Energy balance including Turbulence Effects in Reversed Field Pinch Plasmas

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Ricerca Formazione Innovazione

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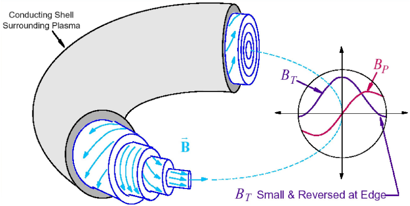
- This suggests the existence of an energy exchange process between different scales.

Other observation from reversed field pinch

RFP configuration characterized by the inversion of the toroidal field at the edge. Configuration sustained mainly by internal current through dynamo mechanism



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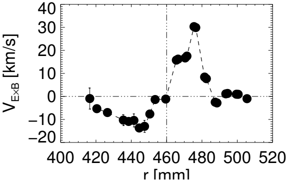

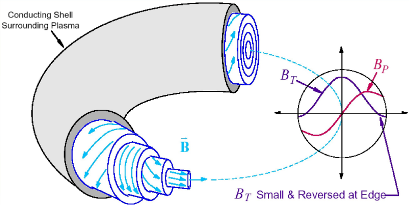

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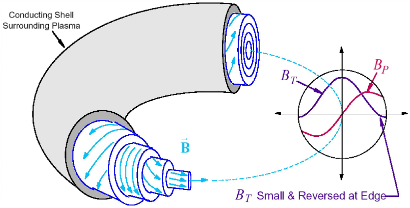

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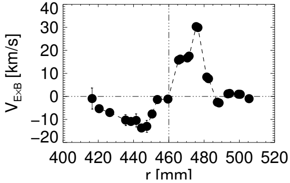
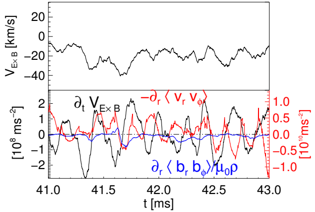


The $\mathbf{E} \times \mathbf{B}$ velocity is sustained by Reynolds stress



What has been observed so far

Presently energy transfer mechanism has been considered between fluctuations and flows

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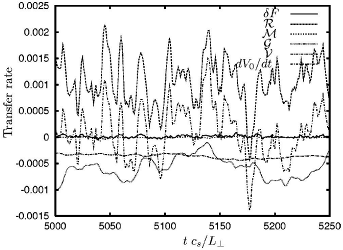
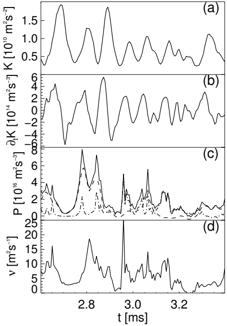
Observed in numerical simulation



V.Naulin *et al*, PoP **12** (2005)

And in experiments



N.Vianello *et al*, PPCF **46** 2006

The model: Assumption and definition

We start from Boltzmann equations for the two species and we use the usual simplifications and common definitions:

Using the following common definition

- $n^i = n^e \stackrel{def}{=} n \qquad q^i = -q^e \stackrel{def}{=} q$
- $\rho = \sum_k m^k n^k \qquad \rho_c = \sum_k q^k n^k$
- $\vec{J} = \sum_k q^k n^k \vec{V}^k \qquad \rho\vec{V} = \sum_k m^k n^k \vec{V}^k$
- $M = \sum m^k \quad m = \frac{m^i m^e}{m^i + m^e} \quad m' = \frac{m^i m^e}{m^i - m^e}$
- $p = p^i + p^e \qquad \pi = \pi^i + \pi^e$ and $\vec{R} = \vec{R}^i = -\vec{R}^e$

The model equation

Usual algebra allows to determine the following equations

Continuity equation $\partial_t \rho + \partial_k(\rho V^k) = 0$

Quasi-neutrality $\partial_k J_k = 0$

Momentum balance

$$\partial_t(\rho V_k) + \partial_j\left(\rho V_j V_k + \frac{m}{q}\frac{J_k J_j}{n}\right) = -\partial_k p - \partial_j \pi_{jk} + \varepsilon_{kjs} J_j B_s$$

Current density

$$\partial_t J_k + \partial_j\left(V_j J_k + J_j V_k - \frac{m}{m'}\frac{J_j J_k}{qn}\right) = \frac{q}{2m'}(\partial_k p + \partial_r \pi_{jk}) + \frac{q^2}{m}\varepsilon_{kjs}\left(nV_j - \frac{m}{qm'}J_j\right)B_s + \frac{q^2}{m}nE_k + \frac{q}{m}R_k$$

Kinetic pressure

$$\partial_t\left(\frac{3}{2}p\right) + \partial_j\left(\frac{3}{2}p\left(V_j - \frac{m}{2m'}\frac{J_j}{qn}\right)\right) + \partial_r\left((p\delta_{js} + \pi_{js})\left(V_s - \frac{m}{2m'}\frac{J_s}{qn}\right) + q_j\right) = \left(-\frac{R_j J_j}{qn} - \frac{m}{2m'}\frac{J_j}{qn}(\partial_j p + \partial_s \pi_{js})\right) - (-V_j \partial_j p - V_s \partial_j \pi_{js})$$

Energy balance in a plasma

Together with Maxwell equations we derive the energy balance equation for Electromagnetic energy (W_{em}), Kinetic energy (K), Thermal energy (U) and Relative kinetic energy ($K^J = \frac{mn}{2}(\frac{J}{qn})^2$)

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Kinetic energy

$$\frac{\partial K}{\partial t} + \partial_j (K V_j + S_j^K) = P_{U K} - P_{K K^j}$$

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$$\frac{\partial U}{\partial t} + \partial_j (U V_j + S_j^U) = P_{K^J U} - P_{UK}$$

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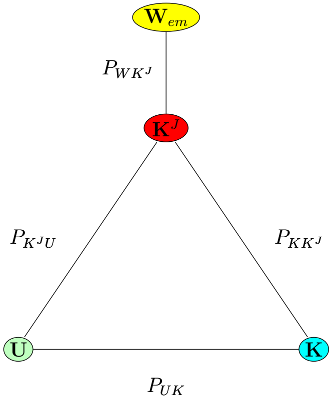
Relative kinetic energy

$$\partial_t K^J + \partial_j \left(K^J V_j + S_j^{K^J} \right) = P_{WK^J} - P_{K^J U} + P_{KK^J}$$

The scheme

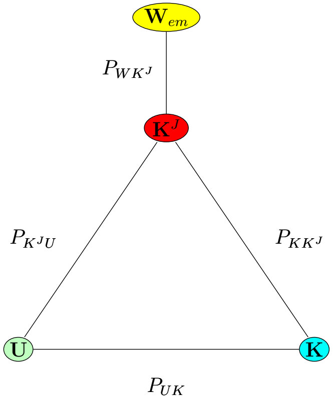
The global energy balance in a plasma including electromagnetic and gradient effects can be consequently summarized as follows



The scheme

But ...

Which are the transfer directions?



The role of fluctuations

Each of the energies (W, K^J, U, K) and of the exchanged powers P is a non-linear function of the six variables $\rho = Mn, \mathbf{V}, \mathbf{J}, \mathbf{E}, \mathbf{B}, T$. In order to understand the role of fluctuations we can easily define the corresponding mean variables $\overline{\rho} = M\overline{n}, \overline{\mathbf{V}}, \overline{\mathbf{J}}, \overline{p} = \overline{nT}, \overline{\mathbf{E}}, \overline{\mathbf{B}}$ and the corresponding fluctuations

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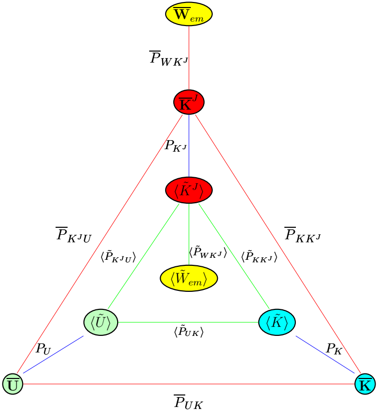
The corresponding ensemble average is easily defined as

$$\langle \tilde{A} \rangle \equiv \langle A - \overline{A} \rangle$$

The global balance

Avoiding the detailed calculation all the equations for mean and fluctuating energies have been derived in order to establish all the power density exchange terms. The complete scheme results as follows:



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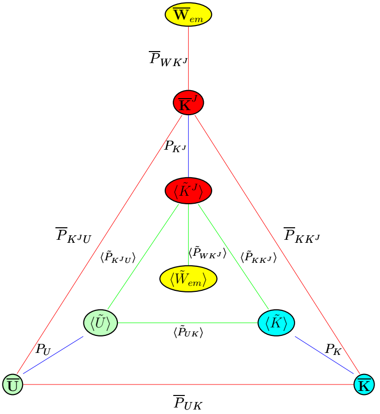


We focus on $\overline{K}$

$$\partial_t \overline{K} + \partial_r (\overline{K} V_r + \overline{S}_r^K) = \overline{P}_{UK} - \overline{P}_{KK^J} - P_K$$

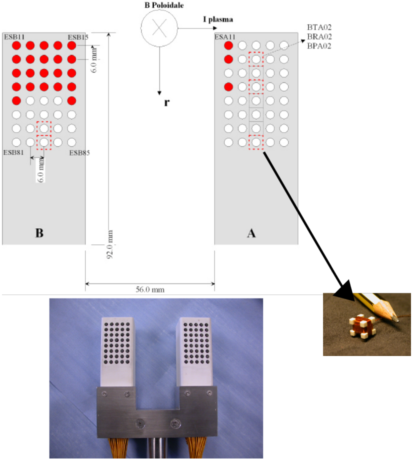
Mean power exchanged: tools

U-probe

Two 2-D arrays of electrostatic probes
5 (toroidal) $\times$ 8 (radial) pins with Δr 6 mm and $\Delta\phi$ 6 mm. Pins with 3 mm $\varnothing$

Two arrays of 7 magnetic probe .
3-axial $(\dot{b}_r, \dot{b}_\theta, \dot{b}_\phi)$ coils. Size 7×6×8 mm, Δr=6mm

5 MHz acquisition sampling with high bandwidth



Experimental measurements: the assumptions

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- First of all we will assume that

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  i.e. the radial component of mean magnetic field and mean current density are almost zero

- Secondary we will assume that

$$\partial_\theta, \partial_\phi \ll \partial_r$$

  i.e poloidal and toroidal symmetry

- We cannot presently evaluate some of the terms, essentially

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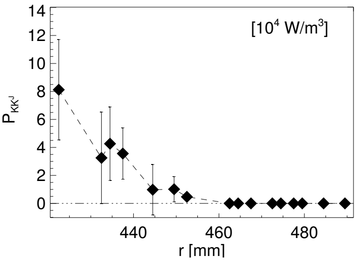
Experimental measurements: Mean kinetic energy balance

$$\overline{P}_{KK^j} = \varepsilon_{rst}\overline{J}_r\overline{V}_s\overline{B}_t - \frac{m}{q^2}\frac{\overline{J}_r\overline{J}_s}{\overline{n}}\partial_r\overline{V}_s \approx -\overline{J}_\theta\overline{V}_r\overline{B}_\phi + \overline{J}_\phi\overline{V}_r\overline{B}_\theta$$



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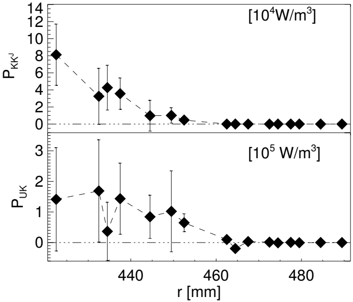
$$\overline{P}_{UK} = -\overline{V}_r \partial_r \overline{p} - \overline{V}_s \partial_r \overline{\pi}_{rs} \approx -\overline{V}_r \partial_r \overline{p}$$



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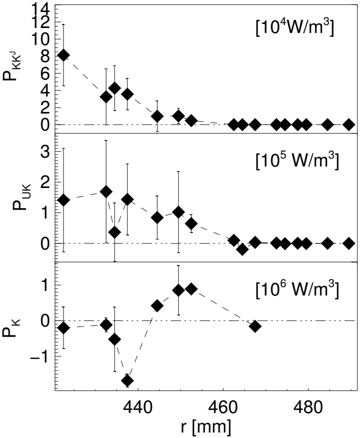
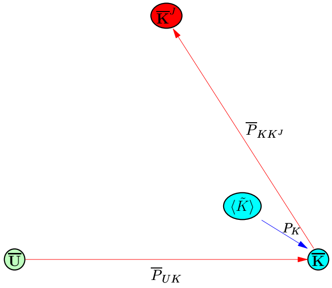
$$P_k$$



Experimental measurements: Mean kinetic energy balance

How the energy flows

- Mean kinetic energy receives power both from mean thermal energy (plasma espansion) and from fluctuations through non-linear term P_K.



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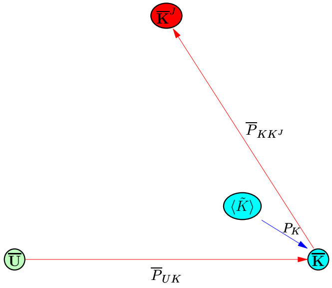

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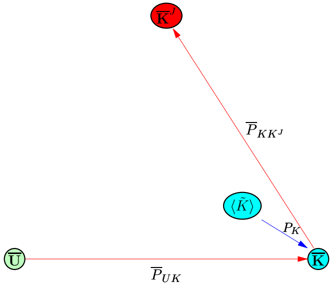
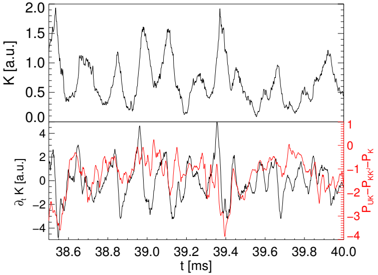
- The unbalanced quantity could go in some sort of dissipation (off-diagonal part of pressure tensor $\longrightarrow$ viscosity)



Temporal evolution

The balance of the energy equation is considered also in their time evolution.



Kinetic energy

Although the strong assumption (in particular the missing non-diagonal terms of pressure tensor) a fairly good agreement between time evolution of $\overline{K}$ and the RHS of its balance equation is observed

Conclusion

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- A complete set of the energy balance equations for electromagnetic, thermal and kinetic energies is determined, including pressure gradients and compressibility.
- The terms which describe the power density exchanged between different energy basins are identified
- Using probe measurements and equilibrium model to determine mean field quantities a first estimate has been done for the mean kinetic energy balance equation in order to identify how energy exchange works between different basins
- The role of kinetic fluctuation in driving mean kinetic energy in the innermost region is retrieved also in the presence of pressure fluctuations

Addendum: P_K

$$P_K = \overline{\rho}\overline{V}_r\langle\tilde{v}_s\partial_s\tilde{v}_r\rangle + \frac{V^2}{2}\partial_r\langle\tilde{\rho}\tilde{v}_r\rangle$$

$$+ mV_s\left\langle\frac{\overline{n}\partial_r\left(n\partial_r\frac{J_r}{qn}\frac{J_s}{qn}\right) - n\partial_r\left(\overline{n}\frac{\overline{J}_r}{q\overline{n}}\frac{\overline{J}_s}{q\overline{n}}\right)}{n}\right\rangle +$$

$$\left[\overline{\rho}\overline{V}_r\left(\left\langle\frac{\partial_r p}{\rho} - \frac{\partial_r\overline{p}}{\overline{\rho}}\right\rangle + \left\langle\frac{\partial_r\pi_{rs}}{\rho} - \frac{\partial_r\overline{\pi}_{rs}}{\overline{\rho}}\right\rangle\right)\right] -$$

$$\varepsilon_{rst}\overline{\rho}\overline{V}_r\left(\left\langle\frac{J_r B_t}{\rho} - \frac{\overline{J}_r\overline{B}_t}{\overline{\rho}}\right\rangle\right)$$

Addendum:kinetic energy

Kinetic energy

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho V^2\right) + \partial_j\left(\frac{1}{2}\rho V^2 V_j + \frac{m}{q^2}\frac{V_s J_s J_j}{n}\right) =$$

$$\left(-V_r \partial_r p - V_s \partial_r \pi_{rs}\right) -$$

$$\left(\varepsilon_{jst} J_j V_s B_t - \frac{m}{q^2}\frac{J_j J_s}{n}\partial_j V_s\right)$$

..or equivalently

$$\frac{\partial K}{\partial t} + \partial_j(K V_j + S_j^K) = P_{UK} - P_{KK^j}$$

Addendum: Electromagnetic energy

Electromagnetic energy

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}\right) + \partial_j\left(\frac{\varepsilon_{jst} E_s B_t}{\mu_0}\right) = -E_s J_s$$

..or equivalently

$$\frac{\partial W_{em}}{\partial t} + \partial_j\left(S_j^W\right) = -P_{WK^J}$$

Addendum Relative kinetic energy

Relative kinetic energy

$$\partial_t \left(\frac{mn}{2} \left(\frac{J}{qn} \right)^2 \right) + \partial_j \left(\frac{mn}{2} \left(\frac{J}{qn} \right)^2 \left(V_j - \frac{m}{m'} \frac{J_j}{qn} \right) \right) = E_s J_s - \left(-\frac{R_j J_j}{qn} - \frac{m}{2m'} \frac{J_j}{qn} (\partial_j p + \partial_s \pi_{sj}) \right) + \left(\varepsilon_{jst} J_j V_s B_t - \frac{m}{q^2} \frac{J_j J_s}{n} \partial_j V_s \right)$$

..or equivalently

$$\partial_t K^J + \partial_j \left(K^J V_j + S_j^{K^J} \right) = P_{WK^J} - P_{K^J U} + P_{KK^J}$$

Addendum: Thermal Energy

Thermal Energy

$$\partial_t \left(\frac{3}{2}p\right) + \partial_j \left(\frac{3}{2}pV_j + S^U_j\right) =$$

$$\left(-\frac{R_j J_j}{qn} - \frac{m}{2m'}\frac{J_j}{qn}(\partial_j p + \partial_s \pi_{js})\right)$$

$$- \left(-V_j \partial_j p - V_s \partial_j \pi_{js}\right)$$

where $S^U_j = -\frac{m}{2m'}\left(\frac{3}{2}p\right)\frac{J_j}{qn} + (p\delta_{js} + \pi_{js})\left(V_s - \frac{m}{2m'}\frac{J_s}{qn}\right) + q_j$

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Thermal Energy

$$\partial_t \left(\frac{3}{2}p \right) + \partial_j \left(\frac{3}{2} p V_j + S_j^U \right) = \left(-\frac{R_j J_j}{qn} - \frac{m}{2m'} \frac{J_j}{qn} (\partial_j p + \partial_s \pi_{js}) \right) - \left(-V_j \partial_j p - V_s \partial_j \pi_{js} \right)$$

where $S_j^U = -\frac{m}{2m'} \left(\frac{3}{2}p \right) \frac{J_j}{qn} + (p\delta_{js} + \pi_{js}) \left(V_s - \frac{m}{2m'} \frac{J_s}{qn} \right) + q_j$

..or equivalently

$$\frac{\partial U}{\partial t} + \partial_j (U V_j + S_j^U) = P_{K^J U} - P_{UK}$$

Addendum: P_{K^J}

$$P_{K^J} = \left[\frac{m}{q}\left(\frac{\overline{J}_r}{q\overline{n}}\partial_s\langle\tilde{v}_s\tilde{j}_r\rangle - \frac{\overline{K}^J}{q\overline{n}}\partial_s\langle\tilde{n}\tilde{v}_s\rangle\right) - \frac{m^2}{m'}\frac{\overline{J}_r}{q\overline{n}}\partial_s\left\langle\frac{J_sJ_r}{q^2n} - \frac{\overline{J}_s\overline{J}_r}{q^2\overline{n}}\right\rangle\right] -$$

$$\frac{\overline{J}_r}{\overline{n}}\langle\tilde{n}\tilde{E}_r\rangle - \left[\frac{\langle\tilde{R}_r\rangle\overline{J}_r}{q\overline{n}} + \frac{m}{2m'}\frac{\overline{J}_r}{q\overline{n}}\langle\partial_r\langle\tilde{p}\rangle + \partial_s\langle\tilde{\pi}_{rs}\rangle\right] +$$

$$\left[-\varepsilon_{rst}\frac{\overline{J}_r}{\overline{n}}(\langle nV_sB_t - \overline{n}\overline{V}_s\overline{B}_t\rangle) - \frac{m}{m'}\varepsilon_{rst}\frac{\overline{J}_r}{q\overline{n}}\langle\tilde{j}_s\tilde{b}_t\rangle + \frac{m}{q}\frac{\overline{J}_r}{q\overline{n}}\partial_s\langle\tilde{j}_s\tilde{v}_r\rangle\right]$$

Addendum: P_U

$$P_U = \overline{U}\left(\frac{\langle\tilde{V}_r\partial_r\tilde{T}_e\rangle}{\overline{T}} + \frac{\partial_r\langle\tilde{n}\tilde{V}_r\rangle}{\overline{n}}\right) + \left\langle\frac{\overline{n}\partial_r S^U_r - n\partial_r\overline{S}^U_r}{n}\right\rangle - \left\langle\frac{\overline{n}P_{K^JU} - n\overline{P}_{K^JU}}{n}\right\rangle + \left\langle\frac{\overline{n}P_{KK^J} - n\overline{P}_{KK^J}}{n}\right\rangle$$