Radial structures and nonlinear excitation of Geodesic Acoustic Modes (GAM)

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GAM continuous spectrum

- In realistic plasmas: $T_e(r)$, $T_i(r)$, $q(r)$
  - $\Rightarrow \omega^2_{GAM} \simeq 2T_i(r)/(m_i R_0^2)\,(7/4 + T_e(r)/T_i(r)) = \omega^2_{GAM}(r)$
  - ω_{GAM} varies radially
  - $\omega^2_{GAM}(r)$ forms a continuous spectrum

- Fluid derivations (kinetic theory later)
  - Quasi-neutrality $\nabla \cdot \delta \mathbf{J} = 0 \;\Rightarrow\; \frac{\partial}{\partial r}\overline{\delta J_r(r,t)} = 0$
  - $\varrho_m \frac{\partial}{\partial t}\delta \mathbf{u} = -\nabla \cdot \delta \mathbf{P} + \delta \mathbf{J} \times \mathbf{B}/c\,;$ $\delta J_r(r,t) = \frac{c}{B}\varrho_m \frac{\partial}{\partial t}\delta u_\theta + \frac{c}{B}\nabla_\theta \delta P_\perp$
  - $\frac{1}{B} \simeq \frac{1}{B_0}\left(1 + \frac{r}{R_0}\cos\theta\right);\, \delta u_\theta \simeq \frac{c}{B}\overline{\delta E_r} \;\Rightarrow\; \overline{\delta J_r(r,t)} = \underbrace{\frac{c^2}{B_0^2}\varrho_m \frac{\partial}{\partial t}\overline{\delta E_r}}_{\text{polarization}} + \underbrace{\overline{\frac{c}{B}\nabla_\theta \delta P_\perp}}_{\text{diamagnetic}}$

- $\overline{\frac{c}{B}\nabla_\theta \delta P_\perp} \simeq -cB_0 \overline{\delta \tilde{P}_\perp \nabla_\theta \left(\frac{1}{B^2}\right)}$

- $\delta \tilde{P}_\perp \simeq (T_e + \gamma_i T_i)\, \delta\tilde{n}_i \; ; \delta\tilde{n}_i = -\frac{i}{\omega} N_0 \nabla_\theta \cdot \left(\frac{c\overline{\delta E_r}}{B}\hat{\theta}\right) = -\frac{i}{\omega} N_0 c \overline{\delta E_r} \nabla_\theta \left(\frac{1}{B^2}\right)$

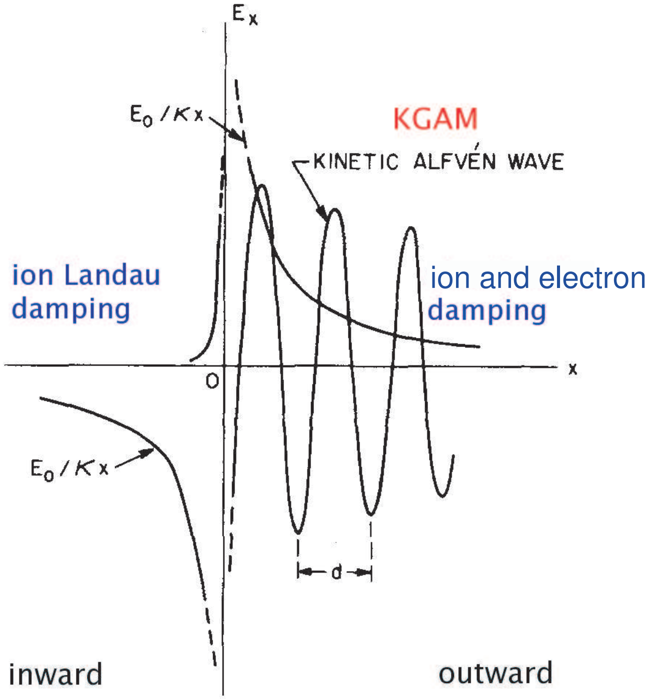
□ $\partial_r \overline{\delta J_r(r,t)} = 0 \Rightarrow$

$$\frac{\partial}{\partial r}\left\{ N_0(r) \left[\omega^2 - \frac{2\left(\gamma_i T_i + T_e\right)(r)}{m_i R_0^2} \right] \overline{\delta E_r} \right\} = 0$$

$\Rightarrow$ Singular solution at $\omega^2 = \omega^2_{GAM}(r)$

$\Rightarrow$ Similar to Alfvén resonance [Chen&Hasegawa POF 1974]

Kinetic GAM



- $\overline{\delta E_r}$ singular at r_0 where $\omega^2 = \omega^2_{GAM}$
  - $\Rightarrow$ $|k_r| \to \infty$ finite ion Larmor radius effects!
  - $\Rightarrow$ Linear mode conversion to Kinetic GAM (KGAM) $\Rightarrow$ propagating radially outward
  - $\Rightarrow$ Similar to, e.g., Kinetic Alfvén Wave (KAW) [Hasegawa&Chen POF 1976]

- Dispersion relation of KGAM
  - Assuming $1 \gg k_r^2\rho_i^2 \gg 1/q^2$ and including higher order $k_r^2\rho_i^2$ corrections in GAM

  $\Rightarrow$ δf_i expansion up to order $O[(\omega_d/\omega)^4]$ terms

$$\omega^2 = \omega^2_{GAM}(r) + Cb_i \qquad C > 0\ , b_i = k_r^2\rho_i^2$$

- $C > 0$, complicated expression, lengthy: can be obtained from [Zonca, Chen, Santoro, Dong PPCF 1998] as a limiting case, using the degeneracy of BAE and GAM spectra [Zonca&Chen 2006] in the long wavelength limit (see later)

  $\Rightarrow$ $b_i > 0$ when $\omega^2 > \omega^2_{GAM}$: propagation

  $\Rightarrow$ $b_i < 0$ when $\omega^2 < \omega^2_{GAM}$: cut-off

- Radial wave equation and mode conversion of GAM

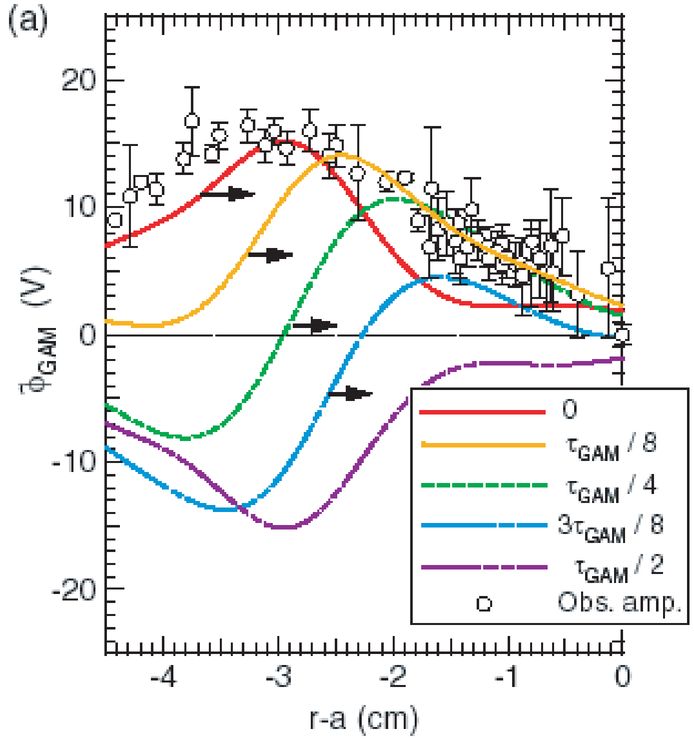
  - In nonuniform plasma $k_r = -i\partial/\partial r$

  $\Rightarrow$ Radial wave equation

$$\frac{\partial}{\partial r}\left\{N_0(r)\left[\rho_i^2(r)C(r)\frac{\partial^2}{\partial r^2}+\omega^2-\omega^2_{GAM}(r)\right]\overline{\delta E_r}\right\}=0$$

  $\Rightarrow$ Same as that for mode conversion of shear Alfvén wave [Hasegawa&Chen POF 1976]

Evidence of outward propagating GAM in JFT-2M [Ido etal. PPCF 2006]



GAM damping (large drift-orbits)

- Quasi-neutrality condition

$$\frac{e}{T_e}\left(\delta\phi - \overline{\delta\phi}\right) = -\frac{e}{T_i}\delta\phi + \langle J_0 \delta g_i \rangle \qquad \overline{\delta\phi} = \oint \frac{d\theta}{2\pi}\delta\phi$$

- Linear gyrokinetic equation for δg_i

$$\left(-i\omega + v_\parallel \frac{\partial}{\partial \ell} + i\omega_d\right)\delta g = -iJ_0 \frac{e}{T} F_0 \omega \delta\phi$$

$$\mathbf{k}_\perp \simeq \hat{\mathbf{r}} k_r \ , \quad J_0 = J_0(\rho_i k_r) \ , \quad \langle \cdots \rangle \equiv \int d^3\mathbf{v} \, (\cdots) \ , \quad \omega_d = v_{dr} k_r = \underbrace{\hat{\omega}_d \sin\theta}_{\text{geodesic}} \ ,$$

$$\hat{\omega}_d = k_r \rho_t \frac{v_t}{R}\left(\frac{v_\perp^2}{2v_t^2} + \frac{v_\parallel^2}{v_t^2}\right) = \omega_{dt}\left(\frac{v_\perp^2}{2v_t^2} + \frac{v_\parallel^2}{v_t^2}\right)$$

- Simple limiting case

$$1 \gg \rho_i^2 k_r^2 \gg 1/q^2 \quad \Rightarrow \left\{ \begin{array}{ccccc} |\omega| & \gg & |\omega_d| & \gg & |v_\parallel \partial/\partial\ell| \\ v_t/R & : & k_r \rho v_t/R & : & v_t/(qR) \end{array} \right.$$

- Solve the linear ion gyrokinetic equation for large magnetic drift orbits

$$\Rightarrow \quad \delta g \simeq \left(1 - \frac{\omega_d}{\omega}\right)^{-1} J_0 \frac{e}{T} F_0 \delta\phi \simeq \left[\left(1 + \frac{\omega_d}{\omega} + \frac{\omega_d^2}{\omega^2}\right) - i\pi\omega\delta\left(\omega - \omega_d\right)\right] J_0 \frac{e}{T} F_0 \delta\phi$$

- Solving quasi-neutrality gives $\delta\tilde{\phi} = \delta\phi - \overline{\delta\phi}$ and GAM D.R.

$$\Rightarrow \quad \delta\tilde{\phi} \simeq \frac{T_e}{T_i} \frac{\omega_{dt}}{\omega} \overline{\delta\phi} \sin\theta \, , \qquad 1 - \left\langle J_0^2 \frac{F_0}{N_0} \right\rangle = \frac{1}{2} \left\langle \left(\frac{\hat{\omega}_d}{\omega}\right)^2 \frac{F_0}{N_0} \right\rangle + \frac{1}{2} \frac{T_e}{T_i} \left(\frac{\omega_{dt}}{\omega}\right)^2$$

$\Rightarrow \quad \omega^2 = \omega_{GAM}^2$, with $\omega_{GAM}^2 = (7/4 + T_e/T_i) v_{ti}^2 / R_0^2$

- Collisionless damping due to resonances with high transit harmonics

$$\gamma_{GAM}/\omega_{GAM} \simeq -\left(1/k_r^2 \rho_i^2\right) \exp\left(-\omega_{GAM}/\omega_{dti}\right) \, , \qquad k_r \rho_i q^2 > 1$$

- For small drift orbits, $k_r \rho_i q^2 < 1$ (known case), $\omega_{ti} = v_{ti}/(qR_0)$

$$\gamma_{GAM}/\omega_{GAM} \simeq -\left(\pi^{1/2}/2\right) q^2 \left(\omega_{GAM}^3/\omega_{ti}^3\right) \exp\left(-\omega_{GAM}^2/\omega_{ti}^2\right)$$

BAE – GAM degeneracy

- Kinetic expression of the GAM dispersion relation is degenerate with that of the low frequency shear Alfvén accumulation point (BAE) in the long wavelength limit (no diamagnetic effects).

- This degeneracy is not accidental [Zonca&Chen PPCF 2006, IAEA 2006, NF 2007]and is due to the identical dynamics of GAM ($n = m = 0$) and s.A. wave near the mode rational surface ($nq \simeq m$) under the action of geodesic curvature, the difference between the two branches is in the mode polarization

- In reference to experimentalobservations of modes at the GAM frequency, besides measuring the mode frequency, it is necessary to measure polarization and toroidal mode number to clearly identify the mode.

- GAM excitation: $n = m = 0 \Rightarrow$ no linear excitation mechanism by spatial non-uniformity. Only instability mechanism is via velocity space: e.g., intense high-speed drifting beam such that $\partial F_b / \partial v_\parallel > 0$ at $v_\parallel \approx q v_{ti}/p$; p =positive integers.

- BAE excitation: $n \simeq m \neq 0 \Rightarrow$ excitation by both energetic ions (at the longest wavelengths) as well as via the AITG mechanism (at the shortest wavelengths) [Zonca etal. POP 1999]. Confirmed by observations on DIII-D [Nazikian etal. PRL 2006].

- BAE – GAM degeneracy can be used to exploit a variety of known results from the kinetic theory of low-frequency shear Alfvén waves in the long wavelength limit $k_r^2\rho_i^2 \ll 1/q^2$, typical of energetic ion driven BAE/EPM and long-wavelength AITG [Zonca etal. PPCF 1998, POP 1999].

- Here, we used $k_r^2\rho_i^2 \gg 1/q^2$ as simple limit for discussions of the KGAM dispersion relation $\omega^2 = \omega_{GAM}^2 + Ck_r^2\rho_i^2$, showing KGAM radially propagates for $\omega^2 > \omega_{GAM}^2(r)$.

- Nonlinear excitations of GAM by ambient Drift Wave (DW) turbulence (see later), requires exploration of shorter wavelengths $k_r\rho_i q^2 > 1$ for correct description of wave collisionless damping for large magnetic drift orbits (this talk).

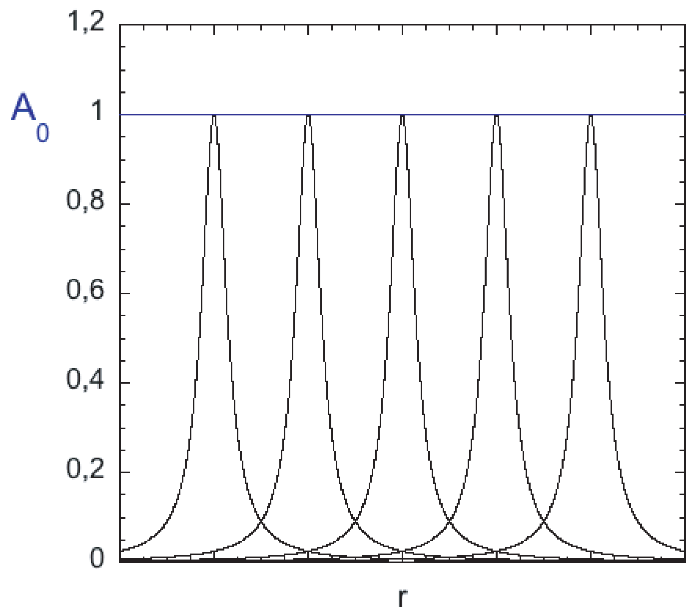
Nonlinear excitations of KGAM

□ Coherent 3-wave interactions [Chen, Lin, White POP 2000]

- Linear parametric instability

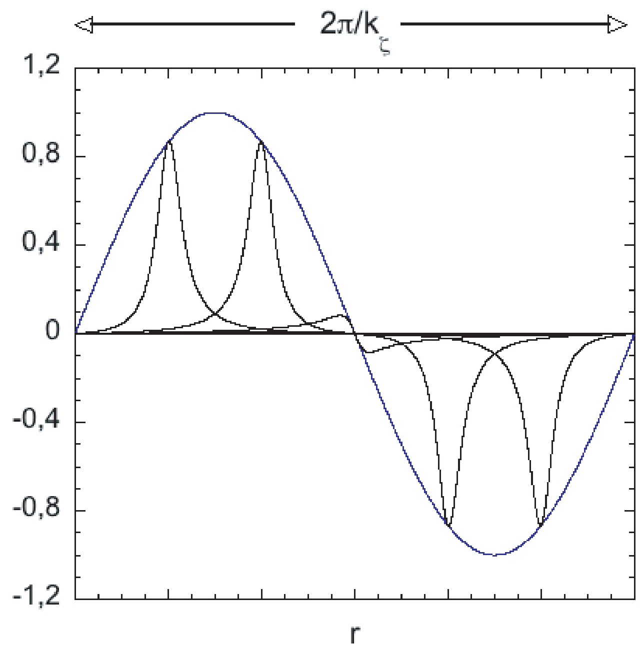
- Pump DW (ITG) $\Rightarrow \delta\Phi_0 : (\omega_0, \mathbf{k}_0)$

$$\delta\Phi_0 = \left[A_0 e^{-in_0\zeta} \sum_m e^{im\theta - i\omega_0 t} \Phi_0(n_0 q - m) + \text{c.c.} \right]$$



- Zonal Mode (KGAM) $\Rightarrow \delta\Phi_\zeta : (\omega_\zeta, \mathbf{k}_\zeta)$ and Pump DW modulation

$$\delta\Phi_\zeta = \left[A_\zeta e^{ik_\zeta r - i\omega_\zeta t} + \text{c.c.} \right]$$



- Lower-sideband DW (ITG) $\Rightarrow \delta\Phi_- : (\omega_-, \mathbf{k}_-)$; $\omega_- = \omega_\zeta - \omega_0$; $\mathbf{k}_- = \mathbf{k}_\zeta - \mathbf{k}_0$;

$$\delta\Phi_- = \left[A_- e^{in_0\zeta + ik_\zeta r - i\omega_- t} \sum_m e^{-im\theta} \Phi_0^*(n_0 q - m) + \text{c.c.} \right]$$

- Resonant decay

  - Frequency mismatch, $\Delta = \omega_0 - \omega_-$, and sideband damping, γ_{ds}
  - Resonant decay condition: $\omega_0 - \omega_- = \omega_{GAM}$; $\omega_\zeta = \omega_{GAM} + i\gamma_\zeta$

$$(\gamma_\zeta + \gamma_g)(\gamma_\zeta + \gamma_{ds}) = \alpha_i \frac{c^2}{2B^2} k_\theta^2 k_\zeta^2 |A_0|^2 \equiv \gamma_{RD}^2$$

  - GAM damping, γ_g, and $\alpha_i = \mathrm{O}(1)$
  - Threshold condition, $\gamma_\zeta = 0 \Rightarrow \gamma_g \gamma_{ds} = (\gamma_{RD}^2)_{th}$
  - Above threshold, $\gamma_{RD}^2 \gg (\gamma_{RD}^2)_{th}$

$$\gamma_\zeta \simeq \gamma_{RD} \propto |k_\zeta||A_0|$$

- Nonlinear excitation favors short (zonal) radial wavelengths ⇒ KGAM is excited
- Nonlinear dynamics

  - $\delta\Phi_-$, $\delta\Phi_\zeta$ growth ⇒ depletes the pump $\delta\Phi_0$

  ⇒ 3-wave nonlinear system with prey-predator self-regulation

$$
\begin{aligned}
(d/dt - \gamma_{0n})\,\delta A_{0n} &= -\frac{c}{B} k_{\theta n} k_g \delta A^*_{-n} \delta A_\zeta \\
(d/dt + \gamma_{-n})\,\delta A_{-n} &= \frac{c}{B} k_{\theta n} k_g \delta A^*_{0n} \delta A_\zeta \\
(d/dt + \gamma_g)\,\delta A_\zeta &= \frac{c}{2B} \alpha_i k_{\theta n} k_g \delta A_{0n} \delta A_{-n}
\end{aligned}
$$

⇒ Driven-dissipative system: limit cycle, period doubling, route to chaos, strange attractor [Wersinger etal PRL 1980].