Multi-scale interactions among macro-MHD, micro-turbulence and zonal flows

A. Ishizawa and N. Nakajima
National Institute for Fusion Science
Background and motivation

- Effects of MHD instabilities and micro-turbulence on plasma confinement have been investigated separately.
- But these instabilities usually appear in the plasma at the same time.
  - Micro-turbulence is observed in Large Helical Device plasmas that usually exhibit MHD activities.
  - MHD activities are observed in reversed shear plasmas with a transport barrier related to zonal flows and micro-turbulence.
    - Takeji, et.al., Nuclear Fusion (2002)
Our goal is to understand multi-scale-nonlinear interactions among micro-instabilities, macro-scale-MHD instabilities and zonal flows.
MHD activities are observed in reversed shear plasmas with a transport barrier related to zonal flows and micro-turbulence.

We will make an initial quasi-equilibrium that corresponds to the equilibrium in the experiment. This equilibrium can be formed by a balance between micro-turbulence and zonal flow.

Takeji, et.al., Nuclear Fusion (2002)
Reduced two-fluid equations

◆ Basic assumptions
  • Flute approximation
  • Large aspect ratio
  • High-beta ordering

◆ Extends the standard four-field model by including temperature gradient effects.

◆ We can describe the nonlinear evolution of tearing modes, interchange modes, ballooning modes and ion-temperature gradient modes.

\[
\frac{k_{\parallel}}{k_{\perp}} \approx \frac{a}{R} \approx \beta \approx \varepsilon
\]
Reduced two-fluid equations

\[
\begin{align*}
\frac{d}{dt} n &= -n_{eq} \frac{\nabla}{\nabla_{\parallel}} + K(n_{eq} \Phi - \tilde{d}_i \beta p_e) \\
 \frac{d}{dt} n_{eq} \frac{\nabla}{\nabla_{\parallel}} &= -\beta \nabla_{\parallel} p \\
 \frac{d}{dt} \frac{Q}{n_{eq}} &= -\nabla_{\parallel} J - \beta K(p) + \tilde{d}_i \beta \nabla_{\perp} \cdot \left[ \nabla_{\perp} \Phi, p_i \right] \\
 \frac{\partial}{\partial t} A &= -\nabla_{\parallel} \Phi + \tilde{d}_i \beta \nabla_{\parallel} p_e + \eta_L v_{e/\parallel} + \eta J \\
 \frac{d}{dt} T_i &= -(\Gamma - 1) T_{eq} \nabla_{\parallel} v_{e/\parallel} - (\Gamma - 1) \kappa_L \tilde{T}_i \\
 &\quad - T_{eq} K((\Gamma - 1)(\tilde{\Phi} + \tilde{d}_i \beta \tilde{T}_i + \tilde{d}_i \beta T_{eq} / n_{eq} \tilde{n})) + \Gamma \tilde{d}_i \beta \tilde{T}_i
\end{align*}
\]

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \left[ \Phi, f \right] = \frac{\partial f}{\partial t} + v_E \cdot \nabla, \quad \Phi: electric potential, \quad v_E = b \times \nabla \Phi
\]

\[
K(f) = 2\varepsilon[r \cos \theta, f], \quad \nabla_{\parallel} f = \varepsilon \partial_{\parallel} f = -[A, f]
\]

\[
f = \sum_{m,n} f_{m,n}(r, t) \exp(im \theta - in \zeta)
\]

\[
Q = \nabla^2 \Phi
\]

\[
J = \nabla^2 A = -J_{\parallel}
\]

\[
T = T_i + T_e
\]

\[
T_{eq} = T_{eq} + \tilde{T}_i
\]

\[
T_e = \tau T_{eq}
\]

\[
n = n_{eq} + \tilde{n}
\]

\[
p = p_i + p_e = Tn = (1 + \tau)n_{eq} T_{eq} + (1 + \tau)T_{eq} \tilde{n} + n_{eq} \tilde{\tilde{T}}_i
\]

\[
p_i = T_i n = n_{eq} T_{eq} + T_{eq} \tilde{n} + n_{eq} \tilde{T}_i
\]

\[
p_e = T_e n = \tau T_{eq} n_{eq} + \tau T_{eq} \tilde{n}
\]

\[
v_{e/\parallel} = \tilde{v}_{\parallel} + \tilde{d}_i \tilde{J} / n_{eq}
\]

\[
\tilde{d}_i = \tilde{\rho}_i / \sqrt{\beta}
\]

\[
\tilde{\rho}_i = \rho_i / a
\]
Initial equilibrium and linear analysis

\[ q, T_{ea}, n_{ea}, \eta_i / 10 \]

\[ \beta = 0.01 \]
\[ \tilde{\rho}_i = 1/80 \]
\[ \nu = 2 \times 10^{-12} \text{m}^4 \]
\[ S = 1.6 \times 10^6 \]
\[ N_m \times N_n \times N_r \]
\[ = 256 \times 128 \times 256 \]
Overview of multi-scale-nonlinear interaction

Electric potential

1st stage
- Balance
- Micro-turbulence
- Zonal flow

2nd stage
- Energy transfer
- Stabilize
- Nonlinear trigger of Macro-MHD

3rd stage
- Alter the balance
- Micro-turbulence
- Zonal flow
Excitation of macro-MHD

Magnetic energy

Equilibrium formed by a balance between micro-turbulence and zonal flow

Nonlinear trigger of macro-MHD

Electric potential

Takeji, et.al., Nuclear Fusion (2002)
Mechanism of the excitation

Feedback loop of nonlinear growth

- Nonlinear growth of two magnetic island chains
- Reduce stabilizing effect of shearing
- Mode locking through Maxwell stress
- Suppression of zonal flow

Helical flux m/n=2

Time evolution of zonal flow

$\log(E_M(n))$

$t v_{ti} / a$

$q$ profile

$Zonal flow$

Helical flux m/n=2

$q$ profile
Macro-MHD changes the balance between turbulence and zonal flow.

Time evolution of zonal flow

Helical flux m/n=2

q profile
Summary

- We find that macro-scale MHD is nonlinearly triggered after a quasi-equilibrium is formed by a balance between micro-turbulence and zonal flow.
- This appearance of macro-MHD can explain the growth of macro-MHD fluctuation observed in tokamak experiment[1].
- This MHD activity alters the balance and spreads the turbulence over the plasma.

Summary of Multi-scale-nonlinear interaction

- Micro-turbulence, zonal flow, and macro-MHD directly interact each other.
  - Nonlinear trigger of macro-MHD activity
  - Macro-scale activity cause fatal effect on a balance between micro-turbulence and zonal flow.

- Future work of multi-scale interaction
  - Effects of the altered balance on transport
  - Energy cascade of the turbulence

[Diagram showing interactions between micro-turbulence, zonal flow, and macro-MHD]
App. 1: Electric potential, helical flux, q-profile

Electric potential

Helical flux m/n=2

q profile

Poincare plot of magnetic field lines
Appendix 2:
Evolution of n=1 double tearing mode
Appendix 3: Macro-MHD does not appear when the MHD is stable against the initial equilibrium.
Appendix 4: Time evolution of energy

Magnetic energy

Kinetic energy