Forced Frequency Sweeping in Plasmas

Roddy Vann

Department of Physics, University of York, UK

H. L. Berk

Institute for Fusion Studies, UT Austin, Texas, USA

"Rapid frequency-sweeping"

- "Rapid frequency-sweeping" means sufficiently fast as not to be due to changes in background plasma parameters
- Interpretation of these events in terms of phase space structures in the particle distribution function $f(x,v,t)$

In this talk, focus on:

- Strongly-driven, non-perturbative regime

Motivation

Context:

- Frequency-sweeping events are typically driven by fast particles – e.g. NBI, fusion-produced alphas

Applications:

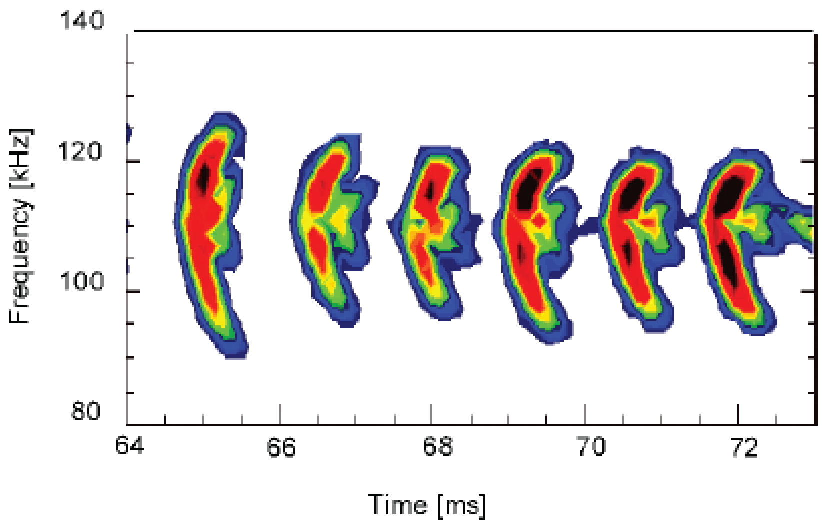
- Develop our understanding of fast particle-driven instabilities and transport
- Use as a diagnostic: sweeping parameters used to deduce plasma parameters
- Apply frequency sweeping mechanism as an energy transfer mechanism (alpha channelling)

Diagnostic example:

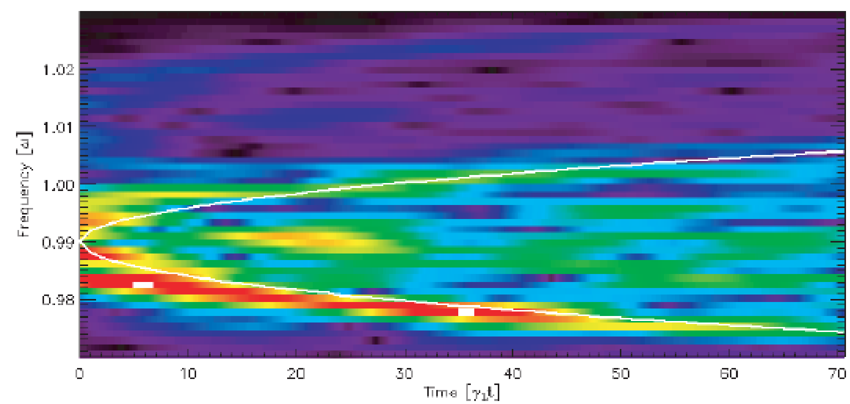
Deducing internal mode amplitude from sweeping parameters

- Theoretical models predict sweeping rate in terms of parameters such as collisionality and gives indication of internal mode amplitude.
- Prediction for a TAE mode in a tokamak using HAGIS code

MAST magnetic fluctuations



HAGIS simulation



Pinches, Berk, Gryaznevich, Sharapov & JET-EFDA team, *PPCF* **46** S47 (2005).

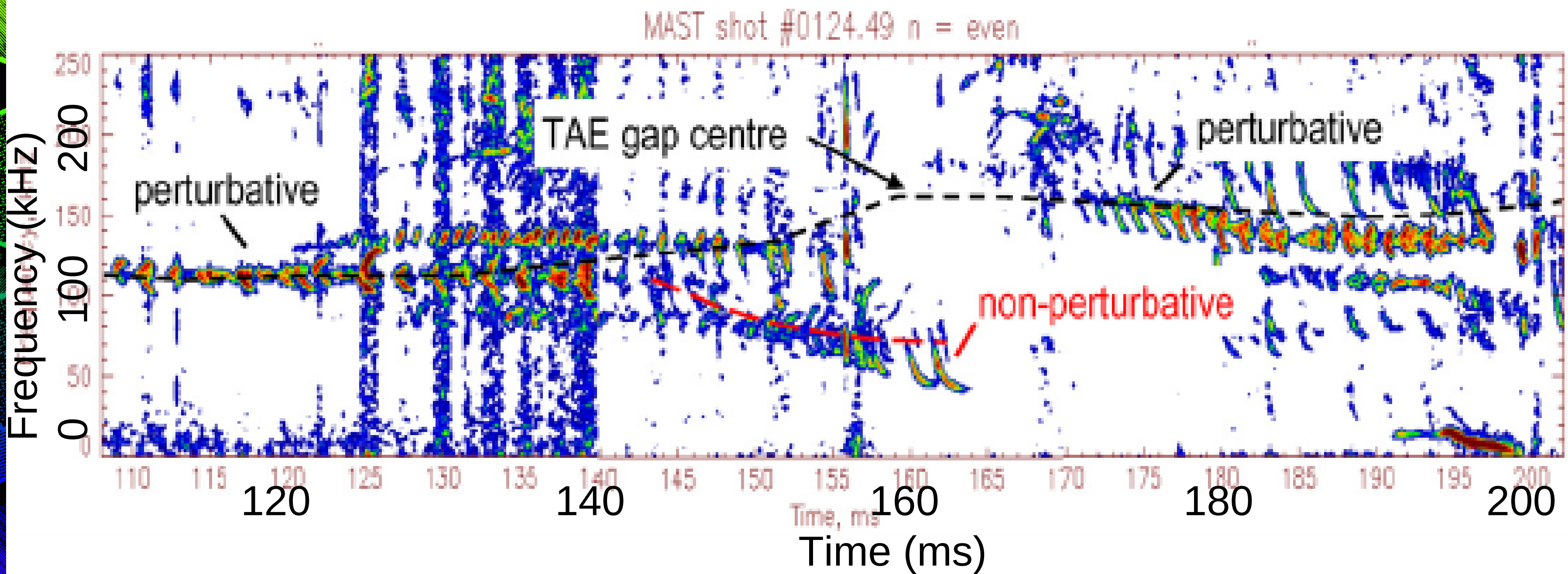
(Non-)Perturbative modes

"Perturbative" or "nonperturbative" describes level of impact of fast particles on modes:

- Perturbative: basic mode structure and frequency given by linear theory of background plasma; kinetic component primarily effects growth rate
- Nonperturbative: ("Energetic Particle Mode"/EPM) linear mode structure and frequency strongly effected determined by fast particle population

(Non-)Perturbative modes

TAE-like sweeping modes can be classified by considering birth frequency



MAST shot 12449, even mode numbers only

Gryaznevich & Sharapov, *Nucl. Fusion*, **46** S942 (2006).

Model equations

- Single distribution *f*(*x*,*v*,*t*) electrostatic model with source injection and collisions:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + E\frac{\partial f}{\partial v} = \nu\left(f - F_{beam}(v)\right) - \nu\left(f - F_{thermal}(v)\right)$$

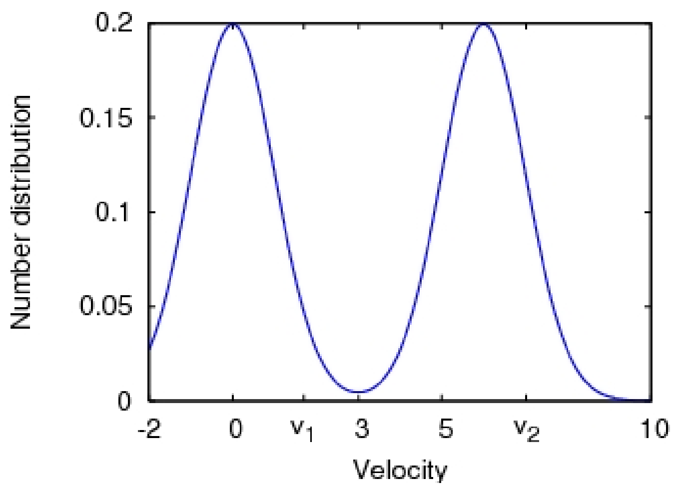
- Field evolution equation includes background damping term acting in a linear resistive manner:

$$\frac{\partial E}{\partial t} + \int (f - f_0)\,d\upsilon = -\gamma_d E$$

- One dimensional ⟶ computationally cheap (although still parallelized)

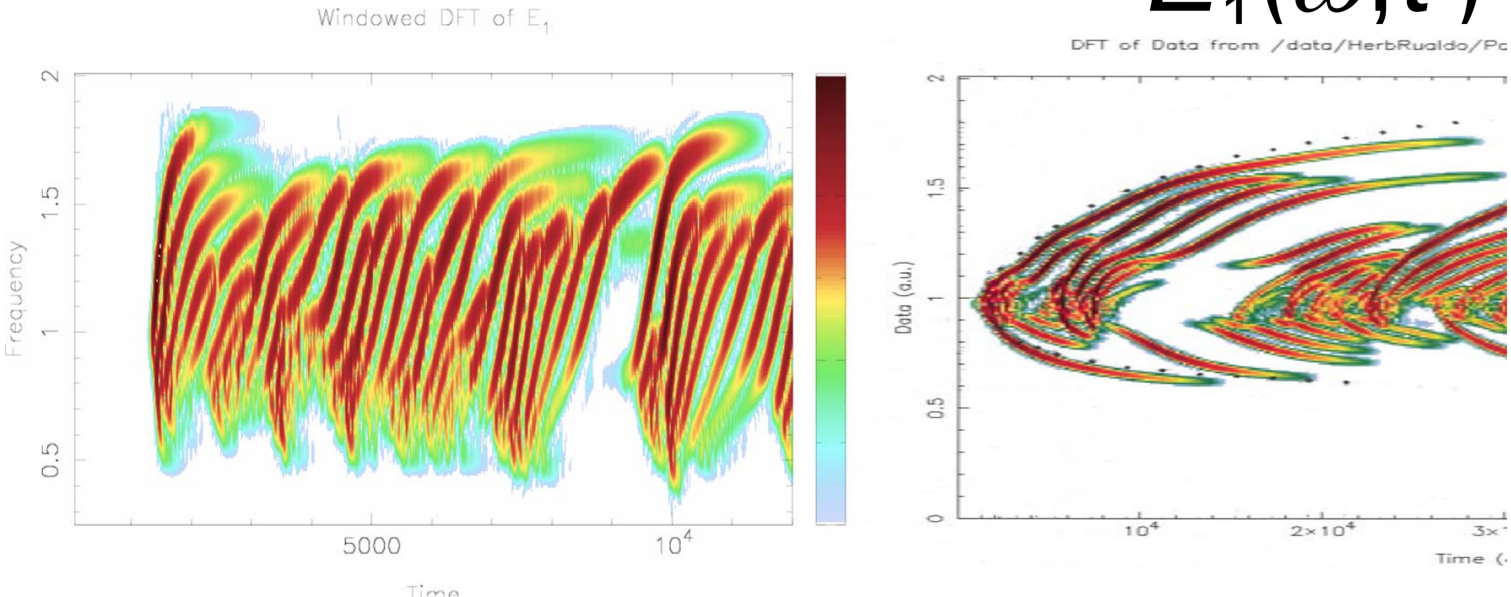
Model equations

- In absence of waves source and 'classical' relaxation would achieve a highly unstable distribution function F_0
- With F_0 chosen as, $F_{beam}(v) = F_{thermal}(v - v_b)$, a non-perturbative unstable mode would arise if distribution could achieve classically predicted level.
- Such a distribution would not be achieved as plasma will find a mechanism to relax.



Field evolution $\tilde{E}_1(\omega,t)$

Frequency (ω/ω_p)
0.5 1.0 1.5



0 $5\text{X}10^3$ 10^4
Time $(\omega_p t)$

Nonperturbative

0 10000 20000
Time $(\omega_p t)$

Perturbative

- Frequency sweeping approximately linear with time $\delta\omega \sim \delta t$ (contrast to perturbative case for which $\delta\omega \sim \delta t^{1/2}$)
- Asymmetric
- Both axes are normalized to underlying wave frequency.
- Collisionality $\nu = 0.0002$; damping rate $\gamma_d = 0.4$

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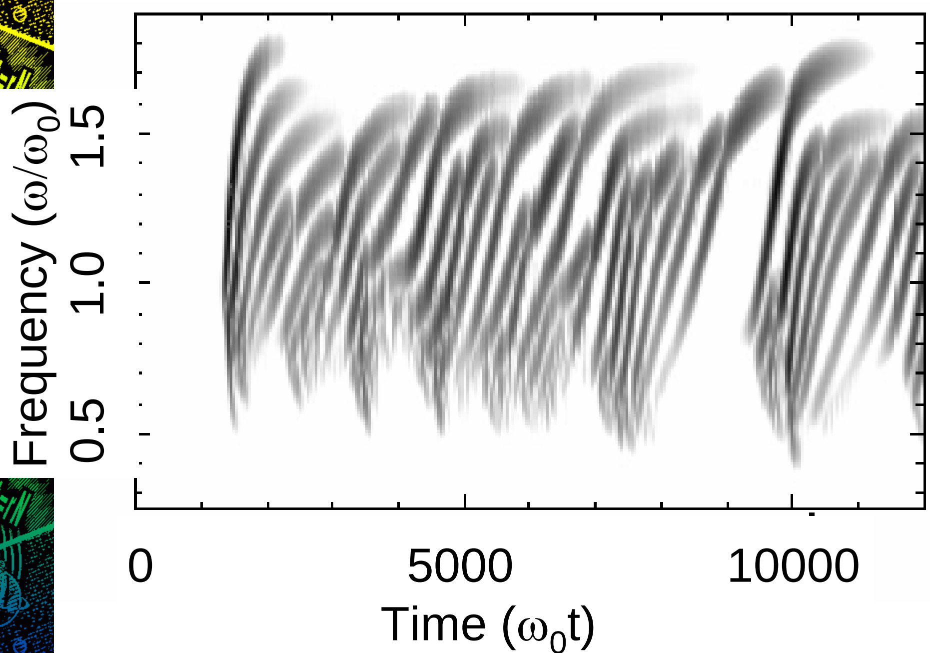
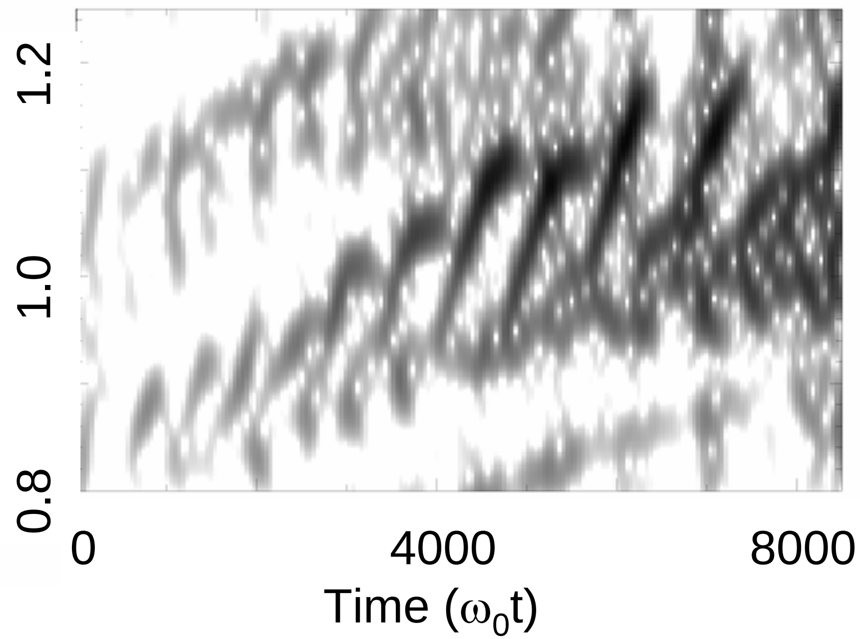
Comparison of Simulation with Experimental Patterns

Simulation



MAST shot 11005



Simulation		Experiment
ω_p	Characteristic frequency ω_0	2π x 100kHz
3.6 x 10^{-4}	Sweeping rate $d\omega/dt$	2.9 x 10^{-4}
0.7	Sweeping extent $\Delta\omega$	0.18

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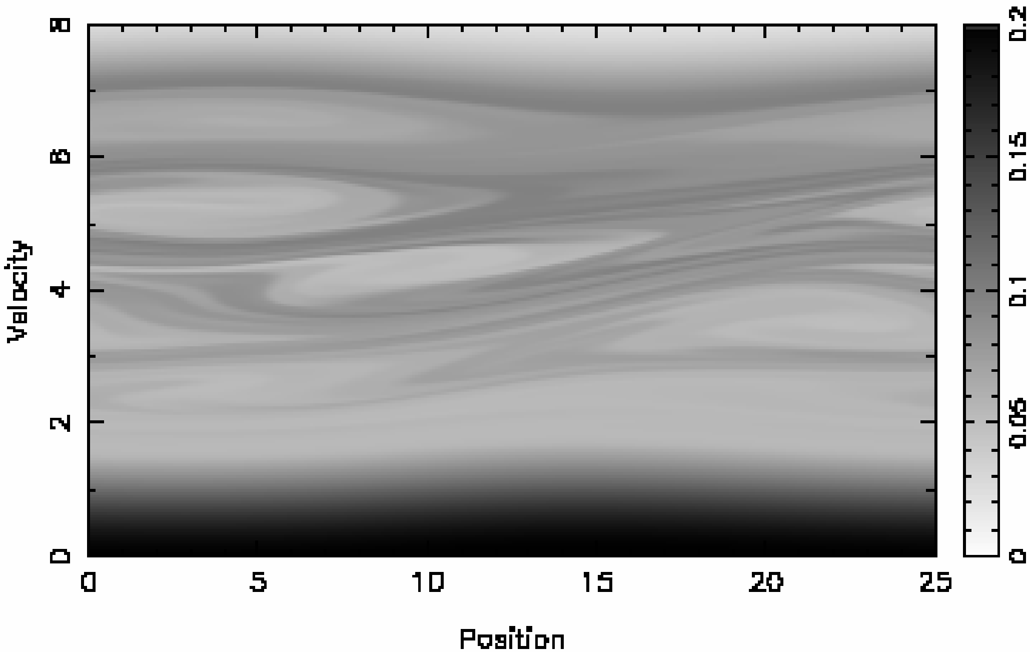
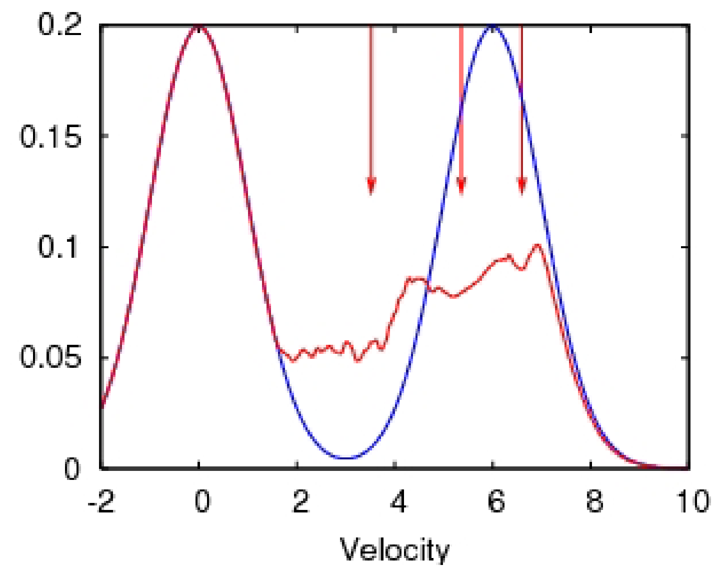
Phase space dynamics

a snapshot at t = 9750

Distribution function $f(x,v)$



Spatial average $f_0(v)$

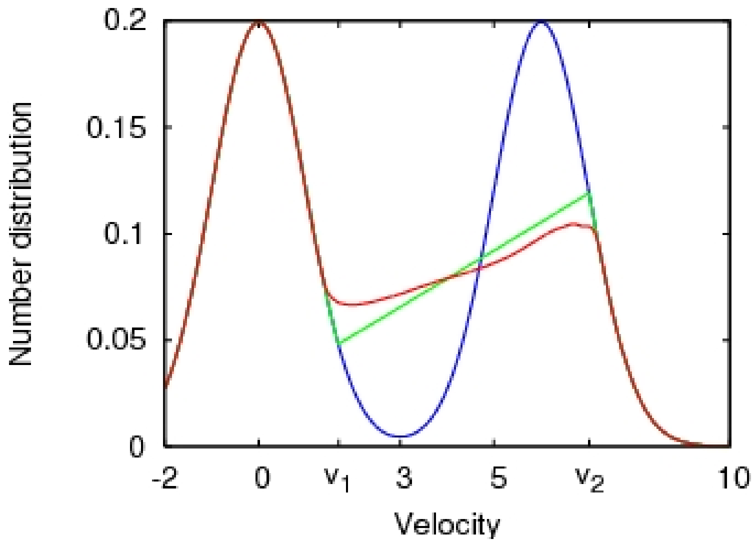


Complex phase space holes are created through resonant interaction with the wave – arrows in plot of $f_0(v)$ correspond to the modes' phase velocities.

The spatial average is remaining far from equilibrium $F_0(v)$

System persists near marginal stability

(i.e. a state in which most unstable mode is marginally stable)



- BLUE is F_0
- RED is long-time average of f_0
- GREEN is candidate marginally-stable distribution

Conclusions:

- Distribution maintains marginally-stable state through the frequency sweeping mechanism
- Resulting distribution energy significantly smaller than what would be predicted without hole evolution (~ 25% in our case)

Fast Chirp theory

- Consider response of background plasma to a trapping region in phase space (a 'bucket') that produces a charge density ρ_{bucket} where distribution function inside bucket taken as the linear phase velocity of marginally stable 'candidate' distribution
- Reactive background plasma response determined from linear dielectric function

$$k^2 \varepsilon_r (\omega_L + \delta\omega)\phi = \rho_{bucket}$$

- Dissipative response replaced by bucket's charge density (new theoretical feature: maintain precise dependence of ε_r to $\delta\omega$, for candidate distribution)
- Background dissipation extracts power from wave. Alternate response to damping is frequency sweeping to extract energy from bucket. Here downward for a clump and upward for a hole

Continuation of Analysis

- To feed background dissipation, power is released by bucket through frequency sweeping mechanism.
- Frequency shift $\delta\omega = \omega - \omega_L$

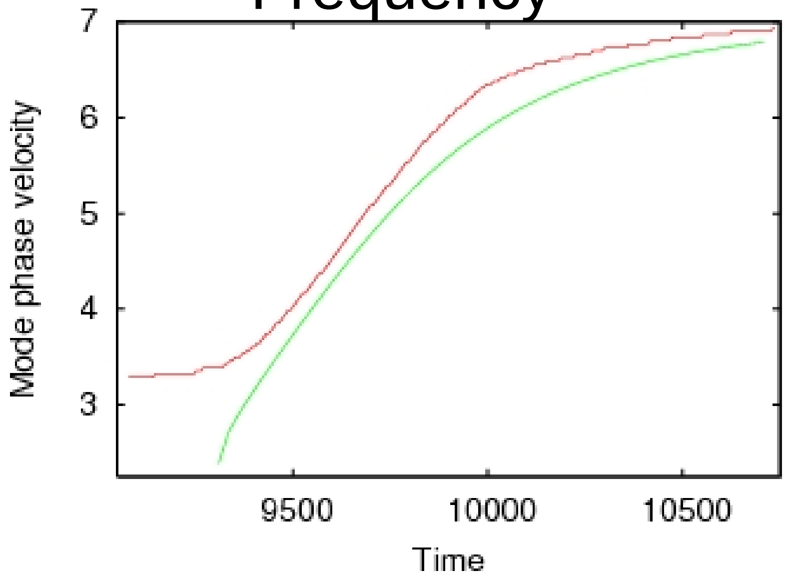
$$\frac{d\,\delta\omega^2}{dt} = \frac{1}{3}\omega_b^2\gamma_d\,\frac{\omega_L}{\omega}\left(\frac{\partial\varepsilon_r(\omega_L,k)}{\partial\omega}\right)\frac{\delta\omega}{\varepsilon_r(\omega,k)}$$

Mode amplitude is best measured in terms of trapping frequency of deeply trapped particle (universal measure of trapping effect which scales to nearly every Hamiltonian system)

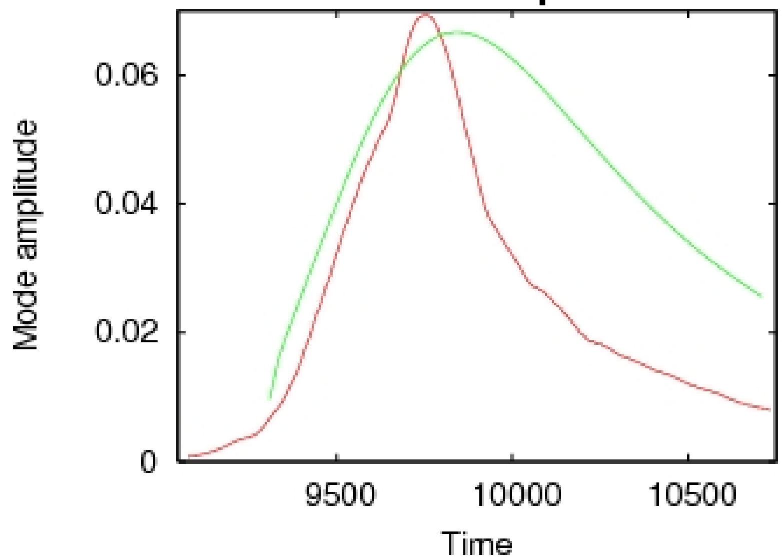
$$\omega_b{}^2 = k^2\phi = \frac{16^2}{9\pi^4}\gamma_L{}^2k^2\left(\frac{f(\frac{\omega}{k}) - f(\frac{\omega_L}{k})}{\frac{\partial f}{\partial\upsilon}(\frac{\omega_L}{k})\varepsilon_r(\omega,k)}\right)^2\left(\frac{\partial\varepsilon_r(\omega_L)}{\partial\omega}\right)^2$$

Evolution of single frequency-sweeping structure

Frequency



Mode amplitude



- **RED** curve extracted from simulation
- **GREEN** curve from non-linear reduced theory
- Only fitting parameter is the time offset (common to both plots)
- Good agreement despite theory not accounting for
    - hole-hole interaction
    - trapping/untrapping of particles

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Discussion

- Deeper insight into hole or clump sweeping dynamics achieved
  - Linear frequency sweeping $\delta\omega \sim \delta t$ of non-perturbative modes
  - System persists near marginal stability
  - Significant reduction in stored fast particle energy
- Quantitative understanding of direction of rapid sweep and enhancement of saturated level
- ~ 50 enhancement of power transfer of beam distribution to background plasma as compared to estimate inferred from original theory (which is based on a perturbative solution)
- Gives viability to prospect of channeling through the intermediary of phase space structures
- We will attempt to understand the power limitations that can maintain sweeping as the relaxation mechanism, as opposed to more violent relaxation with non-linear mode overlap, even when additional linear modes are somehow quenched (an assumption of this modeling)