Extraction of coherent structures from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets

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Motivation

Radial transport at the edge of tokamaks is known to be dominated by turbulent processes.

Extensive studies of turbulent transport of plasma density at the edge of plasma by means of Langmuir probes.

Diagnostics observe a turbulent transport of the plasma density in the scrape-off layer (SOL): superposition of convective events, which are responsible for the transport of matter over long radial distances and of background turbulence.

Different extraction methods have been developed, which are based on signal clipping, see e.g. Antar et al., PoP, 8 (2001).

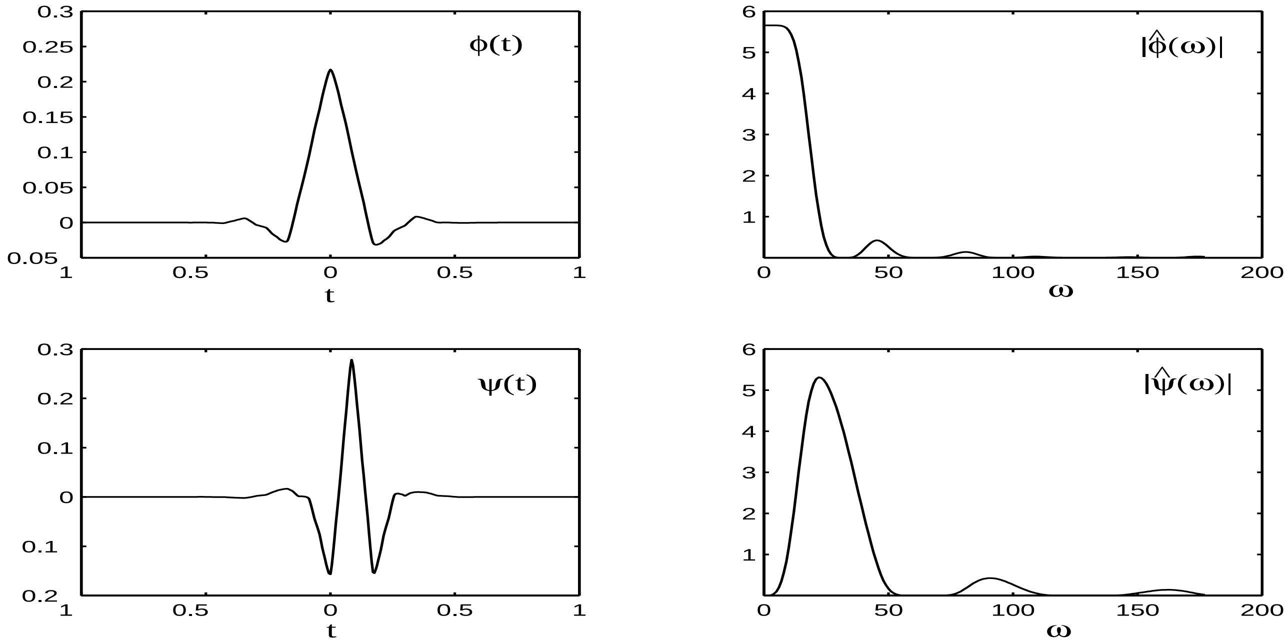
Here: new method to extract coherent bursts from turbulent signals based on the orthogonal wavelet representation.

Orthogonal wavelet representation

The signal is thus developed into an orthogonal wavelet series,

$$S(t) = \overline{S}_{00}\phi_{00}(t) + \sum_{(j,i)\in\Lambda_J} \widetilde{S}_{ji}\,\psi_{ji}(t)$$

where ϕ_{00} is the scaling function and ψ_{ji} the corresponding wavelets, i is the index for the instant t and j the index for the time scale τ.



Coifman 12 wavelet. Top: scaling function $\phi(t)$ and the modulus of its Fourier transform $|\widehat{\phi}(\omega)|$. Bottom: wavelet $\psi(t)$ and the modulus of its Fourier transform $|\widehat{\psi}(\omega)|$.

Iterative algorithm for wavelet denoising

Signal f = Coherent signal + Incoherent noise

Iterative algorithm :

1. The signal f, sampled on N points, is projected on a wavelet basis to obtain the coefficients $\tilde{f}$.
2. Set $n=1$ and $\tilde{f}_I = \tilde{f}$.
3. Compute the threshold $T_n = \left(2 \langle \tilde{f}_I \rangle^2 \ln N\right)^{\frac{1}{2}}$.
4. The incoherent coefficients $\tilde{f}_I$ are those for which $|\tilde{f}| < T_n$.
5. If $T_n \neq T_{n-1}$ go to 3. and set $n=n+1$, else go to 6.
6. The coherent signal f_C is reconstructed from $|\tilde{f}| \geq T_n$.

   The incoherent noise is computed as $f_I = f - f_C$.

Wavelet-based denoising

1. Goal:
   Extraction of coherent vortices
   from a noise which will then be modelled
   to compute the flow evolution.

2. Apophatic principle:
   - **no hypothesis on the vortices**,
   - **only hypothesis on the noise**,
   - simplest hypothesis as our first choice.

3. Hypothesis on the noise:

   $$f_B = f + w$$

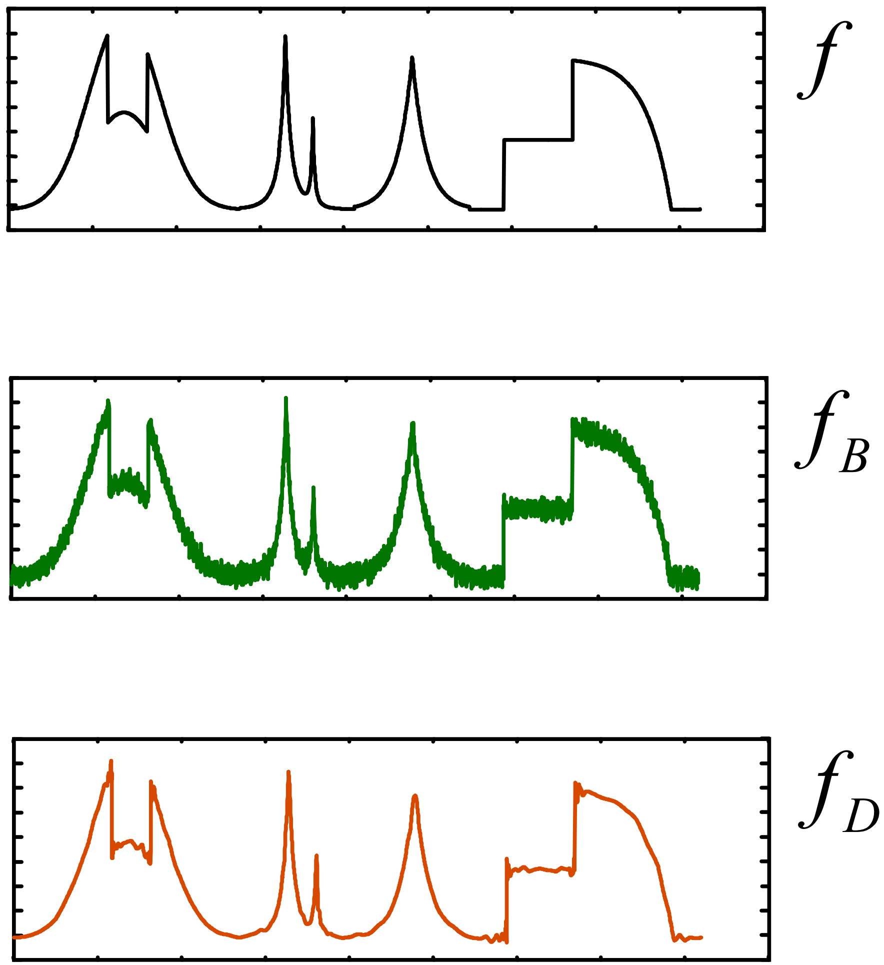
   w : Gaussian white noise,
   σ : variance of the noise,
   N :number of coefficients.

4. Computation of the threshold:

   $$\varepsilon_D = \sqrt{2\sigma \ln(N)}$$

5. Denoised signal:

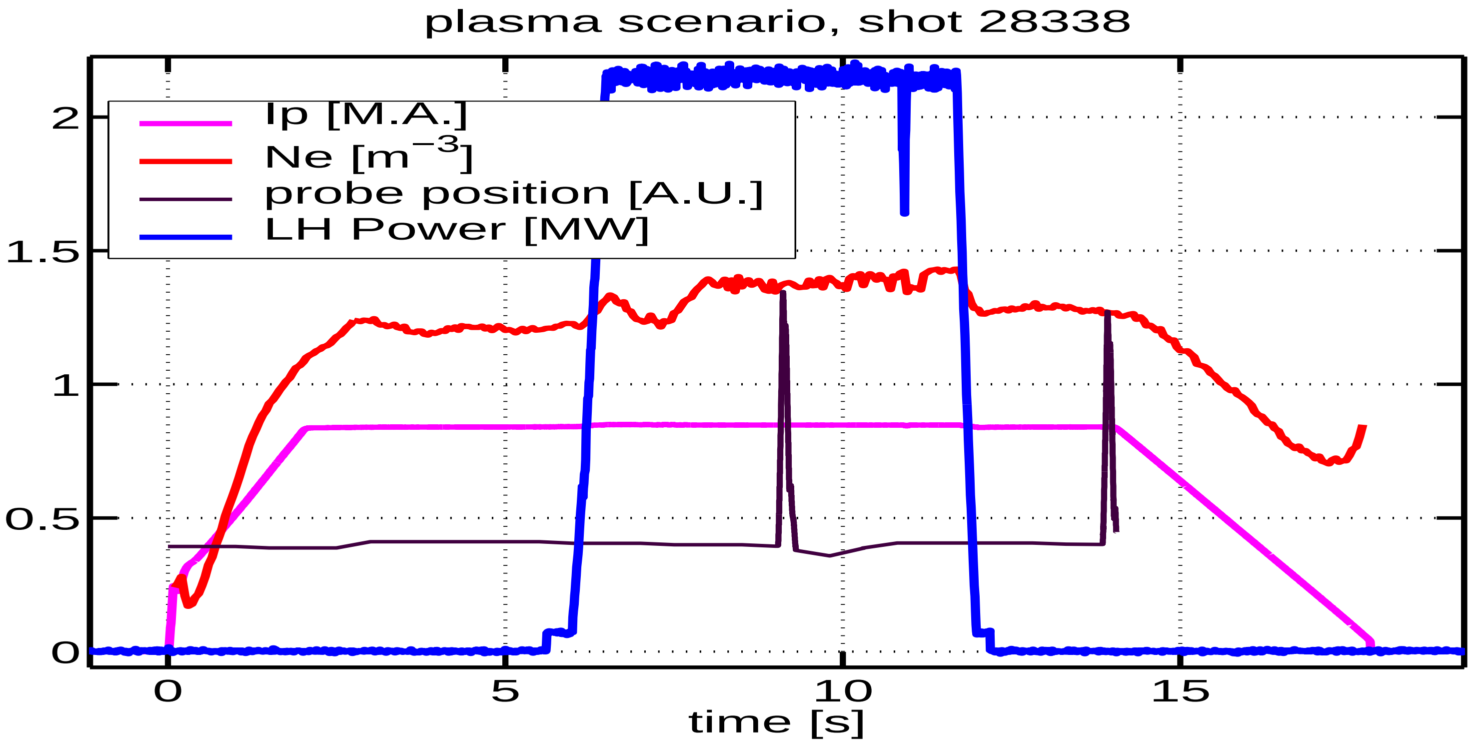
   $$f_D = \sum_{\lambda : |\tilde{f}_\lambda| < \varepsilon} \tilde{f}_\lambda \psi_\lambda$$



*Appl. Comput. Harmonic Analysis, **18** (2), 177-185*

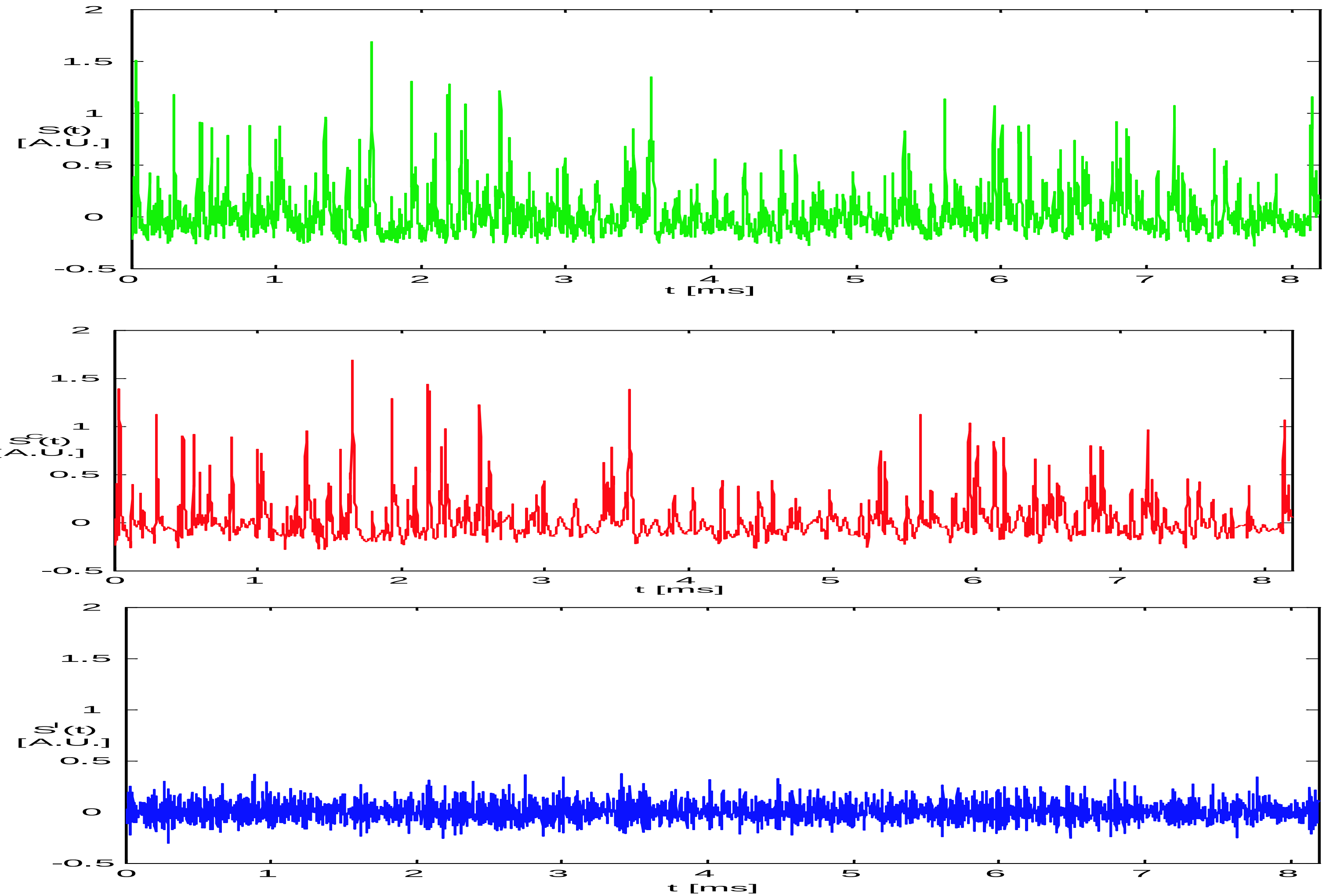
Azzalini, Farge, Schneider
Appl. Comput. Harm. Anal., 2005

Application to turbulent edge plasma



Plasma scenario of the shot 28338 from the tokamak Tore Supra, Cadarache. The duration of the shot is 18 s. The plasma density fluctuations are measured by a fast reciprocating Langmuir probe. When the probe is 2.8 cm away from the LCFS in the SOL, the signal is acquired during time windows of 8 ms.

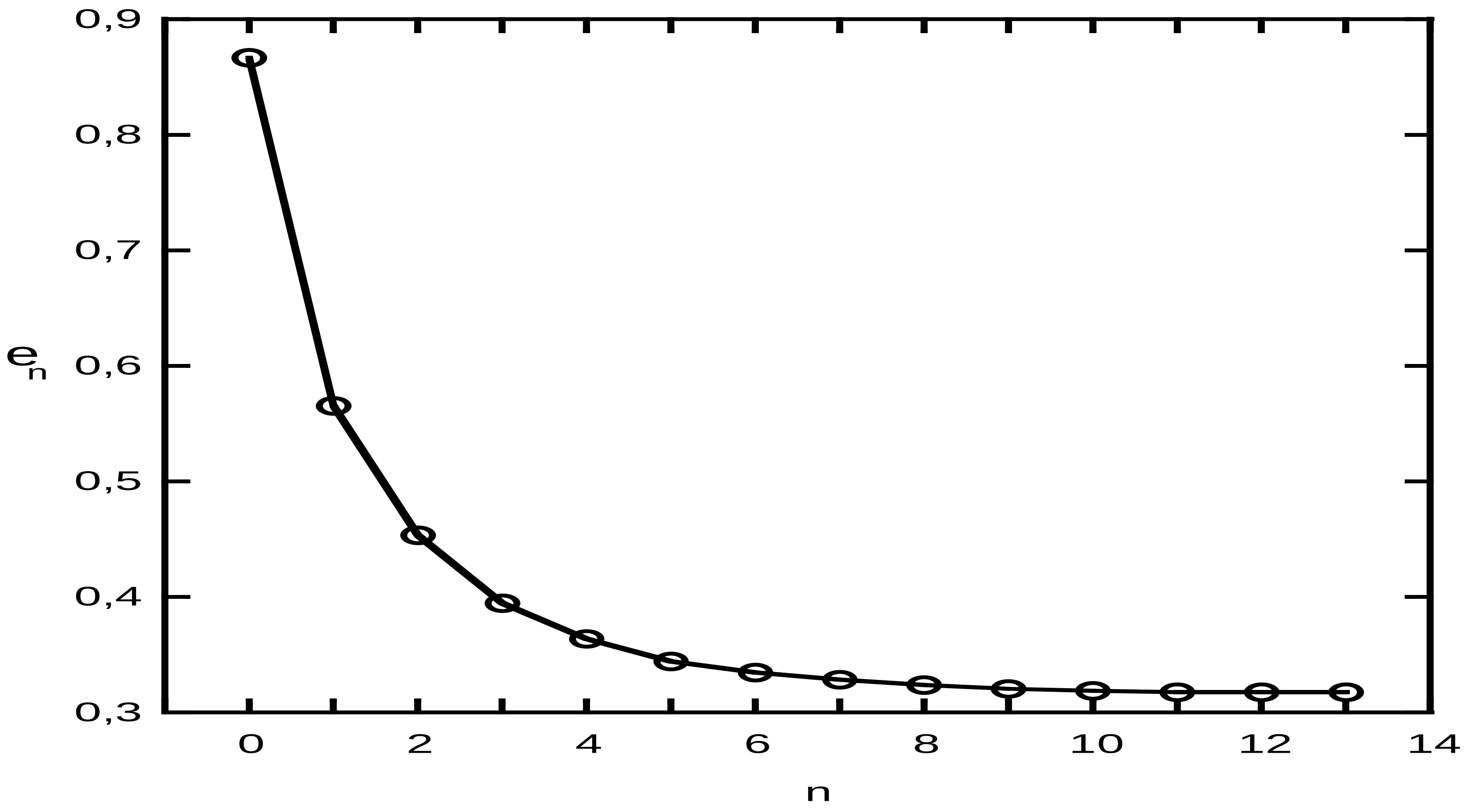
Extraction of coherent bursts(I)



Signal $S(t)$ of duration 8.192 ms, corresponding to saturation current fluctuations measured at 1 MHz in the SOL of the tokamak Tore Supra, Cadarache. Top: total signal S. Middle: coherent part S^C. Bottom: incoherent part S^I.

Extraction of coherent bursts (II)

Optimal threshold value after $n = 12$ iterations of the algorithm.



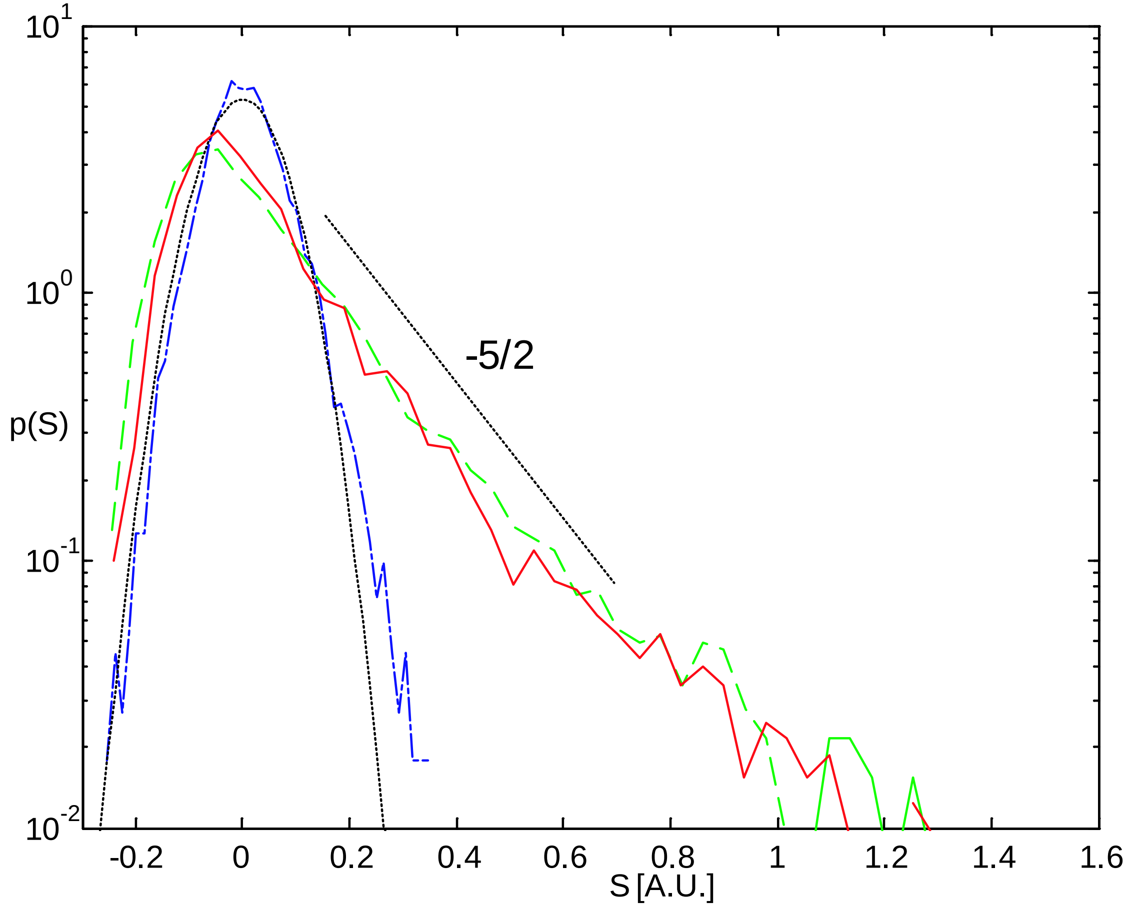
Threshold value ϵ_n versus iteration number n.

Extraction of coherent bursts (III)

Statistical properties of the signal $S(t)$ from the tokamak Tore Supra, Cadarache, for the signal and its coherent and incoherent components using the Coifman 12 orthogonal wavelet.

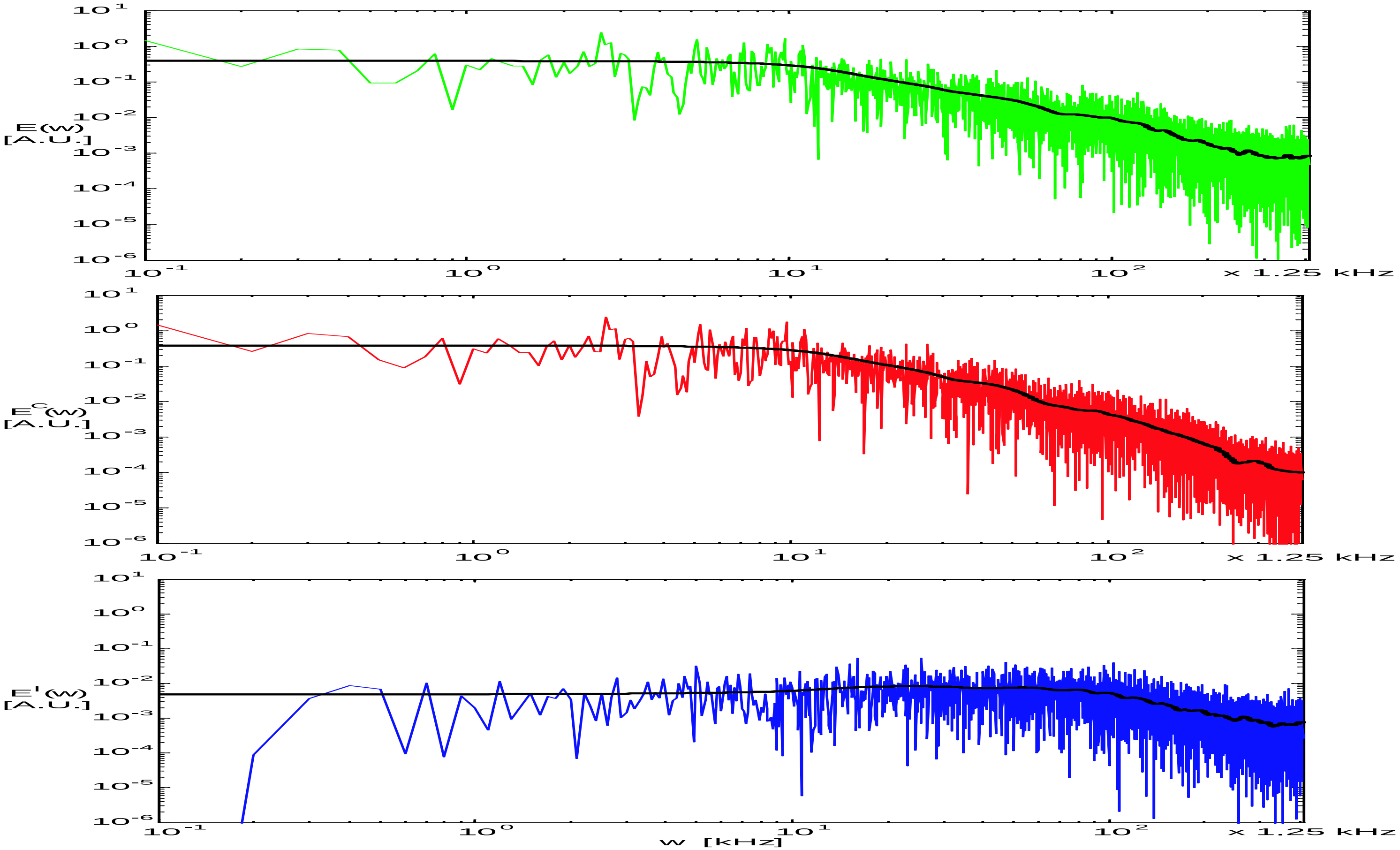
Signal	total S	coherent S^C	incoherent S^I
# of coefficients	8192	479	7713
% of coefficients	100 %	5.8 %	94.2 %
min value	-0.284	-0.282	-0.307
max value	1.689	1.686	0.374
mean value	0.019	0.019	$< 10^{-11}$
Variance σ^2	0.0417	0.0361	0.0056
% of variance	100 %	86.6 %	3.4 %
Skewness	2.564	2.842	0.383
Flatness	12.001	14.224	4.026

Probability Distribution Functions in log–lin coordinates



Probability density function $p(S)$, estimated using histograms with 50 bins. PDF of the total signal S (green dashed line), of the coherent component S^C (red solid line) and of the incoherent component S^I (blue dotted-dashed line), together with a Gaussian fit with variance σ_I^2 (black dotted line).

Fourier spectrum and modified periodogram



Top: total signal $S(t)$, middle: coherent component $S^C(t)$ and bottom: incoherent component $S^I(t)$. The periodogram is plotted in green, red and blue for the total, coherent and incoherent signal, respectively. Superimposed are the modified periodograms by first tapering the data with a raised cosine window (affecting 40 data points at each boundary), and then convolving the periodogram with a Gaussian window (with standard deviation of 40 data points). (black thick line).

Wavelet spectrum (I)

Scalogram: distribution of the variance of the signal scale per scale:

$$\widetilde{E}_j = \frac{1}{2} \sum_{i=0}^{2^j-1} \left(\widetilde{S}_{ji}\right)^2 \ .$$

Relation between the scale index j and the frequency ω: $\omega_j = \frac{\omega_\psi}{2^j}$.

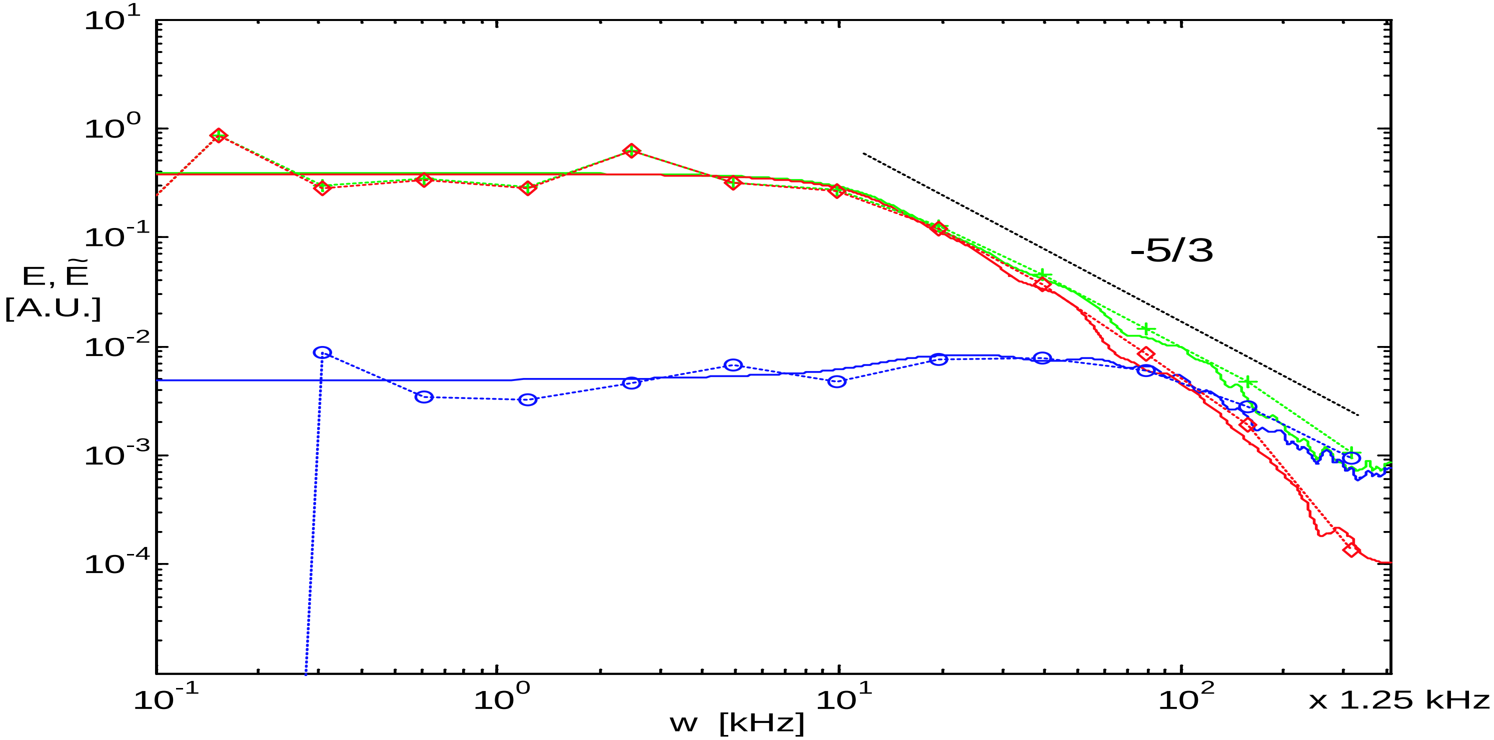
Wavelet spectrum: $\widetilde{E}(\omega_j) = \widetilde{E}_j / \omega_\psi$,

with ω_ψ being the centroid frequency of the mother wavelet whose value is $\omega_\psi = 1.3$ for the Coifman 12 wavelet used here.

Smoothed version of the Fourier spectrum, the smoothing kernel being the square of the Fourier transform of the wavelet, since

$$\widetilde{E}(\omega) = \frac{1}{\omega_\psi} \int_0^{+\infty} E(\omega') \left| \widehat{\psi} \left(\frac{\omega_\psi \, \omega'}{\omega} \right) \right|^2 d\omega' \ .$$

Wavelet spectrum (II)



Wavelet spectra $\widetilde{E}(\omega_j)$ (lines with symbols) and modified periodograms $E(\omega)$ (lines) of the total signal S (green and $+$), of the coherent signal S^C (red and $\diamond$) and of the incoherent signal S^I (blue and $\circ$).

Intermittency: scale dependent flatness (I)

Moments of the wavelet coefficients:

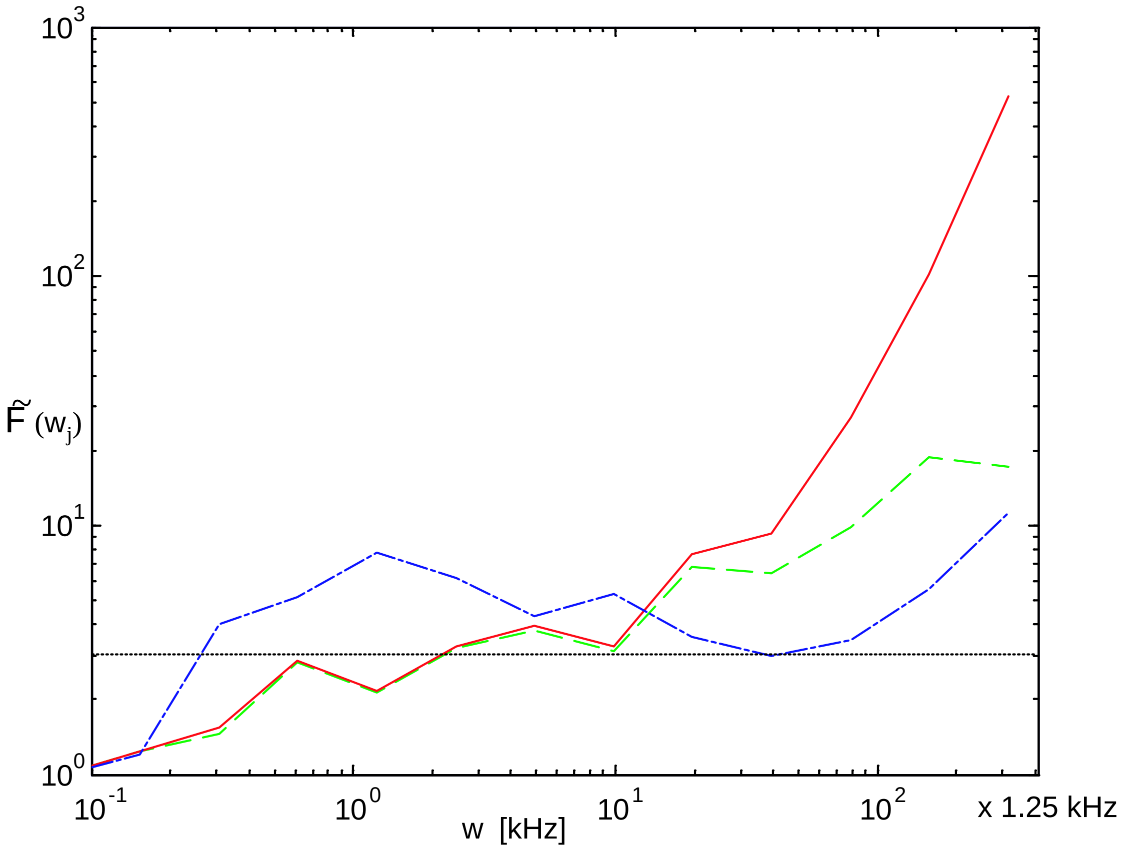
$$\widetilde{\mathcal{M}}_j^p = \frac{1}{2^j} \sum_{i=0}^{2^j-1} \left(\widetilde{S}_{ji}\right)^p \; .$$

The scale dependent flatness is then defined as

$$\widetilde{\mathcal{F}}_j = \frac{\widetilde{\mathcal{M}}_j^4}{\left(\widetilde{\mathcal{M}}_j^2\right)^2} \, .$$

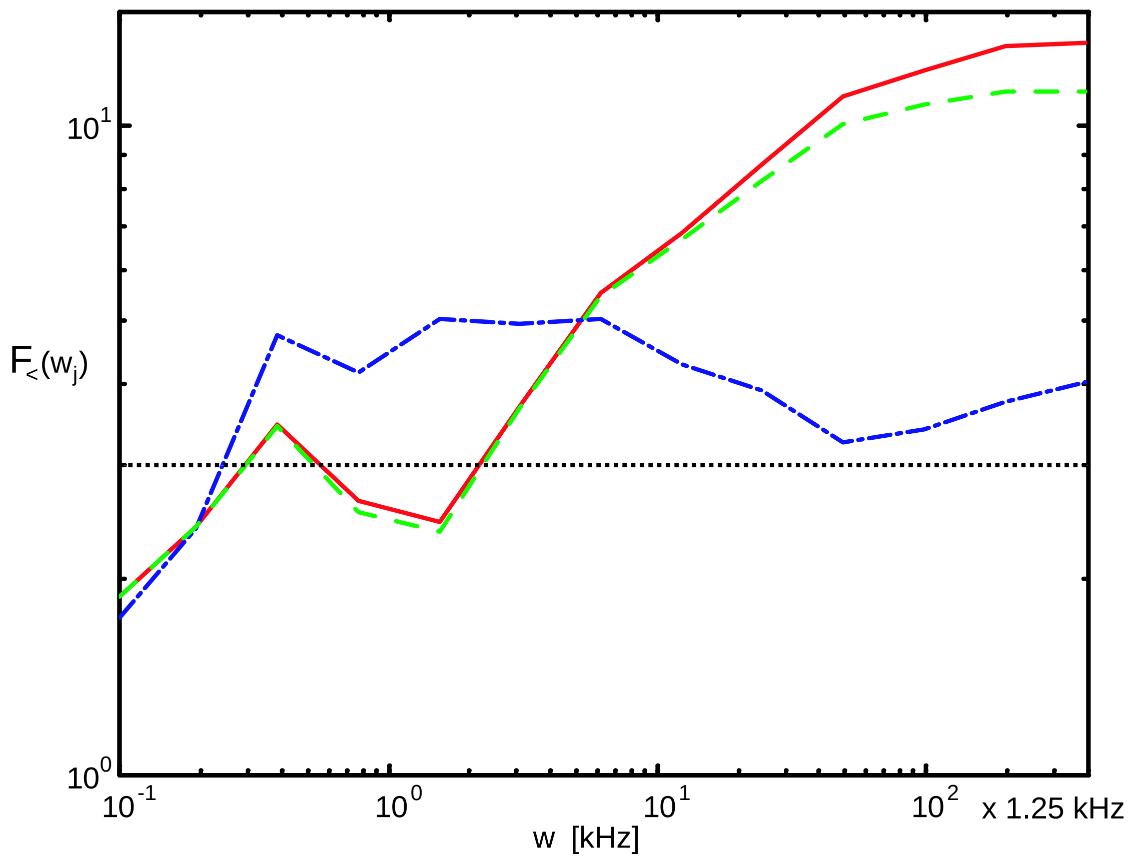
The relation between scale and frequency allows to express the flatness as function of the frequency ω_j, similarly to the wavelet spectrum. Note that Gaussian white noise, which is by definition non–intermittent, would yield a flatness equal to three for all frequencies.

Intermittency: scale dependent flatness (II)



Flatness of the band-pass filtered signal $\widetilde{\mathcal{F}}$ versus frequency ω_j for the total signal S (green dashed line), of the coherent signal S^C (red solid line) and of the incoherent signal S^I (blue dotted-dashed line). The horizontal dotted line $\widetilde{\mathcal{F}}(\omega_j) = 3$ corresponds to the flatness of a Gaussian process.

Flatness $\mathcal{F}_<$ of the low-pass filtered signal



Flatness of the low-pass filtered signal $\mathcal{F}_<$ versus frequency ω_j for the total signal S (green dashed line), of the coherent signal S^C (red solid line) and of the incoherent signal S^I (blue dotted-dashed line). The horizontal dotted line $\mathcal{F}_>(\omega_j) = 3$ corresponds to the flatness of a Gaussian process.

Conclusions

Wavelet-based recursive method to extract coherent bursts out of turbulent signals without any adjustable parameter.

Fast algorithm with linear complexity (based on fast wavelet transform).

Application to ion saturation current measured in the SOL of the tokamak Tore Supra.

Extraction of coherent bursts (containing most of the density variance, correlated with non-Gaussian statistics) from an incoherent background noise (almost decorrelated and quasi-Gaussian).

Non-Gaussianity of the PDF of the coherent component increases with the frequency, which confirms that the bursts are highly intermittent.

In contrast, the incoherent component remains quasi-Gaussian up to high frequencies, which confirms the non intermittency of the background noise.

Conjecture: the coherent bursts are due to organized structures produced by nonlinear interactions and responsible for turbulent transport. The incoherent background corresponds to the turbulent fluctuations which only contribute to turbulent diffusion.

Ref: Farge, Schneider & Devynck. *Phys. Plasmas*, 13, 042304, 2006