Direct drive by cyclotron heating can explain spontaneous rotation in tokamaks

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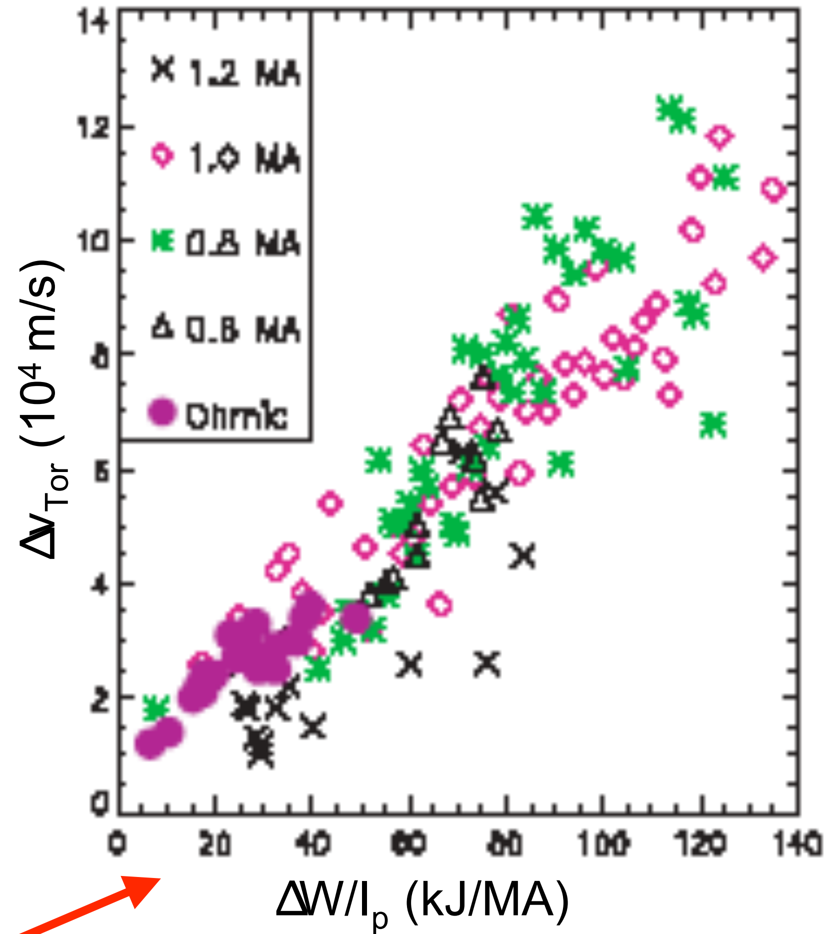
Significance of rotation

- Rotation and velocity shear are important in tokamaks for confinement and stabilization
  - Stabilize resistive wall modes
  - Suppress turbulent transport
- Neutral beams, which provide rotation in current-day tokamaks, may be insufficient in reactor-grade plasmas (e.g., ITER)
  - Short penetration depth
  - Requires high injection energy E, hence modest imparted momentum ~ $P_{inj} / E^{1/2}$
- Hence, considerable interest in the intrinsic toroidal rotation observed to be spontaneously generated, without externally applied torque, in tokamaks heated with waves in the cyclotron frequency range



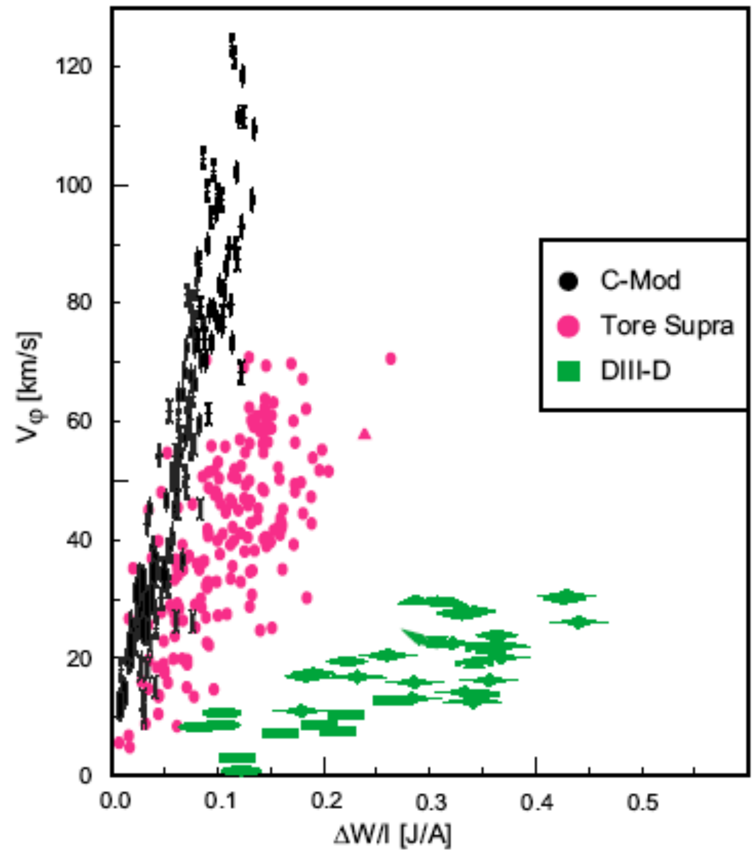
Toroidal rotation with ICRH in C-Mod (Rice et al. 2001)

Experimental observations

- Remarkably similar results for intrinsic toroidal rotation from devices with different heating methods and plasma shapes and conditions
  - **ICRH** in JET, Alcator C-Mod, and Tore Supra: rotation in co-current direction for H-mode or other advanced confinement regime plasmas
  - **ECH** in DIII-D and TCV: counter-current
  - **LH and ECH** in JT-60U

- Experiments find intrinsic rotation velocity is proportional to plasma stored energy (or pressure) and scales inversely with current

- Intrinsic rotation is an appreciable fraction of Alfvén speed
  - Possibly strong enough to stabilize MHD modes in ITER



Toroidal rotation with ICRH and ECH (Rice et al., IAEA 2006)

Earlier theories

- Various theories have been proposed to explain the generation of intrinsic rotation without auxiliary momentum injection, based on:
  - Neoclassical or sub-neoclassical effects [Kim (1991), Rogister (2002)]
  - Radial orbit shifts of cyclotron-heated energetic ions [Chang (1999), Chan (2002), Eriksson (2004)]
  - Turbulence-induced toroidal stress [Shaing (2001)]
  - Electrostatic modes driven by the ion pressure gradient [Coppi (2002)]
  - Blob transport at the plasma edge [Myra (2006)]

- These theories have been carefully compared with experimental observations recently
  - Although each of these theories has its merits, the underlying mechanism for intrinsic rotation is still uncertain
  - Rice (IAEA 2006): "*At present there is no comprehensive, quantitative theoretical explanation of spontaneous/intrinsic rotation*."

- New theory by Gurcan et al. (plenary talk at this meeting)
  - Non-diffusive Reynolds stress and sheared ExB symmetry breaking

New theory

- We propose an alternative explanation, showing that cyclotron wave heating can provide a ***direct*** drive for intrinsic toroidal rotation of the core plasma

- Even though cyclotron heating is applied with a symmetric spectrum of waves, without a preferred toroidal direction, we find that, when the effects of *finite orbit size* and *magnetic field inhomogeneity* are taken into account, the toroidal momentum input due to cyclotron wave heating is actually unbalanced in the toroidal direction, thus causing rotation

- We propose that this type of direct rotation drive could provide an explanation for the spontaneous rotation due to cyclotron heating

Difference with other fast-ion theories

- Earlier fast-ion theories considered the radial electric field (or radial current) produced by a radially outward or inward shift of trapped fast ions as the cause for rotation
- Canonical momentum p_φ is an invariant unless wave-particle resonance occurs.

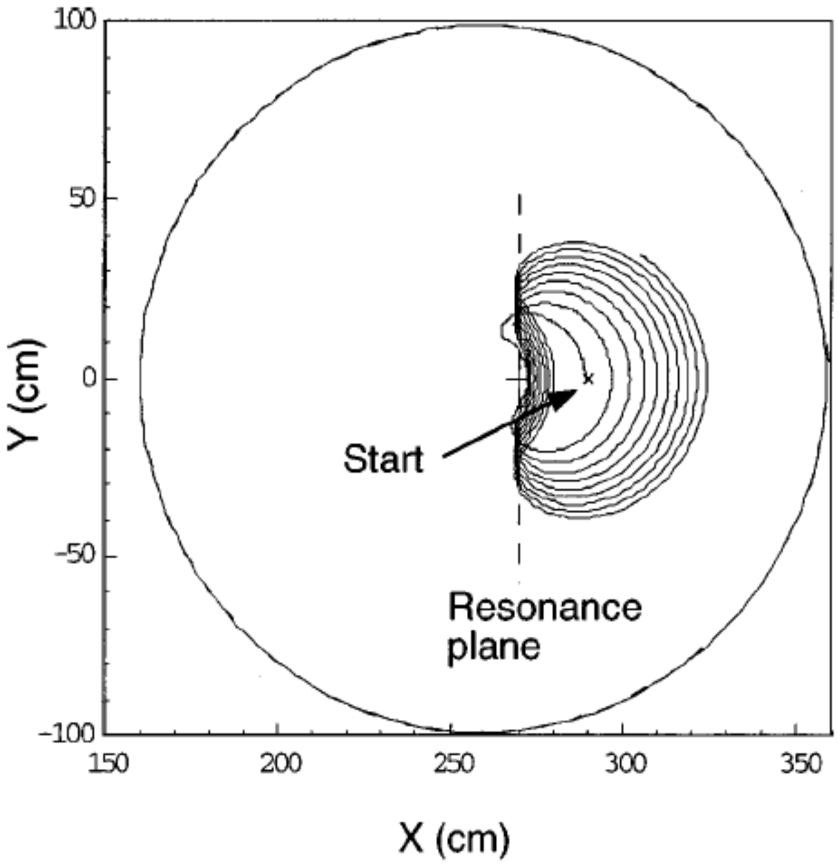
$$p_\varphi = mRv_\varphi - e\psi$$

- With the approximation (near the banana tips, at resonance plane)

$$v_\varphi \cong v_\parallel \rightarrow 0$$

an increase in p_φ leads to radial outward motion

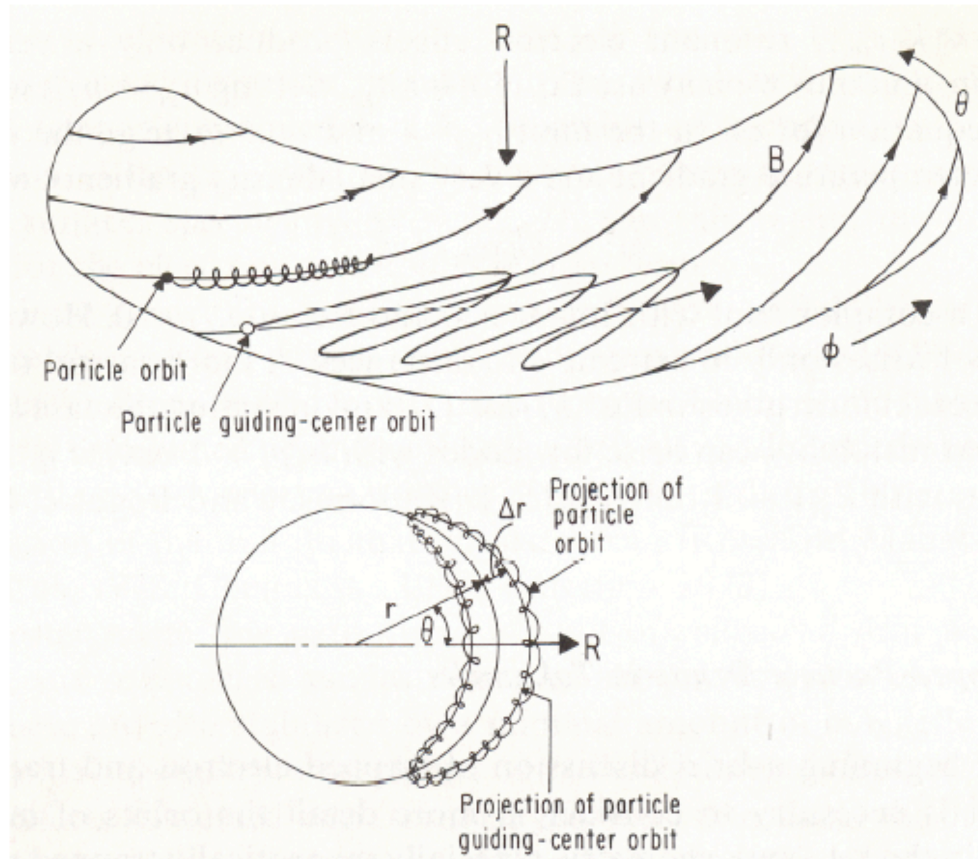
- But this neglects precessional motion



Poloidal projection of guiding center orbit for trapped hydrogen ion with on-axis ICRF perpendicular heating (Chang et al. 1999)

Our theory includes precessional drift motion

- Our theory takes into account the toroidal precessional motion of the trapped ions
  - Since the toroidal precessional drift has a specific direction, trapped ions resonantly interact only with an asymmetric portion of the cyclotron wave spectrum
  - By absorbing energy through cyclotron heating, the rapidly precessing ions increase in number; this causes the toroidal rotation likewise to increase
  - The fast resonant ions can transfer their momentum to the bulk plasma through collisions



Trapped particle orbits in tokamaks

Theoretical approach

- We use quasi-linear theory in action-angle variables (**J**, **θ**) to solve for the perturbed distribution function δf and the slowly diffusing zeroth-order averaged distribution function $\langle F_0 \rangle$:

$$
\begin{aligned}
\frac{\partial \langle F_0 \rangle}{\partial t}(\mathbf{J};t) &= -\frac{\partial}{\partial \mathbf{J}} \cdot \left\langle \delta \dot{\mathbf{J}} \delta f \right\rangle \\
\frac{\partial \delta f}{\partial t} + \boldsymbol{\omega} \cdot \frac{\partial \delta f}{\partial \boldsymbol{\theta}} &= -\delta \dot{\mathbf{J}} \cdot \frac{\partial F_0}{\partial \mathbf{J}}.
\end{aligned}
$$

- For wave absorption, only trapped particles will be considered, since they have more time than passing particles to interact with the applied waves

Solution of quasi-linear equation

- Solution of quasilinear equation

$$\frac{\partial \langle F_0 \rangle}{\partial t} = \frac{\pi e^2}{2m^2|k_\parallel|} \sum_{l=-\infty}^{+\infty} \frac{1}{l^2 v_\perp} \frac{\partial}{\partial v_\perp} v_\perp \left| \delta E^+ J_{l-1} + \delta E^- J_{l+1} \right|^2 \delta\left(v_\parallel - \frac{\omega - k_\varphi \omega_\varphi - l\omega_c}{k_\parallel} \right) \frac{\partial \langle F_0 \rangle}{\partial v_\perp},$$

$$\delta f_l = \frac{\pi e^2}{l\omega_c m^2 c |k_\parallel|} v_\perp \left(\delta E^+ J_{l-1} + \delta E^- J_{l+1} \right) \delta\left(v_\parallel - \frac{\omega - k_\varphi \omega_\varphi - l\omega_c}{k_\parallel} \right) \frac{\partial F_0}{\partial \mu} \exp\{i\mathbf{k}\cdot\mathbf{r}\}.$$

- Parameter definitions:

— Action vector $\mathbf{J} = (M, P, J_\theta)$, with three invariants: the magnetic momentum $M = (m^2/e)\mu$, the canonical angular momentum $P = m\dot{\zeta}R^2 - e\psi$, and the longitudinal invariant $J_\theta = (e/2\pi)\int d\beta\alpha$.

— Corresponding angle vector $\boldsymbol{\theta} = (\theta_g, \varphi, \theta)$, where θ_g is the gyro phase, and where for trapped particles $\varphi = \zeta - q\beta$, and $\theta = (\pi/2)F(\xi,\kappa)/K(\kappa)$. Here, $\kappa = \sin(\theta_t/2)$ with θ_t the turning point, K is the complete elliptic integral of the first kind, and F is the normal elliptic integral of the first kind, with ξ defined by $\kappa\sin(\xi) = \sin(\beta/2)$.

— Frequency vector $\boldsymbol{\omega} = (\omega_c, \omega_\theta, \omega_\varphi)$, with

$$\omega_c = \frac{eB}{m}, \quad \omega_\theta = \frac{\pi v_\perp (r/R)^{1/2}}{2^{3/2} qRK(\kappa)}, \quad \boxed{\omega_\varphi = \frac{v_\perp^2 q}{2Rr\omega_c} D(\kappa),}$$

Toroidal precession frequency (time-averaged)

where

$$D(\kappa) = 4s\frac{E(\kappa) + (\kappa^2 - 1)K(\kappa)}{K(\kappa)} + \frac{2E(\kappa) - K(\kappa)}{K(\kappa)}.$$

Calculation of ICRF-induced torque

- We derive the rate of total momentum input (mechanical) due to cyclotron heating:

$$T_\zeta = \int d^3v m R \omega_\varphi \frac{\partial \langle F_0 \rangle}{\partial t} = \frac{q}{2r\omega_c} D(\kappa) P_w$$

  – We take $\kappa = \sin(\theta_t/2) \sim$ constant, since only trapped particles at the banana tips can have significant absorption of ICRH energy and momentum

- Here, P_w is the energy absorption rate per volume from the RF waves:

$$P_w = \frac{\pi e^2}{4m|k_{||}|} \left|\delta E_x \pm i\,\mathrm{sgn}(\omega)\delta E_y\right|^2 n_{res}$$

(with + sign for ICRH and - sign for ECH)

- Also, n_{res} is the density of resonant trapped particles:

$$n_{res} = \int d^3v \delta \left(v_{||} - \frac{\omega - k_\varphi \omega_\varphi - l\omega_c}{k_{||}} \right) \langle F_0 \rangle$$

- This result seems to explain a number of the experimental observations

Scaling of toroidal rotation velocity

- Convert the rate of total momentum input (torque T) due to cyclotron heating, to toroidal rotation velocity v_φ:

$$T_\zeta = \int d^3v m R \omega_\varphi \frac{\partial \langle F_0 \rangle}{\partial t} = \frac{q}{2r\omega_c} D(\kappa) P_w$$

  - Use T_ζ = n m (dv_φ/dt) and P_w = n d(ΔW)/dt, where ΔW = stored energy (due to ICRF heating).
  - Also, recall that the safety factor q is inversely proportional to plasma current I_p: specifically, $I_p = 2\pi a^2 B_T / R\, q(a)$ in the large aspect ratio limit.

- Derive a formula for the (incremental) toroidal rotation velocity v_φ:

$$v_\phi = \frac{\pi}{e} D(\kappa) \left(\frac{a}{R}\right)\left(\frac{\Delta W}{I_p}\right)$$

  - This reproduces experimental scaling with energy and plasma current

Scaling with temperature ratio

- Formula for the (incremental) toroidal rotation velocity v_φ:

$$v_\phi = \frac{\pi}{e} D(\kappa) \left(\frac{a}{R} \right) \left(\frac{\Delta W}{I_p} \right)$$

- **Experimentally**, DeGrassie et al. (APS/DPP 2006) found that the intrinsic toroidal rotation velocity for the case of ECH, if multiplied by the ratio of central temperatures $T_i(0)$ / $T_e(0)$, provides a better comparison with rotation data from ICRH discharges.

- The **theory** displays this feature, since the stored energy (or power absorbed from the cyclotron waves) is proportional to the central temperature:

$$\Delta W_e = \left(\frac{T_e(0)}{T_i(0)} \right) \Delta W_e$$

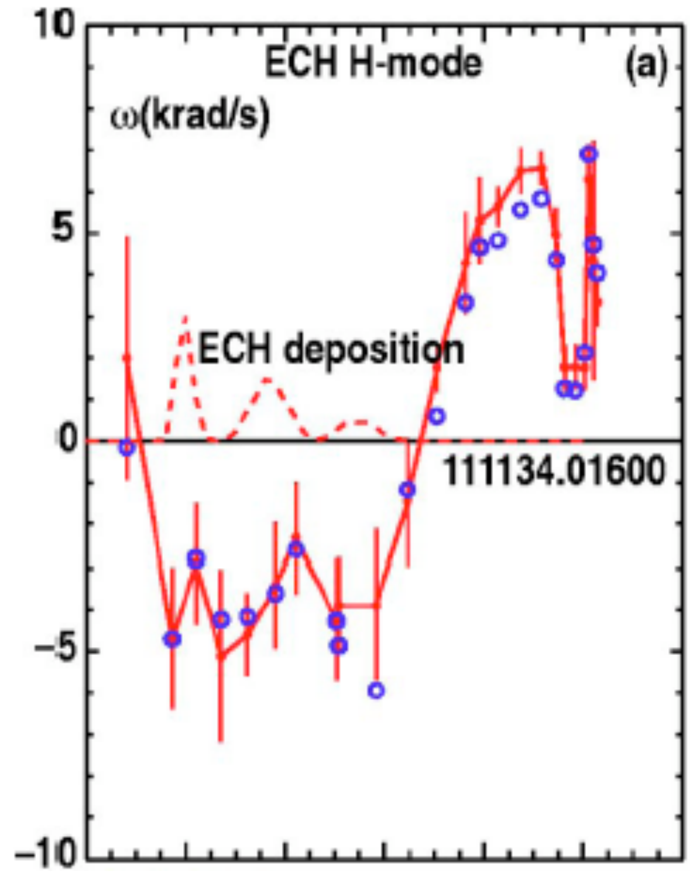
Direction of toroidal rotation velocity

- Theoretical formula for the (incremental) toroidal rotation velocity v_φ:

$$v_\phi = \frac{\pi}{e} D(\kappa) \left(\frac{a}{R}\right) \left(\frac{\Delta W}{I_p}\right)$$

- Consistent with experimental observations:

Experimentally observed rotation	Theoretical v_φ
Direction: – Co-current for ICRF – Counter-current for ECH (i.e., reversed in central region of ECH deposition) – Reverses for either ICRH or ECH when direction of the current is reversed	$\propto (e\ I_p)^{-1}$

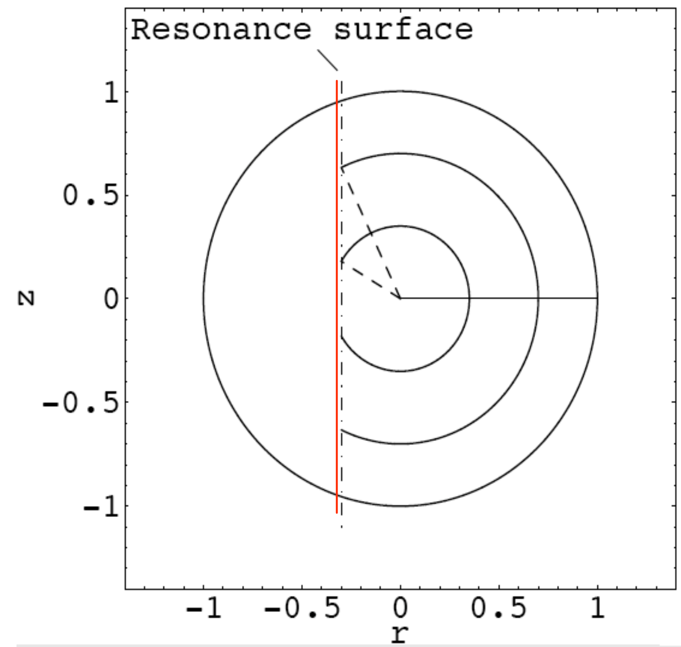


Toroidal rotation profile in DIII-D, with ECH power deposition profile indicated (deGrassie 2004)

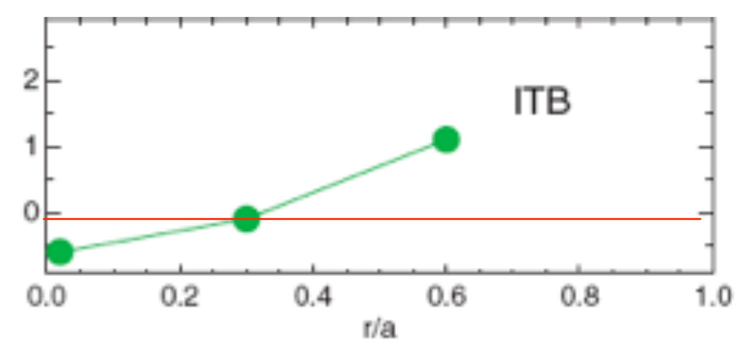
Rotation reversal for off-axis ICRH

- Theoretical formula for the (incremental) toroidal rotation velocity v_φ:

$$v_\phi = \frac{\pi}{e} D(\kappa) \left(\frac{a}{R} \right) \left(\frac{\Delta W}{I_p} \right)$$



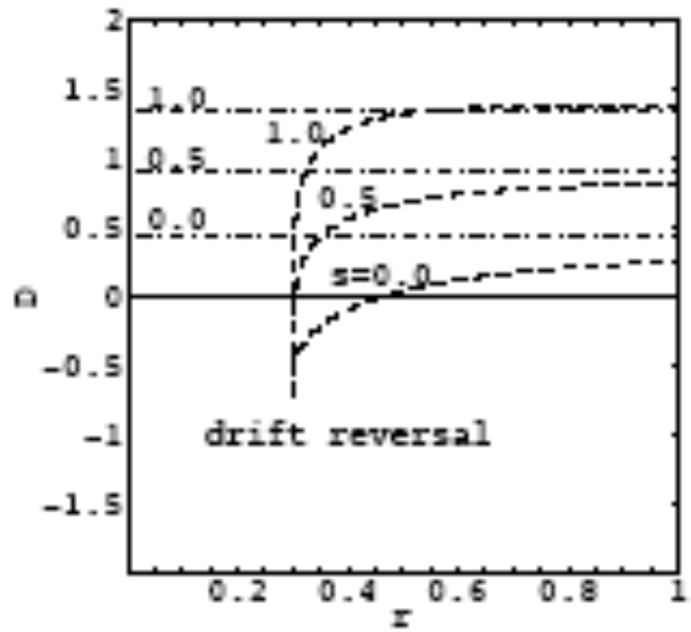
Experimentally observed rotation	**Theoretical v_φ**
Rotation reversal: Near-axis rotation is reversed during off-axis heating in ITB discharge	Drift reversal due to large θ_t at small r



Rotation profile during off-axis heating (Rice 2004)

Inverted radial profile evolution

- **Experimental observations show**:
  - In initial stages, the intrinsic toroidal rotation has a radially inverted profile, being stronger at large radii and weaker toward the center
  - After transition from L-mode to H-mode the rotation and stored energy in the core rapidly increase
- **In other theories** this feature is interpreted to mean that rotation is transported from outer region to center
- **Our theory provides a different explanation**
  - The function D(κ) depends on magnetic shear s, through the banana width effect. For monotonically increasing q(r) profile, the magnitude of D increases with minor radius r and could thus contribute to a radially increasing rotation profile
  - Also, following an L-to-H transition, the central plasma density is known to become peaked; this could lead to enhanced absorption of wave energy in the center, causing central rotation to be increased



D(κ) vs r , with shear s as parameter:
--- off-axis heating
-.-.- on-axis heating

$$v_\phi = \frac{\pi}{e} D(\kappa) \left(\frac{a}{R} \right) \left(\frac{\Delta W}{I_p} \right)$$

Predicted magnitude of rotation drive

- Energy confinement time is shorter than the momentum confinement time; hence, use the linear stage (between red lines) to estimate the ratio of rotation velocity to energy gain

- **Experiment**: Δv_{φ}^{max} [m/s] / ΔW [J] = 0.3

- **Theory**: Δv_{φ}[m/s] / ΔW [J] = 0.1 f_v $\langle q \rangle_v$ / A
  - f_v ~ 2-3 is ratio of central rotation speed to volume-averaged rotation speed
  - $\langle q \rangle_v$ ~ 2-3 is volume-averaged safety factor
  - A = 2 is ratio of ion mass to proton mass

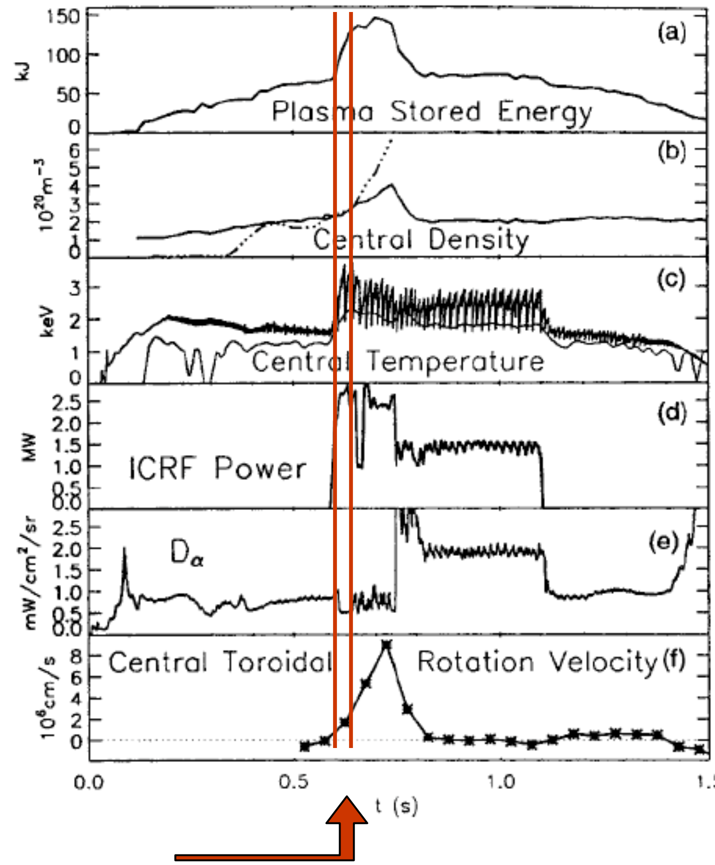
THUS: Δv_{φ}[m/s] / ΔW [J] = 0.2–0.5

- If normalize to plasma current:
  - **Theory gives**

Δv_{φ}[m/s] / {ΔW [J] / I_p [A]} = 3 x 10^5 (τ_{mom}/τ_E)

  - **Experiment finds**

Δv_{φ}[m/s] / {ΔW [J] / I_p [A]} = 10^6

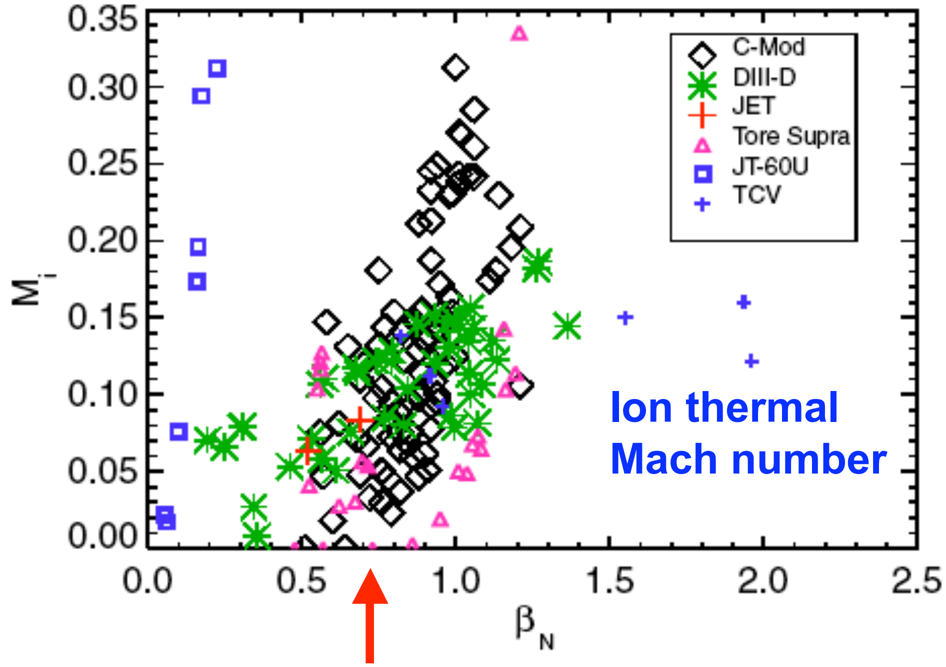


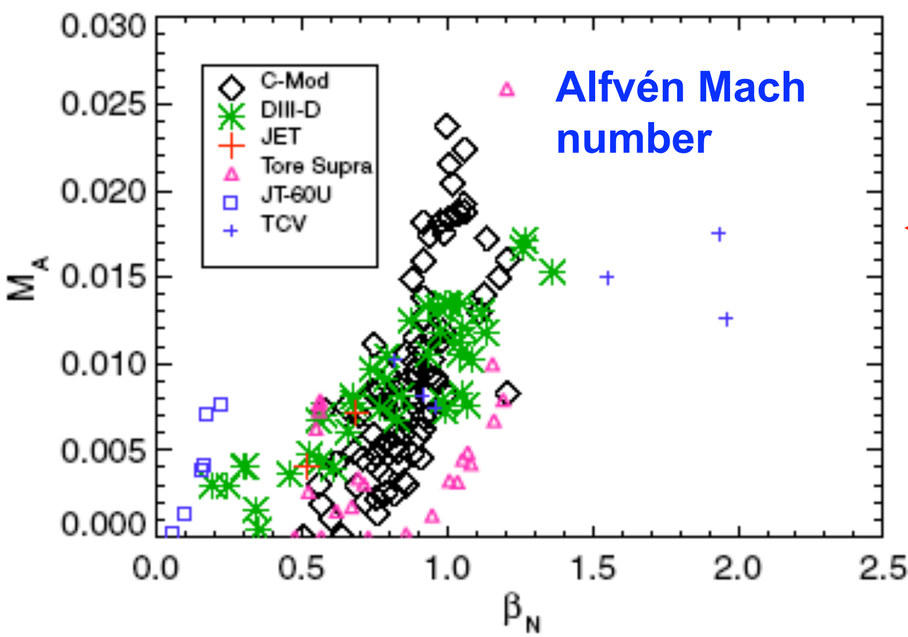
Linear phase

Time evolution of the stored energy, rotation, etc. (Rice 1998)

Scaling in dimensionless variables

- Can incorporate machine size information by recasting the rotation scaling in terms of dimensionless variables
  - Ion thermal or Alfvén Mach number $M_{i,\,A} = v_{\varphi} / v_{i,\,A}$
  - Normalized beta $\beta_N = \beta / (I_p/aB_T)$





- Data from different machines then coincide (**Rice et al., IAEA 2006**)

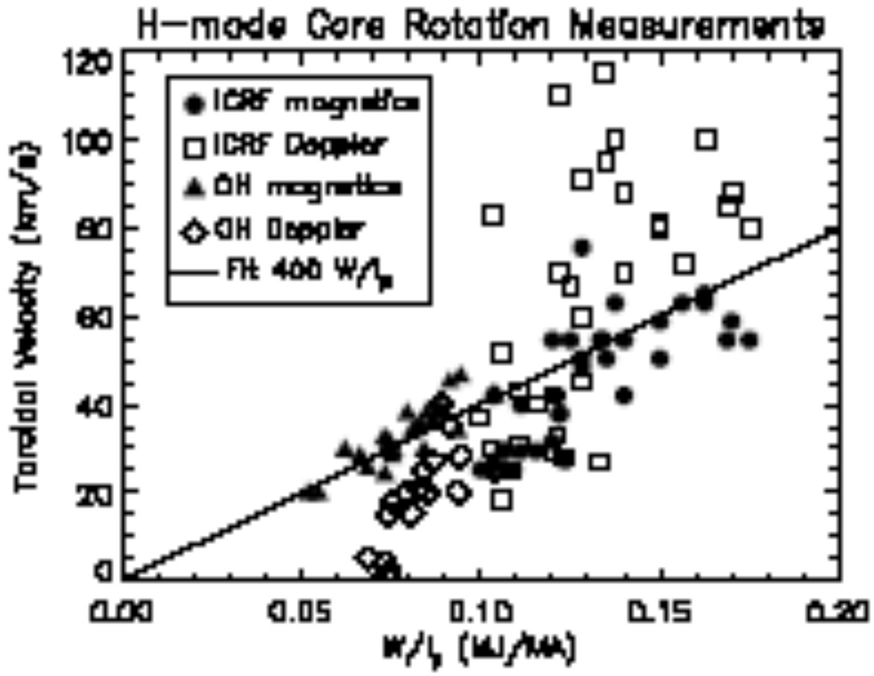
- Our theory is easily rewritten in dimensionless variables:

$$M_A / \beta_N = \pi\, m^{3/2}/eRn^{1/2}$$

Intrinsic rotation in Ohmic plasmas

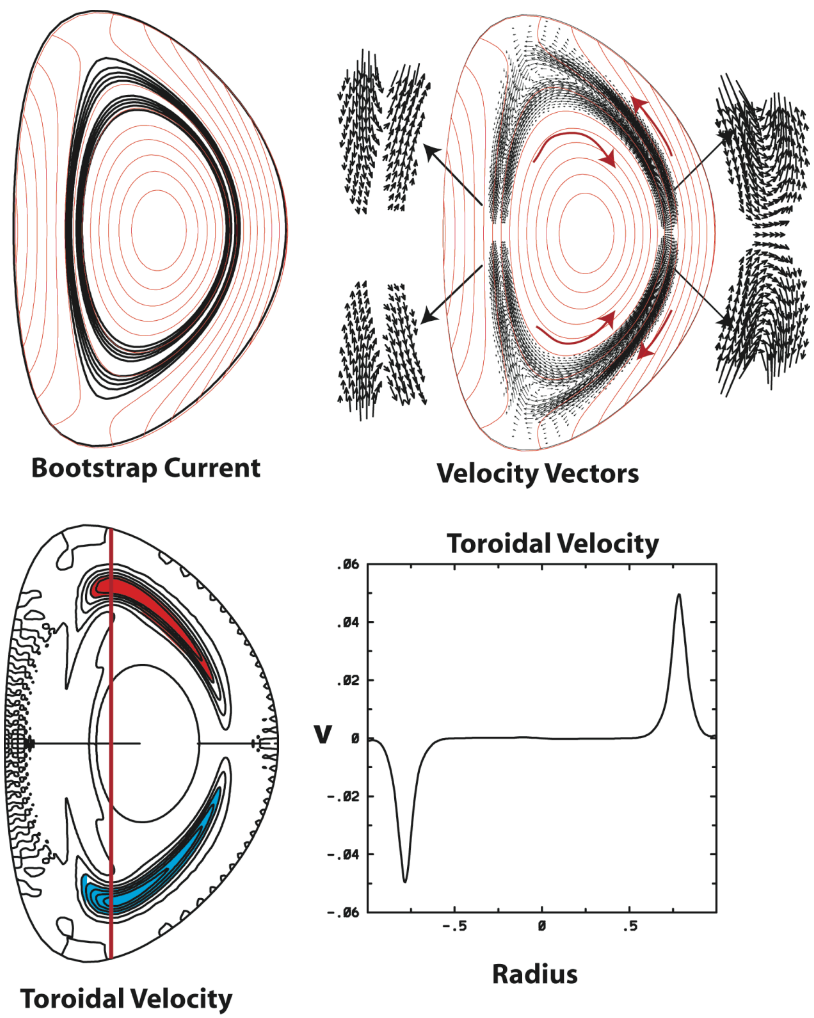
- Interestingly, intrinsic rotation has also been observed in **Ohmically heated plasmas**, without any cyclotron wave heating
  - Scaling of Ohmic rotation with stored energy and plasma current is same as for cyclotron wave heating, although the magnitude is smaller; also the flow is mostly in the SOL region
  - Our theory apparently does not provide an explanation for this. Are there multiple mechanisms?

- Recent work by **Aydemir [APS/DPP 2006]** does seem to explain it
  - Self-consistent flows generated by transport in non-ideal MHD equilibrium (with resistivity, bootstrap current, neoclassical effects)



H-mode rotation velocities, with and without RF heating (Hutchinson et al. 2000)

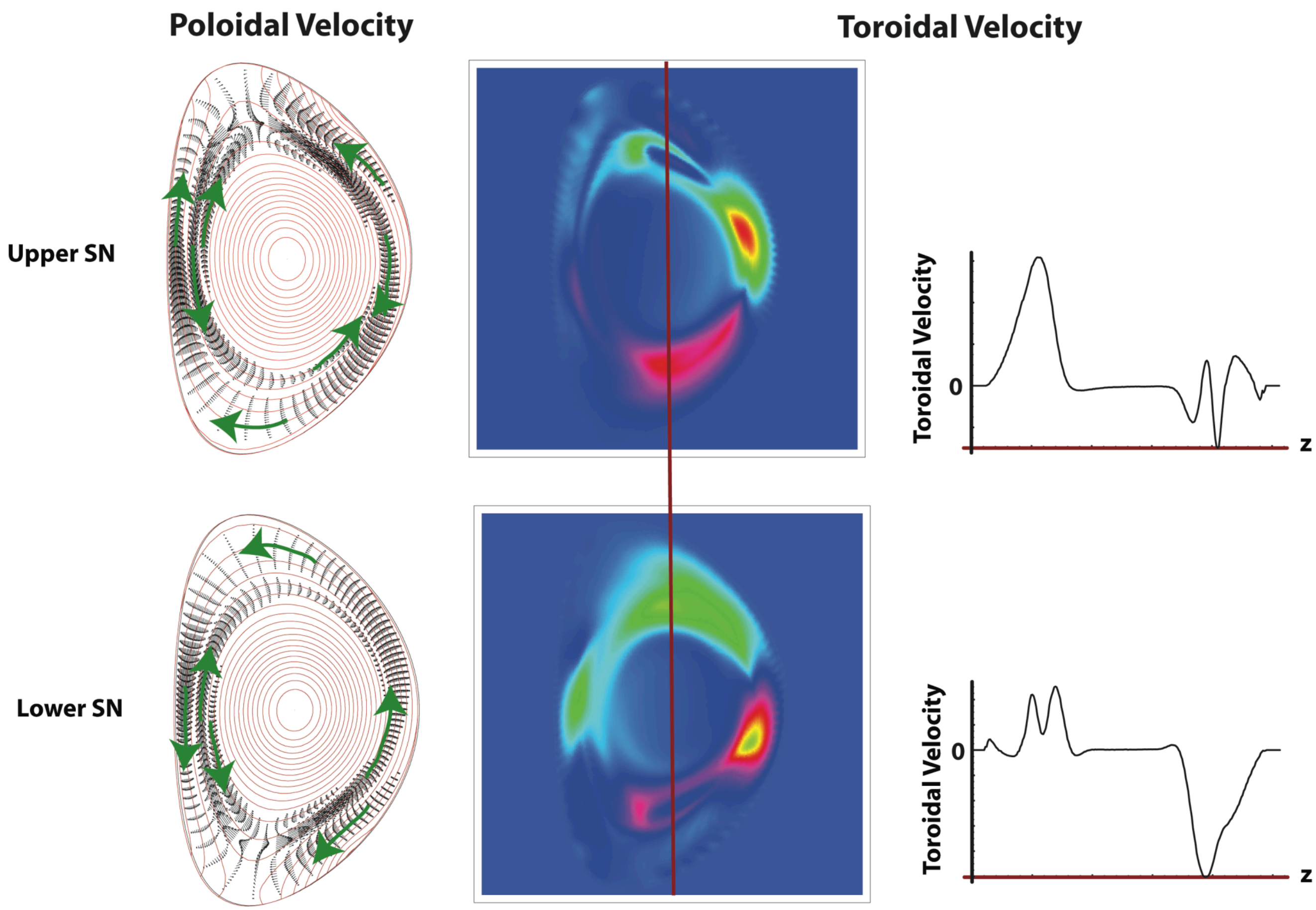
Dipole equilibrium flow pattern in a double-null (DN) configuration



A. Aydemir [APS/DPP 2006]

- A non-uniform "classical resistivity" profile with $S_0=10^6$, and $S_{SOL}=10^2$.
- A simple bootstrap current model, localized to the pedestal region, with $J_{BS}/J_0=0.3$ is used
- Toroidal beta = 5×10^{-3}.
- The resulting flow has an Alfvén Mach number $M\sim10^{-2}$, corresponding to velocities of order 10^4 m/s.
- Induced toroidal flow is also dipolar and has no net toroidal angular momentum.

Equilibrium flows in single-null (SN) configurations



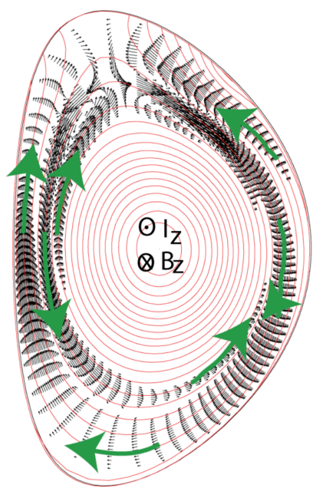
Interaction of the toroidal flow with the X-point transfers momentum to the vessel through the open field lines, leaving a net toroidal angular momentum contribution to the plasma.

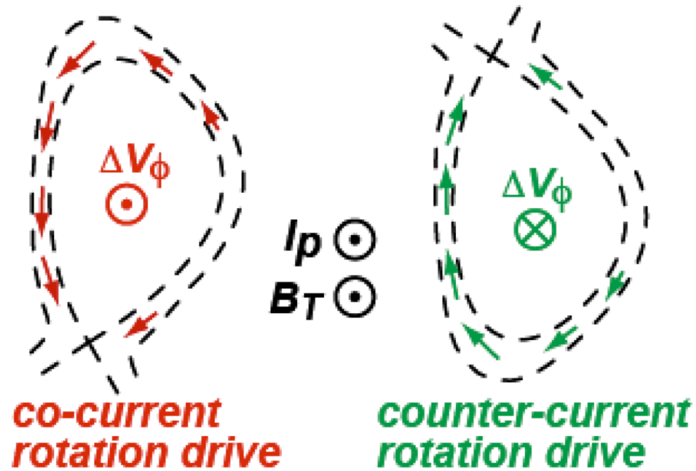
A. Aydemir [APS/DPP 2006]

Comparison with experimentally observed SOL flows

Poloidal Velocity

Upper SN

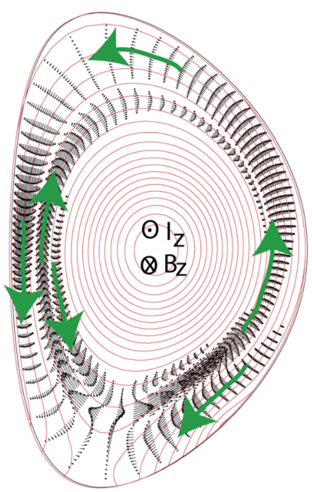




LaBombard, et al., Nucl. Fusion (2004), Phys. Plasmas (2005).

LaBombard et al., Nucl. Fusion (2004)

Lower SN



- Poloidal flow directions and locations appear to be in agreement with experimental observations

- When corrected for different sign of toroidal flux used in numerical calculations, contribution to core rotation is also in agreement

A. Aydemir [APS/DPP 2006]

Summary

- **Proposed a new explanation for intrinsic/spontaneous rotation of core plasma during cyclotron wave heating of tokamaks**
  - Based on asymmetric precessional acceleration of trapped ions by ICRH

- **Find agreement of theoretical predictions with key experimental features, such as:**
  - Scaling of toroidal rotation velocity
  - Direction of rotation
  - Magnitude of rotation
  - Radial profile of rotation

- **This may be one of several mechanisms for intrinsic rotation**
  - Intrinsic rotation observed at plasma edge in purely Ohmically heated discharges apparently requires another mechanism
  - For example, recent work by Aydemir [APS/DPP Annual Meeting 2006] is able to explain the SOL rotation (and its dependence on divertor topology) observed in Ohmic tokamak plasmas (e.g., C-Mod)