Gyrokinetic simulation of toroidal angular momentum transport

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Abstract

Recent experimental results in C-mod (J. E. Rice et al) and DIII-D (deGrassie et al) indicate that tokamaks have an intrinsic *source* of toroidal rotation with an inward or *pinched flow*. This has motivated us to carefully re-examination the formulation of toroidal angular momentum and poloidal rotation continuity equations found in Staebler 2004[1] paper. Our work provides a more explicit formulation for evaluating the turbulent components with the GYRO gyrokinetic code [2] simulations in real tokamak geometry. While GYRO with finite parallel velocity shear providing a Kelvin-Helmholtz drive has been simulating toroidal angular momentum transport since 2003, the radial flow of toroidal angular momentum is now broken into components from the radial-parallel, and radial-perpendicular stress tensors, as well as the convective flow of toroidal angular momentum to better understand the origin of *pinched flows* and how they are affected by ExB velocity shearing. Previous quasilinear estimates in slab geometry treating parallel and ExB shear independently found *pinched flows* possible[3]. Parallel velocity shear and perpendicular ExB shear are of course physically related via radial force balance and the ion pressure gradient profile. Finite parallel velocity (not just sheared parallel velocity) has recently been added to test a possible additional source of pinching in toroidal geometry[4]. Mapping the parametric dependence of core toroidal angular momentum transport and pinch conditions is our key focus. We also use GYRO to evaluate a possible *source* of toroidal angular momentum from the small non-ambipolar component of radial magnetic flutter particle flow. In the core at least, the turbulent viscose transport and source forcing are expected to be small compared to the strong neoclassical magnetic pumping which drags the poloidal rotation to the neoclassical level. However a key point of Ref. 1 is that turbulence can provide some *shift* in the neoclassical poloidal rotation. We use GYRO simulations to determine the size of the shift at finite rho-star.

[1] G.M. Staebler, Phys. Plasmas 11 (2004) 1064
[2] http://fusion.gat.com/theory/Gyro
[3] R.R. Dominguez and G.M. Staebler, Phys. Fluids B5 (1993) 3876
[4] A. G. Peeters, C. Angioni, D. Strintzi, "The toroidal momentum pinch velocity" submitted to PRL Nov 2006

Outline

- Heuristic description of intrinsic toroidal rotation and momentum pinches
- GYRO verification of ExB shear pinch and finite parallel velocity pinch
- Gyrokinetic formulation of toroidal angular momentum transport and poloidal rotation

Heuristic description of intrinsic toroidal rotation

- Toroidal momentum continuity equation:

$$\partial[nv_\phi]/\partial t + r^{-1}\partial[r\Gamma_x^\phi]/\partial r = S_\phi(r)$$

Actually we need to consider the flux surface average of toroidal angular momentum (TAM) continuity $m_i\langle Rnv_\phi\rangle$, but such details are suppressed.

- TAM is a conserved quantity. For intrinsic toroidal rotation with no internal sources, $S_\phi(r)=0[r=0:a]$ the "source" of the momentum is in the banana orbit loses at the edge $S_\phi(r)>0[r=a:a+\rho_{bananna}]$

- At steady state: $\Gamma_x^\phi(r)=(a/r)\Gamma_x^\phi(a)$

At some earlier time $\Gamma_x^\phi(a)<0$ and toroidal angular momentum flowed into the core r< a until it starts to flow out $\Gamma_x^\phi(a)>0$. The edge is both a source and sink (wall neutral CEX). At steady state $\Gamma_x^\phi(a)\sim 0$, we are looking for the *existence of a null transport flow state:* $\Gamma_x^\phi(r)=0[r=0:a]$ with finite values of v_ϕ and $\partial v_\phi(r)/\partial r$.

- In the DIIID case $v_\phi(a)\sim 0$ and $v_\phi(r)$ peaked near r~a/2 [i.e. $\partial v_\phi(a/2)/\partial r\sim 0$]

Heuristic description (cont'd)

- Descriptive (heuristic) transport flow model verified by GYRO simulations:

$$\Gamma_x^{\phi}(r) \equiv -n\eta_{\phi}^{eff}\,\partial \upsilon_{\phi}/\partial r \equiv -n\eta_{\|}^{eff}\,\partial \upsilon_{\|}/\partial r$$

$$= -n\eta_{\|}\partial \upsilon_{\|}/\partial r - n(\eta_{\upsilon}/a)\upsilon_{\|} - n\eta_{E}\partial \upsilon_{ExB}/\partial r + \upsilon_{\phi}\Gamma_x$$

At steady state, the core (even with peaked density profile) will be in "null" plasma flow state $\Gamma_x \sim 0$ and positive only close to edge recycling.
[In any case we ignore convection here.]

$$\upsilon_{\|} = (B_t/B)\upsilon_{\phi} + (B_p/B)\upsilon_{\theta}$$

$$\upsilon_{ExB} = -\upsilon_{*}[\nabla p_i] + (B_t/B)\upsilon_{\theta} - (B_p/B)\upsilon_{\phi}$$

$$\upsilon_{\theta} = \upsilon_{*}^{neo} = -K^{neo} c_s \rho_{*} a\,\partial \ln T_i/\partial r$$

- In toroidal geometry finite $\upsilon_{\|}$ terms in the curvature drifts

A. G. Peeters, C. Angioni, D. Strintzi, "The toroidal momentum pinch velocity" PRL 2007,

show $(\eta_{\upsilon}/a) > 0$ in an ITG adiabatic electron quasilinear model allowing a pinched $\Gamma_x^{\phi}(r) = 0$ state with $-[\partial \upsilon_{\|}/\partial r]/\upsilon_{\|} > 0$

Heuristic description (cont'd)

- In slab geometry (and now GYRO verified in toroidal geometry)

R.R. Dominguez and G.M. Staebler, Phys. Fluids B5 (1993) 3876

showed with an ITG-TEM quasilinear model how ExB shear (η_E -term) can allow null $\Gamma_x^{\phi} \sim 0$ flow states and

[G.M. Staebler et al, BAPS 46 (2001) p221-LP1 17], "Heating Induced Toroidal Rotation and Other Consequences of Anomalous Momentum transport"
argued

"spontaneous toroidal rotation during heating without external torque was shown to follow from the off diagonal nature of the toroidal viscous stress"

[G.M. Staebler TTF 2001 poster]

Note the "stress" $\Pi_{x\phi} = m_i \Gamma_x^{\phi}$ is same as the radial flow of toroidal momentum.

How does intrinsic toroidal rotation result?

- Set $(\eta_\upsilon/a)=0$ and assume for simplicity $(\eta_E/\eta_{||})$ constant in r

Integrating the continuity equation at null flow

$$[\upsilon_{||}+(\eta_E/\eta_{||})\upsilon_{ExB}]\,|_a^r=0\Rightarrow$$

$$\upsilon_\phi(r)=\frac{\{-(B_p/B_t)\upsilon_*^{neo}-(\eta_E/\eta_{||})[\upsilon_*(B/B_t)+\upsilon_*^{neo}]\}\,|_a^r}{\{1-(\eta_E/\eta_{||})(B_p/B_t)\}(r)}+\frac{\{1-(\eta_E/\eta_{||})(B_p/B_t)\}(a)}{\{1-(\eta_E/\eta_{||})(B_p/B_t)\}(r)}\upsilon_\phi(a)$$

If the edge rotation is small, the intrinsic toroidal rotation is diamagnetically scaled, i.e. driven by pressure and temperature gradients and proportional to rho-star.

Note even with $(\eta_E/\eta_{||})=0$: $\upsilon_\phi(r)=\{-(B_p/B_t)\upsilon_*^{neo}\}\,|_a^r+\upsilon_\phi(a)$

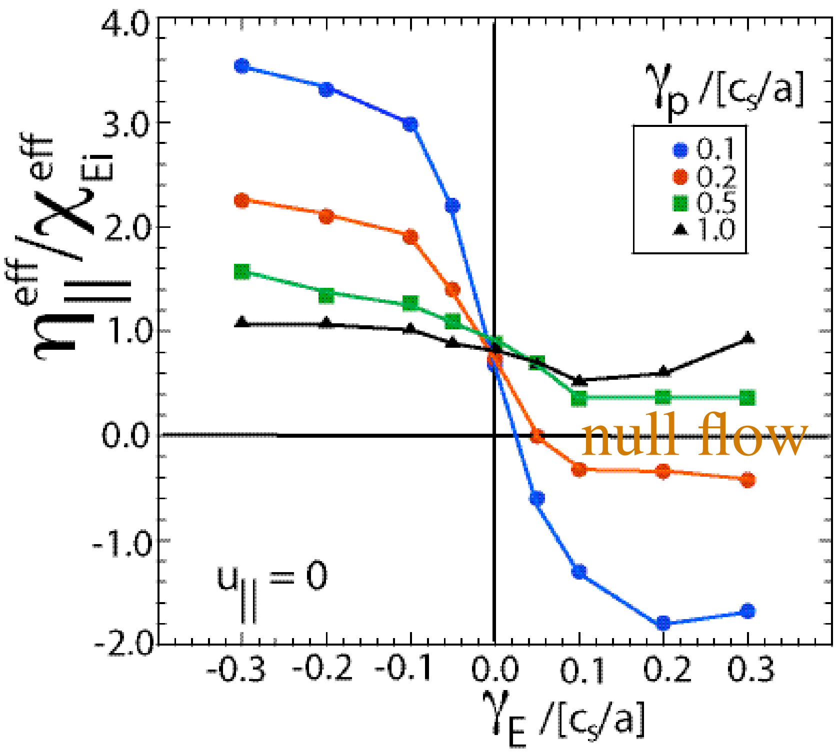
- Assume $(\eta_\upsilon/a)\neq 0$ and $(\eta_E/\eta_{||})=0$, there is a similar solution:

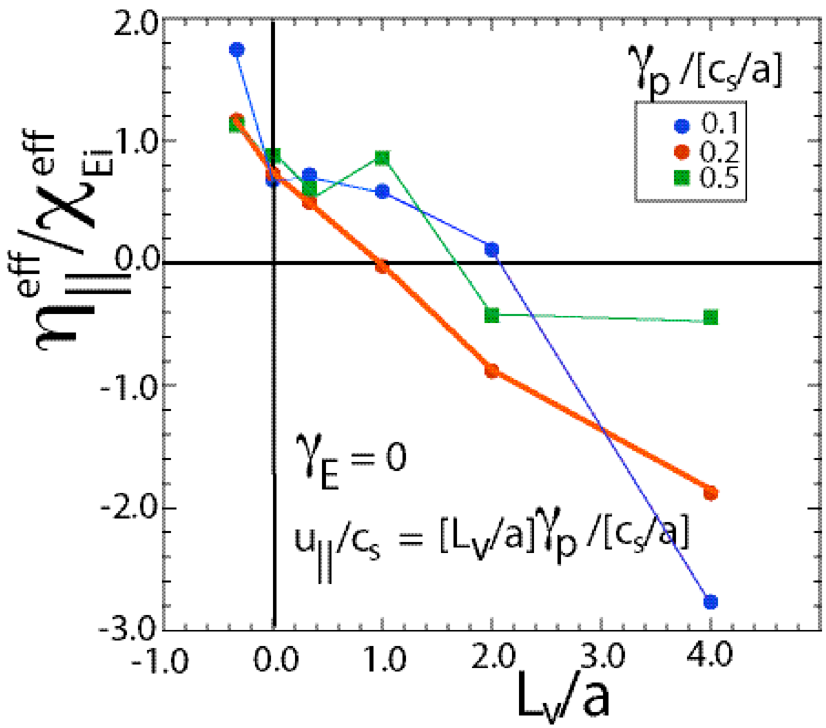
$$\upsilon_\phi(r)=-(B_p/B_t)\upsilon_*^{neo}(r)+\{\upsilon_\phi+(B_p/B_t)\upsilon_*^{neo}\}(a)\exp\{-\textstyle\int_a^r(\eta_\upsilon/\eta_{||})dr'/a\}$$

- We don't really need the pinch effects $(\eta_E/\eta_{||})$ and (η_υ/a) to get intrinsic toroidal rotation, since radial flow is driven by $-\partial\upsilon_{||}/\partial r$ not $-\partial\upsilon_\phi/\partial r$

GYRO simulations verify the pinch effects

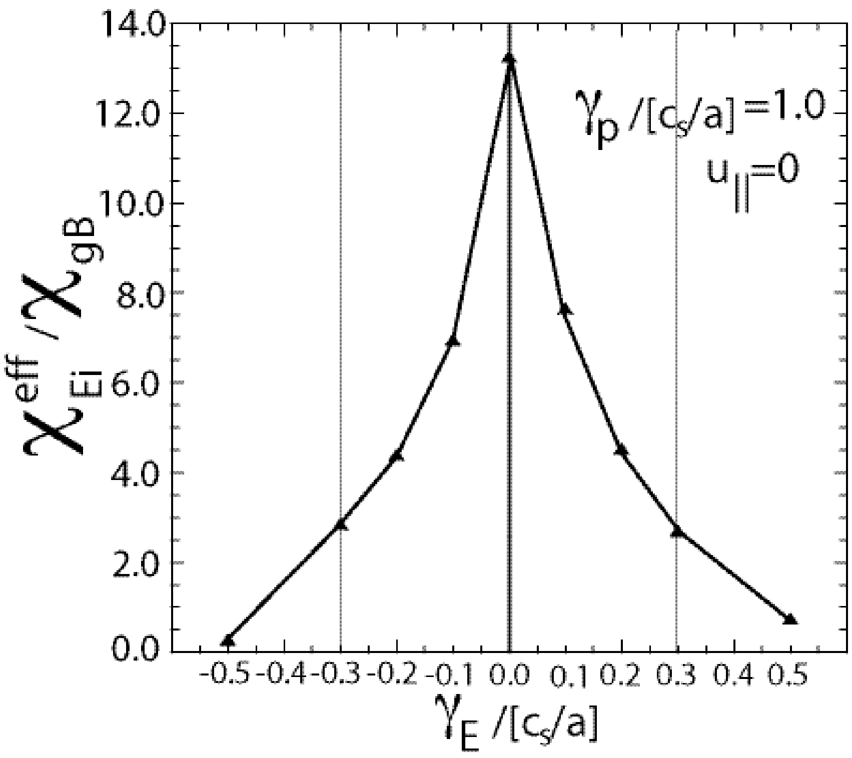
GA standard case with kinetic electrons: q=2,s=1, $a/L_T=3$, $a/L_n=1$, $\nu_*=0$, $\beta=0$
The particle pinch $D^{eff}/\chi^{eff}_{Ei} \sim -0.15$ has been "subtracted". $\gamma_E \equiv (r/q)\partial[(q/r)\upsilon_E]/\partial r$

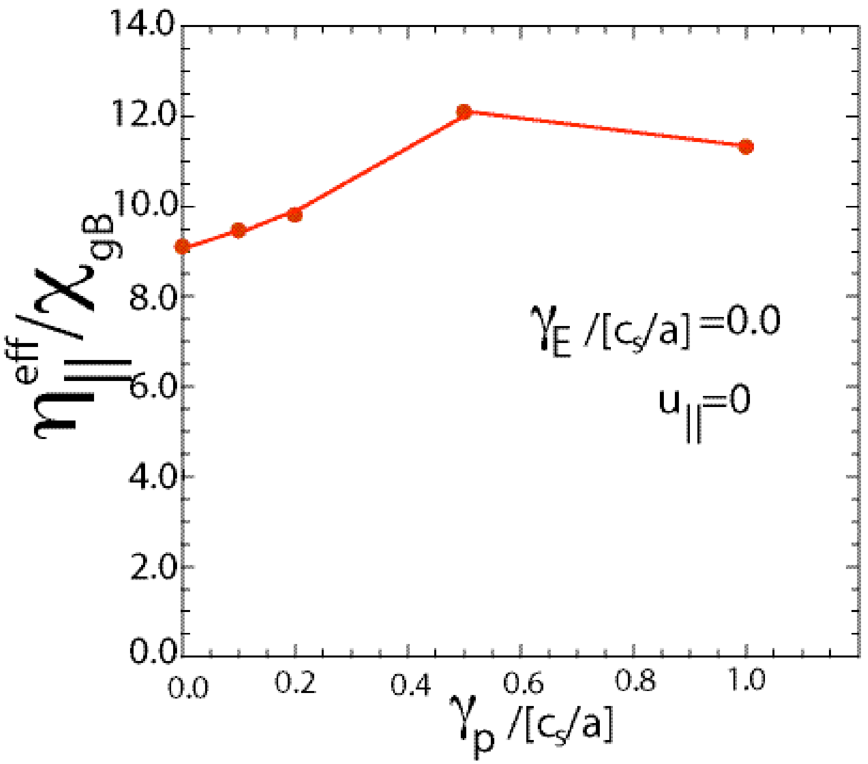




Note that the large parallel rotational shear states $\gamma_p \equiv -R\partial(u_{||}/R)/\partial r \sim 1.0$ as in DIIID unbalance injection have the "normal" $\eta^{eff}_{||}/\chi^{eff}_{Ei} \sim 0.8$ (previously reported in GYRO publications) are far from the pinched (or null) flow $\eta^{eff}_{||}/\chi^{eff}_{Ei} \leq 0$

ExB shear quench





- ExB shear (independent of sign) has usual quench on all low-k transport channels.
- Momentum (and energy) diffusivity increases slightly with parallel velocity shear.

Edge source constraint

- While it appears the null flow profiles could have both co- and counter-current intrinsic rotation as a function of radius $\upsilon_\phi(r_1)<0;\ \upsilon_\phi(r_2)(>0)$ the edge source is only co-current.

- Since toroidal angular momentum is conserved $\int_0^a r'dr'nRM\upsilon_\phi(r')$ must be co-current.

- Ion orbit losses produce co-current momentum into as does the Debye shear Er ExB rotation.

- Hopefully the edge boundary condition $\upsilon_\phi(a)$ is “small” because we can at best put bounds on it:

$\upsilon_\phi^{\max}(a)=(B_t/B)\upsilon_\parallel^{\max}(a)-(B_p/B)\upsilon_\perp(a)$; $(B_t/B)>0$; $(B_p/B)>0$; $j_\phi<0$; $\upsilon_\parallel<0$ ->co

$\upsilon_\perp(a)=\upsilon_E(a)+\upsilon_*(a)$; $-\nabla_x P_i>0$; $E_r(a)=-\nabla_x\phi(a)=-3\nabla_x T_e(a)/e>0$; $\upsilon_\perp(a)>0$ ->co

Empty loss cone gives $\upsilon_\parallel^{\max}\sim-\upsilon_{th}(a/R)/(2\sqrt{\pi})\sim-0.1\upsilon_{th}$ and the $\upsilon_\perp(a)$ projection is much smaller.

In this discussion, it is important to remind that the source is taken to be outside $r=a$, i.e. $a<r<a+\rho_{banana}$. This likely means that $a=r_{ped}$ i. e. the top of the pedestal radius.

Diamagnetically scaled intrinsic toroidal rotation

- Getting the correct non-monotonic profile of $\upsilon_\phi(r)$ will require a very accurate transport model. Getting the finite $\upsilon_{||}$ curvature drift pinch effect in TGLF is straight-forward. Getting the ExB shear properly installed in the TGLF θ–dependent quasilinear mode function is non-trivial.

- However the diamagnetic scaling is testable: Assuming $\upsilon_\phi(a) \sim 0$

$$\upsilon_\phi(scale) \sim \upsilon_* \propto \rho_* c_s \sim c_s^2/a/(eB/mc) \sim T/Ba$$

$$\tau_E \sim 3nTV/P \sim 3nT(\pi a^2 2\pi R\kappa)/P$$

$$\Rightarrow \upsilon_\phi(scale) \propto \tau_E (P/n)/(Ba^3 R\kappa)$$

- Using a collisionless-electrostatic-gyroBohm scaling

$$\tau_E \propto a^2[B^2 a\kappa^2/\hat{\chi}]^{2/5}(\kappa nR/P)^{3/5}$$

$$\hat{\chi}(GYRO) \propto q^2/\kappa \;\; \& \;\; I \propto a^2 B\kappa/Rq$$

$$\upsilon_\phi(scale) \propto a^2[I^2 R^2 \kappa/a^3]^{2/5}(\kappa nR/P)^{3/5}(P/n)/(Ba^3 R\kappa)$$

$$\propto I^{4/5} B^{-1} (P/n)^{2/5} \kappa^0 R^{2/5} a^{-11/5}$$

- For $\upsilon_\phi(scale) \sim \upsilon_* \propto \rho_* c_s$, the shear rates $\propto \rho_* c_s/a$ will much smaller than high-n turbulent growth rates $\propto c_s/a$ in ITER. [May be helpful for stabilizing RWM?]

Gyrokinetic formulation of toroidal angular momentum transport and poloidal rotation used in GYRO

G.M. Staebler, Phys. Plasmas (2004) 1064

- The toroidal angular momentum continuity equation is, (ignoring small or explicitly zero magnetic flutter effects):

$$mn\partial\left\langle R\vec{u}\cdot\hat{\varepsilon}_{\phi}\right\rangle/\partial t+[V'(r)]^{-1}\partial_r V'(r)\{\Gamma^{\phi}+\Gamma^{\phi}_{conv}\}=\left\langle R\hat{\varepsilon}_{\phi}\cdot\vec{F}\right\rangle+\left\langle Rj_x B_p\right\rangle/c$$

$$\Gamma^{\phi}=\left\langle|\nabla r|[\Pi_{xz}(B_t/B)R+\Pi_{xy}(-B_p/B)R]\right\rangle=\Gamma^{\phi}_z+\Gamma^{\phi}_y$$

$$\Pi^A_{xz}\approx m\int d^3\upsilon[\upsilon_z(c\delta E_y/B]\delta f \sim \int d^3\upsilon\,[\sum_k(\delta\upsilon_{Ex})_{-k}m\upsilon_z J_0(k_\perp\rho_\perp)\delta g_k] \sim m\delta\upsilon_{Ex}\delta j^i_z/(en)$$

$$\Pi^A_{xy}\approx m\int d^3\upsilon[\upsilon_y(c\delta E_y/B)\delta f] \sim \int d^3\upsilon\,[\sum_k(\delta\upsilon_{Ex})_{-k}m\upsilon_\perp[ik_x/k_\perp]J_1(k_\perp\rho_\perp)\delta g_k]$$

$$\sim m\int d^3\upsilon\,[\sum_k(\delta\upsilon_{Ex})_{-k}\nabla_x(m\upsilon^2_\perp/2)(c/zeB)J_0(k_\perp\rho_\perp)\delta g_k] \sim m\delta\upsilon_{Ex}\delta j^i_{*y}/ze$$

The perpendicular stress Π^A_{xy} is significantly different from zero with ExB shear.

Gyrokinetic formulation of toroidal angular momentum transport and poloidal rotation used in GYRO (cont'd)

- The poloidal momentum balance is:

$$mn(1+2\bar{q}^2)\partial\langle B_p u_\theta\rangle/\partial t = -3\langle(\vec{b}\cdot\nabla B)^2\rangle\mu_1[(u_\theta - u_\theta^{neo} - u_\theta^A)/B_p + [B_t R/\langle R^2\rangle][\langle\vec{\nabla}\cdot\hat{\varepsilon}_x \Pi_{xy}(-B_p/B)R\rangle$$

$$-\langle\vec{\nabla}\cdot\hat{\varepsilon}_x \Pi_{xz} B\rangle + [B_t R/\langle R^2\rangle][\langle\hat{\varepsilon}_x \Pi_{xz}(B_t/B)R]\rangle]$$

$$+\langle BF_{||}\rangle - [B_t R/\langle R^2\rangle][\langle R\hat{\varepsilon}_\phi\cdot\vec{F}\rangle + \langle\vec{j}\cdot\nabla\psi\rangle/c]$$

(second line with Π_{xz} nearly cancels)

The "magnetic pumping" neoclassical viscosity dominates, dragging the poloidal velocity to the neoclassical value with a small turbulent shift (Staebler 2004):

$$K(\psi) = u_\theta/B_p \Rightarrow u_\theta^{neo}/B_p + u_\theta^A/B_p \quad \text{where} \quad u_\theta^{neo} = [-1.17,\ 0.5,\ 1.7](a/L_T)\rho_* c_s$$

$$u_\theta^A = B_p\langle(\vec{b}\cdot\vec{\nabla}B)\Delta_{zz}^A/2p\rangle/\langle(\vec{b}\cdot\vec{\nabla}B)^2\rangle \quad \text{where} \quad \Delta_{zz}^A = (4/3)\sum_s e_s \int d^3\upsilon\,\upsilon_z \delta E_z \delta f_s = (4/3)\delta E_z \delta j_z$$

$$u_\theta^A \sim O[\rho_*\hat{\chi}(a/L_T)](4/3)(R/a)[\sin(\theta)]\rho_* c_s$$

- For $\gamma_p = 0.2$ GYRO found (for GA-std) $u_\theta^A \sim [3.25\rho_*,\ -0.25\rho_*]\rho_* c_s$ $\quad \gamma_E = [0.2,\ 0.0]$

Gyrokinetic formulation of toroidal angular momentum transport and poloidal rotation used in GYRO (cont'd)

- The radial electric field $E_r = -|\nabla r|\partial\Phi/\partial r$ is given by :

$$\left\langle \vec{u}\cdot R\hat{\varepsilon}_\varphi \right\rangle = \left\langle Ru_\varphi \right\rangle = \omega\left\langle R^2 \right\rangle + K\langle RB \rangle$$

where

$$\omega = -c[\partial\Phi/\partial\psi + (1/ne)\partial P/\partial\psi]$$
$$\partial_\psi = |\nabla r|/(B_p R)\partial_r$$
$$\partial\Phi/\partial\psi = -E_r/(B_p R)$$

- The ion "omega-star" and "omega-drift" terms for a parallel drifted Maxwellian

$$\partial\delta f/\partial t = ...-\{(\partial n_0/\partial r)/n_0 + [(m/2T_0)(v_\perp^2 + v_\parallel'^2) - 3/2](\partial T_0/\partial r)/T_0$$
$$+2(m/2T_0)v_\parallel'[R_0\partial(\left\langle u_\parallel \right\rangle/R_0)/\partial r]\}\nabla r\cdot\hat{b}\times\nabla\left\langle \delta\phi - v_\parallel\delta A_\parallel \right\rangle F_0$$

$$\partial\delta f/\partial t + (c/B)\hat{b}\times[m(v_\parallel'^2 + 2\left\langle u_\parallel \right\rangle v'_\parallel)\hat{b}\cdot\vec{\nabla}\hat{b} + mv_\perp^2/2\vec{\nabla}B/B]\cdot\nabla\delta f + ... = ...$$

where $\gamma_p \equiv -R_0\partial[\left\langle u_\parallel \right\rangle/R_0]/\partial r$ and $v_\parallel'(\theta) = v_{th}\sqrt{\hat{\varepsilon}(1-\lambda B(\theta)} = v_\parallel - u_\parallel$

Summary

- Intrinsic (or spontaneous) toroidal rotation driven by pressure and ion temperature gradients has been long predicted from the ExB shear pinch of toroidal momentum transport (Dominquez and Staebler 1993 & Staebler et al 2001).

- An additional finite parallel velocity - curvature pinch effect has been proposed (Peeters, Angioni, Strinzi 2007)

- GYRO has confirmed both pinch effects can result in a “null” radial flow of toroidal momentum $\eta_{\|}^{eff}/\chi_{Ei}^{eff} \sim 0$ at low parallel velocity shear whereas the normal $\eta_{\|}^{eff}/\chi_{Ei}^{eff} \sim 0.8$ obtains at high parallel velocity shear.

- If $\upsilon_{\phi}(a) << Max[\upsilon_{\phi}(r)]$ the intrinsic toroidal rotation with “null” radial flow will have a diamagnetic scaling $\upsilon_{\phi}(scale) \sim \upsilon_{*} \propto \rho_{*} c_{s}$ leading to ExB shear rates $\propto \rho_{*} c_{s}/a$ much smaller than high-n turbulent growth rates $\propto c_{s}/a$ in ITER.

- The edge source constrains the total intrinsic toroidal angular momentum to be co-current, but we can not predict the size of $\upsilon_{\phi}(a)$ without the source details.

- Accurately predicting the (often) non-monotonic radial profile of the intrinsic toroidal rotation will be a challenge for theory based transport code models.

- GYRO has shown the turbulent shift from neoclassical poloidal rotation is small.