$R_e=5\ 10^5$
Visualization at 256^3

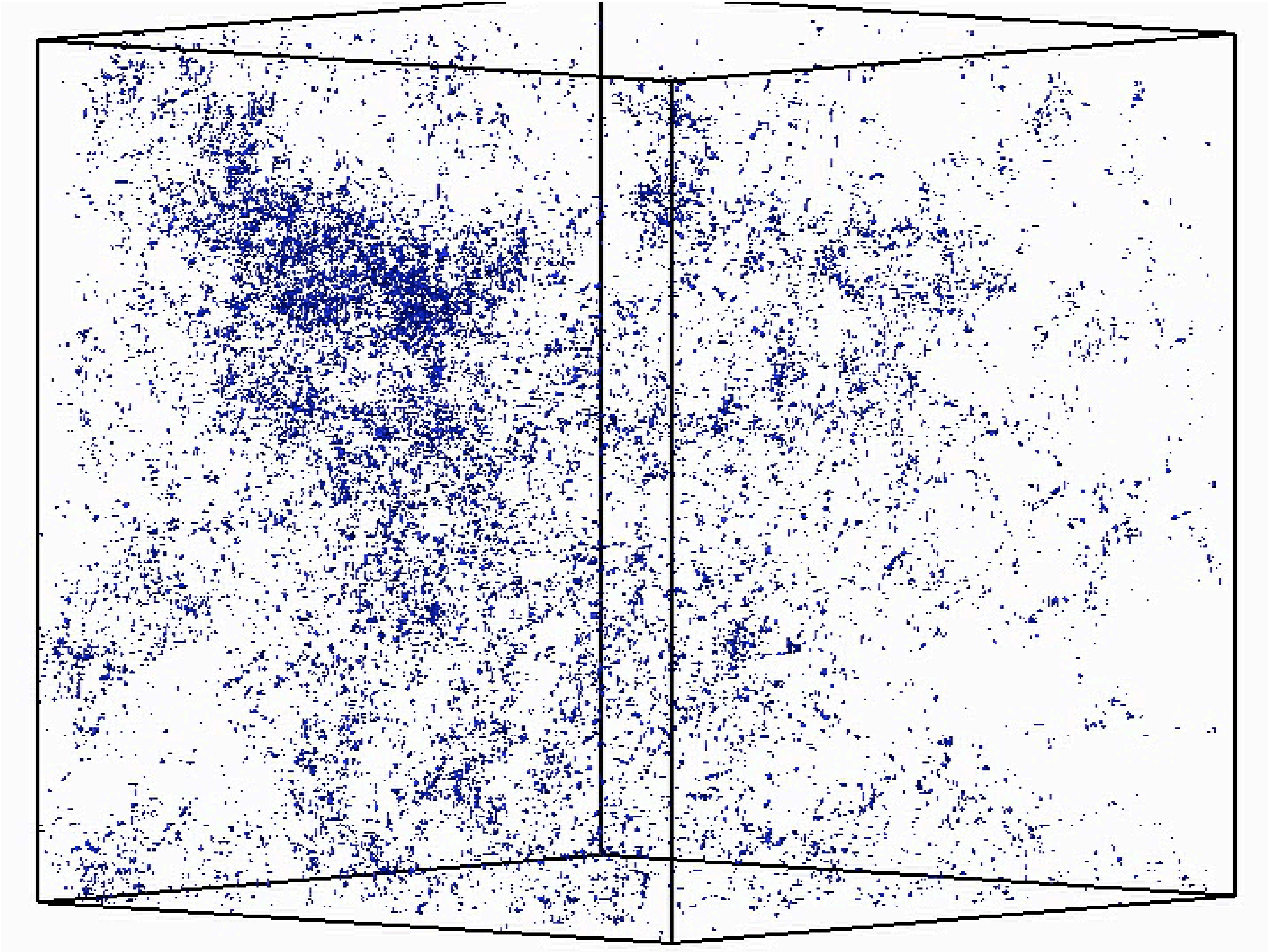
+



$|\omega|=5/3\sigma$

Phys. Fluids, in revision, 2007

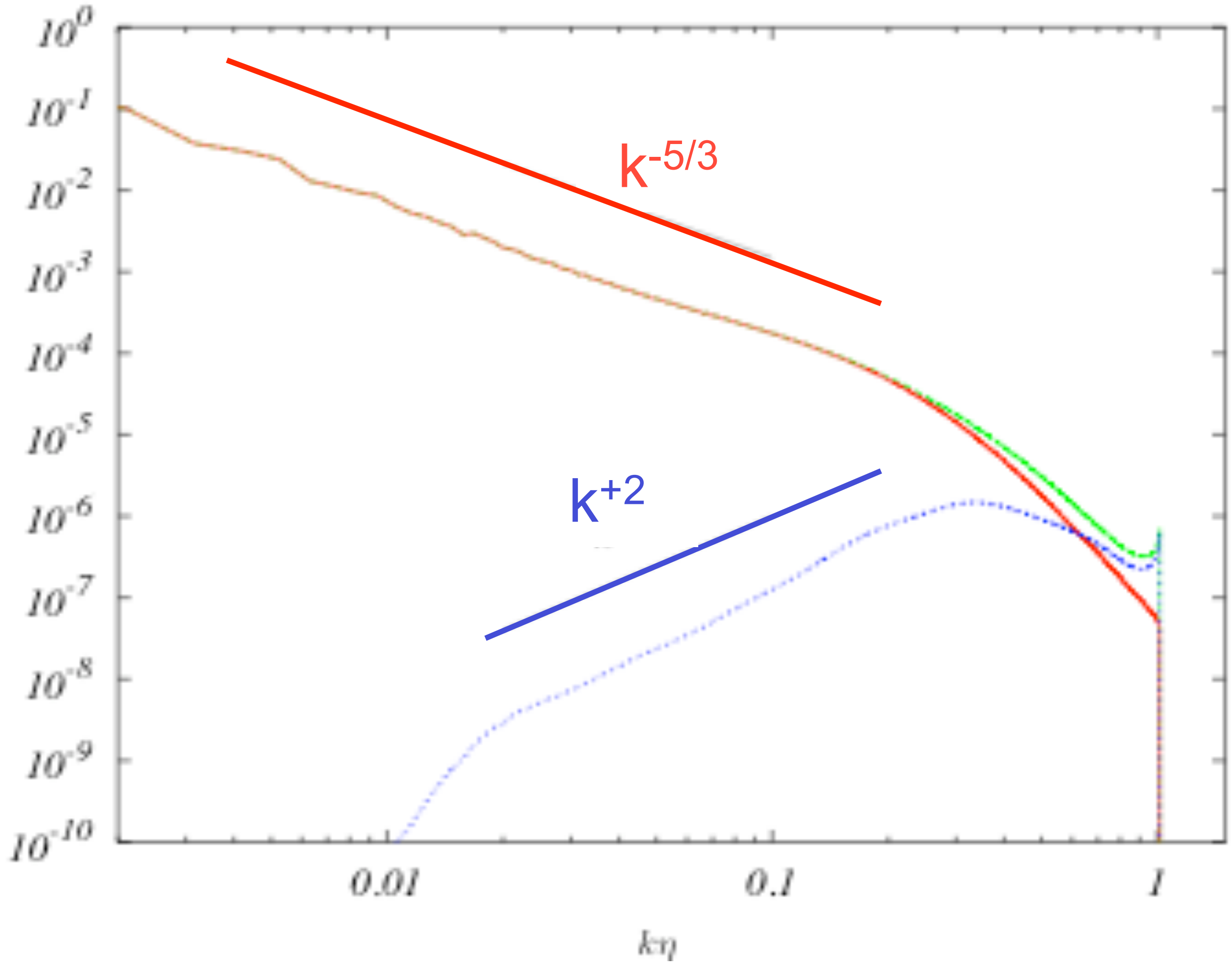




Energy spectrum
in numerical experiment

DNS
N=2048^3

log E(k)



log k

2.6 % N coefficients
80% enstrophy
99% energy

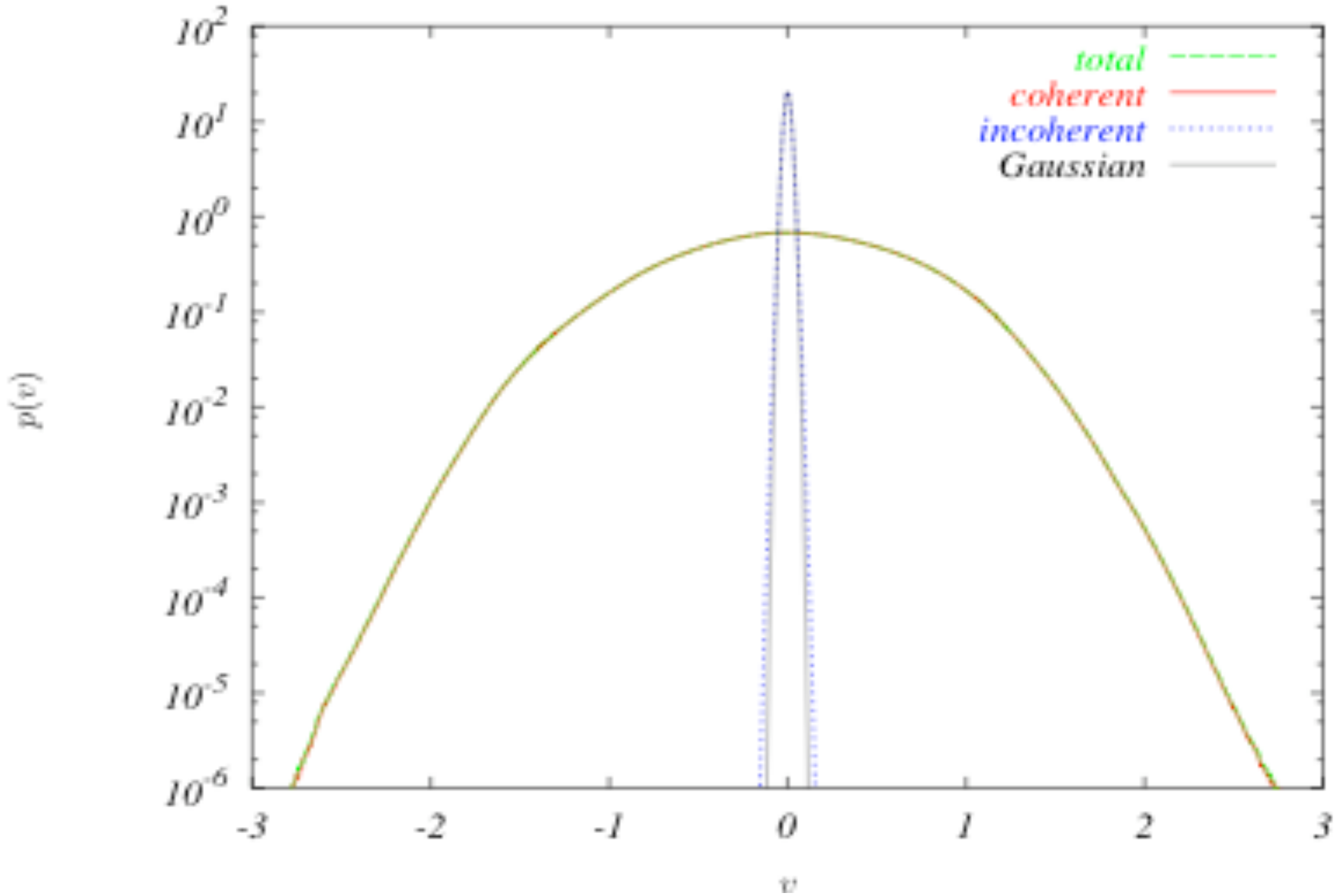
Multiscale Coherent
$k^{-5/3}$ scaling, i.e.
long-range correlation

Multiscale Incoherent
k^{+2} scaling, i.e.
energy equipartition

PDF of velocity

DNS
N=2048³

log p(v)

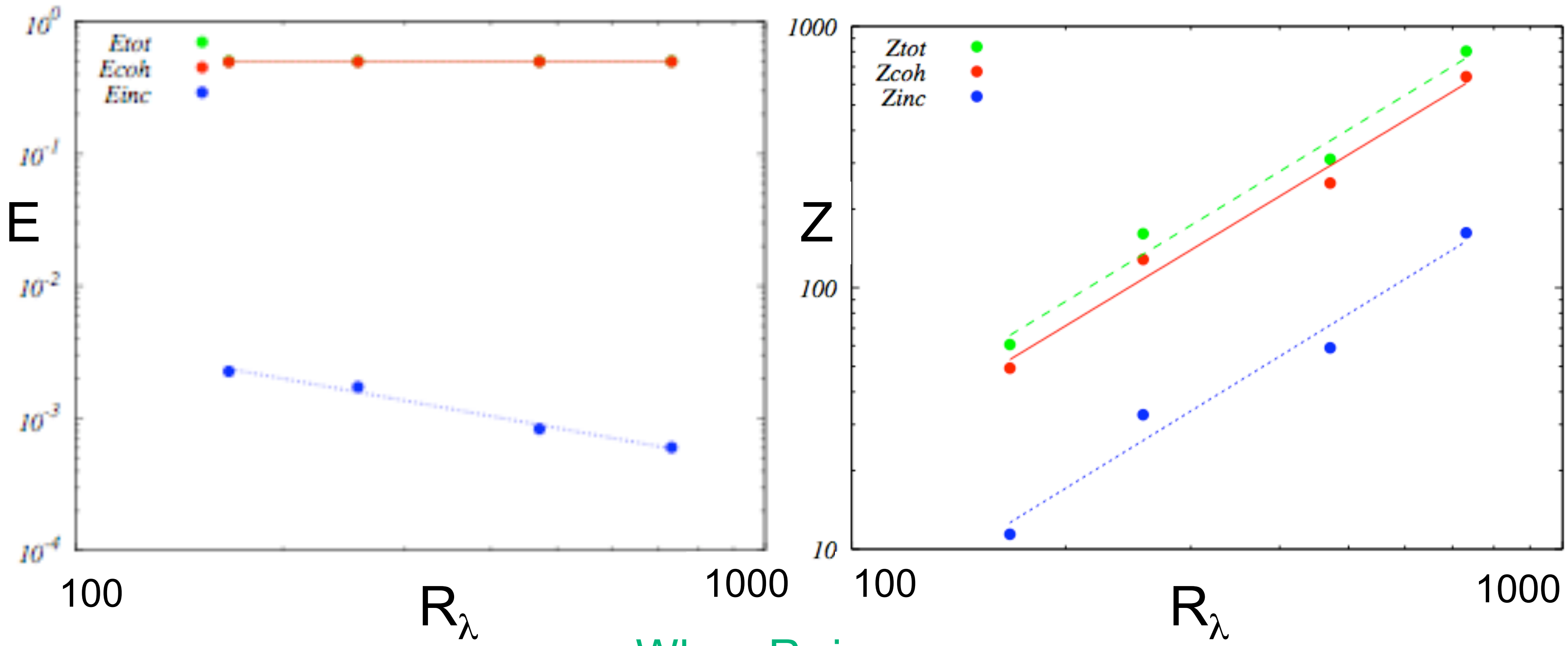


v

2.6 % N coefficients
99% energy

The total and coherent flows have the same extrema.
The incoherent flow has a Gaussian PDF,
therefore its effect should be easy to model

Energy and enstrophy versus R_λ



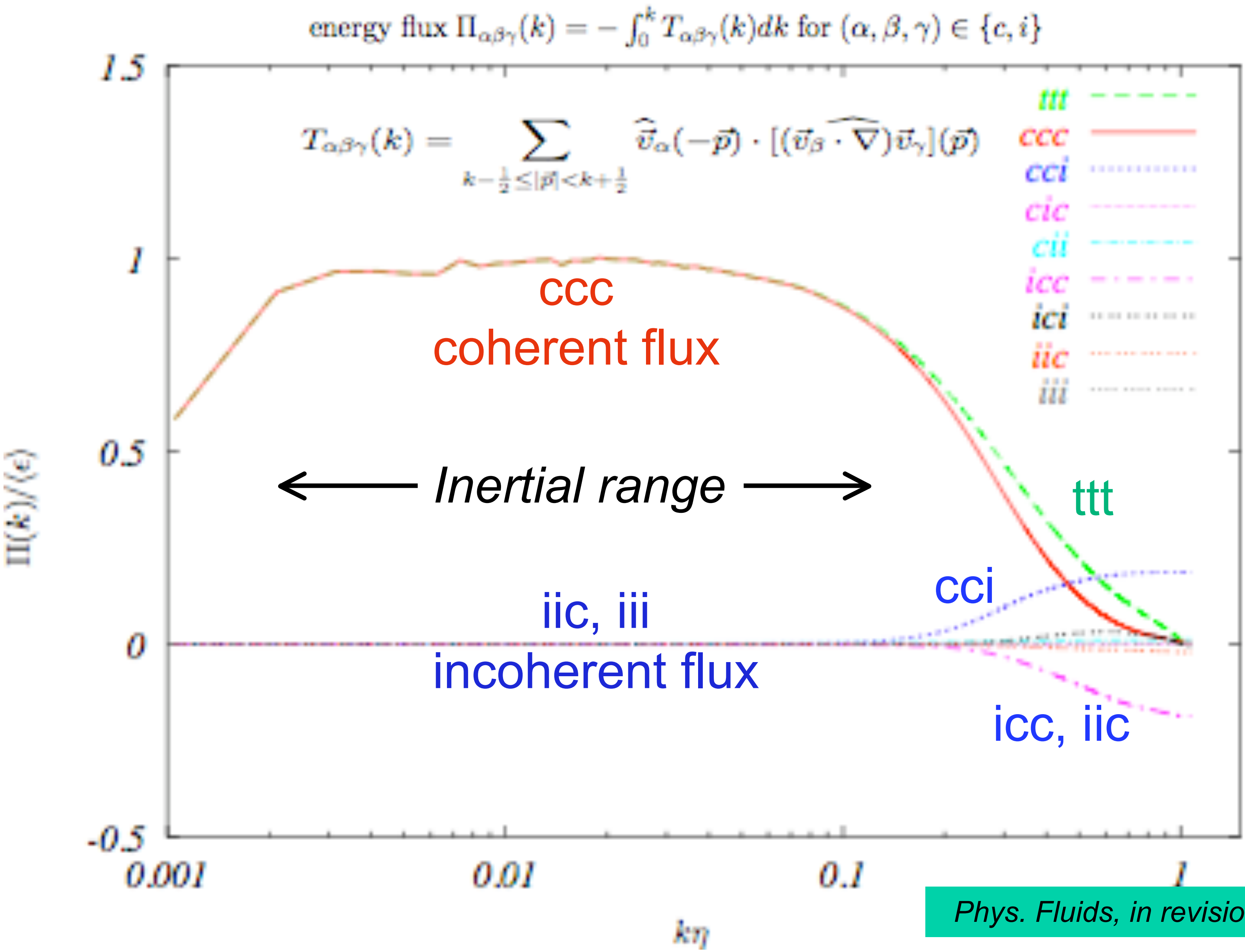
When R_λ increases :

- the incoherent energy decreases,
- the incoherent enstrophy increases.

We conjecture that the incoherent enstrophy quantifies the turbulence level.

Okamoto, Yoshimatsu, Schneider, Farge and Kaneda, Phys. Fluids, in revision, 2007

Nonlinear transfers and energy fluxes



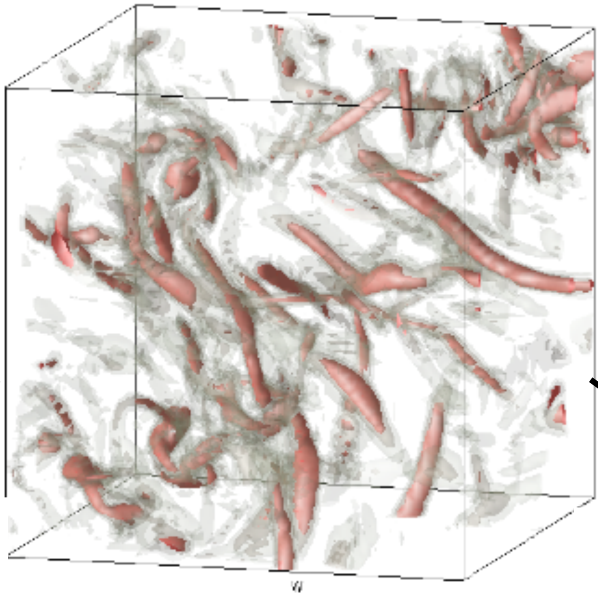
Phys. Fluids, in revision, 2007

Wavelet extraction in 3D vorticity field
in numerical experiment

DNS
$N=256^3$

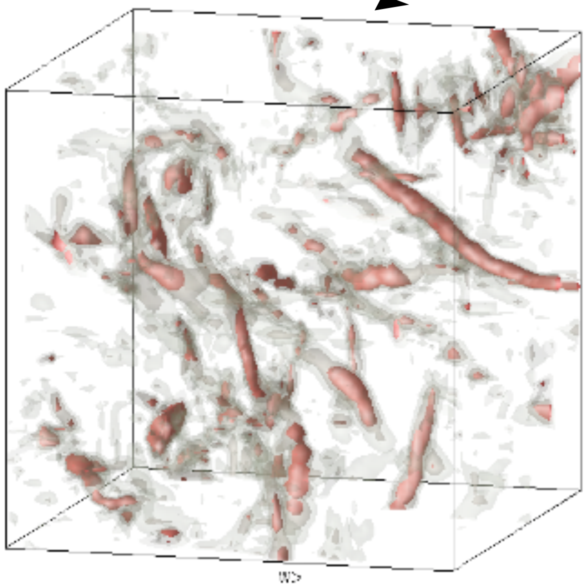
Coherent vorticity

3% N coefficients
99% energy
79% enstrophy



Incoherent vorticity

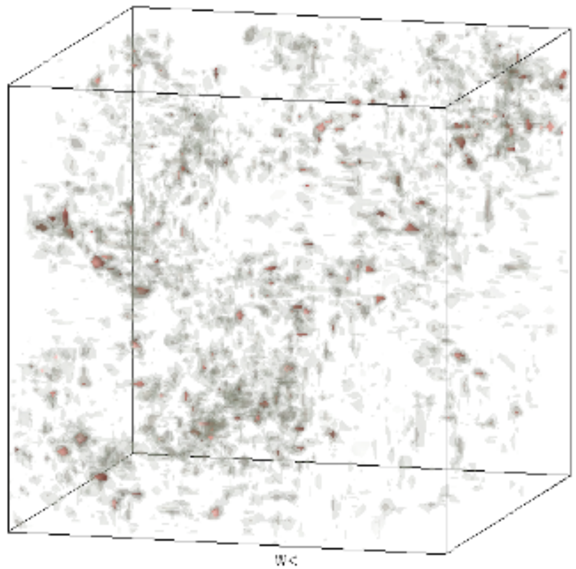
97 % N coefficients
1 % energy
21 % enstrophy



Total vorticity

$R_\lambda=168$
$R_e=3\ 10^4$
Zoom at 64^3

+



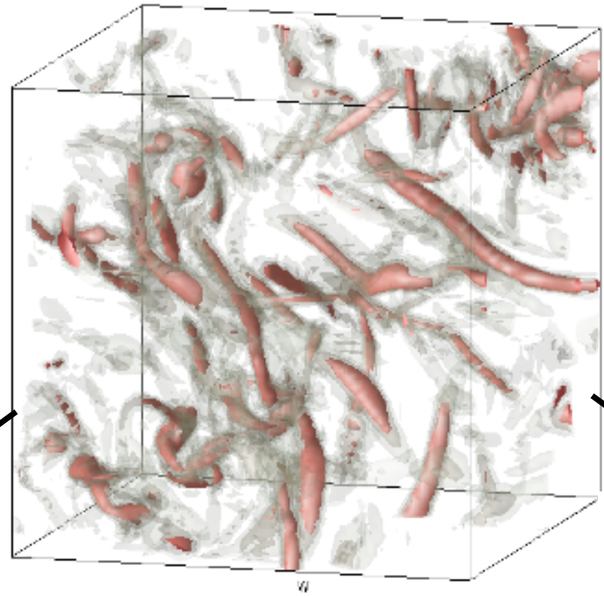
Farge, Pellegrino, Schneider,
*PRL, **87** (55), 2001*

Fourier extraction in 3D vorticity field
in numerical experiment

DNS
$N=256^3$

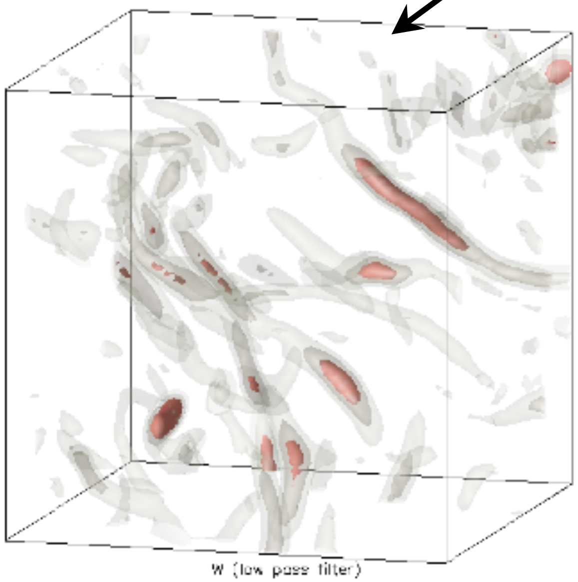
Large scale vorticity

3 % of coefficients
99% of kinetic energy
71% of enstrophy



Small scale vorticity

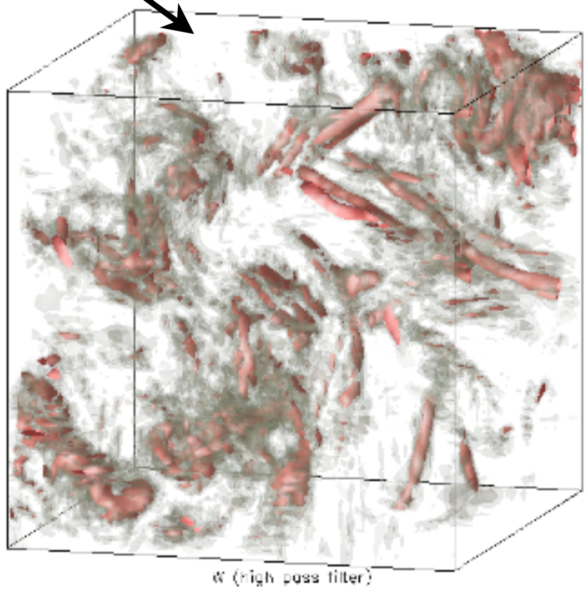
97% of coefficients
1% of kinetic energy
29% of enstrophy



Total vorticity

$R_\lambda=168$
$R_e=3\ 10^4$
Zoom at 64^3

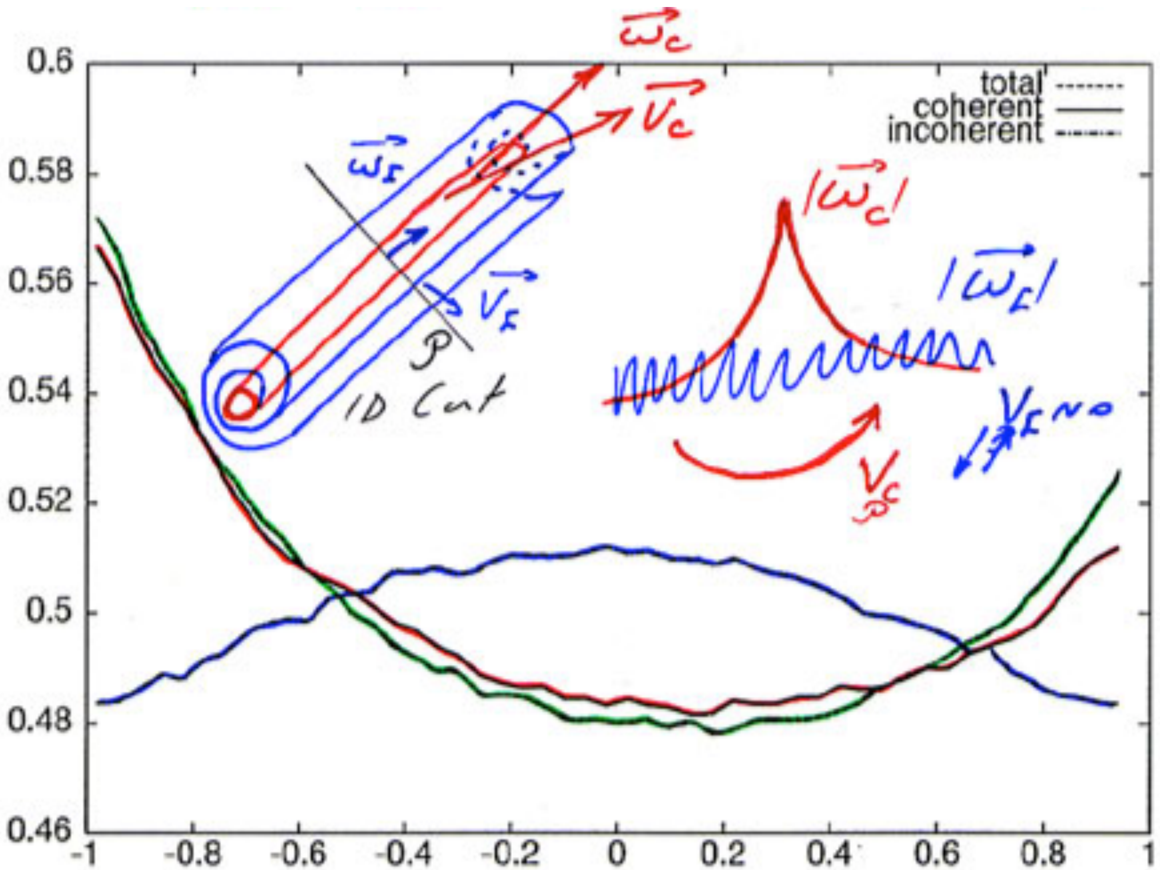
+



Farge, Schneider et al., Phys. Fluids, ***15*** *(10), 2003*

Relative helicity
in numerical experiment

DNS
N=256^3



$$h = \frac{\vec{V} \cdot \vec{\omega}}{|\vec{V}||\vec{\omega}|}$$

*Farge, Pellegrino, Schneider, PRL, **87** (55), 2001*

Coherent

Velocity parallel to vorticity

$\Rightarrow$ *tubes*

Incoherent

Velocity orthogonal to vorticity

$\Rightarrow$ *sheets*

Interpretation of the turbulent cascade

'The terms "scale of motion" or "eddy of size l " appear repeatedly in the treatments of the inertial range. One gets an impression of little, randomly distributed whirls in the fluid, with the ***cascade process consisting of the fission of the whirls into smaller ones****, after the fashion of Richardson's poem. This picture seems drastically in conflict with what can be inferred about the qualitative structures of high-Reynolds number turbulence from laboratory vizualization techniques and from the application of the Kelvin's circulation theorem'.*

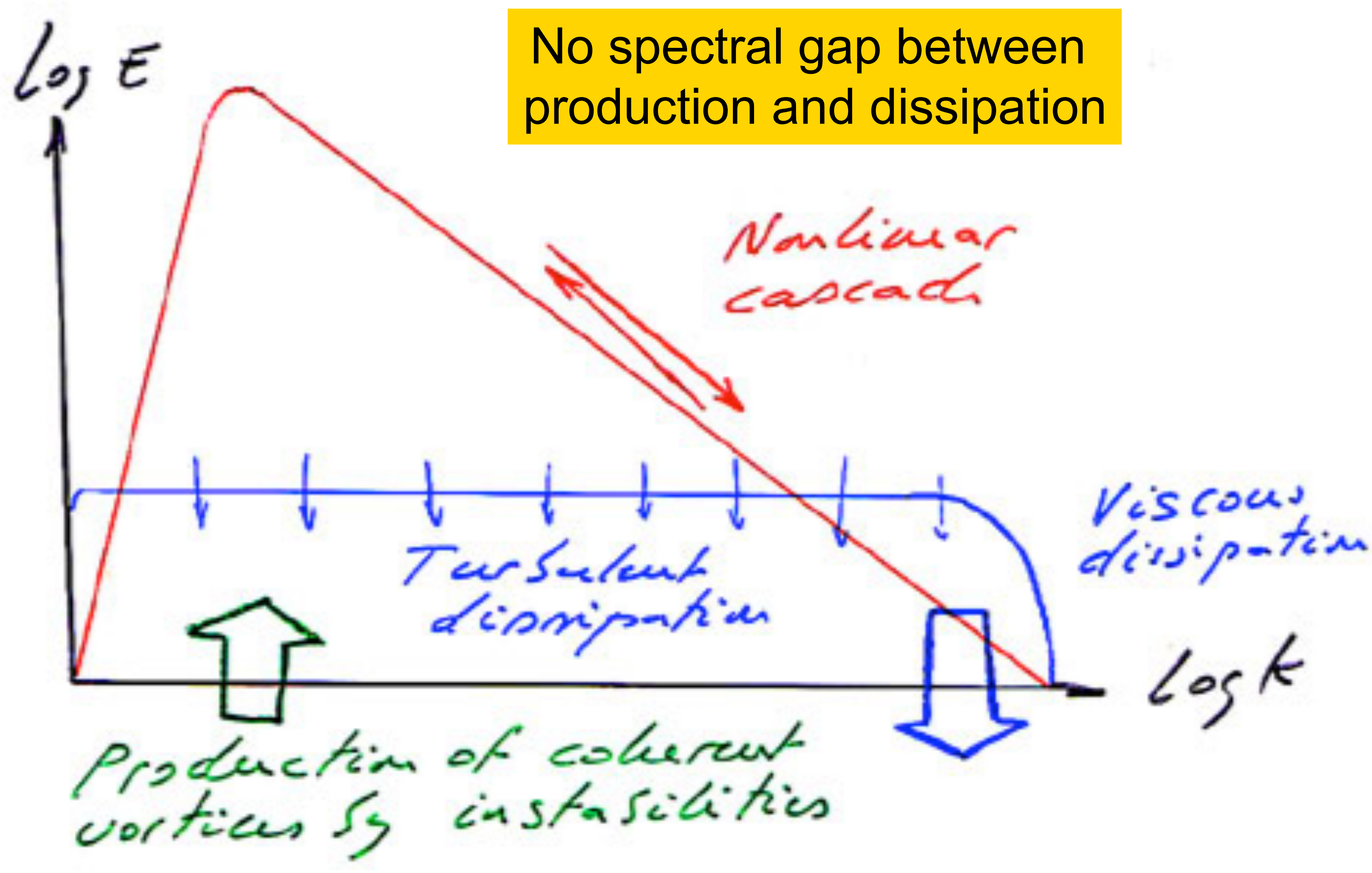
Robert Kraichnan,
On Kolmogorov's inertial range theories,
J. Fluid Mech., 62, 305-330, 1974

We should find a **new interpretation of the turbulent cascade**, **taking into account** the nonlinear dynamics of Navier-Stokes equations and the **formation of coherent vortices** in regions of strong shear.

New interpretation of the energy cascade

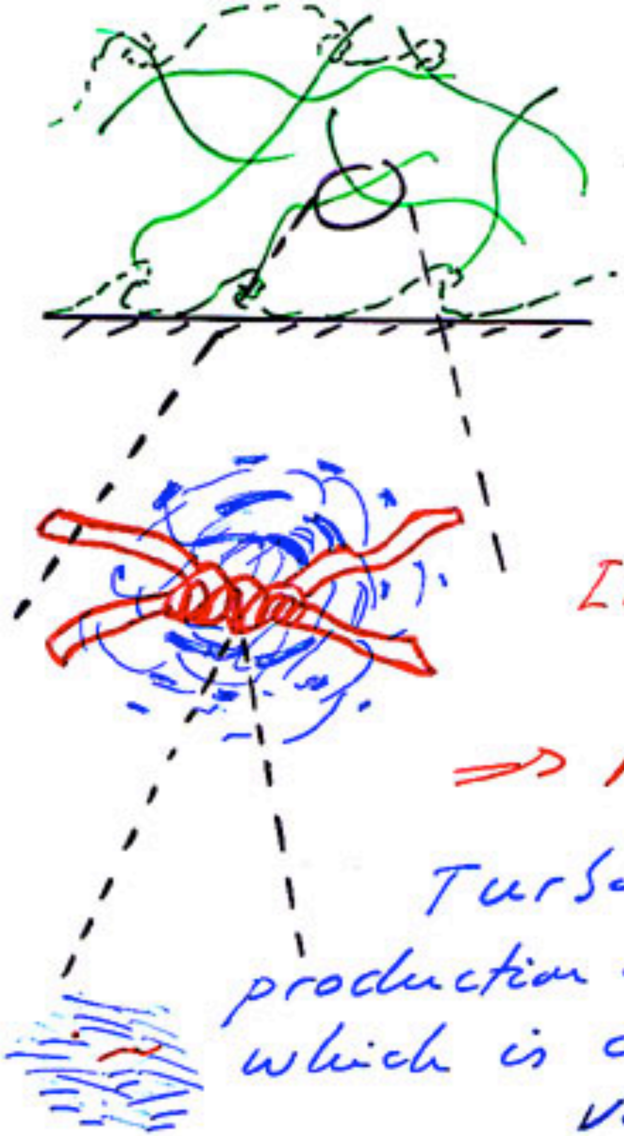
Fourier space viewpoint

No spectral gap between production and dissipation



New interpretation of the energy cascade

Physical space viewpoint



PRODUCTION

Webs of vortex tubes produced by instabilities in shear layers

INERTIAL RANGE

Intermittent vortex interactions

⇒ Nonlinear cascade

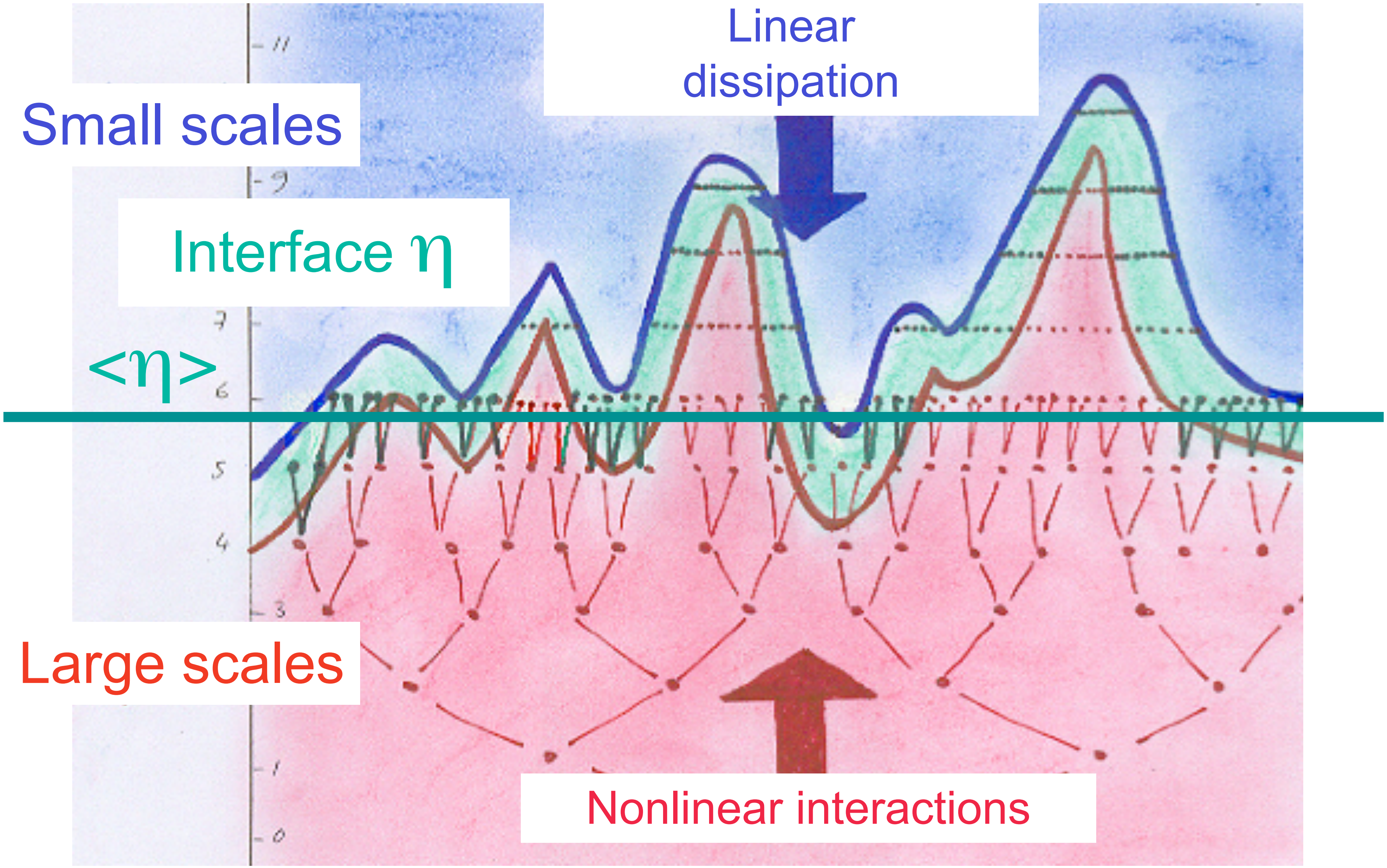
+

Turbulent dissipation

production of background noise which is damped in the viscous range

New interpretation of the energy cascade

Wavelet space viewpoint



Conclusion

The description of turbulent flows in terms of **mean value plus random fluctuations** seems eroded since there is **no scale separation** in the fully-developed turbulent regime.

We have developed a **wavelet-based filter** to separate the coherent fluctuations from the incoherent ones. The algorithm works in any space dimension, has no adjustable parameter and is fast, requiring only **order N operations**.

In each flow realization fluctuations are thus split into two orthogonal components which exhibit different statistics:

- **coherent fluctuations**, which are long-range correlated and non-Gaussian, and correspond to coherent vortices,
- **incoherent fluctuations**, which are decorrelated and Gaussian, and correspond to a random background flow.

Perspectives

The nonlinear wavelet filter
disentangles two different dynamics :

- a **nonlinear** dynamics, corresponding to **transport by the coherent vortices**,
- a **linear** dynamics corresponding to **turbulent dissipation**.

We conjecture that
discarding the incoherent flow
may be sufficient to **model turbulent dissipation**

$\Longrightarrow$ **Coherent Vortex Simulation (CVS)**.

Development of the CVS model equations (I)

Two–dimensional Navier–Stokes equations in vorticity–velocity formulation

$$\begin{aligned} \partial_t \omega + \vec{v} \cdot \nabla \omega - \nu \nabla^2 \omega &= \nabla \times \vec{f} \qquad (1) \\ \nabla \cdot \vec{v} &= 0 \\ \omega(0, \vec{x}) &= \omega_0 \end{aligned}$$

with the vorticity $\omega = \nabla \times \vec{v}$, and the velocity $\vec{v}$; $\vec{f}$ is a given forcing, and $\nu > 0$ the kinematic constant viscosity.
Completed with periodic boundary conditions.
The velocity can be reconstructed from the vorticity using the Biot-Savart relation,

$$\vec{v} = \nabla^{\perp} (\nabla^2)^{-1} \omega \qquad (2)$$

with $\nabla^{\perp} = (-\partial y, \partial x)$.

Development of the CVS model equations (II)

Using the decomposition of the flow into coherent and incoherent componenents, i.e.

$$\omega = \omega_C + \omega_I \quad \text{and} \quad \vec{v} = \vec{v}_C + \vec{v}_I \ , \tag{3}$$

and projecting (1) onto coherent and incoherent components we obtain an equivalent system

$$\partial_t \omega_C + (\vec{v}_C \cdot \nabla \omega_C + \vec{v}_C \cdot \nabla \omega_I + \vec{v}_I \cdot \nabla \omega_C + \vec{v}_I \cdot \nabla \omega_I)_C - \nu \nabla^2 \omega_C \ = \ \nabla \times \vec{f}_C$$

$$\partial_t \omega_I + (\vec{v}_C \cdot \nabla \omega_C + \vec{v}_C \cdot \nabla \omega_I + \vec{v}_I \cdot \nabla \omega_C + \vec{v}_I \cdot \nabla \omega_I)_I - \nu \nabla^2 \omega_I \ = \ \nabla \times \vec{f}_I$$

$$\nabla \cdot \vec{v}_C = 0 \quad \text{and} \quad \nabla \cdot \vec{v}_I = 0$$

$$\omega_C(0, \vec{x}) = (\omega_0)_C \quad \text{and} \quad \omega_I(0, \vec{x}) = (\omega_0)_I$$

which describes the evolution of the coherent and incoherent flow and their coupling in the spirit of nonlinear Galerkin methods, see e.g. (P. Constantin and C. Foias, 1988).

Estimations for the magnitude of the different terms (II)

This yields the following orders of magnitude of the different terms:

- $O(1)$ terms: ∇w_I and $\nabla^2 \omega_I$

- $O(\varepsilon)$ terms: ω_I

- $O(\varepsilon^2)$ terms: $\vec{v}_I$

The terms for the coherent flow $(\omega_C, \nabla\omega_C, \nabla^2\omega_C, \vec{v}_C)$ are all by construction of $O(1)$. Neglecting terms of increasing orders we obtain a hierarchy of CVS models.

Development of the CVS model equations (V)

Retaining only terms containing the coherent flow we obtain the following model equations:

CVS $O(1)$

$$\begin{aligned} \partial_t \omega_C + (\vec{v}_C \cdot \nabla \omega_C)_C - \nu \nabla^2 \omega_C &= \nabla \times \vec{f}_C \\ \nabla \cdot \vec{v}_C &= 0 \\ \omega_C(0, \vec{x}) &= (\omega_0)_C \end{aligned} \tag{5}$$

Note that in this system the influence of the incoherent flow is completely neglected and only the time evolution of the coherent flow is computed.

Development of the CVS model equations (VI)

Retaining the $O(1)$ and additionally the $O(\varepsilon)$ terms we obtain more complete equations:

CVS $O(\varepsilon)$

$$\begin{aligned} \partial_t \omega_C + (\vec{v}_C \cdot \nabla \omega_C + \vec{v}_C \cdot \nabla \omega_I)_C - \nu \nabla^2 \omega_C &= \nabla \times \vec{f}_C \qquad (6) \\ \partial_t \omega_I + (\vec{v}_C \cdot \nabla \omega_C + \vec{v}_C \cdot \nabla \omega_I)_I - \nu \nabla^2 \omega_I &= \nabla \times \vec{f}_I \qquad (7) \end{aligned}$$

$$\nabla \cdot \vec{v}_C = 0 \quad \text{and} \quad \nabla \cdot \vec{v}_I = 0$$

$$\omega_C(0, \vec{x}) = (\omega_0)_C \quad \text{and} \quad \omega_I(0, \vec{x}) = (\omega_0)_I$$

We observe that the equation for ω_I is a linear advection–diffusion equation, where the coherent velocity $\vec{v}_C$ is given.

Results for the CVS $O(\varepsilon)$ model (I)

Term	ε_0	ε_1	ε_{it}
$(\vec{v}_C \cdot \nabla\omega_C)_C$	1	1	1
$(\vec{v}_C \cdot \nabla\omega_C)_I$	$7.69 \cdot 10^{-1}$	$5.50 \cdot 10^{-1}$	$9.51 \cdot 10^{-2}$
$(\vec{v}_C \cdot \nabla\omega_I)_C$	$4.59 \cdot 10^{-1}$	$2.97 \cdot 10^{-1}$	$6.37 \cdot 10^{-2}$
$(\vec{v}_C \cdot \nabla\omega_I)_I$	$9.47 \cdot 10^{-1}$	$5.34 \cdot 10^{-1}$	$8.22 \cdot 10^{-2}$
$(\vec{v}_I \cdot \nabla\omega_C)_C$	$7.93 \cdot 10^{-2}$	$7.78 \cdot 10^{-3}$	$3.29 \cdot 10^{-4}$
$(\vec{v}_I \cdot \nabla\omega_C)_I$	$6.25 \cdot 10^{-2}$	$9.99 \cdot 10^{-3}$	$2.81 \cdot 10^{-4}$
$(\vec{v}_I \cdot \nabla\omega_I)_C$	$3.03 \cdot 10^{-2}$	$1.40 \cdot 10^{-3}$	$1.69 \cdot 10^{-5}$
$(\vec{v}_I \cdot \nabla\omega_I)_I$	$6.95 \cdot 10^{-2}$	$3.77 \cdot 10^{-3}$	$2.25 \cdot 10^{-5}$

Relative norms of the nonlinear terms with respect to $(\vec{v}_C \cdot \nabla\omega_C)_C$ using different thresholds ε. The values are averaged in time over 20τ.

Coherent Vortex Simulation (CVS)

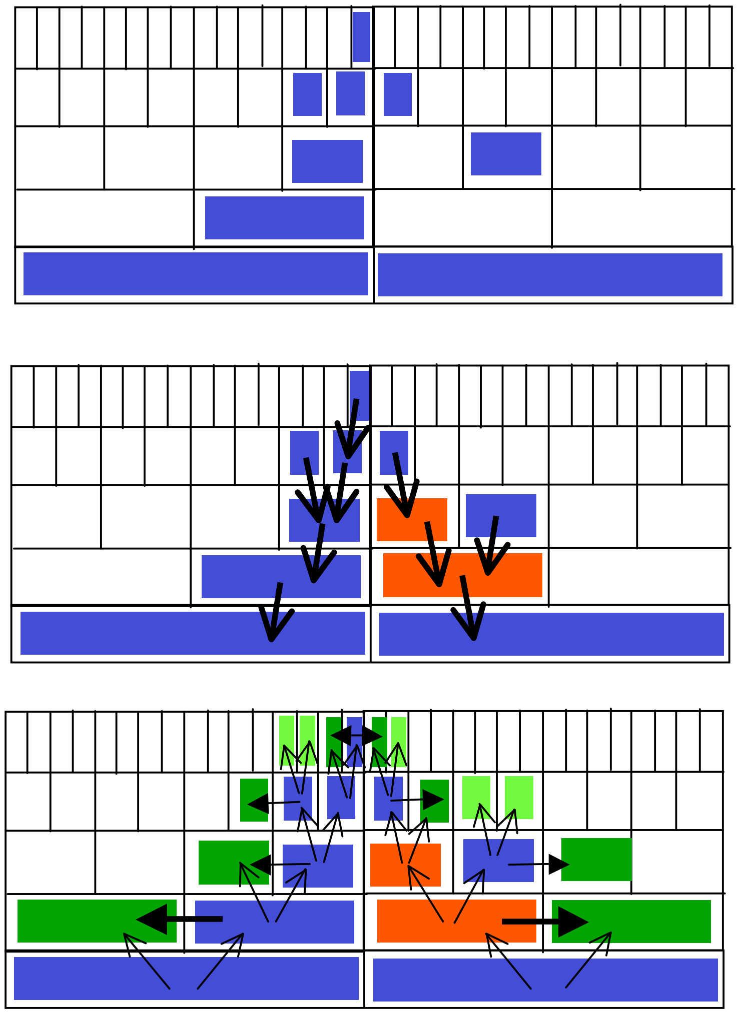
1. Projection of vorticity onto an orthogonal wavelet basis.
2. Extraction of coherent vortices using orthogonal wavelets.
3. Computation of the coherent vorticity by inverse transform.
4. Computation of the coherent velocity using Biot-Savart's law.
5. Addition of a security zone in wavelet space.
6. Integration of Navier-Stokes of in the reduced wavelet basis.
7. Use volume penalization to describe solid walls and obstacles.

*Farge, Schneider and Kevlahan, Phys. Fluids, **11** (8), 1999*

*Farge and Schneider, Flow Turbulence and Combustion, **66**, 2001*

*Schneider, Farge, Azzalini and Ziuber J. Turbulence, **7** (44), 2006*

Adapted wavelet basis



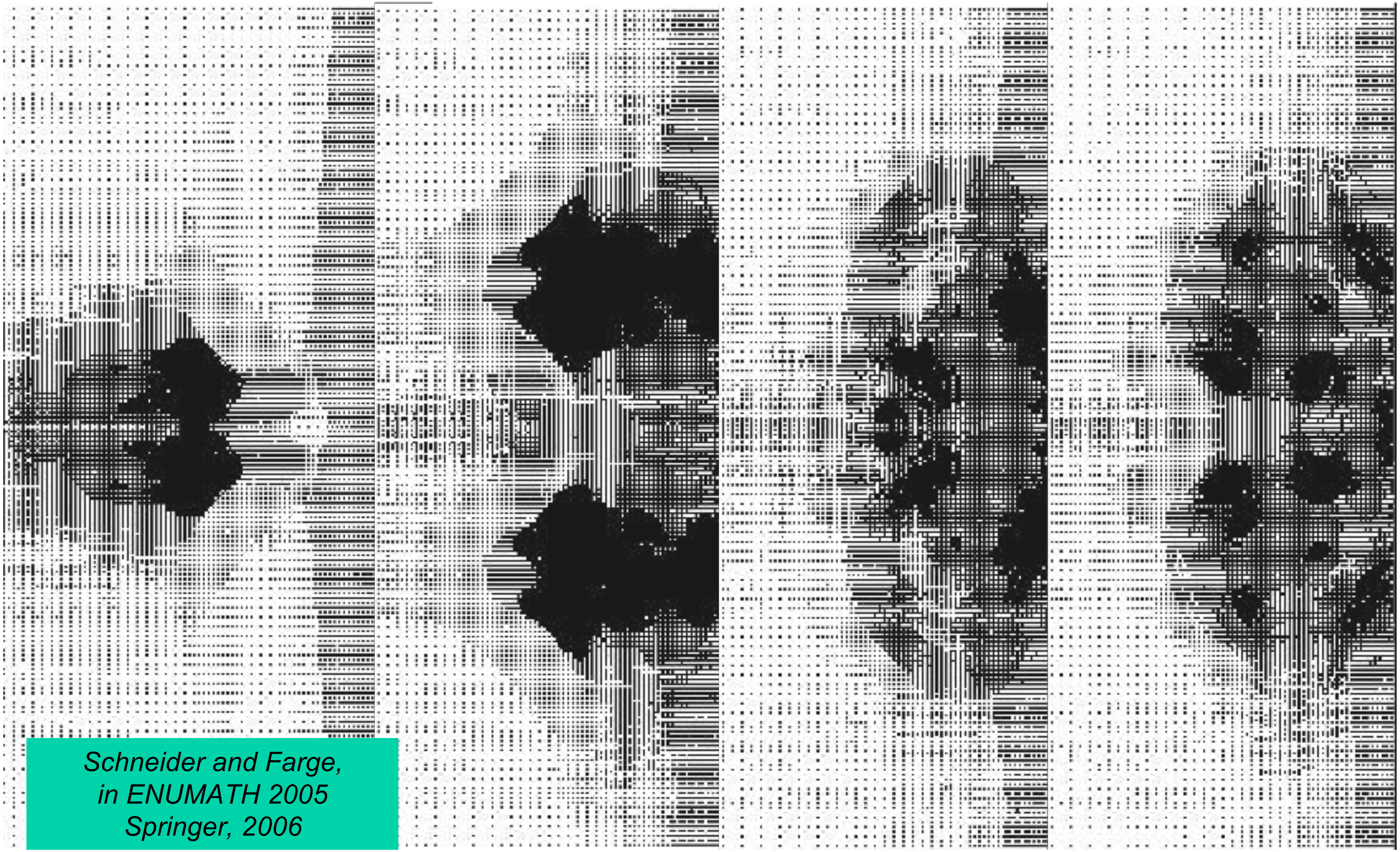
1. Selection of the wavelet coefficients whose modulus is larger than the threshold.
2. Construction of a 'graded-tree' which defines the 'interface' between the coherent and incoherent wavelet coefficients.
3. Addition of a 'security zone' which corresponds to dealiasing.

Dipole impinging on a wall at Re= 1000

Time evolution of vorticity computed by CVS

Adapted grid generated from wavelets

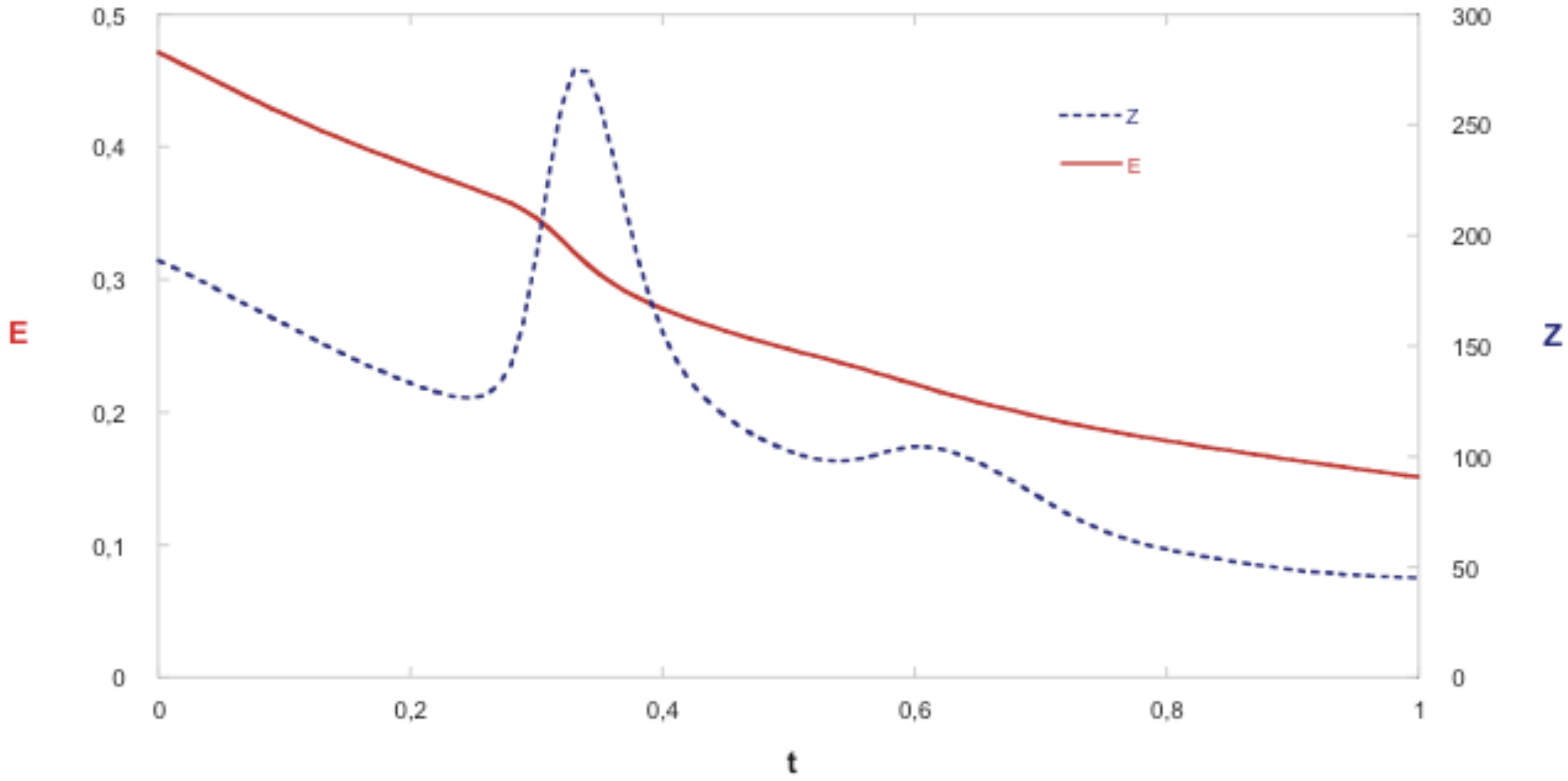
Time evolution of the grid computed by CVS



Schneider and Farge,
in ENUMATH 2005
Springer, 2006

Dipole impinging on a wall at Re= 1000

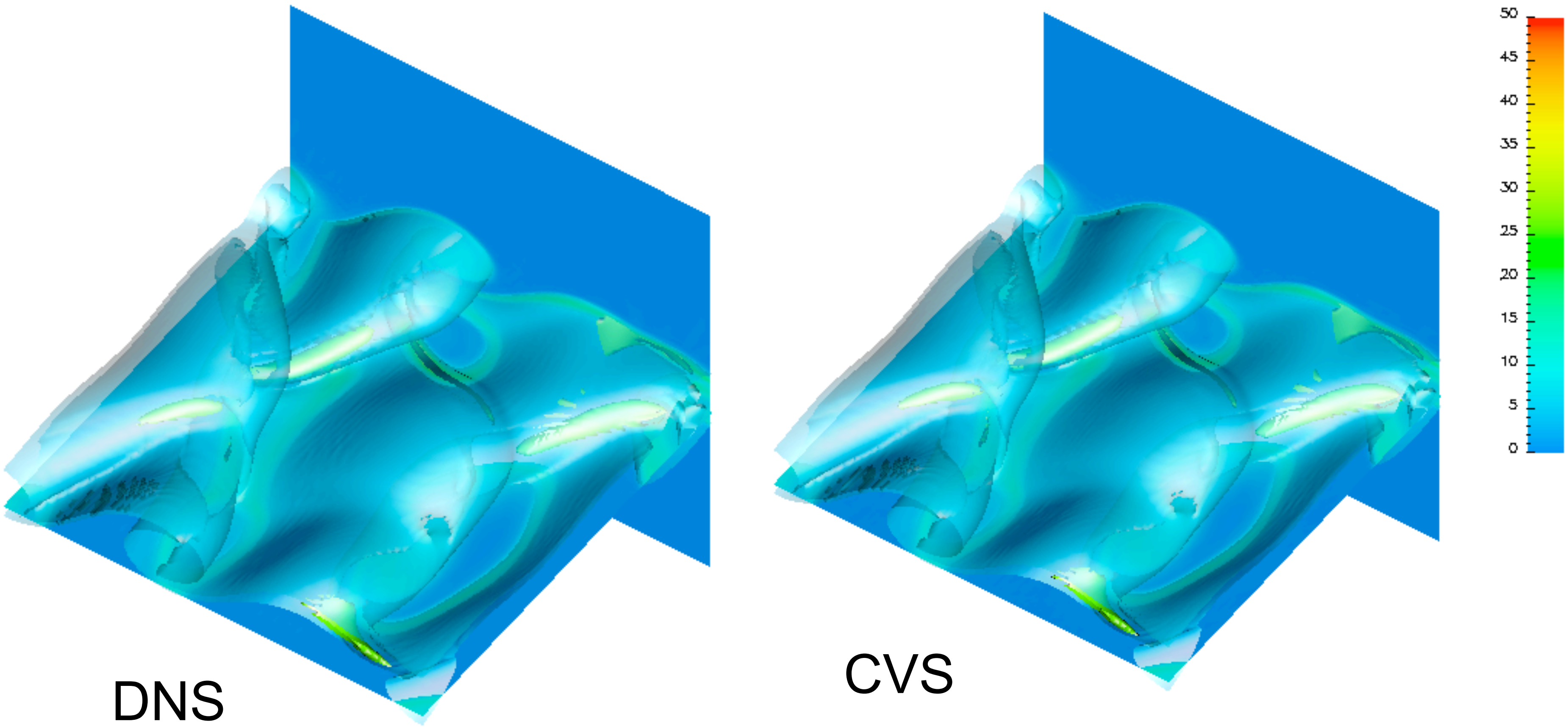
Time evolution of E and Z computed by CVS



Time evolution of energy (red) and enstrophy (blue).

Schneider and Farge, in ENUMATH 2005, Springer, 2006

3D mixing layer
Comparison between DNS and CVS

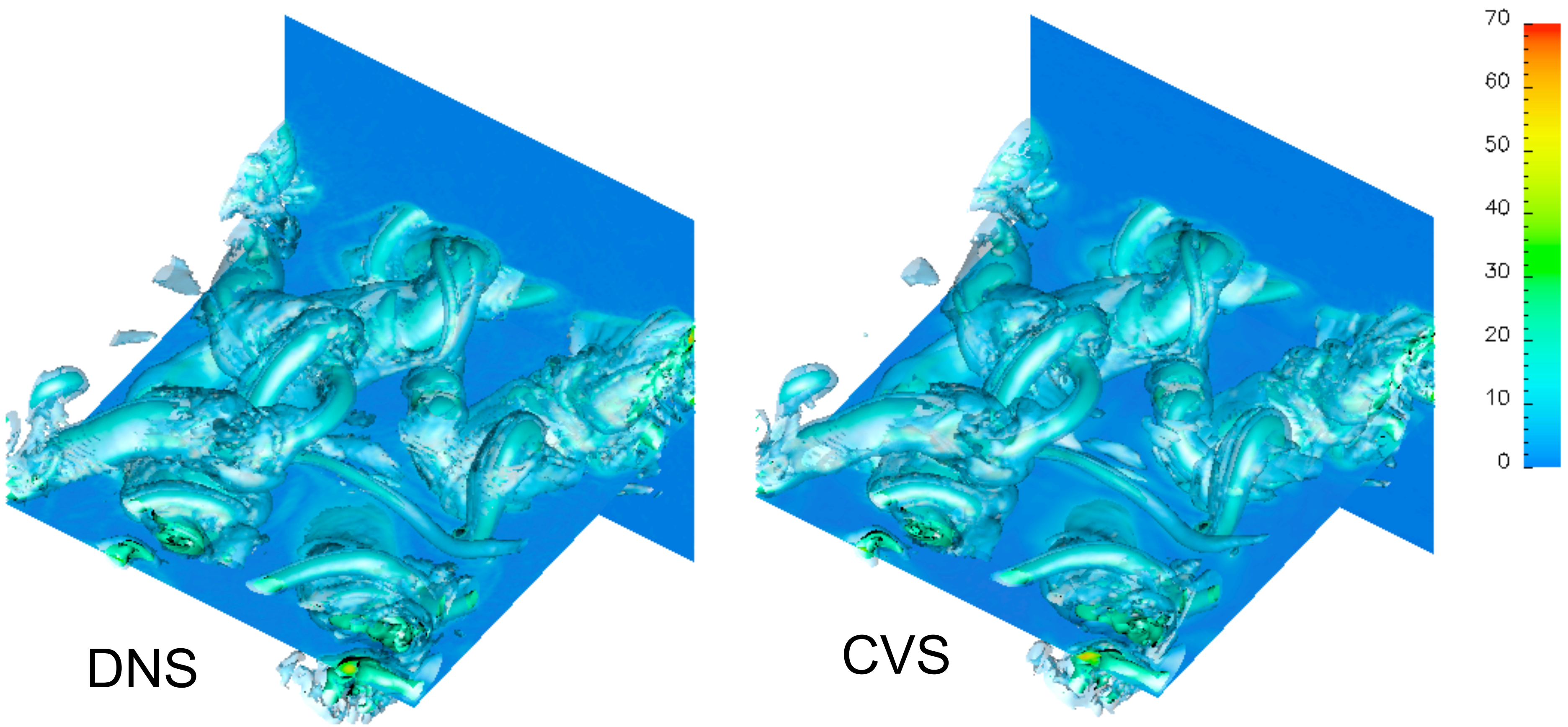


4 eddy turnover times

Schneider, Farge, Pellegrino, Rogers, J. Fluid Mech., 534, 2005

3D mixing layer

Comparison between DNS and CVS

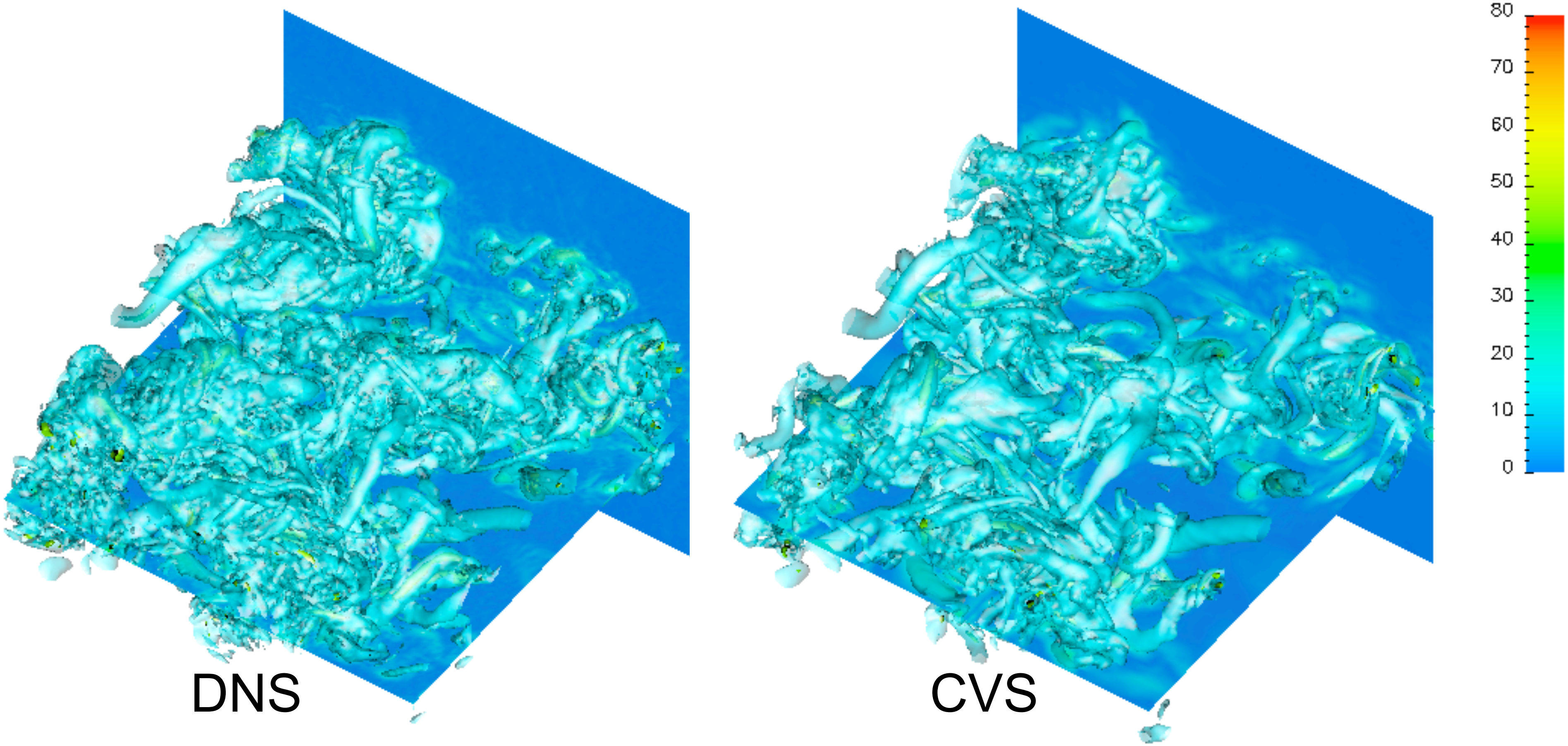


8 eddy turnover times

Schneider, Farge, Pellegrino, Rogers, J. Fluid Mech., 534, 2005

3D mixing layer

Comparison between DNS and CVS



12 eddy turnover times

Schneider, Farge, Pellegrino, Rogers, J. Fluid Mech., 534, 2005

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*Annual Review of Fluid Mechanics, **24**, 395-457*

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Wavelet filtering to study mixing in 2D irotropic turbulence
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Marie Farge and Kai Schneider, 2001
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*Flow, Turbulence and Combustion, **66**, 393-426*

Kai Schneider, Marie Farge, Alexandre Azzalini and Jörg Ziuber, 2006
Coherent vortex extraction and simulation of 2D isotropic turbulence
*J. of Turbulence, **7**, **44**, 1-24*

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