ION Particle Transport in the TOKAMAK Edge Plasma

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Momentum balance requires a pinch-diffusion relation among Γ_r , $\partial p_i / \partial r$ and $V_{pinch}\left(\upsilon_\phi, \upsilon_\theta, E_r\right)$.

Combining the pinch-diffusion relation for Γ_r and the continuity equation yields a generalized diffusion equation with off-diagonal elements and convection (pinch).

Particle & Momentum Balances Must Be Satisfied

Particle Balance

$$\nabla \cdot \Gamma_j \equiv \nabla \cdot n_j \upsilon_j = S_j^{ion} \qquad (1)$$

Momentum Balance

$$\nabla \cdot \left(n_j m_j \upsilon_j \upsilon_j\right) + \nabla p_j + \nabla \cdot \boldsymbol{\pi}_j = n_j e_j \left(\upsilon_j \times \boldsymbol{B}\right) + n_j e_j \boldsymbol{E} + \boldsymbol{F}_j + \boldsymbol{M}_j - n_j m_j \nu_{elcxj}^{j} \upsilon_j \qquad (2)$$

Radial Component

$$E_r^0 = \frac{1}{n_j^0 e_j} \frac{\partial p_j^0}{\partial r} + \upsilon_{\phi j}^0 B_\theta^0 - \upsilon_{\theta j}^0 B_\phi^0 \qquad (3)$$

Toroidal Component

$$n_j^0 m_j \nu_{jk}^0 \left(\left(1 + \beta_j\right) \upsilon_{\phi j}^0 - \upsilon_{\phi k}^0\right) = n_j^0 e_j E_\phi^A + e_j B_\theta^0 \Gamma_{rj} + M_{\phi j}^0 \qquad (4)$$

where

$$\beta_j \equiv \frac{\nu_{gvj}^0 + \nu_{\perp j}^0 + \nu_{anomj}^0 + \nu_{nj}^0 + \nu_{elcxj}^0 + \nu_{ionj}}{\nu_{jk}^0} \equiv \frac{\nu_{dj}^*}{\nu_{jk}^0} \qquad (5)$$

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Various Processes for Radial Transport of Toroidal Angular Momentum can be written in the form $RnmvV_{\phi}$

Neoclassical

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle = \left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle_{gv} + \left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle_{\perp} \qquad (6)$$

where GYROVISCOUS

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle_{gv} = -\left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^3 h_p \eta_4 \frac{\partial}{\partial l_p}\left(\upsilon_\phi / R\right)\right)\right\rangle \qquad (7)$$

and "Perpendicular"

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle_{\perp} = -\left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^3 h_p \eta_2 \frac{\partial}{\partial l_\psi}\left(\upsilon_\phi / R\right)\right)\right\rangle \qquad (8)$$

make a low order Fourier expansion $X(r,\theta) = X^0(r)\left[1 + X^c\cos\theta + X^s\sin\theta\right]$

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle_{gvj} \approx \frac{1}{2}\eta_{4j}\frac{r}{R_0}\left(L_n^{-1} + L_T^{-1} + L_{\upsilon_\phi}^{-1}\right)\left[\left(4+\tilde{n}_j^c\right)\tilde{\upsilon}_{\phi j}^s + \tilde{n}_j^s\left(1-\tilde{\upsilon}_{\phi j}^c\right)\right]\upsilon_{\phi j} \qquad (9)$$

$$\equiv R_0 n_j^0 m_j \nu_{gvj}\upsilon_{\phi j}$$

and

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle_{\perp j} \approx R_0\eta_{2j}\left[L_{\upsilon_\phi}^{-1}\left(\frac{1}{r} - L_{\eta_2}^{-1}\right) - \frac{1}{\upsilon_{\phi j}}\frac{\partial^2\upsilon_{\phi j}}{\partial^2 r}\right]\upsilon_{\phi j} \equiv R_0 n_j^0 m_j \nu_{\perp j}\upsilon_{\phi j} \qquad (10)$$

where the $\tilde{n}_j^c \equiv n_j^c/\varepsilon$, etc. are poloidal asymmetry coefficients

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VARIOUS PROCESSES ... (CONTINUED)

TURBULENT (ANOMALOUS) VISCOSITY

usually assumed to be of the form of Eqs. (8) and (10) with an enhanced viscosity coefficient η_{anom} ,

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\Pi\right\rangle_{anomj}\approx R_0\eta_{anomj}\left[L_{\upsilon_\phi}^{-1}\left(\frac{1}{r}-L_{\eta_2}^{-1}\right)-\frac{1}{\upsilon_{\phi j}}\frac{\partial^2\upsilon_{\phi j}}{\partial^2 r}\right]\upsilon_{\phi j}\equiv R_0 n_j^0 m_j\nu_{anomj}\upsilon_{\phi j} \qquad (11)$$

INERTIAL TORQUE (using Eq. 1)

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\left(n_j m_j \mathbf{\upsilon}_j\mathbf{\upsilon}_j\right)\right\rangle=\left\langle R^2\nabla\phi\cdot n_j m_j\left(\mathbf{\upsilon}_j\cdot\nabla\right)\mathbf{\upsilon}_j\right\rangle+R_0 n_j m_j\nu_{ionj}\upsilon_{\phi j} \qquad (12)$$

and

$$\left\langle R^2\nabla\phi\cdot n_j m_j\left(\upsilon_j\bullet\nabla\right)\upsilon_j\right\rangle\simeq\frac{1}{2}\Big(\frac{\upsilon_{rj}}{R_o}\left\{\varepsilon\left(1+\tilde{n}_j^c+\tilde{\upsilon}_{\phi j}^c\right)-2R_o L_{\upsilon_\phi j}^{-1}\right\}-$$

$$\varepsilon\frac{\upsilon_{\theta j}^0}{R_o}\left\{\tilde{\upsilon}_{\phi j}^s\left(1+\tilde{n}_j^c+\tilde{\upsilon}_{\theta j}^c\right)-\tilde{\upsilon}_{\theta j}^s\left(1+\tilde{\upsilon}_{\phi j}^c\right)-\tilde{\upsilon}_{\phi j}^c\tilde{n}_j^s\right\}\Big)n_j m_j R_0\upsilon_{\phi j}^0\equiv R_0 n_j m_j\nu_{nj}\upsilon_{\phi j}^0 \qquad (13)$$

CHARGE-EXCHANGE & ELASTIC SCATTERING

$$R_0 n_j^0 m_j\nu_{cxel}\upsilon_\phi$$

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PINCH-DIFFUSION TRANSPORT RELATION

Combining the radial and toroidal components of the momentum balance equations—Eqs. (3) and (4)--yields a generalized pinch-diffusion relation for the radial particle flux

$$\Gamma_{rj} \equiv \left\langle n_j \upsilon_{rj} \right\rangle = n_j D_{jj} \left(L_{nj}^{-1} + L_{Tj}^{-1} \right) - n_j D_{jk} \left(L_{nk}^{-1} + L_{Tk}^{-1} \right) + n_j \upsilon_{pj} \qquad (14)$$

“diffusion coefficients”

$$D_{jj} \equiv \frac{m_j T_j \left(\nu_{dj}^* + \nu_{jk} \right)}{\left(e_j B_\theta \right)^2} \quad , \quad D_{jk} \equiv \frac{m_j T_k \nu_{jk}}{e_j e_k (B_\theta)^2} \qquad (15)$$

“pinch velocity”

$$n_j \upsilon_{pj} \equiv -\frac{M_{\phi j}}{e_j B_\theta} - \frac{n_j E_\phi^A}{B_\theta} + \frac{n_j m_j \nu_{dj}^*}{e_j B_\theta} \left(\frac{E_r}{B_\theta} \right) + \frac{n_j m_j f_p^{-1}}{e_j B_\theta} \left(\left(\nu_{jk} + \nu_{dj}^* \right) \upsilon_{\theta j} - \nu_{jk} \upsilon_{\theta k} \right) \qquad (16)$$

where $f_p^{-1} \equiv B_\phi / B_\theta$.

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PINCH-DIFFUSION ... (CONTINUED)

CONFIRMATION

Equation (14) constrains the edge pressure gradient, hence density gradient

$$\frac{-1}{n_i}\frac{\partial n_i}{\partial r} \equiv L_{ni}^{-1} = L_{pi}^{-1} - = \frac{\upsilon_{ri} - \upsilon_{p,i}}{D_i} - L_{Ti}^{-1} \qquad (17)$$

When υ_{ri} is determined from the continuity equation, υ_{pi} is evaluated from measured quantities, L_{Ti}^{-1} and is inferred from experiment, integration of Eq. (17) predicts the measured density profile in the edge of several DIII-D H-mode shots (e.g. PoP, 11, 5487, 2004).

GENERALIZED RADIAL DIFFUSION EQUATION

Substitute the pinch-diffusion relation Eq. (14) into the continuity equation, Eq. (1)

$$-\frac{\partial}{\partial r}\left(D_{jj}\frac{\partial n_j}{\partial r}\right)-\frac{\partial}{\partial r}\left(D_{jk}\frac{\partial n_k}{\partial r}\right)-\frac{\partial}{\partial r}\left(D_{jj}\frac{n_j}{T_j}\frac{\partial T_j}{\partial r}\right)- \frac{\partial}{\partial r}\left(D_{jk}\frac{n_j}{T_k}\frac{\partial T_k}{\partial r}\right)+\frac{\partial\left(n_j\upsilon_{pj}\right)}{\partial r}=S_j \qquad (18)$$

COMMENTS

Only the first, “self-diffusion” term is usually included in edge codes, with a diffusion coefficient fit to match experimental density profiles.

The “other species” diffusion term, the temperature diffusion terms, and the pinch convective term are usually neglected.

Our experience indicates that the pinch convective term, which must be evaluated from the rotation velocities, is dominant in the plasma edge.

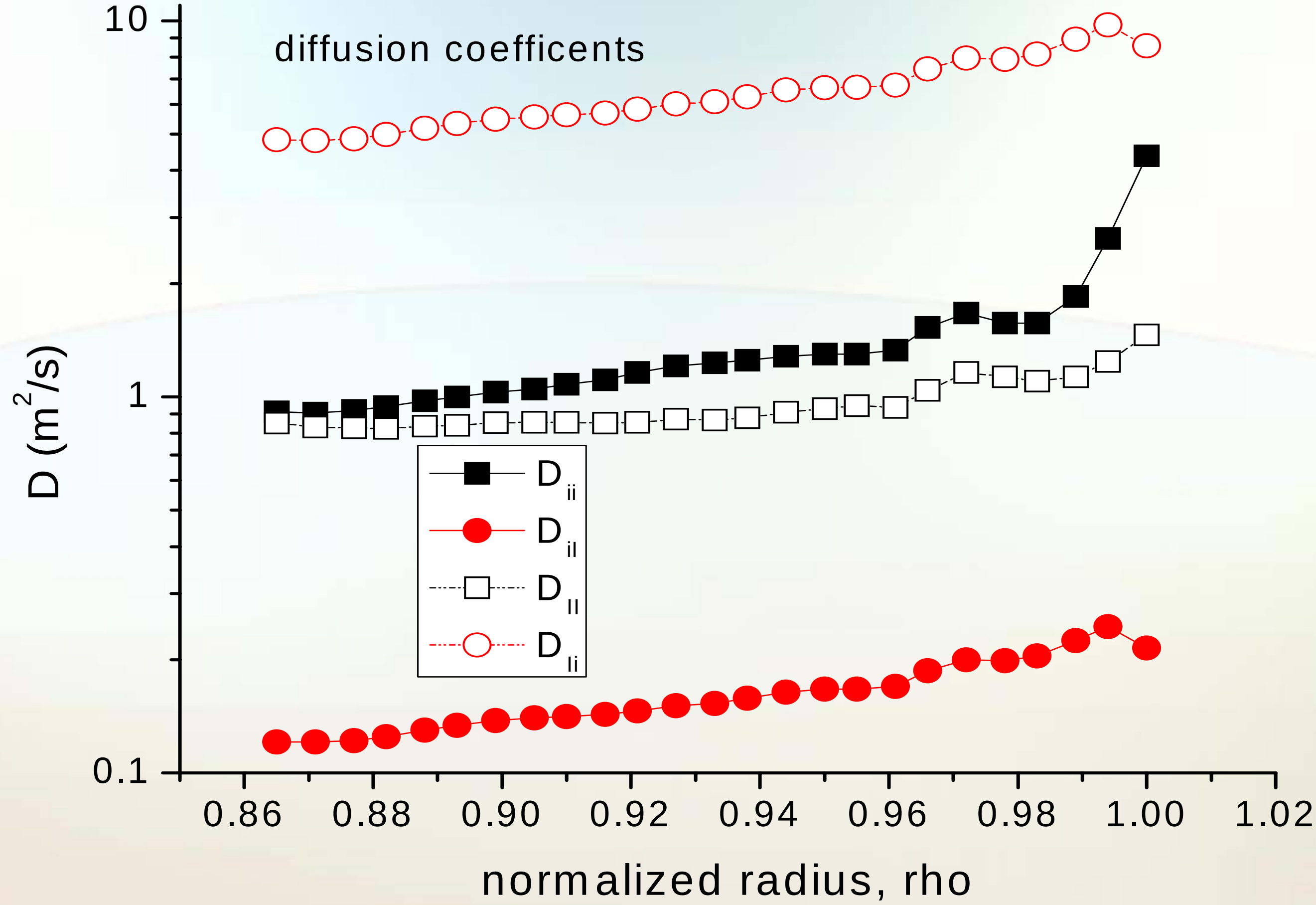

Figure 1: Generalized diffusion coefficients in the edge of DIII-D H-mode shot 92976

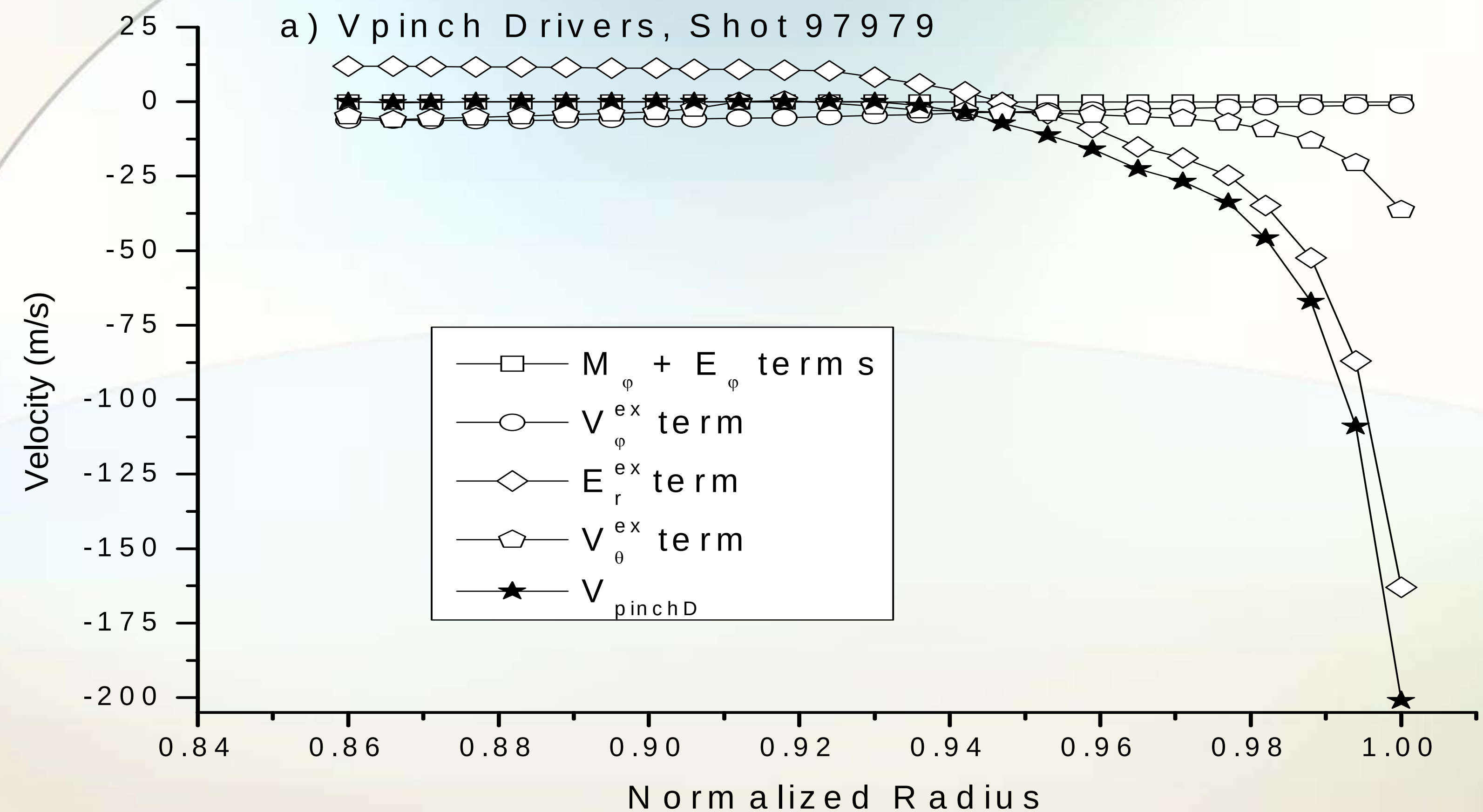

Figure 2 Pinch velocity in the edge of DIII-D H-mode shot 97979

Another form for the pinch velocity that uses measured carbon toroidal rotation $\upsilon_{\phi I}$ is

$$\upsilon_{p,i} = \frac{\left[-M_{\phi i} - n_i e_i E_{\phi}^{A} + n_i m_i \left(\nu_{iI} + \nu_{di}^{*} \right) \left(f_p^{-1} \upsilon_{\theta i} + E_r / B_{\theta} \right) - n_i m_i \nu_{iI} \upsilon_{\phi I} \right]}{n_i e_i B_{\theta}}$$

CONCLUSIONS

Momentum (rotation) equations must be solved along with continuity equation to determine particle transport in edge.

If a diffusion equation is used to calculate particle transport in the edge, then it should contain the “off-diagonal” and convective (pinch) terms required by momentum balance.