Two fluid global simulations of ITB and ETB formation and relaxation phenomena in Tokamaks

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Outline

- Introduction
- A global cylindrical electromagnetic (EM) two fluids model of plasma transport
- Numerical results of nonlinear EM two fluids simulation of a JET plasma using the CUTIE code
- CUTIE results on L-H transition and ELMs phenomena in COMPASS-D
- Conclusion and future work

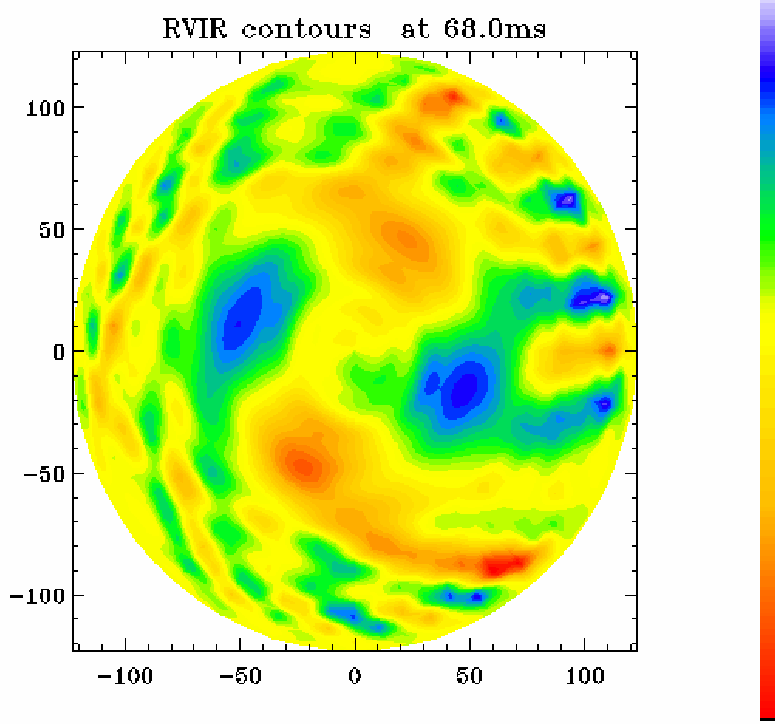
Introduction

- The transport of energy and particles in tokamaks is controlled by the non linear interaction of fluctuating plasma fields at scales of frequency ranging from the Alfven frequency to the drift frequency and space scales ranging from the plasma minor radius (MHD) to the electron Larmor radius.

- Due to the complexity of the task, the dynamics on these disparate scales is often decoupled: the microscale (ion-electron Larmor radius) is described by gyrokinetic equations while the macroscale is described by MHD equations.

- Transition phenomena such as the formation of ITBs or ETBs, L-H transition and ELMs are linked to energy flowing from small scale structures into large scale flows and vortices. They can be described by taking into account the non linear coupling between small scale turbulence, MHD and every intermediate scale lengths.

A global cylindrical electromagnetic (EM) two fluids model of plasma transport



Two-Fluid Evolution Equations

$$\frac{\partial n}{\partial t} + \nabla.(n\mathbf{v}) = S_p$$

$$m_i n \frac{d\mathbf{v}}{dt} = -\nabla(p_i + p_e) + \mathbf{j}\times\mathbf{B} + F_{eff}$$

$$\frac{3}{2} n \left[\frac{\partial T_{i,e}}{\partial t} + \mathbf{v}_{i,e}.\nabla T_{i,e}\right] + nT_{i,e}\nabla.\mathbf{v}_{i,e} = -\nabla.q_{i,e} + P_{i,e}$$

$$\mathbf{E} + \mathbf{v}_e \times \mathbf{B} = -\nabla p_e / en + \mathbf{R}_e$$

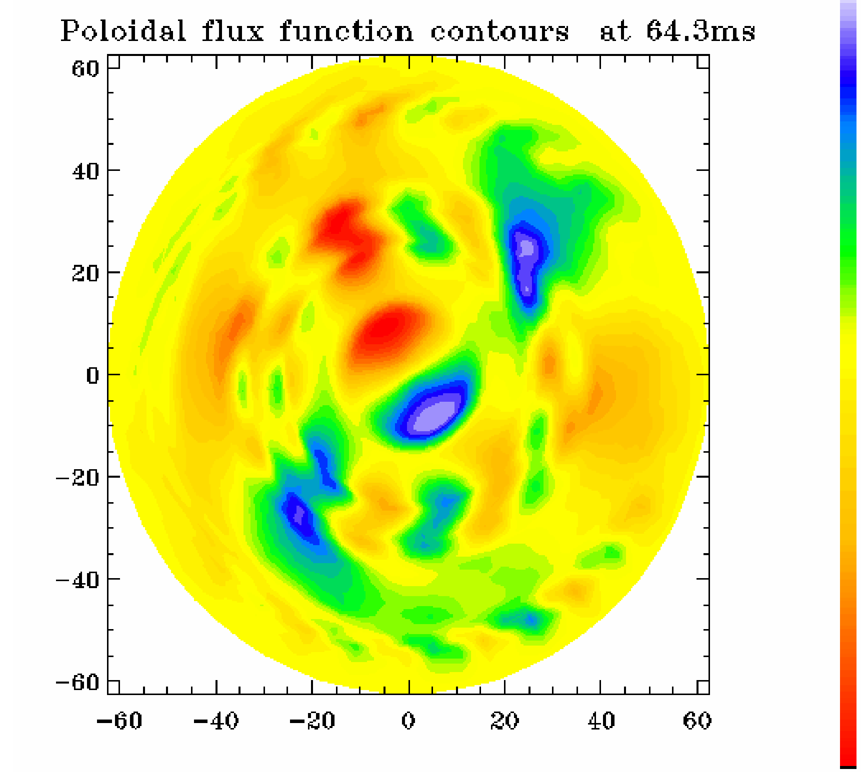
A global cylindrical electromagnetic (EM) two fluids model of plasma transport

Generic Transport Equation & Flux

$$\frac{\partial F_0}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_F) + S_F$$

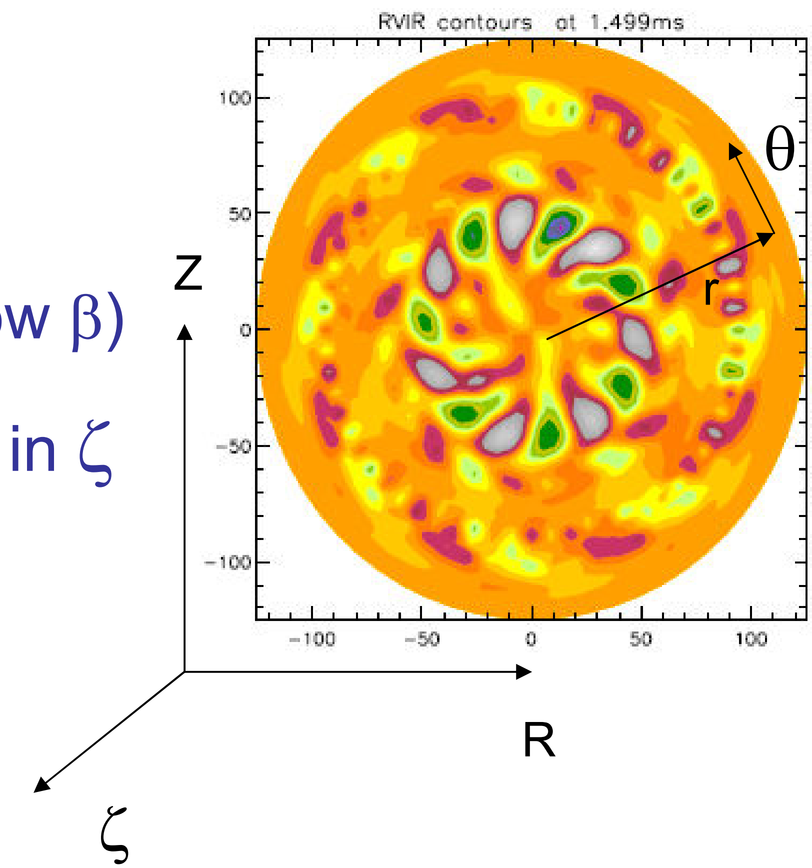
$$\Gamma_F(r,t) = \Gamma_F^{\rm coll} + \Gamma_F^{\rm turb}$$

$$\Gamma_F^{\rm turb} = - < \tilde{F} \times \frac{c}{B}(\frac{1}{r}\frac{\partial\tilde{\Phi}}{\partial\theta}) >_{\theta,\zeta}$$



A global cylindrical electromagnetic (EM) two fluids model of plasma transport

- Time independent cylindrical magnetic surfaces: concentric circles in the poloidal plane (low β)
- Periodic boundary conditions in ζ
- Toroidal curvature effects included in the fluctuation equations
- q profile evolved but Ip and B toroidal fixed



A global cylindrical electromagnetic (EM) two fluids model of plasma transport

The plasma fields are expanded in terms of the m, n components

$$n^* = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \hat{n}_{m,n}(r,t)\exp(im\theta + in\zeta)$$

The evolution of the flux surface averages (e.g. n_{00} (r, t) component) is calculated through the mean equations

$$\frac{\partial n_{e0}}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r\left(\left(D_{nc} + D_{turb}\right)\frac{\partial n_{e0}}{\partial r} + \left\langle \tilde{v}_r^E \tilde{n} \right\rangle + n_{e0} V_{ware} \right) + S$$

$$\frac{3}{2}\frac{\partial n_0 T_{e,i0}}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r\left(\left(\chi_{nc}^{e,i} + \chi_{turb}^{e,i}\right)\frac{\partial n_0 T_{e,i0}}{\partial r} + \frac{3}{2}\left\langle \tilde{v}_r^E \tilde{p}_{e,i} \right\rangle + \frac{3}{2} n_0 T_{e,i0} V_{ware} \right) + H_{e,i}$$

$$\frac{\partial v_{\theta 0}}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r\left\langle \tilde{v}_r^E \tilde{v}_\theta \right\rangle + \frac{\left\langle \tilde{j}_\zeta \tilde{B}_r \right\rangle}{m_i n_{e0}} - \mu_{nc}\left(v_{\theta 0} - v_{nc}\right)$$

$$\frac{\partial v_{\zeta 0}}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r\left(\left\langle \tilde{v}_r^E \tilde{v}_\zeta \right\rangle + \chi_{turb}^{i}\frac{\partial v_{\zeta 0}}{\partial r} \right) + F_\zeta$$

$$\tilde{v}_r^E = -\frac{1}{rB_0}\frac{\partial \tilde{\phi}}{\partial \theta}$$

$$\tilde{B}_r = \frac{1}{r}\frac{\partial \tilde{\psi}}{\partial \theta}$$

A global cylindrical electromagnetic (EM) two fluids model of plasma transport

The electric and magnetic field average components are

$$\frac{\partial B_{\theta 0}}{\partial t} = \frac{\partial}{\partial r}\left(\eta_{nc}\left(j_{\zeta 0} - j_{bs}\right) - \left\langle \tilde{v}_r \tilde{B}_\theta - \tilde{v}_\theta \tilde{B}_r \right\rangle\right)$$

$$E_{r0} = -\frac{1}{e n_{e0}}\frac{\partial p_{i0}}{\partial r} + v_{\zeta 0} B_{\theta 0} - v_{\theta 0} B_{\zeta 0}$$

$$j_{\zeta 0} = \frac{1}{r \mu_0}\frac{\partial}{\partial r} r B_{\theta 0}$$

The initial profiles are evolved for a given set of sources keeping constant the value of the fields at the boundary of the radial domain (*r=a plasma minor radius*)

- Red box: Neoclassical transport coefficients including trapped particles effects
- Green box: Prescribed sources
- Blue box: Terms calculated separately in the fluctuation equations
- Yellow box: Turbulent diffusion coefficients prop. To the fluctuations

Fluctuation equations

The dynamics of the remaining m, n components of the fluctuating fields is calculated by solving numerically the fluctuation equations in Gaussian units

$$\begin{aligned}
\Theta &= \nabla.(\frac{n_0(r,t)}{n_0(0,t)}\nabla_\perp \frac{c\phi}{B_0}) \\
\frac{c\phi}{B_0} &= \bar{V}_{th}\rho_s\phi^* \\
\frac{\psi}{B_0} &= \rho_s\beta^{1/2}\psi^* \\
\Theta &= \frac{\bar{V}_{th}}{\rho_s}\Theta^* \\
n^* &= \delta n_e/N^* \\
\lambda^*_{i,e} &= \delta T_{i,e}/T^* \\
\xi^* &= n_0(r,t)\delta v_{\|}/\bar{\xi}
\end{aligned}$$

$$\begin{aligned}
\mathbf{u}_0 &= -\frac{cE_{r0}}{B}\mathbf{e}_\theta + \mathbf{b}_0 v_{||0} \\
\mathbf{u}_{e0} &= -\frac{cE_{r0}}{B}\mathbf{e}_\theta + \mathbf{b}_0(v_{||0} - j_{||0}/en_0) \\
\mathbf{v}_0 &= \mathbf{u}_0 + \frac{c}{en_0B}\frac{\partial p_{i0}}{\partial r}\mathbf{e}_\theta \\
\Theta_0 &= \frac{1}{r}\frac{\partial}{\partial r}(\frac{rn_0(r,t)}{N^*}\frac{c}{B_0}\frac{\partial \Phi_0}{\partial r}) \\
\mathbf{v}_{e0} &= -\left[\frac{cE_{r0}}{B} + \frac{c}{en_0B}\frac{\partial p_{e0}}{\partial r}\right]\mathbf{e}_\theta
\end{aligned}$$

Several advection velocities are used in the theory

Fluctuation equations

$$\frac{\partial \Theta^*}{\partial t} + \mathbf{v}_0.\nabla\Theta^* + \bar{V}_A\nabla_{||}\rho_s^2\nabla_\perp^2\psi^* = \bar{V}_A\rho_s\frac{1}{r}\frac{\partial\psi^*}{\partial\theta}\frac{4\pi\rho_s}{cB_0}j_0' + \bar{V}_{th}\rho_s\frac{1}{r}\frac{\partial(\psi^*,\rho_s^2\nabla_\perp^2\psi^*)}{\partial(r,\theta)}$$
$$+\bar{V}_{th}\rho_s\left[\frac{1}{r}\frac{\partial(\Theta^*,\phi^*)}{\partial(r,\theta)} + (\frac{N^*}{n_0})\frac{1}{r}\frac{\partial(\Theta^*,p_i^*)}{\partial(r,\theta)}\right]$$
$$-\frac{2\bar{V}_{th}\rho_s}{R_0}\left[\frac{\cos\theta}{r}\frac{\partial p^*}{\partial\theta} + \sin\theta\frac{\partial p^*}{\partial r}\right] + \rho_s^2\Theta_0'\frac{1}{r}\frac{\partial\phi^*}{\partial\theta}$$
$$+\Sigma_\Theta^*$$

Vorticity equation

- Tearing and microtearing terms
- Curvature terms
- Viscous term (small scale turbulence cut off)
- Coefficients Coupled with the mean fields

Generalized Ohm's law

$$\frac{\partial\psi^*}{\partial t} + \mathbf{v}_{e0}.\nabla\psi^* + \bar{V}_A\nabla_{||}\phi^* = \bar{V}_A(\frac{N^*T_{e0}}{n_0T^*})\nabla_{||}n^*$$
$$+\bar{V}_{th}\rho_s\left[\frac{1}{r}\frac{\partial(\psi^*,\phi^*)}{\partial(r,\theta)} - (\frac{N^*T_{e0}}{n_0T^*})\frac{1}{r}\frac{\partial(\psi^*,n^*)}{\partial(r,\theta)}\right] + \Sigma_\psi^*$$

Fluctuation equations

$$\frac{\partial n^*}{\partial t} + \mathbf{u}_{e0}.\nabla n^* + \bar{V}_A \nabla_{||}\rho_s^2\nabla_\perp^2\psi^* + \bar{V}_{th}\nabla_{||}\xi^* = \bar{V}_A\rho_s\frac{1}{r}\frac{\partial\psi^*}{\partial\theta}\frac{4\pi\rho_s}{cB_0}j_0' + \bar{V}_{th}\rho_s\frac{1}{r}\frac{\partial(\psi^*,\rho_s^2\nabla_\perp^2\psi^*)}{\partial(r,\theta)}$$
$$+\bar{V}_{th}\rho_s\frac{1}{r}\frac{\partial(n^*,\phi^*)}{\partial(r,\theta)} + \bar{V}_{th}\rho_s(\frac{n_0'}{N^*})\frac{1}{r}\frac{\partial\phi^*}{\partial\theta}$$
$$-\frac{2\bar{V}_{th}\rho_s}{R_0}\left[\frac{\cos\theta}{r}\frac{\partial p_e^*}{\partial\theta} + \sin\theta\frac{\partial p_e^*}{\partial r}\right]$$
$$+\frac{2\bar{V}_{th}\rho_s}{R_0}(\frac{n_0}{N^*})\left[\frac{\cos\theta}{r}\frac{\partial\phi^*}{\partial\theta} + \sin\theta\frac{\partial\phi^*}{\partial r}\right]$$
$$+\Sigma_n^*$$

Electron continuity equation

Electron inertia is neglected in this system

$$\frac{\partial\xi^*}{\partial t} + \mathbf{u}_0.\nabla\xi^* + \bar{V}_{th}(\frac{T_{e0}+T_{i0}}{T^*})\nabla_{||}n^* = (\frac{n_0 v_{||0}'}{N^*})\rho_s\frac{1}{r}\frac{\partial\phi^*}{\partial\theta} + \bar{V}_{th}\rho_s\frac{1}{r}\frac{\partial(\xi^*,\phi^*)}{\partial(r,\theta)}$$
$$-\bar{V}_{th}\rho_s\beta^{1/2}(\frac{p_0'}{P^*})\frac{1}{r}\frac{\partial\psi^*}{\partial\theta} - \bar{V}_{th}\rho_s\beta^{1/2}\frac{1}{r}\frac{\partial(p^*,\psi^*)}{\partial(r,\theta)}$$
$$-\bar{V}_{th}(\frac{n_0}{N^*})\nabla_{||}(\lambda_i^* + \lambda_e^*) + \Sigma_\xi^*$$

Total parallel momentum

Fluctuation equations

$$\frac{\partial n^*}{\partial t} + \mathbf{u}_{e0}.\nabla n^* + \bar{V}_A \nabla_\parallel \rho_s^2 \nabla_\perp^2 \psi^* + \bar{V}_{th} \nabla_\parallel \xi^* = \bar{V}_A \rho_s \frac{1}{r}\frac{\partial \psi^*}{\partial \theta}\frac{4\pi\rho_s}{cB_0} j_0' + \bar{V}_{th}\rho_s \frac{1}{r}\frac{\partial(\psi^*, \rho_s^2\nabla_\perp^2\psi^*)}{\partial(r,\theta)}$$

$$+ \bar{V}_{th}\rho_s \frac{1}{r}\frac{\partial(n^*,\phi^*)}{\partial(r,\theta)} + \bar{V}_{th}\rho_s\left(\frac{n_0'}{N^*}\right)\frac{1}{r}\frac{\partial \phi^*}{\partial \theta}$$

$$- \frac{2\bar{V}_{th}\rho_s}{R_0}\left[\frac{\cos\theta}{r}\frac{\partial p_e^*}{\partial \theta} + \sin\theta \frac{\partial p_e^*}{\partial r}\right]$$

$$+ \frac{2\bar{V}_{th}\rho_s}{R_0}\left(\frac{n_0}{N^*}\right)\left[\frac{\cos\theta}{r}\frac{\partial \phi^*}{\partial \theta} + \sin\theta\frac{\partial \phi^*}{\partial r}\right]$$

$$+ \Sigma_n^*$$

Electron continuity equation

Electron inertia is neglected in this system

$$\frac{\partial \xi^*}{\partial t} + \mathbf{u}_0.\nabla \xi^* + \bar{V}_{th}\left(\frac{T_{e0}+T_{i0}}{T^*}\right)\nabla_\parallel n^* = \left(\frac{n_0 v_{\parallel 0}'}{N^*}\right)\rho_s \frac{1}{r}\frac{\partial \phi^*}{\partial \theta} + \bar{V}_{th}\rho_s\frac{1}{r}\frac{\partial(\xi^*,\phi^*)}{\partial(r,\theta)}$$

$$-\bar{V}_{th}\rho_s\beta^{1/2}\left(\frac{p_0'}{P^*}\right)\frac{1}{r}\frac{\partial \psi^*}{\partial \theta} - \bar{V}_{th}\rho_s\beta^{1/2}\frac{1}{r}\frac{\partial(p^*,\psi^*)}{\partial(r,\theta)}$$

$$-\bar{V}_{th}\left(\frac{n_0}{N^*}\right)\nabla_\parallel(\lambda_i^* + \lambda_e^*) + \Sigma_\xi^*$$

Total parallel momentum

Fluctuation equations

Energy equations

$$\frac{3}{2}\left[\frac{\partial\lambda_i^*}{\partial t}+\mathbf{u}_0.\nabla\lambda_i^*\right]+\bar{V}_{th}(\frac{N^*T_{i0}}{n_0T^*})\nabla_{||}\xi^* = \frac{3}{2}\bar{V}_{th}\rho_s\left[\frac{1}{r}\frac{\partial(\lambda_i^*,\phi^*)}{\partial(r,\theta)}+(\frac{T'_{i0}}{T^*})\frac{1}{r}\frac{\partial\phi^*}{\partial\theta}\right]$$
$$+(\frac{T_{i0}}{T^*})\frac{2\bar{V}_{th}\rho_s}{R_0}\left[\frac{\cos\theta}{r}\frac{\partial p_i^*}{\partial\theta}+\sin\theta\frac{\partial p_i^*}{\partial r}\right]$$
$$+(\frac{T_{i0}}{T^*})\frac{2\bar{V}_{th}\rho_s}{R_0}\left[\frac{\cos\theta}{r}\frac{\partial\phi^*}{\partial\theta}+\sin\theta\frac{\partial\phi^*}{\partial r}\right]$$
$$+\Sigma_{\lambda_i}^* + \nabla\cdot\left(\chi_i\vec{b}\cdot\nabla\lambda_i^*\right)$$

$$\frac{3}{2}\left[\frac{\partial\lambda_e^*}{\partial t}+\mathbf{u}_{e0}.\nabla\lambda_e^*\right]+\bar{V}_{th}(\frac{N^*T_{e0}}{n_0T^*})\nabla_{||}\xi^* = \frac{3}{2}\bar{V}_{th}\rho_s\left[\frac{1}{r}\frac{\partial(\lambda_e^*,\phi^*)}{\partial(r,\theta)}+(\frac{T'_{e0}}{T^*})\frac{1}{r}\frac{\partial\phi^*}{\partial\theta}\right]$$
$$-(\frac{N^*T_{e0}}{n_0T^*})\bar{V}_A\nabla_{||}\rho_s^2\nabla_\perp^2\psi^* + \nabla\cdot\left(\chi_{\|e}\vec{b}\cdot\nabla\lambda_e^*\right)$$
$$-(\frac{T_{e0}}{T^*})\frac{2\bar{V}_{th}\rho_s}{R_0}\left[\frac{\cos\theta}{r}\frac{\partial p_e^*}{\partial\theta}+\sin\theta\frac{\partial p_e^*}{\partial r}\right]$$
$$+(\frac{T_{e0}}{T^*})\frac{2\bar{V}_{th}\rho_s}{R_0}\left[\frac{\cos\theta}{r}\frac{\partial\phi^*}{\partial\theta}+\sin\theta\frac{\partial\phi^*}{\partial r}\right]$$
$$+\Sigma_{\lambda_e}^*$$

Magnetic fluctuations are important for the electron energy fluctuation dynamic

The numerical solution

- The system is solved numerically by CUTIE on a grid of 100x 64 x 32 points (r, m, n). Boundary condition on the fluctuation fields is 0 value at the r=a.
- The linear terms are solved in Fourier space and the nonlinear terms are calculated in real space and Fourier transformed (pseudo-spectral)
- The nonlinear solution is iterative at each time step using a block tri-diagonal radial solver

JET discharge #59137

Cutie

Simulation initial central values at t=48.6 s

Bt=2.55 T

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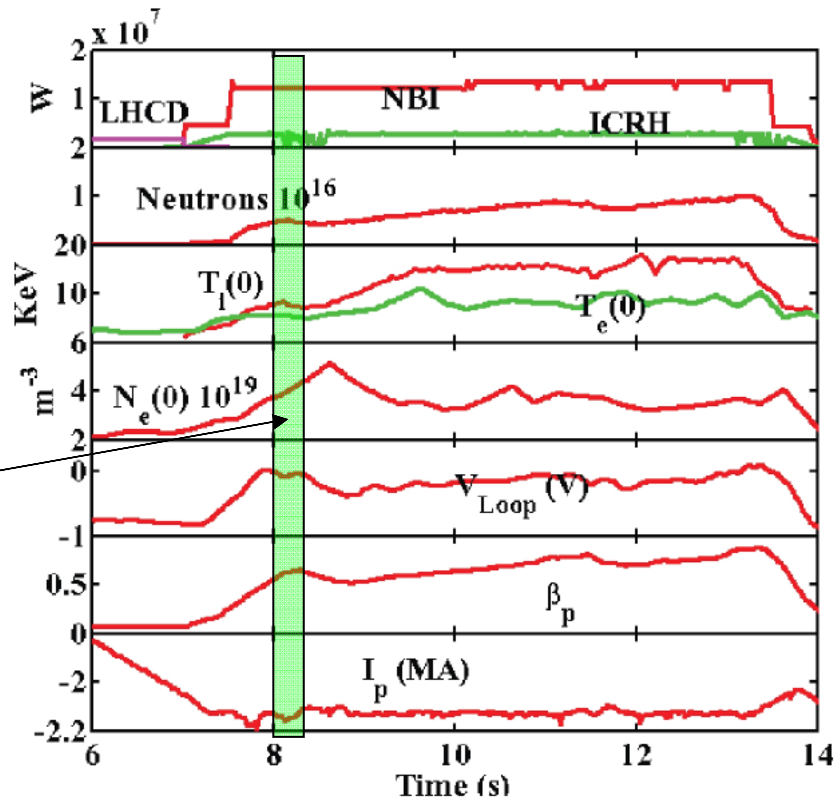
59137



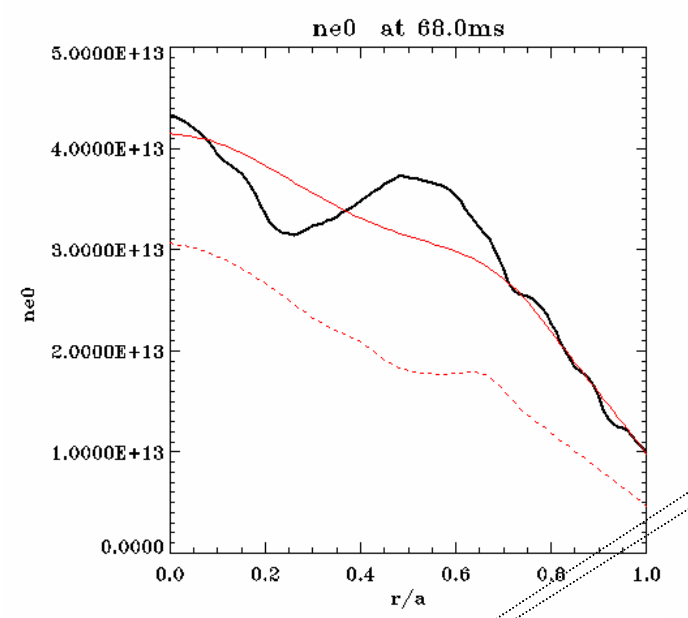
A “typical” JET Hybrid scenario discharges with a small amount of ICRH (≈3MW)

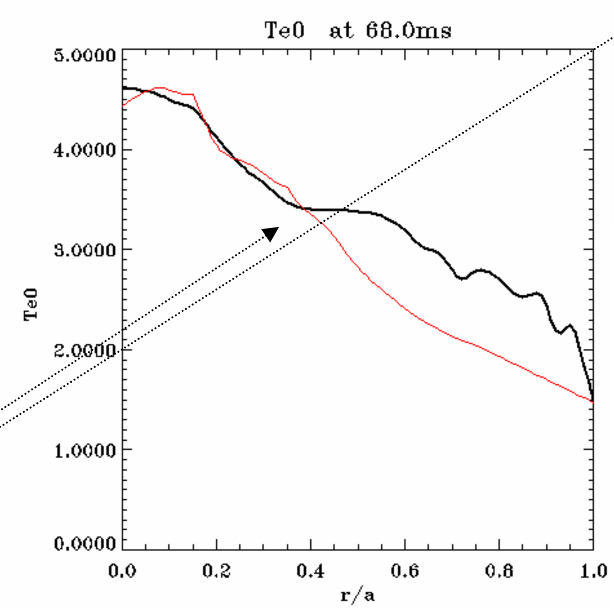
High ion and electron temperatures but low density

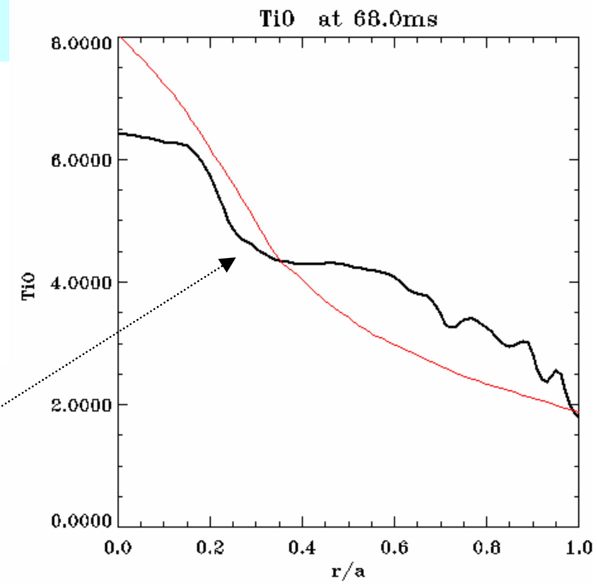
Configuration steady for almost 5 sec

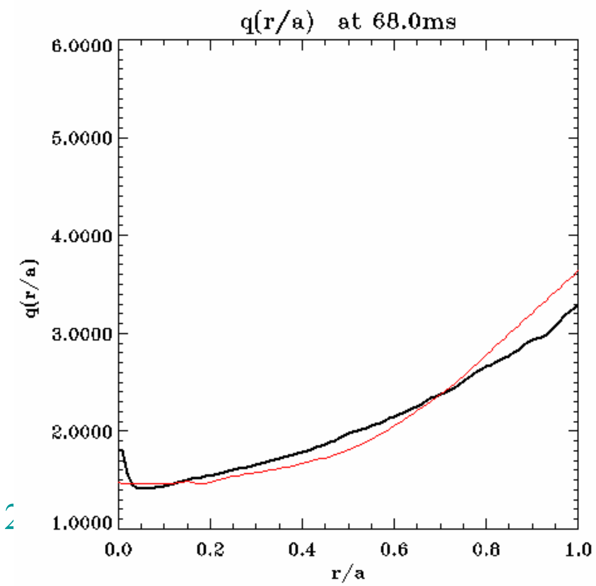
CUTIE simulation of #59137

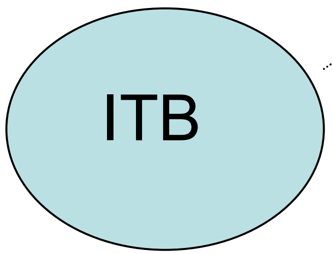
Shot #59137 @ t=48.6 s; 68 ms simulation (grid r=100, m=64, n=32); CUTIE profiles compared with JETTO transport analysis (red); **initial profiles taken parabolic**





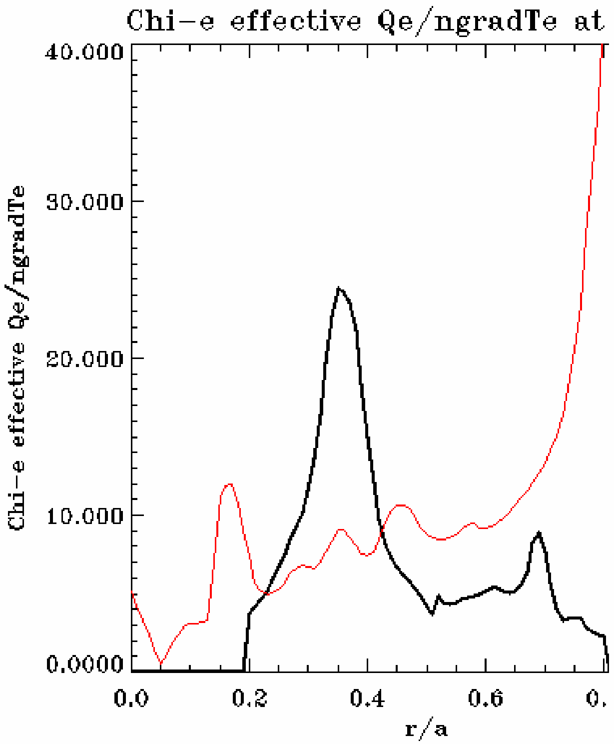


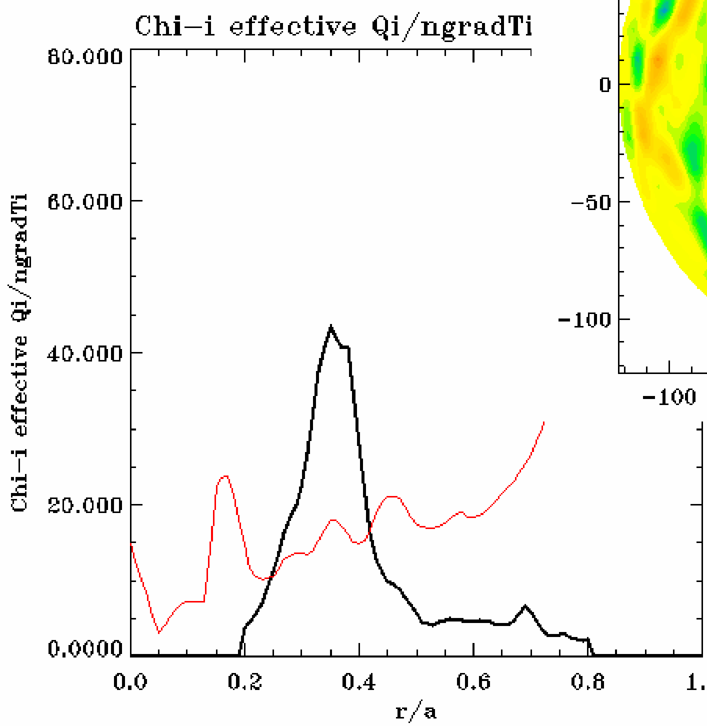


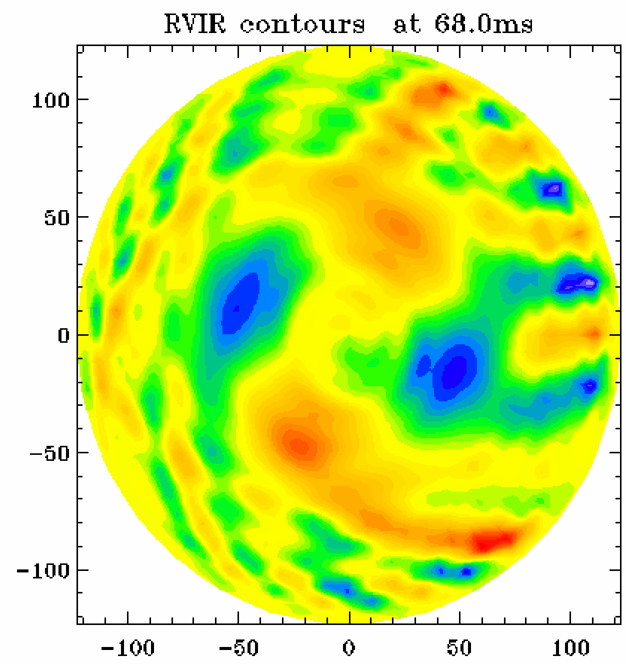


CUTIE simulation of #59137

Transport barrier : m=2 mode -> local flattening of the profiles and transfer of energy from high k modes into low k modes (MHD)







CUTIE simulation of compass

Summary of CUTIE results on COMPASS #26363

(cf. Valovič et al 26th EPS Maastricht, 1999)

- With *constant power and current* , starting with an L-mode (indicated by absence of ELM's and profiles) increasing line-average density "step-wise" leads to a **continuous transition** to an ELM-free state followed a few ms later to strongly ELMing edge.

- When the simulation is continued at high enough density, there appears to be a "re-transition" to L-mode via increased turbulence.

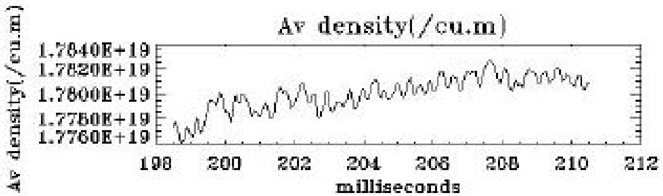
- The behaviour with a linear " density ramp" leads to a continuing ELM-free H-mode, which, at high enough density re-transitions to an L-mode. Edge ballooning modes appear to be controlled by dissipational effects (at high density and low temperature).

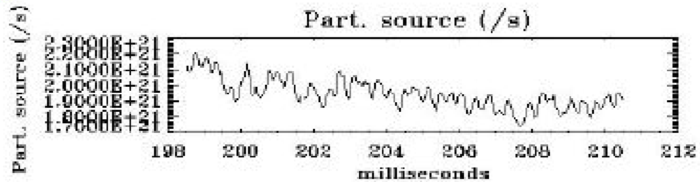
- Thus calculations suggest that the H-mode has two distinct states (ELM-free and ELMing) with the same macroscopic parameters, but determined by history.

CUTIE simulation of compass

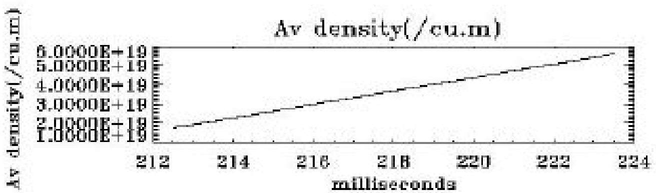
COMPASS#26363- CUTIE wave forms-I

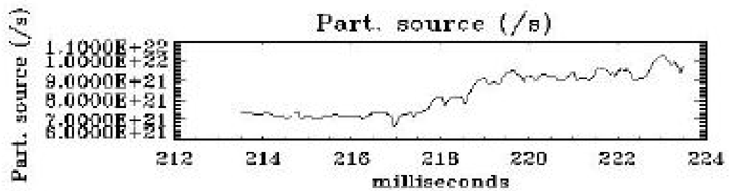
L-mode



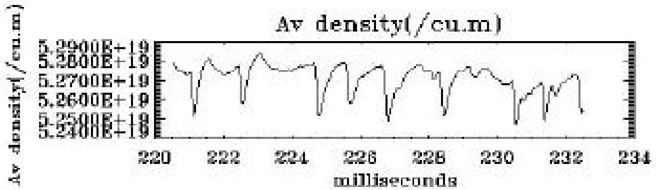


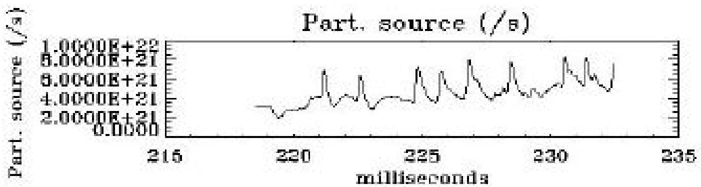
H-mode, ramped <n> "ELM-free"





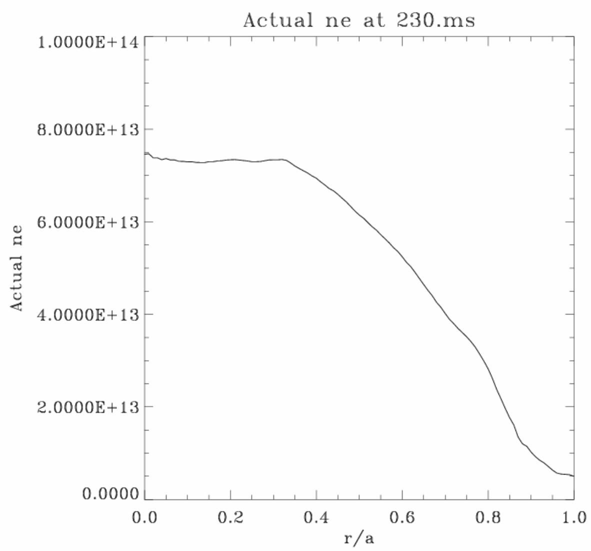
H-mode, stepped <n> "ELMs"



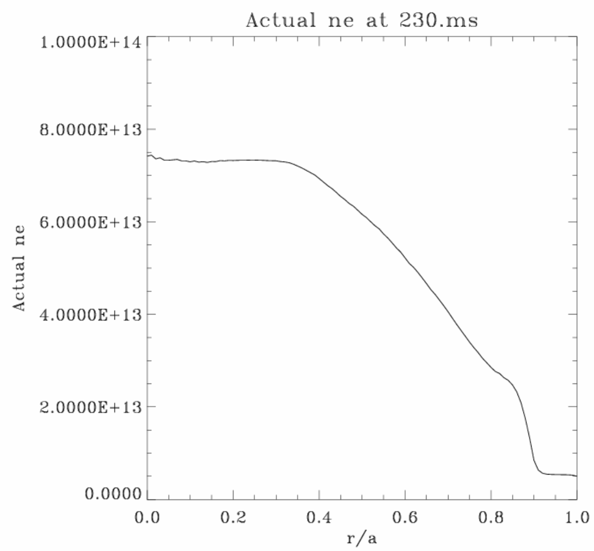


CUTIE simulation of compass

After the “ELM”



Before the “ELM”



Conclusion and future work

- Nonlinear simulation and linear microstability analysis of JET ITB in hybrid scenario seem to indicate that the barrier arise due to steepening of the gradients driven by the appearance of long wavelength nonlinear structures localised at the foot of the barrier

- Future work goes in the direction of improving the fluid simulation by "coupling" a linear gyrokinetic code (e.g. Kinezero) to CUTIE to provide a better treatment of the high k components of the spectrum of fluctuations which are not solved in CUTIE.

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