Dynamics of the L-H and H-L Transitions, and Implications for the Pedestal

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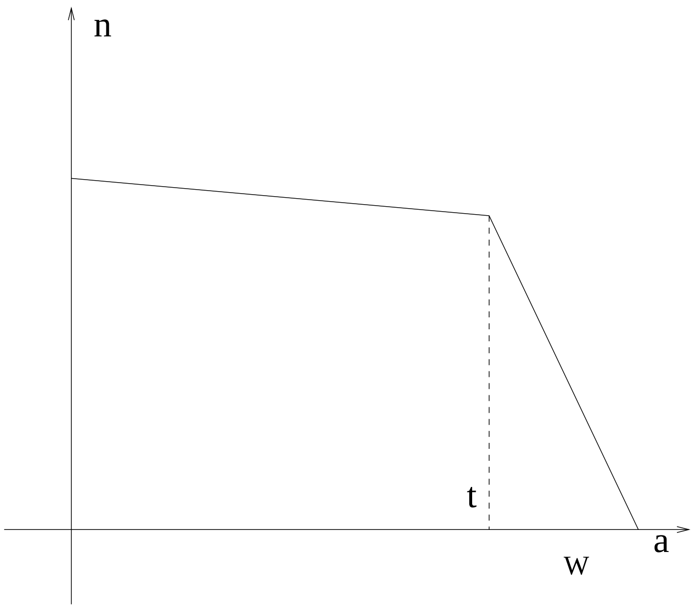
Transport barrier



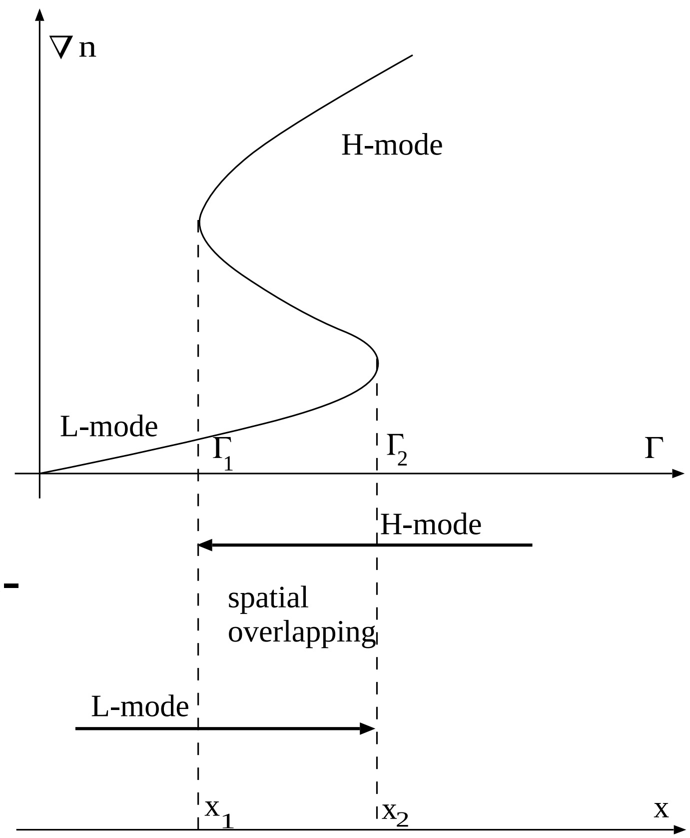
- region of reduced (turbulent) transport relative to surroundings
- evident profile steepening

-definition

- Key issues:

-threshold

-width/extent -pedestal



Transport bifurcation evolution

- bistable flux model (minimal)

– Hinton ’91, Heat flux

$$-Q = \left(\chi_{nc} + \frac{\chi_T}{1 + \alpha\,(du_E/dx)^2} \right) \nabla T$$

χ_{nc} neoclassical –H-mode survivor, χ_{nc}, $\chi_{Turbulent}$ both const.
u_E - from radial force balance, $\alpha \sim 1/\gamma^2$

- two stable branches, H-mode gradient MHD-stability limited
- phase coexistence region
- transition may occur in *co-existence region* at any point
- key question: where (when) does *it actually* occur?
- flux suppression factor depends on both pressure and density gradients, suggests two field model at least (p, n)

Two field problem → Minimal Acceptable Model

- use two component model introduced by Hinton and Staebler, '93
- two equations for diffusive particle and energy transport
- flux suppression factors originating from $\mathbf{E} \times \mathbf{B}$ flow shear

particles:

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial x}\left[D_0 + \frac{D_1}{1+\alpha\,(du_E/dx)^2}\right]\frac{\partial n}{\partial x} = S(x)$$

heat:

$$\frac{3}{2}\frac{\partial p}{\partial t} - \frac{\partial}{\partial x}\left[\chi_0 + \frac{\chi_1}{1+\alpha\,(du_E/dx)^2}\right]\frac{\partial p}{\partial x} = H(x)$$

(ideally should be supplemented with the toroidal momentum transport, impurites...)

- S (fueling) is concentrated at the edge, $x \simeq a$ –edge fueling
- H (heating) at plasma center ($x = 0$) on -axis deposition
- equations are coupled because

$$\frac{du_E}{dx} \simeq -\frac{c}{eBn^2}\frac{\partial n}{\partial x}\frac{\partial p}{\partial x} \rightarrow E_r' \; coupling$$

$\chi_{1,} D_1 \rightarrow$ pre-transition,
$\chi_0, D_0 \rightarrow$ post-transition ($\neq$ neo, necessarily)

Reduction of the Model

$$g_1 = -\frac{dn}{dx},\; g_2 = -\frac{dp}{dx},$$

quasi-stationary situation:
–*exact* relation between gradients of p and n

$$g_2 = \frac{QD_1 g_1}{\chi_1 \Gamma - (D_0 \chi_1 - \chi_0 D_1)\, g_1}$$

arrive at effectively one field evolution

$$\frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2} \left[g + \frac{\lambda g}{1 + \beta(x) g^4 (1 + \theta g)^{-2}} - \Gamma_1(x) \right]$$

$\lambda = D_1/D_0$, $\beta = \alpha Q^2 D_1^2 / \chi_1^2 \Gamma^2$, $\theta = (\chi_0 D_1 - D_0 \chi_1)/\chi_1 \Gamma$, $\Gamma_1 = \Gamma/D_0$

-*decoupled equations*

2 field $\rightarrow$ 1 field (but more complex functional form)

N.B. *Analytical Part* of Hinton - Stabler '93

$$\Rightarrow \frac{\chi_1}{\chi_0} = \frac{D_1}{D_0}$$

so, $g_2 = QD_1 g_1 / \chi_1 \Gamma$

but

$$? \quad \frac{\chi_1}{\chi_0} = \frac{D_1}{D_0} \quad ?$$

For ES turbulence: $\chi_1 \sim D_1$
Post transition:

$$\chi_0 \sim \chi_{neo}$$

(with squeezing modes)

$$D_1 \ll \chi_{neo}$$

$\lambda = D_1/D_0 > \lambda_{crit} \rightarrow 2-8$ depending on $\theta = (\chi_0 D_1 - D_0 \chi_1)/\chi_1 \Gamma$
and physics of D_1 uncertain....
$\Rightarrow$ non-ELM particle transport in pedestal ??

stationary solutions

$\rightarrow$what is required for phase co-existence?
need to find roots of the equation

$$g + \frac{\lambda g}{1 + \beta(x) g^4 (1+\theta g)^{-2}} = \Gamma_1$$

$\Rightarrow$phase coexistence criterion

$$\Pi_- < \sqrt{Q\Gamma} < \Pi_+$$

$$\Pi_\pm \equiv \left(\frac{y_\pm}{\alpha}\right)^{1/4} \frac{1+\lambda+y_\pm}{1+y_\pm} D_0 \sqrt{\frac{\chi_1}{D_1}}, \quad y_\pm = \frac{3\lambda}{2} - 1 \pm \frac{3}{2}\sqrt{\lambda\left(\lambda - \frac{16}{9}\right)}$$

Solution requires regularization$\Rightarrow$ Coexistence $\not\Rightarrow$ Actual Transition

where **is** transition?

Hyperdiffusion regularization- Reduced Transition Model

$$\frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2}\left[g + \frac{\lambda g}{1+\beta(x)g^4\left(1+\theta g\right)^{-2}} - \Gamma_1(x) - \varepsilon^2 \frac{\partial^2 g}{\partial x^2}\right] \rightarrow \textbf{Maxwell}$$

Other Approaches to Regularization:

Variational approach

$$\frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2}\frac{\delta\Lambda}{\delta g}$$

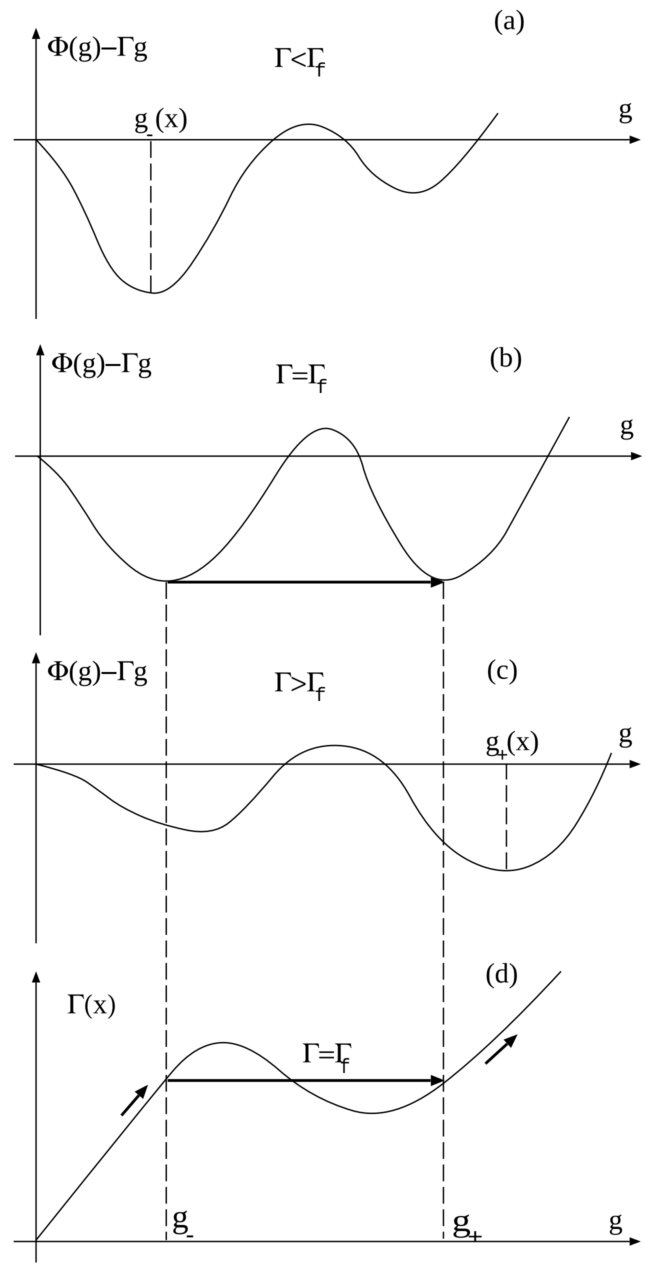
where

$$\Lambda = \int [\Phi(g) - \Gamma_1 g]\, dx$$

One can verify that

$$\frac{d\Lambda}{dt} \leq 0$$

so that the “true” stationary solution requires a global minimum of Λ. This leads to the Maxwell rule as well.



Another route to regularization....

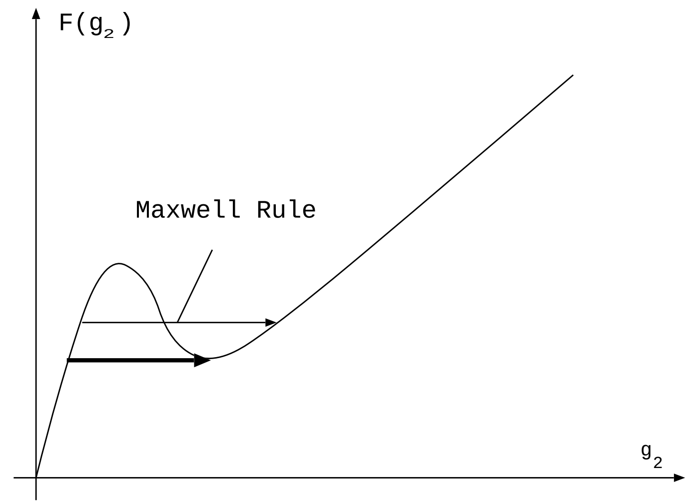
$\Rightarrow$ add physics to break factorization of u'_E...

Curvature effects of the pressure profile

- second derivative of the pressure profile

$$\frac{du_E}{dx} \simeq -\frac{c}{eBn^2}\frac{\partial n}{\partial x}\frac{\partial p}{\partial x} + \frac{c}{eBn}\frac{\partial^2 p}{\partial x^2}$$

bifurcation problem (reduction to one field still works)



$$F\left(g_2, \mu\right) \equiv$$

$$\chi_0 g_2 + \frac{\chi_1 g_2}{1+\left(\frac{\sigma g_2^2}{1+\kappa g_2} + \mu \frac{dg_2}{dx}\right)^2} = Q\left(x\right)$$

$$\sigma = \sqrt{\alpha}\frac{c}{eBn^2}\frac{\Gamma\chi_1}{QD_1};$$

$$\kappa = \frac{D_0\chi_1 - \chi_0 D_1}{D_1 Q};\ \mu = \sqrt{\alpha}\frac{c}{eBn}$$

Conclusions

- ⇒hyperdiffusion regularization, variational principle and noise lead to the <u>Maxwell rule</u>
- ⇒new rule for barrier location is established: in the finite pressure curvature case it occurs at the <u>lowest possible value of thermal flux</u> (for co-existence)
- in the core plasma, the curvature of the pressure profile is shown to be able to produce an L→H transition even if the density profile is flat (i.e. stable)
- △curvature driven transition is different from the standard case in which the density and pressure barriers are coupled

⇒What Does this All Mean, in Practice....

⇒for "standard" minimal model:

- ∃ co-existence region
- scale↔ λ_N (*tiny* in ITER)

- $P_{crit} \leftrightarrow D_0$ (*very poorly understood*)

- hysteresis $O(1/2)$ expectation i.e. Maxwell back-transition naive back transition (not good news)

$\Rightarrow$including <u>pressure curvature:</u>

-transition for $Q_0 = Q_{min}$

-hysteresis uncertain

-transition possible for weak flat $\nabla n \rightarrow$ beat λ_N??

-*dynamics* require further study.

Zero dimension

L-H transition model, Kim and Diamond (2003). Operates on four variables in local approximation.

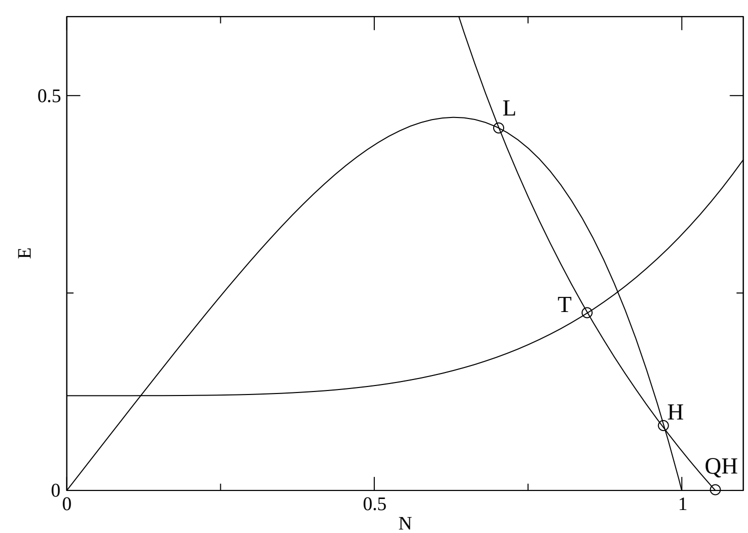
- drift wave turbulence level E
- drift wave driving temperature gradient N
- zonal flow velocity $\sqrt{U}$
- mean flow shear follows instantly the temperature gradient, $V \propto N^2$, where d is a constant

$\triangle$dynamical system, driver $q(\tau)$ (heat source) – main control parameter

$$\frac{dE}{dt} = \left(N - N^4 - E - U\right) E$$

$$\frac{dU}{dt} = \vartheta\left(\frac{E}{1+\zeta N^4} - \eta\right) U$$

$$\frac{dN}{dt} = q(t) - (\rho + \sigma E) N$$



L- and H-modes: $U = 0$ but $E \neq 0$, T-mode: $E \neq 0$ and $U \neq 0$, QH-mode: $U = E = 0$.

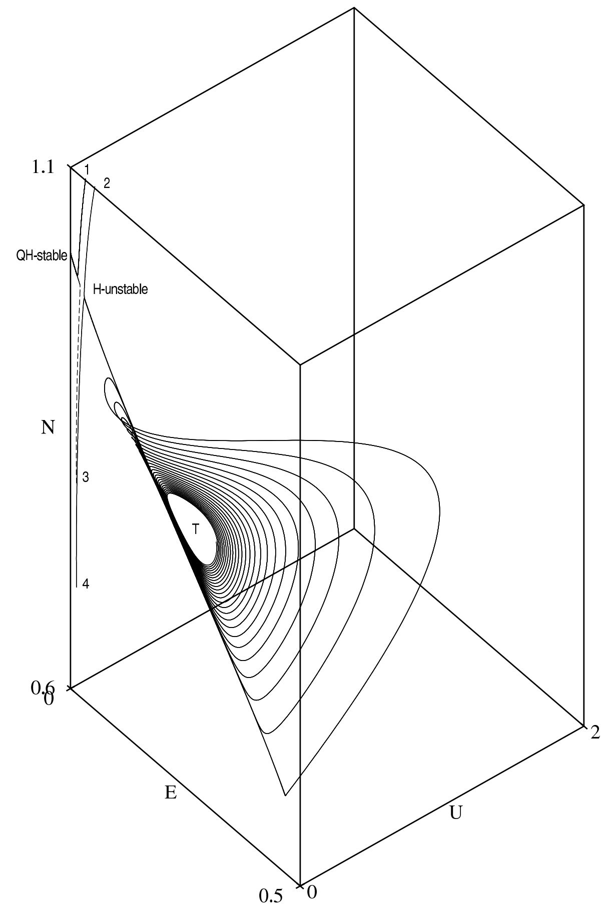
key aspects of dynamics

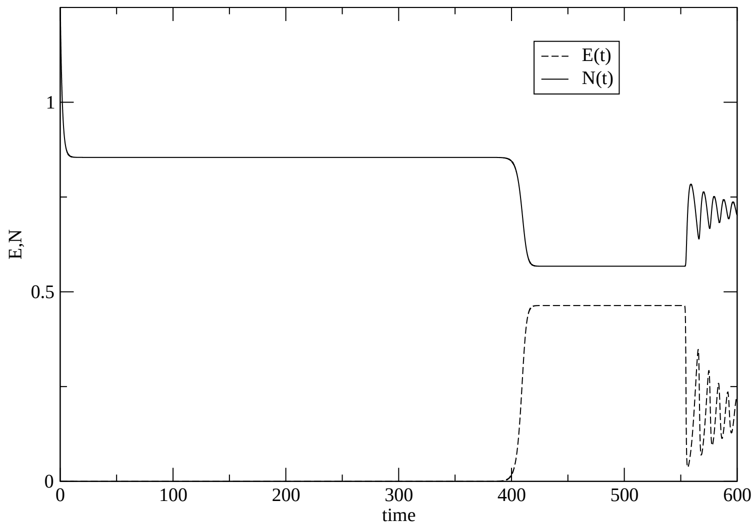
- Hopf bifurcation of T-mode into limit cycle on a center manifold of the system

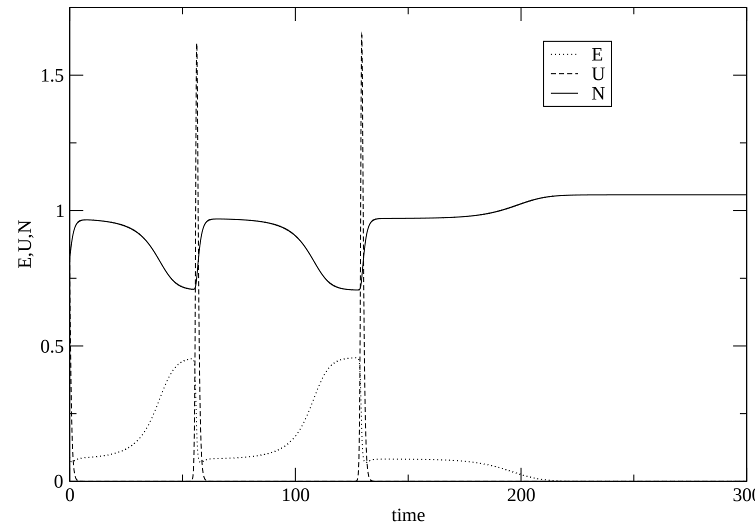
- The center manifold is two dimensional attractor of three-dimensional system formed by eigenspace spanned on the two purely imaginary complex conjugated eigenvalues.

- The third eigenvalue has $\Re\lambda < 0$ which ensures local attraction to the center manifold

- T→QH system leaves the center manifold




E(t)
N(t)
E,N
time
0
0.5
1
0
100
200
300
400
500
600


E
U
N
E,U,N
time
0
0.5
1
1.5
0
100
200
300

Conclusions (0-D)

- local LH transition model is analyzed
- four singular points of dynamical system are identified
- stability conditions obtained
- two central manifolds described
- bifurcation scenarios studied
- the range of hysteretic behavior identified and turned out to be narrow (< 0.1 of the control parameter value)