Low-q Resonances, Transport Barriers, and Secondary Electrostatic Cells

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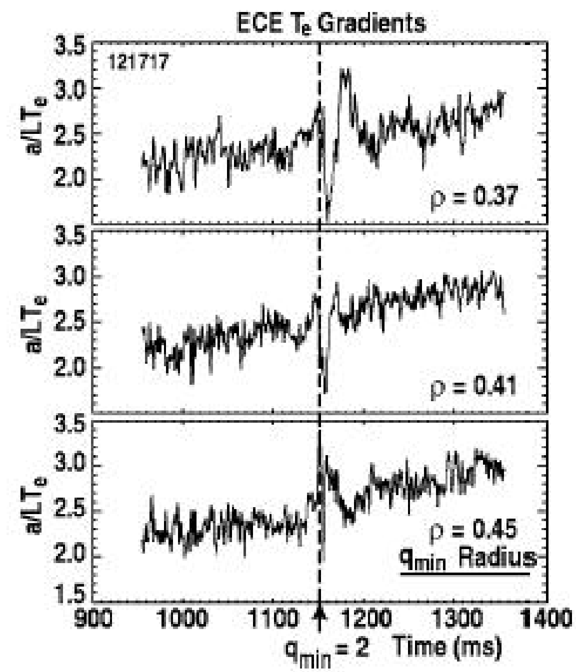
Recent Experimental Observations on DIII-D

- No magnetic signal detected at ITB formation
- Corrugations centered around ∇T_e profile event, i.e.
  - steepening before and after $q_{\min} = 2$
  - flattening at $q_{\min} = 2$



(Austin et. al. '06)

- GYRO simulations (Waltz, et. al '06):
  - exhibit profile 'corrugations' at q-resonance
  - indicate "zonal flows" correlated with corrugation structure
  - suggest zonal flows as ITB trigger

Critical Issues

- Zonal flow hypothesis forces the question:
- Why are zonal flows linked to resonant-q surfaces?
- Answer must address coexistence of:
  1. region of profile flattening at resonant surface
     - region of localized mixing, transport
  2. barrier formation nearby resonance
     - neighboring region of strong shear flow
- In particular, spatial profile of turbulence critical to shear flow generation

→ Necessary to understand response of turbulence to appearance of low-q surface

Secondary Convective Cell

- Finite-m analogue of zonal flow
  - Describes localized electrostatic convective cell excited by ambient background turbulence
  - Damped by synergism between collisional damping and resistive field line bending
  - $v_r \neq 0$, hence convective cell introduces strong mixing near resonance
    - $\rightarrow$provides robust mechanism for shaping mean profiles
- Excitation of cell linked to appearance of low-q rational surfaces
  - Most unstable in regions of weak magnetic shear
- Hence provides natural means of linking trigger of ITBs to off-axis low-q surfaces

Basic Equations I

- Minimal description requires two components

a.) Dynamical model for large scales

  - Gyrokinetics provides useful framework for describing evolution of convective cell
  - Separating fields into mean and fluctuating components: $\psi = \bar{\psi} + \tilde{\psi}$

- Electrostatic gyrokinetic equation is given by

$$\begin{aligned} \frac{\partial \bar{f}_i}{\partial t} + U\hat{b}\cdot\nabla\bar{f}_i &= J_0(\lambda)\left\{-\frac{c}{B}\hat{b}\times\nabla\bar{\phi}\cdot\nabla F_{0i} + \frac{e}{m_i}\hat{b}\cdot\nabla\bar{\phi}\frac{\partial F_{0i}}{\partial U}\right\} \\ &- \left\langle\frac{c}{B}\hat{b}\times J_0(\lambda)\nabla\tilde{\phi}\cdot\nabla\tilde{f}_i\right\rangle + C(f_i) \end{aligned}$$

Small Scale stresses

Collisions

- Here mean field nonlinearity subdominant to stresses exerted by small scales

Basic Equations II

- Considering weak magnetic shear and $k_\perp \rho_s < 1$
  - To first order in $k_\perp^2 \rho_s^2$, vorticity equation may easily be derived from gyrokinetic equation:

$$\left(\frac{\partial}{\partial t} + \gamma_d - \nu_c \nabla_\perp^2\right) \nabla_\perp^2 \bar{\phi} = -\frac{v_A^2}{\eta}\left(1 - i\frac{\omega_d^*}{\omega}\right) \nabla_\parallel^2 \bar{\phi} - \omega_d^* \nabla_\perp^2 \bar{\phi} - \frac{c}{B}\left\langle \hat{b} \times \nabla\tilde{\phi} \cdot \nabla \nabla_\perp^2 \tilde{\phi} \right\rangle$$

b.) Small scale evolution

- Wave kinetics provides convenient framework for treating evolution of small scales

$$\frac{\partial \langle N \rangle}{\partial t} = \frac{\partial}{\partial k_x}\left(D_k \frac{\partial \langle N \rangle}{\partial k_x}\right) + \frac{\partial}{\partial x}\left(D_x \frac{\partial \langle N \rangle}{\partial x}\right) + \gamma_k \langle N \rangle - \Delta\omega_k \langle N \rangle^2$$

$$D_k = k_y^2 \sum_q R(k,q)\, q_x^4 \left|\phi_q\right|^2, \qquad D_x = \sum_q R(k,q)\, q_y^2 \left|\phi_q\right|^2$$

Excitation Threshold I

- Two scale analysis + wave kinetics for drift wave turbulence yields:

$$\frac{v_A^2 q_y^2}{\eta L_s^2}\left(1 - i\frac{\omega_d^*}{\omega}\right)\frac{d^2\phi_q}{dq_x^2} = \left\{\left[\nu_c + \nu_T\left(q_x\right)\right] q_x^2 + \left[\gamma_d - i\left(\omega - \omega_d^*\right)\right]\right\} q_x^2 \phi_q$$

$$\nu_T = c_s^2 \sum_k R\left(k, q\right) \frac{\rho_s^2 k_y^2}{\left(1 + \rho_s^2 k_\perp^2\right)^2} k_x \frac{\partial \left\langle N \right\rangle}{\partial k_x}$$

- Drive -> 'negative viscosity' from modulational instability
- Damping -> friction, collisional damping
- Localization -> field line bending <-> magnetic shear, non-axisymmetric component

$$v_T\left(q_x\right) < 0 \quad \text{for} \quad \frac{\partial \left\langle N \right\rangle}{\partial k_x} < 0$$

Excitation Threshold II

- Eigenvalue provides estimate for fluctuation intensity threshold to excite cell

frictional damping

magnetic shearing

$$N \approx \Gamma + \left(\frac{\frac{3\pi}{2} + \delta}{1 - \epsilon} \right)^{2/3} \frac{\nu_c^{2/3}}{\eta^{1/3}} \left(\frac{v_A q_y}{L_s} \right)^{2/3} \frac{1}{\gamma_k}$$

$$\epsilon \equiv \frac{3}{4}\alpha \left(\ln\left(\frac{16}{\alpha}\right) - \frac{N - 2\Gamma}{N - \Gamma} \right), \quad \alpha \equiv \frac{1}{4}\frac{N}{(N - \Gamma)^2}\hat{\nu}_c$$

- Notice that magnetic shear & collisional viscosity work in synergy
  - strong magnetic shear forces vortex to be strongly localized in space
  - thinner cell more strongly damped by collisional viscosity (note scale dependence in turbulent viscosity)
- For weak magnetic shear, saturation asymptotes to that found previously for zonal flows
  - Cell most important in regions of weak magnetic shear

Cell Structure

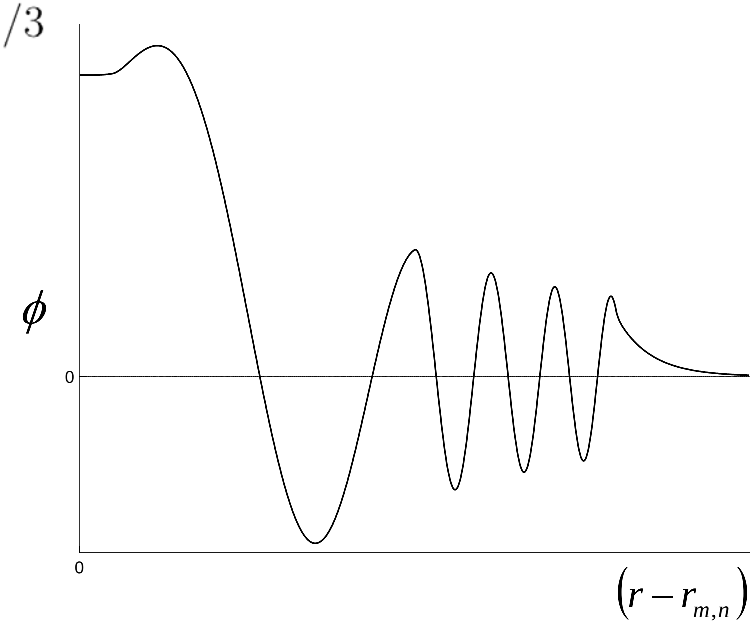
- Neglecting the scale separation for simplicity, a simple asymptotic form in real space can be written as:

$$\phi(x) \sim \frac{1}{x^{3/4}} \exp\left(i\frac{2}{3} \left(\frac{x}{\Delta x} \right)^{3/2} \right)$$

- Note the intensity of the shear flow scales as:

$$\Delta x \equiv \left(\left| \nu_T(0) \right| \eta \right)^{1/6} \left(L_s / \left(v_A q_y \right) \right)^{1/3}$$

- Shearing strongest away from resonant surface, $\left| v_y' \right| \sim x^{1/4}$
- Vortex satisfies dual criteria of:
  - strong mixing near resonant surface
  - peak of shearing adjacent to rational surface



Power Threshold

- Saturation criteria can be understood to correspond to the critical intensity of turbulence necessary to excite cell
- Thus, power threshold can be easily derived via power balance, i.e.

$$Q_{\mathrm{crit}} = -\chi_{\mathrm{crit}} \frac{\partial T_i}{\partial r} \approx v_{th} T_i \eta_i \epsilon_T^{-1/2} \tau^2 N_{\mathrm{crit}}$$

- Where a simple standard model for ITG turbulence has been used (Romanelli, ‘88)
- The power threshold is then given by

$$P_{\mathrm{in}} \sim R r_b Q_{\mathrm{crit}} \sim R r_b v_{th} T_i \eta_i \epsilon_T^{-1/2} \tau^2 N_{\mathrm{crit}}$$

- Where the notation is standard, and

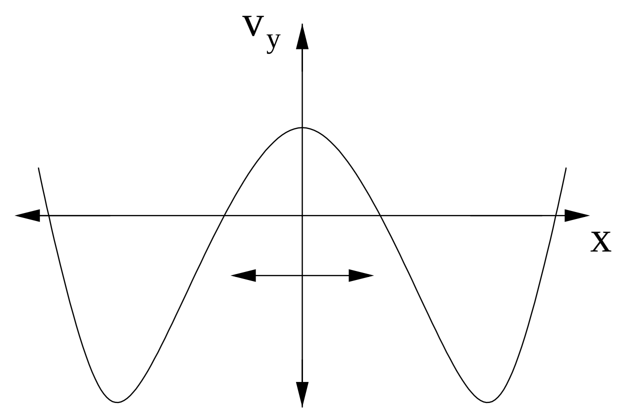
$$N_{\mathrm{crit}} \approx \Gamma + \left(\frac{\frac{3\pi}{2} + \delta}{1 - \epsilon} \right)^{2/3} \frac{\nu_c^{2/3}}{\eta^{1/3}} \left(\frac{v_A q_y}{L_s} \right)^{2/3} \frac{1}{\gamma_k}$$

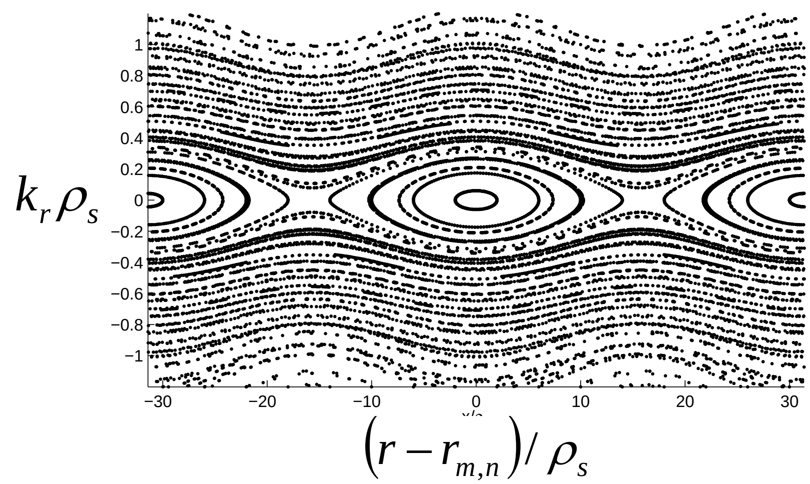
- Clearly, the power threshold increases for stronger friction, viscosity, and decreases for weaker magnetic shear

Nonlinear Evolution I

- Finite amplitude flow capable of "trapping" wave quanta (driftons) in maximum of shear flow
- Trapped driftons undergo closed orbits in phase space
  - nonlinear transfer of energy to shear flow pattern quenched! (Kaw et. al. '02)
- Integrability of system made possible by two integrals of motion (considering cross-section of torus):

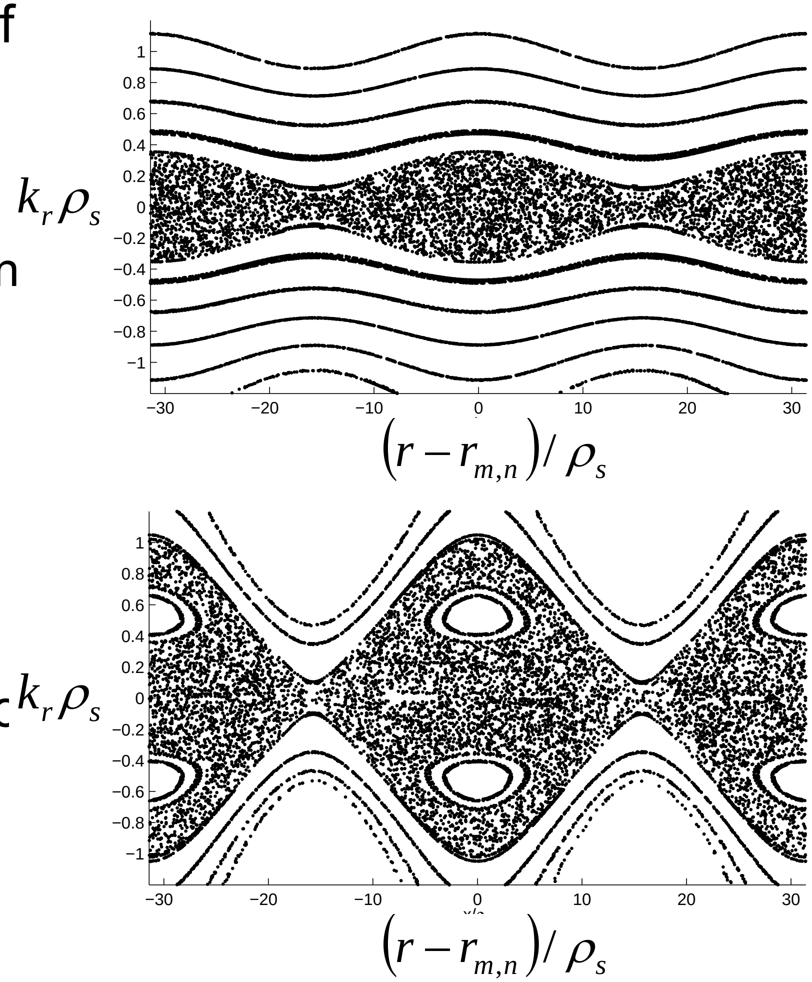
$$\delta\omega = \omega_k + \vec{v}_0 \cdot \vec{k}, \quad k_\theta$$





Nonlinear Evolution II

- Non-axisymmetric component of convective cell removes k_θ as integral of motion
  - Integrable orbits vanish
  - Replaced with stochastic motion
- Ray chaos prevents cell saturation by ray trapping
- Nonlinear wave trapping circumvented as nonlinear cell saturation mechanism!
- Result may be easily extended to generalized non-axisymmetric flow structures
- Stochasticity induced by breaking of axisymmetry similar to Tokamak->Stellarator



Conclusion

- Electrostatic convective cells likely to play strong for ITB transition in minimum-q profiles
- Cell formation satisfies:
  - profile flattening or “corrugation” at the resonant surface
  - barrier formation nearby the rational surface
- Non-axisymmetric component of convective cell allow nonlinear wave trapping to be circumvented
  - relevant for Reynolds stress driven flows with non-vanishing mean