Nonlinear Refractive Suppression of Turbulence and Transport by Strong Magnetic Shear

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Outline

Transport barriers:

- Fusion plasmas
- Turbulence intermittency

Intermittency in interstellar turbulence

Shear Alfvén and kinetic Alfvén wave cascades

Refractive suppression of turbulence mixing in KAW turbulence

- Mixing of current
- Mixing of density

Modeling probability distribution functions

Simulation results

Discussion

Transport barriers are essential for improved confinement in fusion

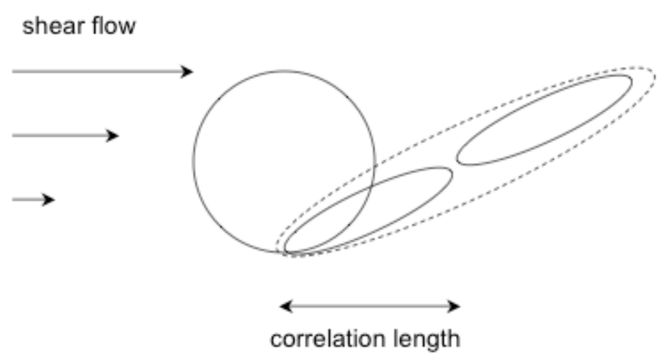
H mode

Edge barrier tied to a region of very strong shear flow

Suppression of turbulence by shear flow widely studied

Nonlinear: turbulent eddies driven by ∇T, ∇n sheared apart

$$\omega_{shear} > (\tau_{correlation})^{-1}$$

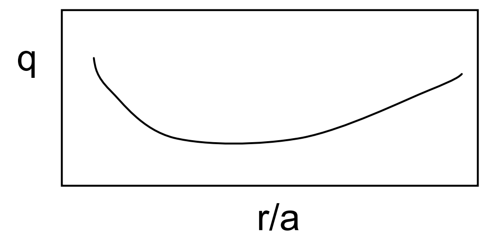


Linear: shear stabilizes ∇T, ∇n driven instabilities

$$\omega_{shear} > \gamma_{linear}$$

Internal transport barriers

Suppression by combination of flow shear (as above) and reversed magnetic shear



Magnetic shear understood as linear stabilization mechanism

Magnetic shear also suppresses via nonlinear mechanism

Linear mechanism:

Impose sheared magnetic equilibrium - turn crank - get growth rate

Growth rate sensitive to profiles ⇒ repeat for every new profile

Nonlinear mechanism:

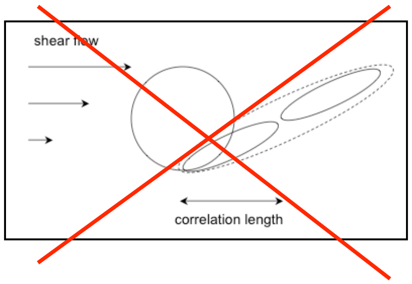
Not sensitive to profile ⇒ more universal

Mathematically equivalent to shear flow mechanism: $\omega_{shear} > (\tau_{correlation})^{-1}$

Physically very different

No flow

No shearing a la cartoon



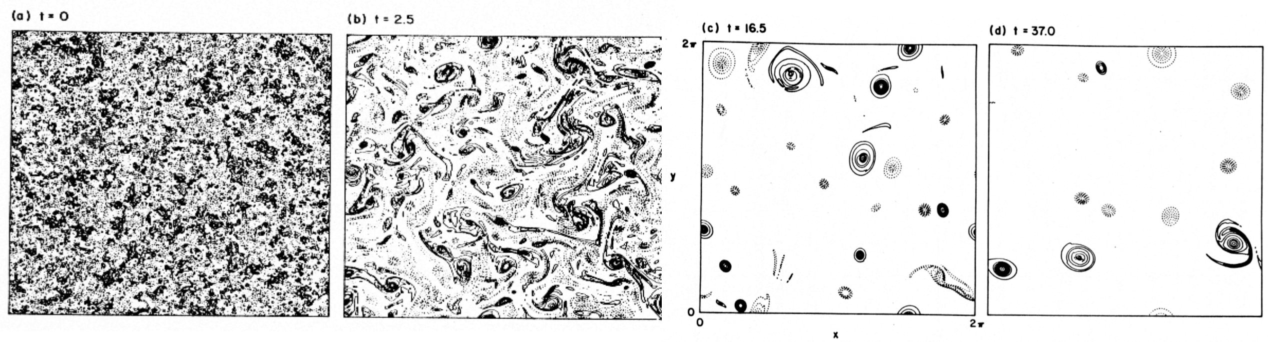
Explore mechanism in context of turbulence intermittency:

Transport suppression allows special fluctuations to be coherent

Two types of intermittency explained by two turbulence suppression mechanisms (flow shear, magnetic shear)

Flow shear ↔ Coherent vortices in decaying 2D Navier-Stokes turbulence

Simulations: long lived vortices emerge from Gaussian initial pdf (McWillaims, JFM 84)



High vorticity structures live forever; average vorticity structures die

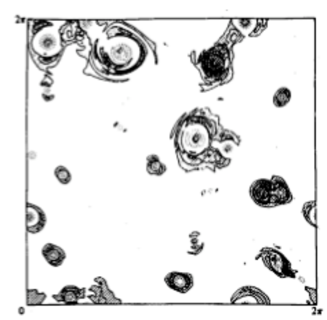
Theory: coherent vortices have edge shear flow ⇒ barrier to mixing (Terry, 89)

Nonlinear suppression mechanism $\omega_{shear} > (\tau_{correlation})^{-1}$ maps onto structure profiles in Gaussian curvature (GC)

GC << 1 strong vortex, no shear, no turbulence

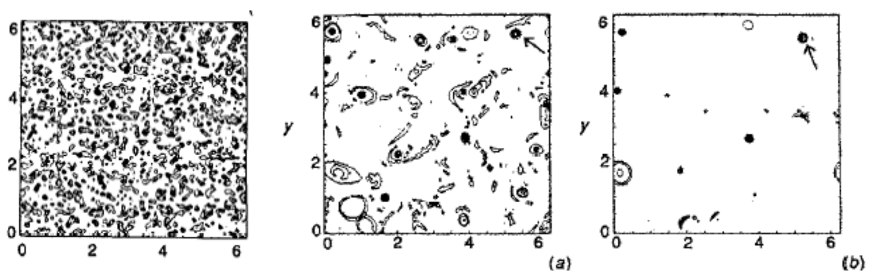
GC >> 1 strong vortex, strong shear, no mixing

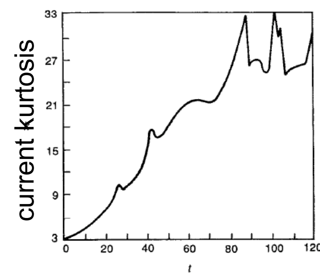
GC ≈ 1 vorticity, shear balanced ⇒ turbulent region with strong mixing



Magnetic shear ↔ Coherent current filaments in decaying kinetic Alfvén wave turbulence

Simulation: long lived current filament emerge from Gaussian initial pdf (Craddock 91)





High current structures live forever; average current structures die

Filaments strikingly similar to vortices of 2D Navier Stokes

What quantity has strong edge shear?

Answer: magnetic field

Does magnetic shear do to KAW turbulence what flow shear does to NS turbulence?

Magnetic shear suppresses mixing by refraction of KAW turbulence
$\Rightarrow$ coherent density filaments form in interstellar turbulence

Intermittency in interstellar turbulence:

Pulsar scintillation scaling $\Rightarrow$ intermittent electron density in ISM (Boldyrev 05)

Pulsar signal dispersed by electron density fluctuations in ISM

Pulse width scales as R^4 (R: distance from source)

If pdf of electron density fluctuations Gaussian, signal width ~ R^2

If pdf is Levy distributed (long tail), can recover R^4 scaling

Key questions:

- Does current intermittency of KAW turbulence support coherent density structures?
- How does it all work?
- How can structures form without flow?
  - (all prior intermittency mechanisms involve flow)
- What differentiates structures from surrounding flow?

Shear Alfvén and kinetic Alfvén cascades

How does electron density evolve in magnetic turbulence?

Basic model: RMHD + compressible electron continuity

$$\frac{\partial \hat{\psi}}{\partial t} + \nabla_{\parallel} \hat{\phi} = \eta \hat{J} + \nabla_{\parallel} \hat{n} + \frac{C_s}{V_A} n_0^{-1} \nabla \hat{\psi} \times z \cdot \nabla n_0, \quad (1)$$

$$\frac{\partial}{\partial t} \nabla_{\perp}^2 \hat{\phi} - \nabla \hat{\phi} \times z \cdot \nabla \nabla_{\perp}^2 \hat{\phi} = -\nabla_{\parallel} \hat{J}, \quad (2)$$

$$\frac{\partial \hat{n}}{\partial t} - \nabla \hat{\phi} \times z \cdot \nabla \hat{n} + \nabla_{\parallel} \hat{J} - \frac{C_s}{V_A} n_0^{-1} \nabla \hat{\phi} \times z \cdot \nabla n_0 = 0, \quad (3)$$

where

$$\nabla_{\parallel} = \frac{\partial}{\partial z} + \nabla \hat{\psi} \times z \cdot \nabla,$$

$$\hat{J} = \nabla_{\perp}^2 \hat{\psi},$$

Field	Term (large scale) Turbulent Alfvén wave	Term (small scale) Kinetic Alfvén wave
Electron density	Elec. advection (⊥ flux): $v \cdot \nabla n$	ǁ compr. (ǁ flux): $B \cdot \nabla J$
Flow	Lorentz force: $B \cdot \nabla J$	Ion advection: $v \cdot \nabla v$
Magnetic field	Parallel electric field: $B \cdot \nabla \phi$	Elec. pressure: $B \cdot \nabla\ n$

Shear Alfvén and kinetic Alfvén cascades

Fluctuations change character across the gyroradius scale
(Fernandez et al.)

Large scales ($k\rho_i << 1$): Alfvén waves

Coupling: B and v

Density: advected passively (no reaction back on B or v)

Intermittency: Primary structure is current filament

Ancillary structure in vorticity

Density tracks flow; flow is integral of vorticity

$\Rightarrow$ density not strongly intermittent

Small scales ($k\rho_i > 0.1$ in ISM): Kinetic Alfvén waves

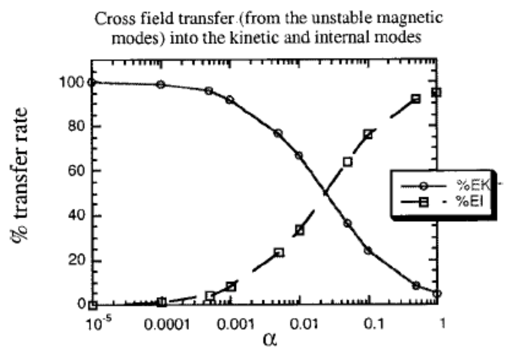
Coupling: B and n

Flow: Dominated by self advection, decouples from B and n

Intermittency: Under study

Shear Alfvén and kinetic Alfvén cascades

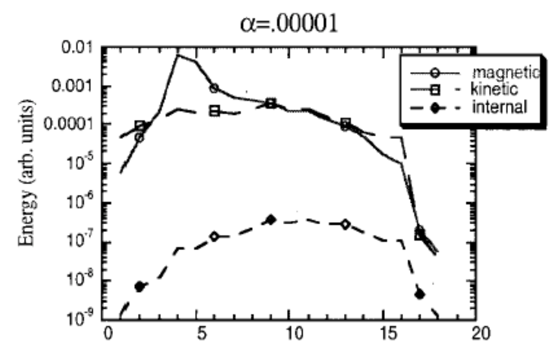
Electron density fluctuations increase in kinetic Alfvén regime as *n* equipartitions with *B*



Spectral energy transfer:

$k\rho_i$ << 1: transfer dominated by *v* ⇔ *B*

$k\rho_i$ ≥ 1: transfer dominated by *n* ⇔ *B*



Low k:

v and B equipartitioned

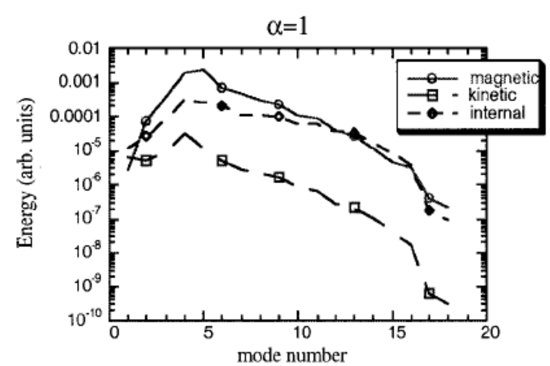
Density at level dictated by $v \cdot \nabla n_0$

High k:

n and B equipartitioned, even if no linear or external drive of density

v and B decouple

Low k - high k crossover at $k\rho_i < 1$



Refractive suppression of mixing in KAW turbulence

Kinetic Alfvén wave turbulence modeled by reduced fluid system for *B* and *n*

$$\frac{\partial \psi}{\partial t} - \mathbf{B} \cdot \nabla n = \eta \nabla^2 \psi$$ Ohms law for flux, with electron pressure force

$$\frac{\partial n}{\partial t} + \mathbf{B} \cdot \nabla \nabla^2 \psi = \upsilon \nabla^2 n$$ Density continuity with parallel compression

where

$\mathbf{B} = B_0 \hat{\mathbf{z}} + \nabla \psi \times \hat{\mathbf{z}}$ Field, $\mathbf{J} = \nabla^2 \psi \, \hat{\mathbf{z}}$ Current

Fluctuations in plane perpendicular to guide field (like reduced MHD)

No ion flow; ions are fixed, neutralizing background

Regimes analogous to low ($\upsilon<\eta$), high ($\upsilon>\eta$) magnetic Prandtl number govern temporal decay

Refractive suppression of mixing in KAW turbulence

Kinetic Alfvén waves propagate along guide, filament fields

Along guide field

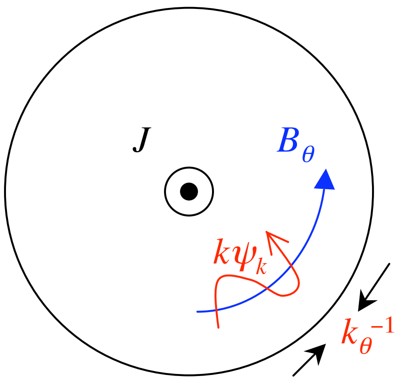
$\omega = k_z k$ ←wavenumber perpendicular to guide field

↑ wavenumber along guide field

Fluctuating field, density equipartition: $-k\psi_k = ib_k = n_k$

Along field of a current filament (isolate by setting $k_z = 0$)

$$\omega = \frac{B_\theta(k_f)}{B_0} k_\theta k = \frac{B_\theta(k_f)}{B_0} k_\theta (k_\theta^2 + k_r^2)$$



Dispersion depends only on wavenumbers ⊥ to guide field

To isolate refraction of KAW by filament field, set $k_z = 0$

Refractive suppression of mixing in KAW turbulence

Filament, turbulence evolve on two time scales

$$\psi = \psi_0(r,\tau) + \tilde{\psi}(r,\vartheta,t)$$
$$n = n_0(r,\tau) + \tilde{n}(r,\vartheta,t)$$

← KAW turbulence: evolves on rapid time t, has variation in θ

↑Coherent structure: evolves on slow time τ, uniform in θ
Assume there is density component - see if it survives mixing

Filament localized $J_0(r) = J_0(r=0)\left(1 - \frac{r^2}{a^2}\right)$; $\partial\psi_0/\partial r = B_\theta(r)$; $\partial^2\psi_0/\partial r^2 = J_0(r)$

- Structure is nonuniform, stationary refracting medium for KAW turbulence
- Turbulence mixes structure by turbulent stresses (averaged over t)
- Seek conditions under which mixing of structure becomes very slow
- Expect slow mixing when refraction of KAW turbulence by structure becomes very strong $\Rightarrow$ shear in B_θ very strong
- Localization of $J_0 \Rightarrow B_\theta$ shear zero in filament core, strong in filament edge

Mixing suppression in KAW turbulence similar to 2D NS (but different fields are involved and meaning changes)

No flow, but localized *J* of coherent structure creates inhomogeneous *B* (V_A) that refracts turbulence away from structure

	KAW	2D Navier Stokes
Localized coherent structure (origin at center of structure)	$J_z(r) = \mu_0(\nabla \times \mathbf{B})_z$ (current)	$\omega_z(r) = (\nabla \times \mathbf{V})_z$ (vorticity)
Inhomogeneous azimuthal "flow" of structure	$B_\vartheta(r) \rightarrow V_A(r)$	$V_\vartheta(r)$
Turbulence	Kinetic Alfven waves $\tilde{b}_r, \tilde{b}_\vartheta, \tilde{n}$	Turbulent eddies $\tilde{v}_r, \tilde{v}_\vartheta$
Turbulence source	$n_0(r), J_z(r)$ (structure)	$\omega_z(r)$ (structure)
Agent that prevents mixing	Inhomogeneity of B_θ refracts KAW turbulence	Shear flow V_θ of vortex shears eddies

Slow time evolution: stresses of rapidly evolving KAW turbulence mix structure

$$\frac{\partial n_0}{\partial \tau} + \frac{1}{2\pi i}\int_{-i\infty+\hat{\gamma}}^{i\infty+\hat{\gamma}} d\gamma' \sum_{m'} \left\langle \left[\tilde{b}_r(-m',-\gamma')\frac{\partial}{\partial r} + \tilde{b}_\theta(-m',-\gamma')\frac{im'}{r} \right] \nabla^2 \tilde{\psi}(m',\gamma',r) \right\rangle = \upsilon \nabla^2 n_0$$

$$\frac{\partial \psi_0}{\partial \tau} - \frac{1}{2\pi i}\int_{-i\infty+\hat{\gamma}}^{i\infty+\hat{\gamma}} d\gamma' \sum_{m'} \left\langle \left[\tilde{b}_r(-m',-\gamma')\frac{\partial}{\partial r} + \tilde{b}_r(-m',-\gamma')\frac{im'}{r} \right] \tilde{n}(m',\gamma') \right\rangle = \eta \nabla^2 \psi_0$$

↑Turbulent stresses

Turbulence: refracted by magnetic shear of filament

$$\gamma \tilde{\psi}_{\gamma,m} + \mathbf{B}(r)\cdot\nabla\, \tilde{n}_{\gamma,m} + \tilde{\mathbf{b}}\cdot\nabla\tilde{n} + \tilde{\mathbf{b}}\cdot\nabla n_0(r) = 0$$

$$\gamma \tilde{n}_{\gamma,m} + \mathbf{B}(r)\cdot\nabla\, \tilde{j}_{\gamma,m} + \tilde{\mathbf{b}}\cdot\nabla\tilde{j} + \tilde{\mathbf{b}}\cdot\nabla j_0(r) = 0$$

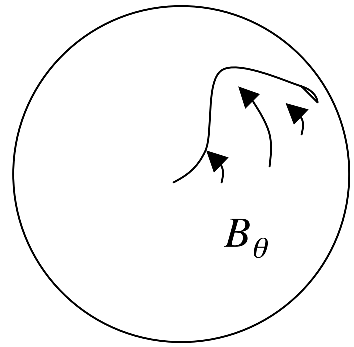
↑Refraction: KAW propagation in sheared structure field

Refractive suppression of mixing in KAW turbulence

Large magnetic shear ⇒ strong refraction of turbulence away from filament core

Refraction in cylindrical field governed by shear:

$$j' = \frac{d}{dr}\left[\frac{B_\vartheta(r)}{r}\right]$$



j' measures distortion of KAW phase fronts from linear rays

For $B_\theta \sim r$ rays are linear

Expand B_θ/r in Taylor series about edge location r_0

When j' large, system remains turbulent only if nonlinearity balances shearing

⇒ classic asymptotic boundary layer of small width Δr when j' large:

$$m\Delta r \frac{\partial}{\partial r}\left(\frac{B_\vartheta}{r}\right)\Bigg|_{r_0} \tilde{n} \approx \tilde{b}_r \frac{1}{\Delta r}\tilde{n} \quad \Longrightarrow \quad \Delta r^2 = \frac{\tilde{b}_r}{mj'}$$

$\Delta r << a$ when j' large

$\Rightarrow B_\theta >> \tilde{b}_r$

Refractive suppression of mixing in KAW turbulence

Asymptotic boundary layer analysis yields spatial, temporal scales of turbulence in boundary layer

Boundary layer must properly account for KAW dynamics

Closure theory keeps track of KAW dynamics

Closure theory is Gaussian - apply because Gaussian-breaking shearing field is stationary on fast scale - makes turbulence inhomogeneous

Non Gaussian features develop on slow scale

Closure:

- Complicated: 6 separate diffusivities of differing orders
- Enter in same order when paired with corresponding derivatives
- System is 8th order PDE in r, not amenable to WKB
- Treat with dimensional analysis $\Rightarrow$ find spatial, temporal scales in terms of parameters; functional forms of variation not found

Dimensional analysis $\Rightarrow$ $\partial/\partial r \to 1/\Delta r$, PDE $\to$ algebraic equation

Refractive suppression of mixing in KAW turbulence

Dimensional solutions of closure equations: refraction limits turbulence to filament edge

Apply boundary layer asymptotic analysis:

j' formally ordered large

Δr formally order small

Unique functional relationship yields consistent balance

$$\frac{1}{(\Delta r)^2} = \frac{(imj'\hat{d}_2 - \gamma \hat{d}_3)}{2\hat{d}_1^2} + \frac{1}{2\hat{d}_1^2}\left[(imj'\hat{d}_2 - \gamma \hat{d}_3)^2 - 4\hat{d}_1^2(m^2 j'^2 + \gamma^2)\right]^{1/2}$$

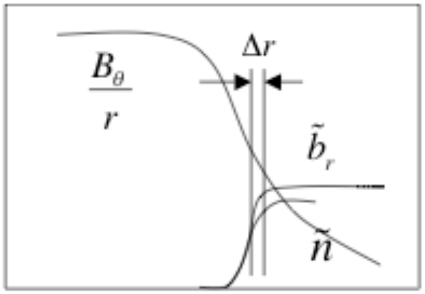
γ: turbulence decay rate in boundary layer - order j'

$\hat{d}_n$: Closure diffusivities - all of same order with $\hat{d}_n \sim \dfrac{\tilde{\psi}}{a}$

Layer width narrow when shear large:

$$\Delta r \sim \sqrt{\frac{\hat{d}_n}{mj'}}$$

Layer at edge in strong shear region



Refractive suppression of mixing in KAW turbulence

Dimensional solutions of closure equations: refraction is strong when filament field is large compared with surrounding values

2. Condition for strong refraction

$$\frac{\Delta r}{a} \sim \sqrt{\frac{d|_{r>a}}{a^2 m j'}} \sim \sqrt{\frac{\tilde{\psi}|_{r>a}}{a^3 m j'}} << 1$$

Magnetic analog of BDT flow shear suppression criterion

since $j' = \frac{d}{dr}\left(\frac{B_\theta}{r}\right) \approx \frac{B_\theta}{a^2}$ and $\frac{\tilde{\psi}|_{r>a}}{a} \approx \tilde{b}_\theta|_{rms}$

$$\frac{\Delta r}{a} \sim \sqrt{\frac{\tilde{b}_\theta|_{rms}}{B_\theta}}$$

⇒ Refraction strong, refractive boundary layer small when filament field is significantly larger than rms turbulent field

Dimensional solutions of closure equations: refraction enhances turbulent decay in filament

3. Decay rate of turbulence in filament

   Balance of refraction rate, turbulent diffusion rate ⇒ fast decay

$$\gamma \sim imj'$$

Compared to $\tau_{turb} \sim a^2 \Big/ \tilde{b}_\theta\Big|_{rms}$ (turbulent decay time outside filament):

$$\gamma\tau_{turb} \sim \frac{imj'}{\tilde{b}_\theta\Big|_{rms} / a^2} \sim \frac{B_\theta}{\tilde{b}_\theta\Big|_{rms}} >> 1$$

⇒ Turbulence in filament decays rapidly relative to turbulence outside

Refractive suppression of mixing in KAW turbulence

Filament decay time from Reynolds stress-like turbulent correlations is very small

Evaluate stresses like asymptotic dimensional analysis:

$$\int_{-i\infty+\hat{\gamma}}^{i\infty+\hat{\gamma}} d\gamma' \sum_{m'} \left\langle \left[\tilde{b}_r(-m',-\gamma') \frac{\partial}{\partial r} + \tilde{b}_\theta(-m',-\gamma') \frac{im'}{r} \right] \nabla^2 \tilde{\psi}(m',\gamma',r) \right\rangle$$

$$\Rightarrow \quad \tau_n \gamma = \tau_{\nabla^2\psi} \gamma = \left(\frac{a}{\Delta r} \right)^3 \sim \left(\frac{B_\theta}{\tilde{b}_\theta\big|_{rms}} \right)^{3/2}$$

Boundary layer mixing time slow compared to turbulent decay time

$$\tau_n / \tau_{turb} = \tau_{\nabla^2\psi} / \tau_{turb} = \frac{a}{\Delta r} \sim \left(\frac{B_\theta}{\tilde{b}_\theta\big|_{rms}} \right)^{1/2}$$

Mixing time slow compared to external turbulence times

By time mixes across a few layer widths, turbulence has decayed away

⇒ Filament lifetimes arbitrarily large

Refractive suppression implies characteristic profile for Gaussian curvature (for comparison with simulations)

Gaussian curvature is topological construct

$$C_T = \left[\frac{\partial A_x}{\partial x} - \frac{\partial A_y}{\partial y}\right]^2 + \left[\frac{\partial A_y}{\partial x} + \frac{\partial Ax}{\partial y}\right]^2 - \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]^2$$

For total magnetic field (filament + turbulence) in KAW:

$$C_T = \left[r\frac{d}{dr}\left(\frac{\tilde{b}_r}{r}\right) - \frac{1}{r}\frac{\partial}{\partial\theta}\tilde{b}_\theta\right]^2 + \left[r\frac{d}{dr}\left(\frac{B_\theta + \tilde{b}_\theta}{r}\right) - \frac{1}{r}\frac{\partial}{\partial\theta}\tilde{b}_r\right]^2 - \left[J_0 + \tilde{j}\right]^2$$

Refractive suppression ⇒

Filament core: J_0 is largest field; $d/dr(B_\theta)\rightarrow 0$ ⇒ C_T large and negative

Filament edge: $d/dr(B_\theta)$ is largest field; $J_0 \rightarrow 0$ ⇒ C_T large and positive

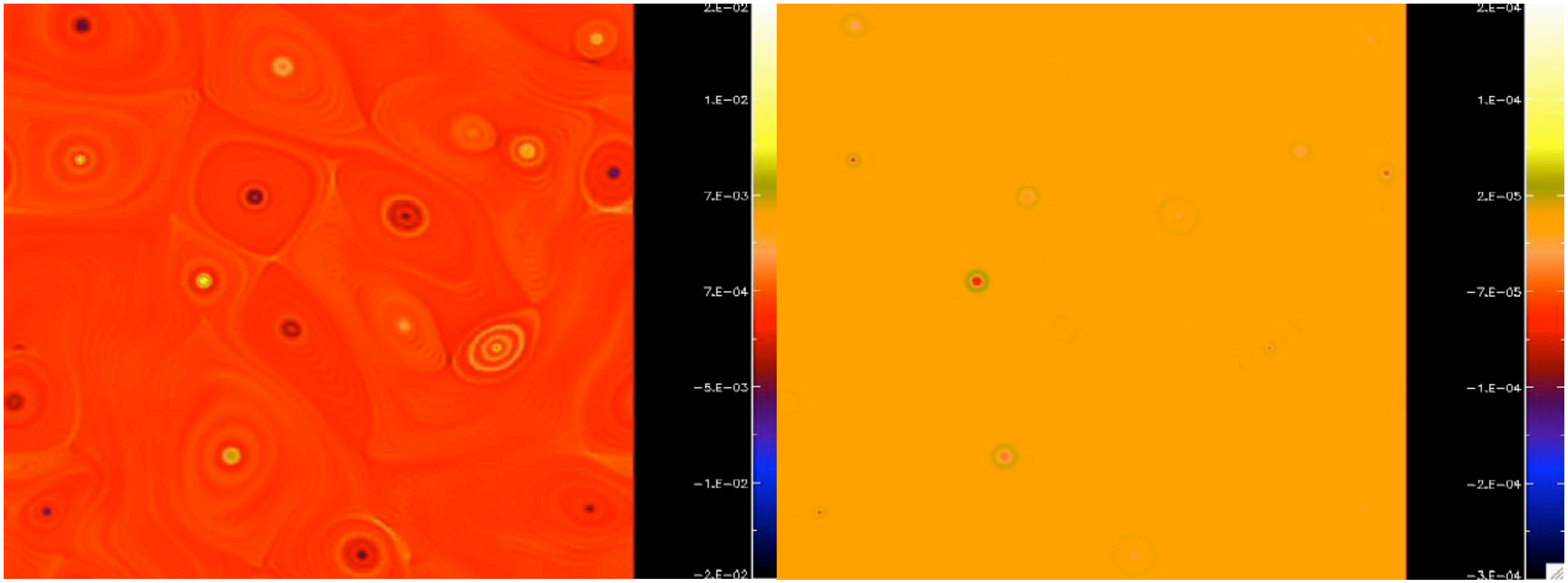
Outside filament: J_0 and B_θ small; stresses, current balance

⇒ C_T near zero everywhere

This profile is observed in simulations

Simulation

Gaussian curvature is zero except at location of filaments



Gaussian curvature is negative in filament core, positive in edge

True for filaments of either sign

Strongly confirms that intermittency due to refraction of KAW turbulence in large shear of filament

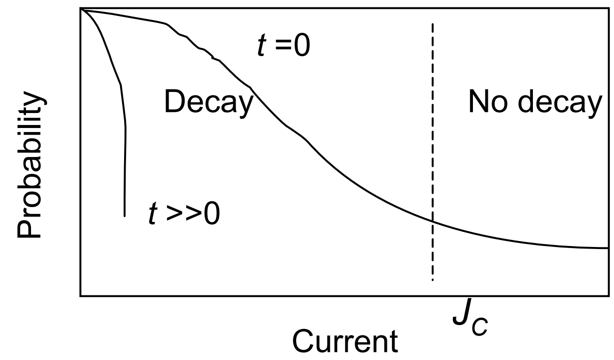
Probability distribution functions become non Gaussian as turbulence decays

Current fluctuations above a critical value of current do not decay, everything else does

$J < J_C$: PDF collapses onto $\delta(J)$

$J > J_C$: No evolution

$\Rightarrow$ greatly enhanced tail



Kurtosis (related to 2nd order moment)

Gaussian: $\kappa = 3$

Current PDF: $$\kappa = \frac{3}{2}\left(\frac{l}{a}\right)^2\left[1 + \mathrm{O}\left(\frac{j_{rms}(t=0)}{J_C}\right)\right] >> 3$$

l is mean filament separation

Modeling probability distribution functions

Density is also non Gaussian, but kurtosis need not be large

Current is localized

Magnetic field extends beyond filament with 1/*r* falloff

In KAW, density and magnetic field in equipartition

$\Rightarrow$ density extends beyond filament with 1/*r* falloff

With $n \propto 1/r$ and $P(n)dn = 2\pi r dr$

the PDF, after several decay times, is $P(n) \cong C_n \frac{1}{n^3}$

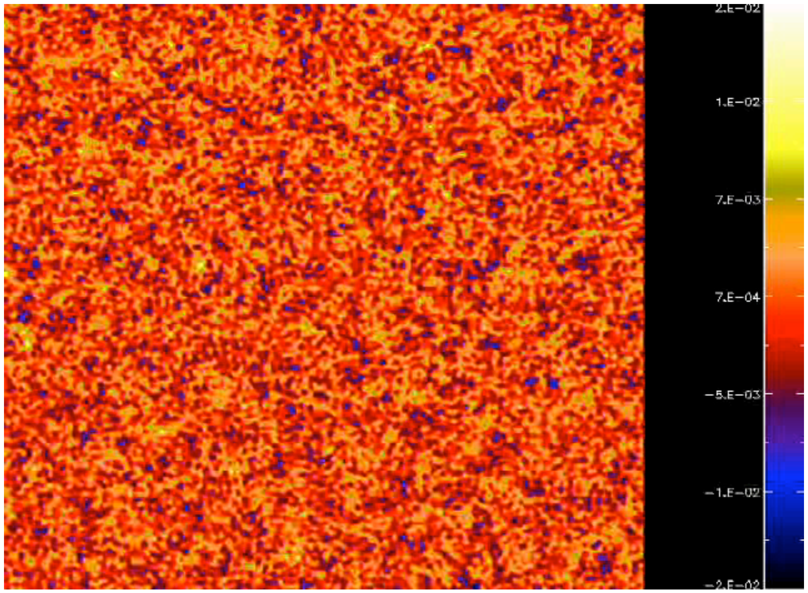
Pulsar *rf* waves are scattered by density gradient with $\nabla n \propto 1/n^2$

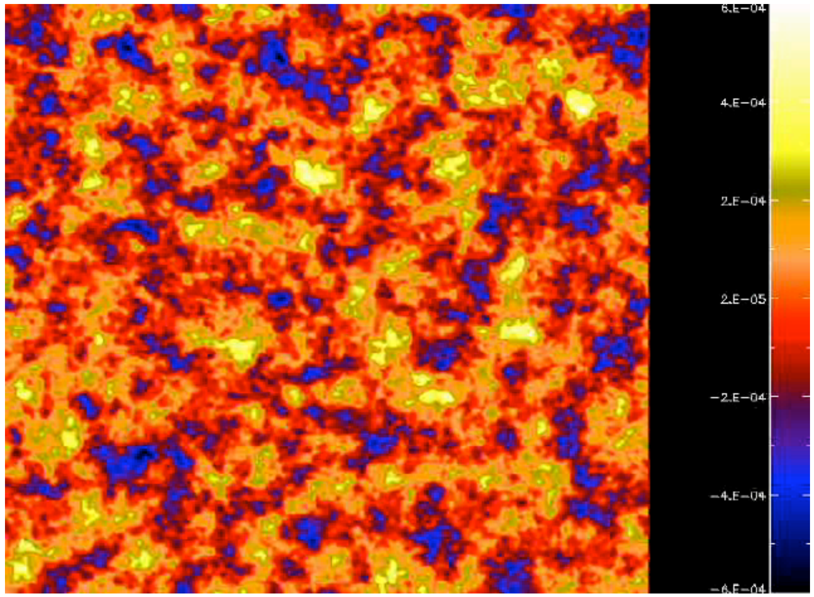
$\Rightarrow \quad P(\nabla n) = C_{\nabla n} \frac{1}{n^2}$ Levy distribution

Consistent with inferences from pulsar width scaling

Simulation

With η =0 current evolves from initial Gaussian state to highly intermittent state





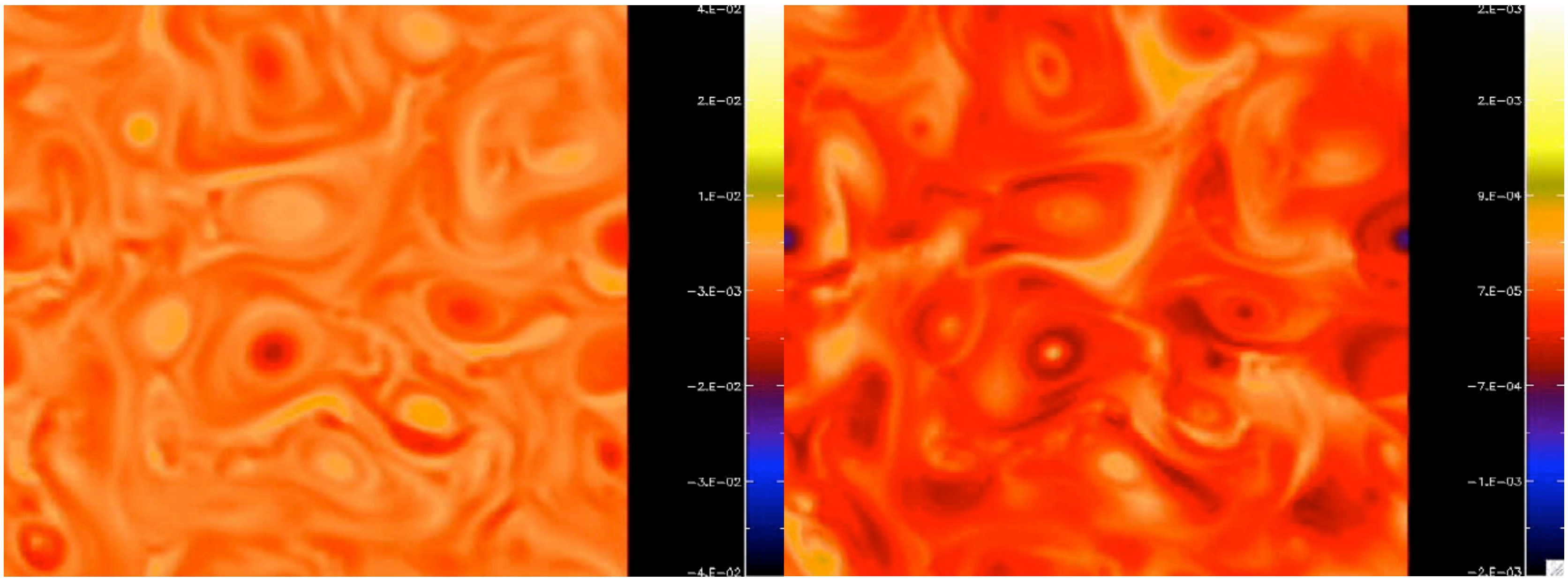
Filaments are circular

Kurtosis reaches ~ 10^2

Density kurtosis remains close to 3

Simulation

Density filaments develop in region of current filaments when $\upsilon = 0$



1 to 1 mapping from current to density filaments

Density structures are less pronounced - consistent with non localization

Conclusions

Shear-flow induced transport barriers (H mode) have magnetic analog

Strong magnetic field inhomogeneity refracts turbulence if it is amenable to refraction. i.e., if turbulence is wavelike and sensitive to B field

Kinetic Alfvén wave turbulence is example

- Refraction explains observation of coherent current filaments in simulations of KAW turbulence

Theoretical calculation shows refractive effect

- Predicts observed spatial behavior of Gaussian curvature
- Predicts Levy distribution for density gradient
- Consistent with observations

Other kinds of strong inhomogeneities that don't drive instability may be capable of forming transport barriers