Nonlinear excitation and damping of Zonal Flows using a renormalized polarization response

F.L. Hinton and P.H. Diamond

University of California at San Diego

TTF2007

Motivation

- Zonal Flows suppress ITG turbulence
- They are not damped by collisionless linear mechanisms (Landau damping)
- The mechanism for the nonlinear damping of Zonal Flows is therefore an important subject for research
- Other suggested mechanisms, e.g. the tertiary instability, do not seem to work
- We suggest another approach to understanding the damping mechanism: the renormalized polarization response

Outline

- Review of the Zonal Flow polarization calculation
- Consider a simpler example: dressed test-particles in gyrokinetic plasmas
- Renormalized dielectric response from coherent mode-coupling
- Potential fluctuation spectrum from renormalized dielectric response to particle discreteness (noise)
- Conclusions

Zonal Flows

Drift-kinetic equation for the nonadiabatic part of the ion distribution function:

$$\frac{\partial g_{\vec{q}}}{\partial t} + v_{\|}\hat{b}\cdot\nabla g_{\vec{q}} + i\vec{q}\cdot\vec{v}_d g_{\vec{q}} = \frac{e}{T_i}F_i\frac{\partial \phi_{\vec{q}}}{\partial t} - \frac{1}{B}[\phi, g]_{\vec{q}} \tag{1}$$

where $\vec{q} = (0, 0, q)$, and where the $\vec{E}\times\vec{B}$ nonlinearity has been written as

$$[\phi, g] \equiv \hat{b}\times\nabla\phi\cdot\nabla g \tag{2}$$

Assume ion bounce time is shortest time scale, so

$$g_{\vec{q}} = \exp(-iQ)h_{\vec{q}} \tag{3}$$

where $Q = qv_{\|}/\Omega_p$ and $\hat{b}\cdot\nabla h_{\vec{q}} = 0$. After bounce averaging,

$$\frac{\partial h_{\vec{q}}}{\partial t} = \frac{e}{T_i}F_i\overline{(e^{iQ})}\frac{\partial \phi_{\vec{q}}}{\partial t} - \overline{\left(e^{iQ}\frac{1}{B}[\phi, g]_{\vec{q}}\right)} \tag{4}$$

UCSD

A simpler problem: dressed test-particles in gyrokinetic plasmas

Ion drift-kinetic equation, unsheared slab model:

$$\left(\frac{\partial}{\partial t} + v_{\parallel}\frac{\partial}{\partial z}\right) f - \frac{e}{m_i}\left(\frac{\partial \phi}{\partial z}\frac{\partial F}{\partial v_{\parallel}} + \frac{1}{\Omega_i}\frac{\partial \phi}{\partial y}\frac{\partial F}{\partial x}\right) = -\frac{1}{B}\hat{z} \times \nabla\phi \cdot \nabla f \tag{5}$$

Initial conditions:

$$f(\vec{x}, v_{\parallel}, 0) = \sum_i w_{i0}\delta(\vec{x} - \vec{x}_{i0})\delta(v_{\parallel} - v_{\parallel i0}) \tag{6}$$

Quasineutrality with Boltzmann electrons:

$$\frac{n_o e^2}{T_e}\phi = \frac{n_o e^2}{T_i}\rho_i^2 \nabla_{\perp}^2 \phi + e\int dv_{\parallel} f \tag{7}$$

UCSD

Fourier-Laplace transform:

$$(-i\omega + ik_z v_\parallel) f_{\vec{k},\omega} - \mathcal{L}_{\vec{k}} \phi_{\vec{k},\omega} = f_{\vec{k}}(v_\parallel, 0) + \mathcal{N}_{\vec{k},\omega}[\phi, f] \qquad (8)$$

where $p = -i\omega$ is the Laplace transform parameter, the linear operator is

$$\mathcal{L}_{\vec{k}} = \frac{ie}{m} \left(k_z \frac{\partial F}{\partial v_\parallel} + \frac{k_y}{\Omega} \frac{\partial F}{\partial x} \right) \qquad (9)$$

and the ExB nonlinearity is

$$\mathcal{N}_{\vec{k},\omega}[\phi, f] = \frac{i}{B} \sum_{k',\omega'} (\vec{k} \cdot \vec{k}' \times \hat{z}) \phi_{\vec{k}',\omega'} f_{\vec{k}-\vec{k}',\omega-\omega'} \qquad (10)$$

Initial conditions:

$$f_{\vec{k}}(v_\parallel, 0) = \sum_i w_{i0} \exp[-i\vec{k} \cdot \vec{x}_{i0}] \delta(v_\parallel - v_{\parallel i0}) \qquad (11)$$

Dupree-Tetreault renormalization

Assuming the Fourier modes are statistically independent, the nonlinearity can be approximated by its coherent part:

$$\mathcal{N}_{\vec{k},\omega}[\phi, f] \simeq \beta_{\vec{k},\omega}\phi_{\vec{k},\omega} - d_{\vec{k},\omega} f_{\vec{k},\omega} \tag{12}$$

where

$$\beta_{\vec{k},\omega} = \frac{1}{B^2}\sum_{\vec{k}',\omega'} \frac{(\vec{k}\cdot\vec{k}'\times\hat{z})^2}{[i(\omega-\omega') - i(k_z - k'_z)v_\parallel]}\phi_{\vec{k}',\omega'} f^*_{\vec{k}',\omega'} \tag{13}$$

and

$$d_{\vec{k},\omega} = \frac{1}{B^2}\sum_{\vec{k}',\omega'} \frac{(\vec{k}\cdot\vec{k}'\times\hat{z})^2}{[i(\omega-\omega') - i(k_z - k'_z)v_\parallel]}|\phi_{\vec{k}',\omega'}|^2 \tag{14}$$

Hence,

$$f_{\vec{k},\omega} \simeq \frac{(\mathcal{L}_k + \beta_{\vec{k},\omega})\phi_{\vec{k},\omega} + f_{\vec{k}}(v_\parallel, 0)}{(-i\omega + ik_z v_\parallel + d_{\vec{k},\omega})} \tag{15}$$

Nonlinear equation for the potential:

Quasineutrality:

$$\frac{n_0 e}{T_e}\phi_{\vec{k},\omega} = -\frac{n_o e^2}{T_i}k_\perp^2\rho_i^2\phi_{\vec{k},\omega} + e\int dv_\parallel f_{\vec{k},\omega} \tag{16}$$

Using the expression for $f_{\vec{k},\omega}$, this becomes

$$\frac{n_o e}{T_e}\epsilon(\vec{k},\omega)\phi_{\vec{k},\omega} = i\int dv_\parallel \frac{f_{\vec{k}}(v_\parallel, 0)}{(\omega - k_z v_\parallel + i d_{\vec{k},\omega})} \tag{17}$$

where

$$\epsilon(\vec{k},\omega) = 1 + k_\perp^2\rho_s^2 - i\frac{T_e}{n_o}\int dv_\parallel \frac{\mathcal{L}_{\vec{k}} + \beta_{\vec{k},\omega}}{(\omega - k_z v_\parallel + i d_{\vec{k},\omega})} \tag{18}$$

is the renormalized dielectric. The total damping is $\gamma_{\vec{k}} = -\epsilon_I \Big/ \frac{\partial \epsilon_R}{\partial \omega_{\vec{k}}}$, which will involve $\beta_{\vec{k},\omega}$ as well as $d_{\vec{k},\omega}(\approx k_\perp^2 D)$.

UCSD

Drift waves

Assuming $k_z v_{Ti}/\omega \ll 1$,

$$\begin{aligned}
\epsilon(\vec{k},\omega) &= 1 + k_\perp^2\rho_s^2 - \frac{\omega_{*e}}{(\omega + i\gamma_L)} - \frac{k_z^2 c_s^2}{\omega^2} && (19)\\
&\quad - \left(1 - \frac{\omega_{*e}}{\omega} - \frac{2k_z^2 c_s^2}{\omega^2}\right)\frac{i d_{\vec{k},\omega}}{\omega} && (20)\\
&= \epsilon_R(\vec{k},\omega) + i\epsilon_I(\vec{k},\omega) && (21)
\end{aligned}$$

where γ_L is the linear damping. Using the linear dispersion relation $1 + k_\perp^2\rho_s^2 - \omega_{*e}/\omega - k_z^2 c_s^2/\omega^2 = 0$ the imaginary part of the dielectric is

$$\epsilon_I(\vec{k},\omega_{\vec{k}}) = \frac{\omega_{*e}\gamma_L}{\omega_{\vec{k}}^2} + \left(k_\perp^2\rho_s^2 + \frac{k_z^2 c_s^2}{\omega_{\vec{k}}^2}\right)\frac{d_{\vec{k},\omega}}{\omega_{\vec{k}}} \qquad (22)$$

The total damping is $\gamma_{\vec{k}} = -\frac{\omega_{\vec{k}}^2}{\omega_{*e}}\epsilon_I$. Because the $\beta_{\vec{k},\omega}$ term is included, there is a near cancellation, leading to the small factor multiplying $d_{\vec{k},\omega}$.

Potential fluctuation spectrum

Using

$$\left\langle \phi_{\vec{k}\omega}\phi_{\vec{k}'\omega'}\right\rangle = (2\pi)^4 \left\langle \phi^2\right\rangle_{\vec{k}\omega} \delta(\vec{k}+\vec{k}')\delta(\omega+\omega') \tag{23}$$

and

$$\left\langle f_{\vec{k}}(v_{\|},0) f^*_{\vec{k}}(v'_{\|},0)\right\rangle = \sum_i w_{i0}^2 F(v_{\|})\delta(v_{\|}-v'_{\|}) \tag{24}$$

we find

$$\left\langle \phi^2\right\rangle_{\vec{k}\omega} = const \sum_i w_{i0}^2 \frac{\int dv_{\|} F(v_{\|})\delta(\omega - k_z v_{\|})}{|\epsilon(\vec{k},\omega)|^2} \tag{25}$$

The potential fluctuation spectrum is given as the renormalized dielectric response to discrete particle noise.

Conclusions

- The linear form for the fluctuation spectrum becomes invalid even when the plasma is stable, but close to marginal stability.
- Therefore, the Fluctuation-Dissipation Theorem cannot be expected to have the linear form for linearly unstable plasmas.
- Analytical models of noise damping effects require some care – simple $\boldsymbol{k^2 D}$ type estimates are invalid.
- Zonal Flows driven by noise and turbulence require some care in calculating, using knowledge gained from simpler examples.
- ZF potential should be renormalized polarization response to noise and incoherent mode-coupling from drift waves