Physics of Intrinsic Rotation and Transport of Toroidal Momentum: A Resonant Particle - Quasi Particle Approach (Wave)

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Outline

- → Motivation
- → Results
  - Momentum Theorem
  - Wave Momentum Flux
  - Resonant Particle Flux
  - Symmetry ?
- → Implications → Scenarios
- → Extensions

Issues in Theory

- recent works
  - Gurcan, et.al. → $\langle V_E \rangle'$ induced residual stress
  - Hahm, et.al. → momentum pinch TEP + Thermoelectric

- observations:
  → severe (model / mode) dependency, "messy"....
  → 'liberties' with cross phase
    - "believeable" results fluid ....
    - resonant particles? → EP's .....
    stiff → marginal
    - wave momentum? → useful for insight
    - structure interaction?

∴

- Resonant Particle + Quasi-Particle Formulation (Wave kinetic)
- 2 Phase System]
  → treat { resonant particles accurately / fluid effects
  → reveal wave-particle momentum balance
    analyze
  → new, simple look at { problem / previous results
  → new results/effects ....

⇒ Model and Key Issues

- slab GK, electrostatic
- full parallel dynamics → $\left\{ \begin{array}{l} \text{radial scattering} \\ \text{parallel acceleration} \end{array} \right.$
- QL with parallel quasi-linearity

⇒ $\quad WMD = NRPMD$

$$\frac{\partial}{\partial t}\left(RPMD + WMD\right) = 0 \qquad \left\{ \begin{array}{l} \text{Momentum} \\ \text{Budget} \end{array} \right.$$

Field MD → 0

⇒ RPMD → resonant particle kinetics

WMD → wave kinetics !

∴

$$\underbrace{\frac{\partial}{\partial t}\langle P_\parallel \rangle_R}_{\text{resonant particle momentum}} + \underbrace{\frac{\partial}{\partial r}\langle \tilde{V}_r \tilde{P}_\parallel \rangle_R}_{\text{flux of resonant particle momentum (easy)}} = \underbrace{q\langle \tilde{E}_\parallel (\tilde{n}_i - \tilde{n}_e) \rangle_R}_{\text{total force on resonant particles}}$$

$$RHS = \sum_{\underline{k}} \frac{k_\parallel}{\omega_k} Q_{\underline{k}}$$ → connects to wave momentum evolution

$$Q_{\underline{k}} = \omega_k \epsilon_{IM}(k, \omega_k) \frac{|E_k|^2}{8\pi} \quad , \quad N_{\underline{k}} = \frac{\partial \epsilon}{\partial \omega} \frac{|E_k|^2}{8\pi}$$

⇒ Wave kinetics, 'Poynting' Thm, etc., etc.

Momentum Theorems

$P_{\parallel} = k_{\parallel} N$

resonant particle flux

wave momentum flux

$\Rightarrow$

$$\frac{\partial}{\partial t}\langle P_{\parallel}\rangle + \frac{\partial}{\partial r}\langle \tilde{V}_r \tilde{P}_{\parallel}\rangle_R + \frac{\partial}{\partial r}\left\langle \left(V_{g,r} P_{\parallel}^{w}\right)\right\rangle$$

$$= \int d\underline{k} \left\{ \left\langle \frac{dk_{\parallel}}{dt} \right\rangle \langle N\rangle - D_{k_{\parallel},k_{\parallel}} \frac{\partial\langle N\rangle}{\partial k_{\parallel}} \right\}$$

refractive force

- coherent
- stochastic

GAM straining !?
ZF side-band

$\therefore$ integrated momentum ...

$$\frac{\partial}{\partial t} P_{Total} = -\left(\langle \tilde{V}_r \tilde{P}_{\parallel}\rangle_R + \langle V_{g,r} P_{\parallel}^{w}\rangle \right)\Big|_a$$

boundary flux

$$+ \int_0^a dx \int d\underline{k} \left\{ \left\langle \frac{dk_{\parallel}}{dt} \right\rangle \langle N\rangle - D_{k_{\parallel},k_{\parallel}} \frac{\partial\langle N\rangle}{\partial k_{\parallel}} \right\}$$

- $\dot{P}_T > 0 \rightarrow$ outflow of $P_{\parallel}^{w} < 0$

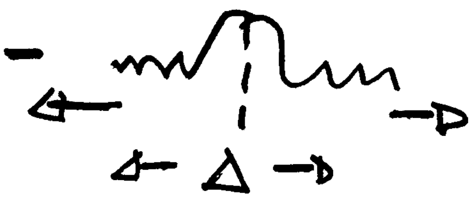
  inflow of $P_{\parallel}^{w} > 0$

  wave momentum <u>explicitly</u> enters spin-up evolution (why not ?..)

$\rightarrow$ refractive force "<u>integrates</u>" $\rightarrow$ concentrated at boundary

- poloidal asymmetries
- GAMs, ZF sidebands ...

→ How Calculate $\Pi_{r,\parallel}^{w} = \int dk\, v_{gr} k_\parallel N$?
localized modes



$v_{gr} \gtrless 0$ equi-probable

$\Delta / L_{\text{intensity}} \lesssim 1$

⇒ akin large optical depth radiation hydro → see Mihalas and Mihalas
for reading fun....

⇒ Chapman-Enskog type journey
via $C(N)$ → wave interactions (L'vov, et. al.)

$$\Pi_{r,\parallel}^{w} = \int d\underline{k}\; v_{g,r}\, k_\parallel \left\{ \overset{①}{-\frac{v_{g,r}}{\gamma_T}\frac{\partial \langle N\rangle}{\partial r}} + \overset{②}{\frac{k_\theta \langle V_E\rangle'}{\gamma_T}\frac{\partial\langle N\rangle}{\partial k_r}} \right\}$$

① → diffusive scattering of quasiparticles
<u>down</u> intensity gradient (<u>edge</u> peaked)
→ influx of Q-P momentum from
edge, $\nabla\langle N\rangle$ driven $\gamma_T \sim 1/\tilde{\tau}_{ck}$

② → off-diagonal flux — $\nabla_k\langle N\rangle$ driven
→ shearing is essential physics ↔ <u>refraction</u>
a la' Zonal flow problem
— symmetry breaking required

$\langle k_\parallel\rangle \neq 0$ — <u>no</u> <u>free</u> <u>lunch</u> !!

$\Rightarrow$

$$\Pi^{w}_{r,\parallel} = \underbrace{-D}_{\text{radiative diffusivity}} \frac{\partial}{\partial r}\Big(\langle k_\parallel\rangle \underbrace{I}_{\text{intensity}}\Big) \quad (1)$$

$$+\,\alpha_{sh}\langle k_\parallel\rangle \Big\langle \underbrace{\frac{\partial v_{gr}}{\partial k_r}}_{\to\, V_{*e,i}\text{ enters sign}}\Big\rangle \frac{I(r)}{\nu_T}\frac{\partial\langle V_E\rangle}{\partial r} \quad (2)$$

$D \sim v_{gr}^2/\nu_T \sim D_{GB}$

more than coincidence?

$\to \langle k_\parallel\rangle \to$ symmetry breaking.... ?

$\to$ how formulate a general approach?

$$\frac{\partial\langle k_\parallel\rangle}{\partial t} = -\int dk \underbrace{\left(\frac{\partial k_\parallel}{\partial k_r}\right)}_{\text{B-shear}} \frac{\partial}{\partial r}\big(k_\theta\langle V_E\rangle\big)\langle \bar{N}\rangle$$

$\to \begin{cases} k_\theta/L_s\,{\scriptstyle \mathcal{U}} & \text{- slab} \\ \nabla/k_\theta\hat{s} & \text{- torus}\end{cases}$

$$-\int d\underline{k}\left\{\underbrace{D_{k_\parallel,k_\parallel}}_{\sim\langle q_\parallel^2(\tilde{\omega}_k+k\cdot\tilde{V})^2_{\underline{q}}\rangle}\frac{\partial\langle\bar{N}\rangle}{\partial k_\parallel} + D_{k_\parallel,k_\theta}\frac{\partial\langle\bar{N}\rangle}{\partial k_\theta}\right\}$$

$\to$ GAM field straining

$$+\int d\underline{k}\; \underbrace{k_\parallel\langle\bar{N}\rangle\gamma_{\underline{k}}}_{\text{growth asymmetry, and ultimate high }\underline{k}\text{-damping}}$$

→ <u>So Many Transitions</u> ... <u>So Little Time</u> ...

$$\Pi^{w}_{r,\parallel} = -D_{rad.}\frac{\partial}{\partial r}\left(\langle k_\parallel\rangle I\right) + \alpha_{sh}\langle k_\parallel\rangle\left\langle\left(\frac{\partial v_{gr}}{\partial k_r}\right)\right\rangle\frac{I}{v_T}\frac{\partial\langle V_E\rangle}{\partial r}$$

① ②

ⓐ for $\nabla I \sim 0$ → "core"

$\Pi^{w}_{r,\parallel} \sim$ ②

→ mode population

→ $\langle V_E\rangle'$

transition via:

→ $V_{*i} \leftrightarrow V_{*e}$ change (TCV limited?!)

→ $\frac{\partial\langle V_E\rangle}{\partial r}$ ↑ → ~ ITB

→ $\langle k_\parallel\rangle$ change ~ TBD

→ magnetic shear !

ⓑ for $\nabla I \neq 0$ → "edge"

$\Pi^{w}_{r,\parallel} \sim$ ① + ②

transition by:

→ $\langle V_E\rangle'$ ↑ → ② amplified

→ $\langle k_\parallel\rangle$ amplified

② >> ①

$\boxed{\text{L→H à la C-Mod}}$

ⓒ ∇I significant

$\partial\langle V_E\rangle/\partial r$ modest

⇒ $\langle k_\parallel\rangle$ ↑ ~ $\frac{\partial\langle V_E\rangle}{\partial r}$

~ GAMs

scaling with $\langle V_E\rangle'$ significant.

$\Pi^{w}_{r,\parallel} \sim$ ①, with $\langle k_\parallel\rangle$ changing ...

here <u>not</u> linked $\langle \tilde{E}_r \rangle$ 'exclusively'

→ <u>Resonant Particles</u>

$$\Pi_R = -\chi_\phi \frac{\partial \langle V_\parallel \rangle}{\partial r} + V \langle V_\parallel \rangle + S$$

→ $\sim \nabla n, \nabla T$

⇝ resonant population size critical → how far out on tail...

$$\sim \exp\left(-\omega_k^2 / k_\parallel^2 v_{Th}^2\right)$$

→ $\chi_\phi \sim \chi_i$; $\chi_\phi \neq \chi_i$ ratio ⟺ $\omega/k_\parallel$ spectr.

→ if $\langle k_\parallel \rangle \neq 0$; $\underbrace{\frac{\omega}{k_\parallel} > \langle V_\parallel \rangle \ ; \ S \gg V \langle V_\parallel \rangle}_{\underline{S \text{ dominates}}}$

$S < 0$ for $\omega/k_\parallel < 0$

if $\langle k_\parallel \rangle = 0$, $S = 0$ → need symm. breaking!!

→ if: $\langle k_\parallel \rangle = 0$ (no symmetry breaking in turbulence)

$V \neq 0$, sign? ⟺ 'it depends'...

can have $\left\{ \begin{matrix} V \neq 0 \\ V < 0 \end{matrix} \right\}$ for ITG, in some cases... → quantitative ?

⇒ interesting interplay resonant, Π^W ... ?

→ deviation from marginality !?

⟺ more transitions ... ?

→ Looking Ahead

- improve model, geometry, $\langle V_E \rangle'$ for resonant part.
  ⇒ develop from Hahm GK eqn. ⟺ new couplings

- ES → EM ?!

  ES ⇒ WMD = NRPMD

  EM ⇒ WMD = NRPMD + Field MD

  → drift-Alfven turbulence

  → Alfvenic EP modes → KSAW

  Abraham force enters field → plasma interaction .... something new!

- clear, 'clean' treatment of resonant particles enables extension to EP modes, EP momentum transport

- use WMD to constrain/test WTT → STT transition → exploit separation R.P.'s, waves

More risqué:

→ C.-E. convergence → fractional kinetics of wave-packets ?!

→ extend to encompass turbulence spreading ....

Fun Ahead!