Flow Shear Effects on Resistive MHD Instabilities in Tokamaks

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Motivation

•Success (or lack thereof) of magnetic confinement experiments largely due to design being based on ideal MHD.

•Control of magnetic confinement experiments has evolved to real time stabilization and avoidance of instabilities

•As experiments operate closer to the ideal stability limit resistive instabilities play an increasingly important role.

•Linear models are not entirely experimentally relevant.

•Non-linear non-ideal neoclassical two-fluid MHD with flow, mode coupling and error fields really needed for a (semi) complete picture. Issue for control

- •Real-time analysis inaccessible (long time to come)
- •Predictive understanding of extended MHD instabilities is needed

Here we address linear and nonlinear non-ideal MHD with mode coupling specifically to begin a careful study of the effects of toroidal flow shear.

Outline

- Brief review of recent work
  - Analysis of the onset of 2/1 in DIII-D Hybrid discharges - what we can do ignoring flow shear.
- Highlights of some relevant work with flow shear
  - Differential Flow vs. Shear Flow
- Description of current study
  - Flow shear in linear analysis
  - NIMROD simulations of 3/2 driven by 1/1 in flow
- Results and Summary Discussion

Highest β Tokamak Discharges, Including Hybrid Discharges, Typically Terminated by 2/1

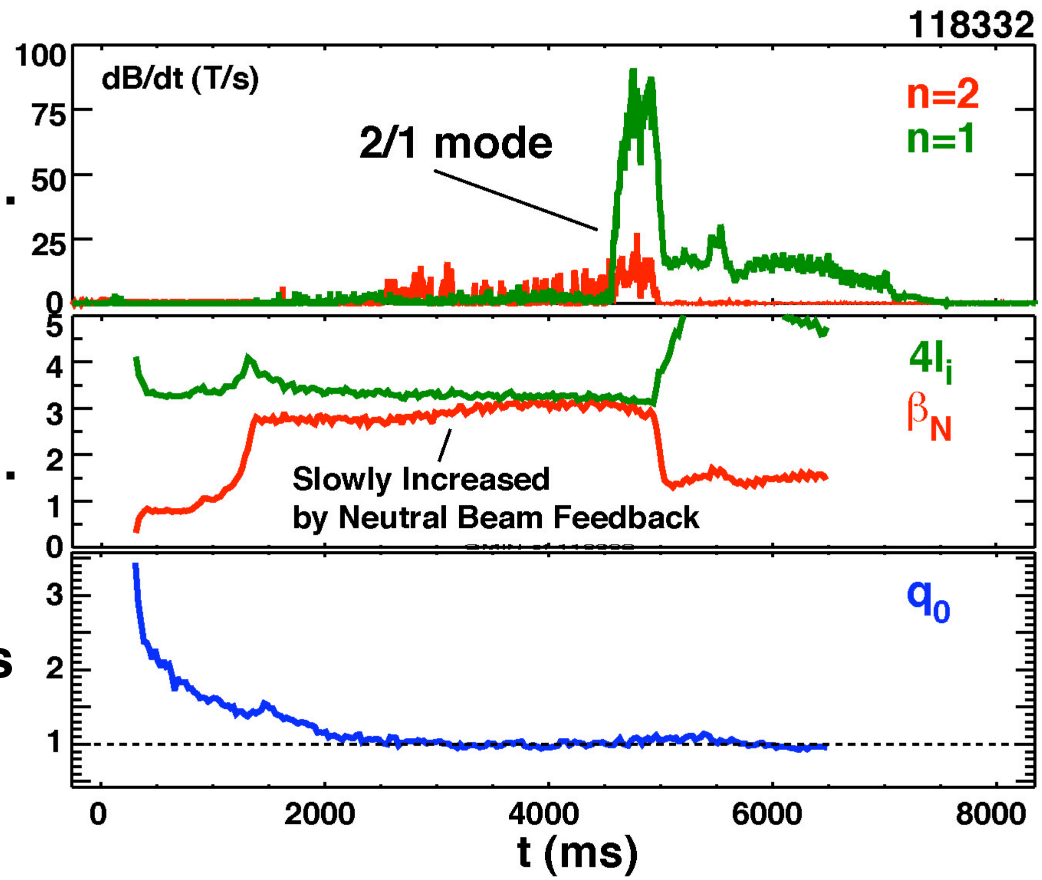
In Hybrid Discharges q_0 approaches and hovers near 1.

Resonance induced negative current drive sustains $q_0 \geq 1$.

Little to no 1/1 mode observed.

Steady state 3500ms-4500ms.

2/1 resistive mode often grows and terminates the discharge.



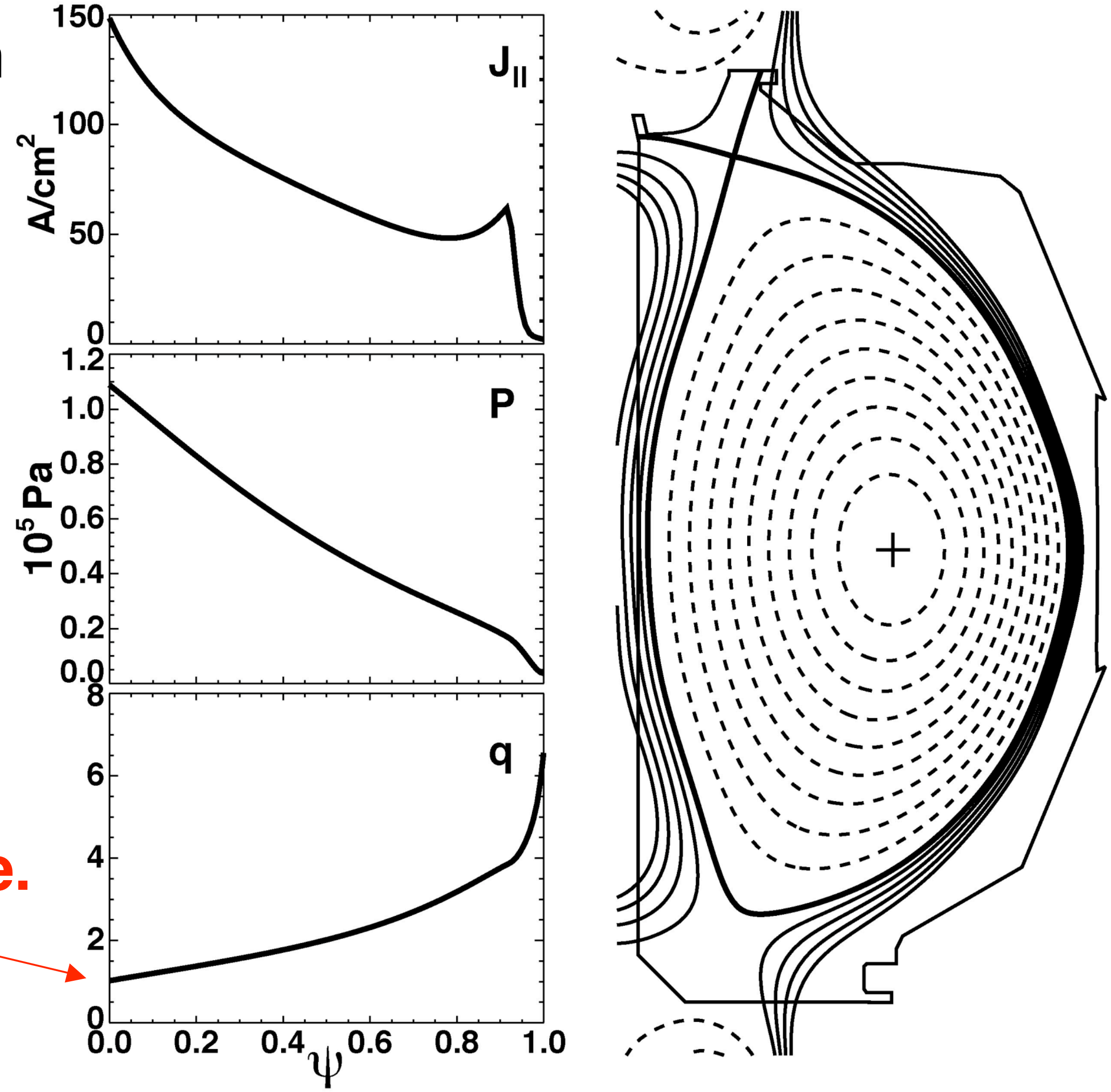
Are the q=1 resonance and the 2/1 mode onset related?

Equilibrium Reconstruction Just Before 2/1 Onset Used as Basis for “Family” of Equilibria to Examine Stability

Accurate equilibrium reconstruction uses B_z, $T_{e,i}$ and Density profile data.

β_N=3.09
l_i=0.849
$\beta_N/4l_i$=0.91

q_0~1 has moderate shear, giving small radius of q=1 surface.



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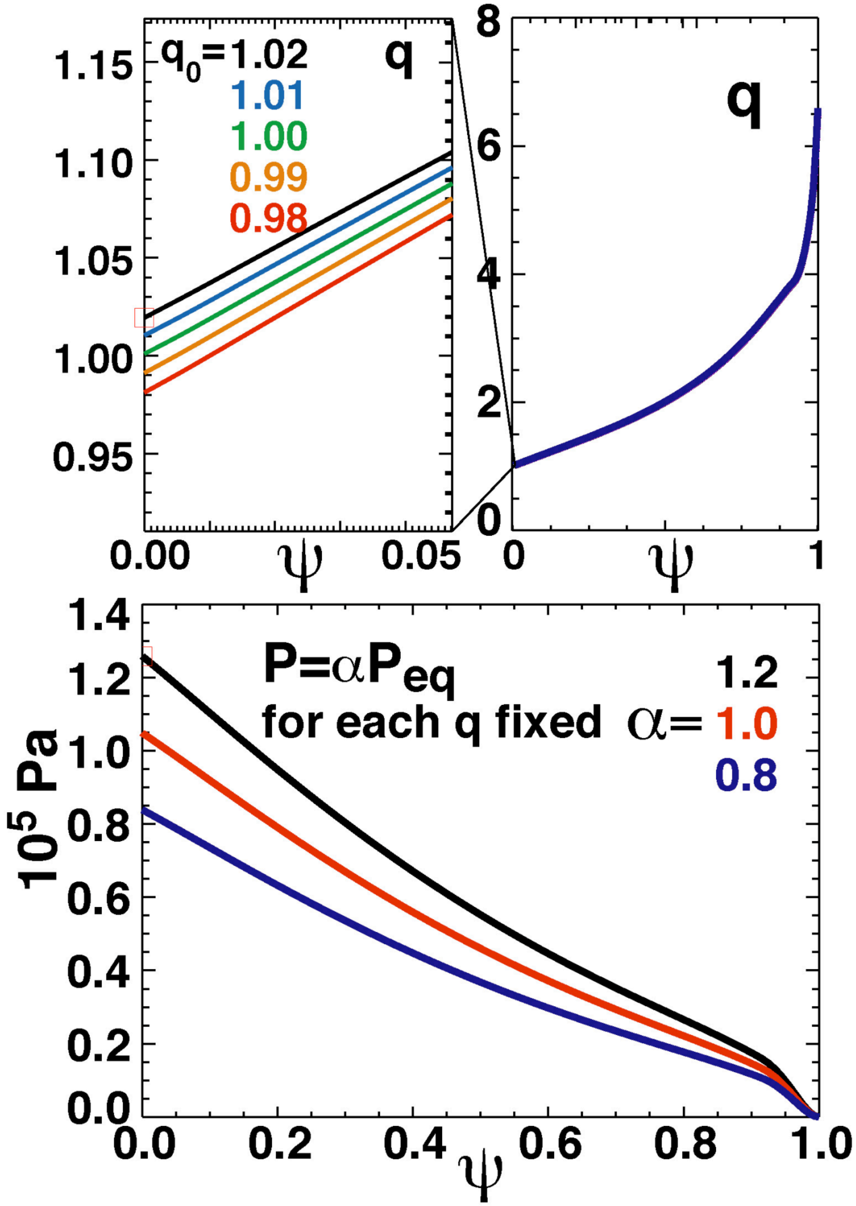
q_0 Constrained to ~2% by Data: Investigate Role in Stability by Varying P for Series of Fixed q_0

Constraint on q_0 varied within uncertainty of reconstruction, $0.98<q_0<1.02$, with little change in equilibrium near q=2 and elsewhere.



For each q_0 stability of 2/1 mode as function of β is computed.

Pressure increases with T_e at fixed density, which affects inner layer via resistivity and equilibrium pressure.

Complete Linear Stability at Rational Surfaces is Described by Matrix Dispersion Relation

$$\boxed{det[\mathbf{D}' - \mathbf{D}(Q)] = 0}$$

Solve for Q=γτ normalized growth rate.

PEST-III Outer Ideal Solution

Inner Layer Solution

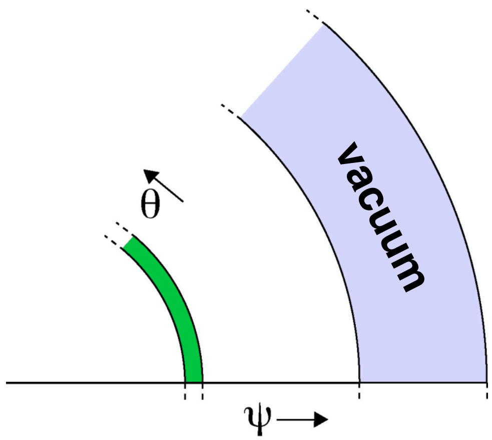
Pure Interchange Parity

$$\mathbf{D}' \equiv \frac{1}{2}\begin{bmatrix} \mathbf{A}' & \mathbf{B}' \\ \mathbf{\Gamma}' & \mathbf{\Delta}' \end{bmatrix} \qquad \mathbf{D} \equiv \frac{1}{2}\begin{bmatrix} \mathbf{A}(Q) & 0 \\ 0 & \mathbf{\Delta}(Q) \end{bmatrix}$$

Coupling

Pure Tearing Parity

We study the single resonant surface 2/1. High flow shear between surfaces shields coupling.



Comparison Between Tearing Parity Analysis and Coupled Tearing and Interchange Clarifies Sensitivity

The Glasser, Greene and Johnson (1975) analytic inner layer is compared to the numerical result from Galkin, Turnbull, Greene and Brennan, (2002).

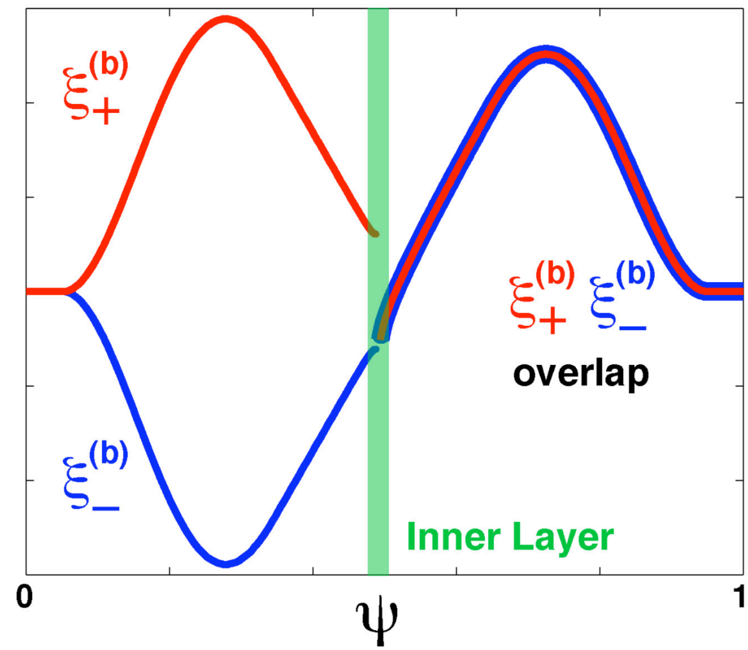
	parity		parity
A′	++	**A(Q)**	++
B′	+-		
Γ′	-+		
Δ′	- -	**Δ(Q)**	- -

$$\begin{bmatrix} \mathbf{A}' & \mathbf{B}' \\ \mathbf{\Gamma}' & \mathbf{\Delta}' \end{bmatrix} - \begin{bmatrix} \mathbf{A}(Q) & 0 \\ 0 & \mathbf{\Delta}(Q) \end{bmatrix} = 0$$

$$\mathbf{\Delta}' - \mathbf{\Delta}(Q) = 0$$

$$\mathbf{\Delta} = 2\pi \frac{V_s}{X_o} \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} Q^{5/4} (1 - \frac{\pi D_R}{4Q^{3/2}})$$

Large Solution (b)



Galkin 2002 solves the problem numerically finding both Δ(Q) and A(Q).

GGJ Solves the problem analytically for Δ(Q) alone, not A(Q).
Includes interchange drive through D_R.

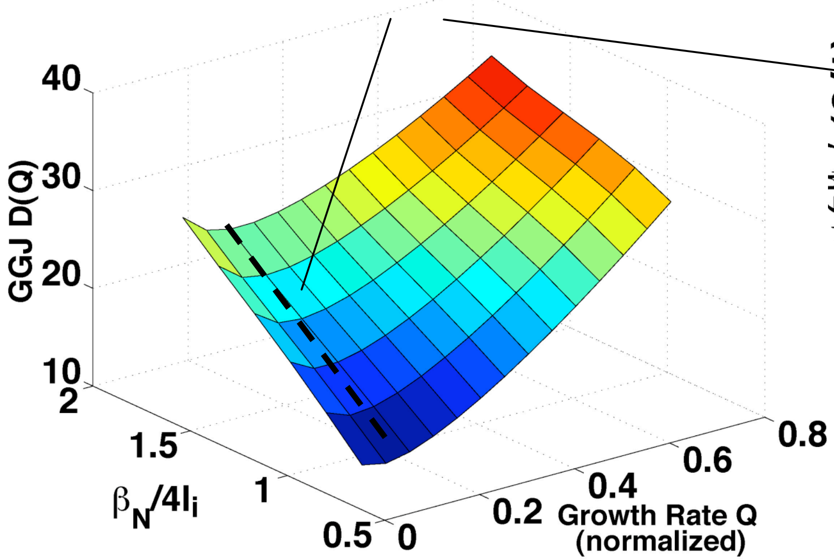
Inner Layer Analysis Indicates $\Delta(Q)$ is Large, GGJ Result Implausible

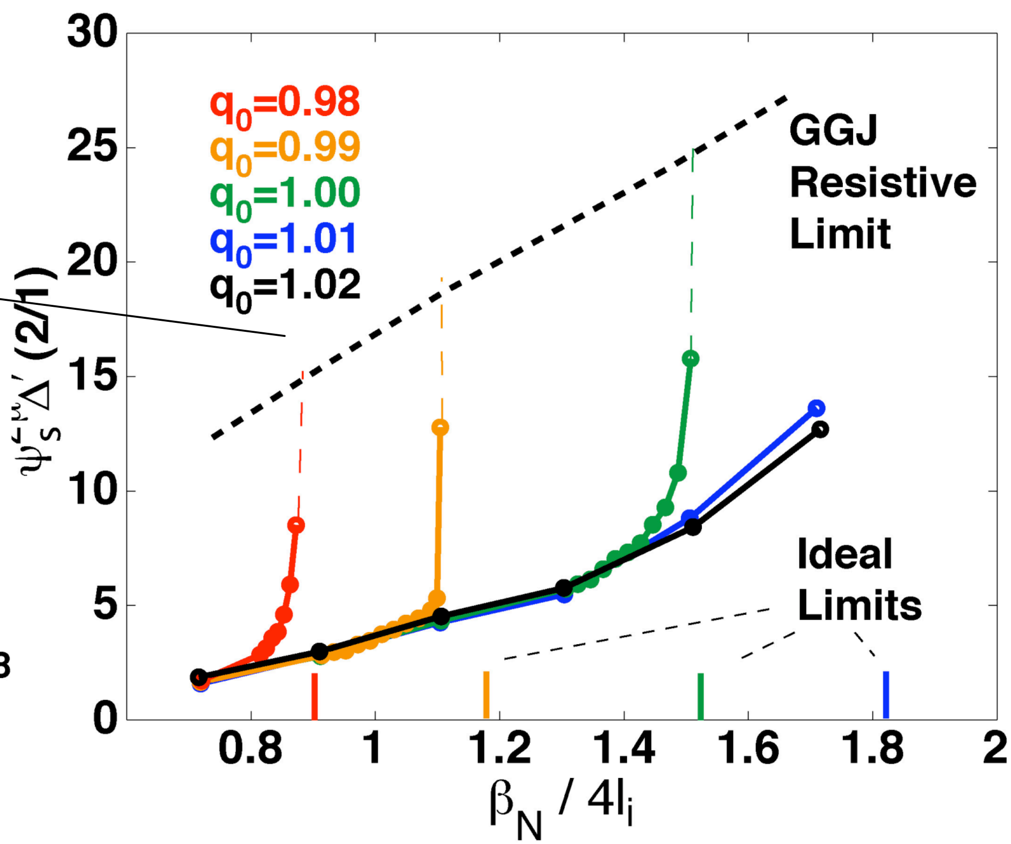
Very Large $\Delta'>\Delta(Q)$ Needed For Onset

Onset point extremely close to ideal limit, suggesting resistive instability not accessible to experiment.

$$\Delta' - \Delta(Q) = 0$$

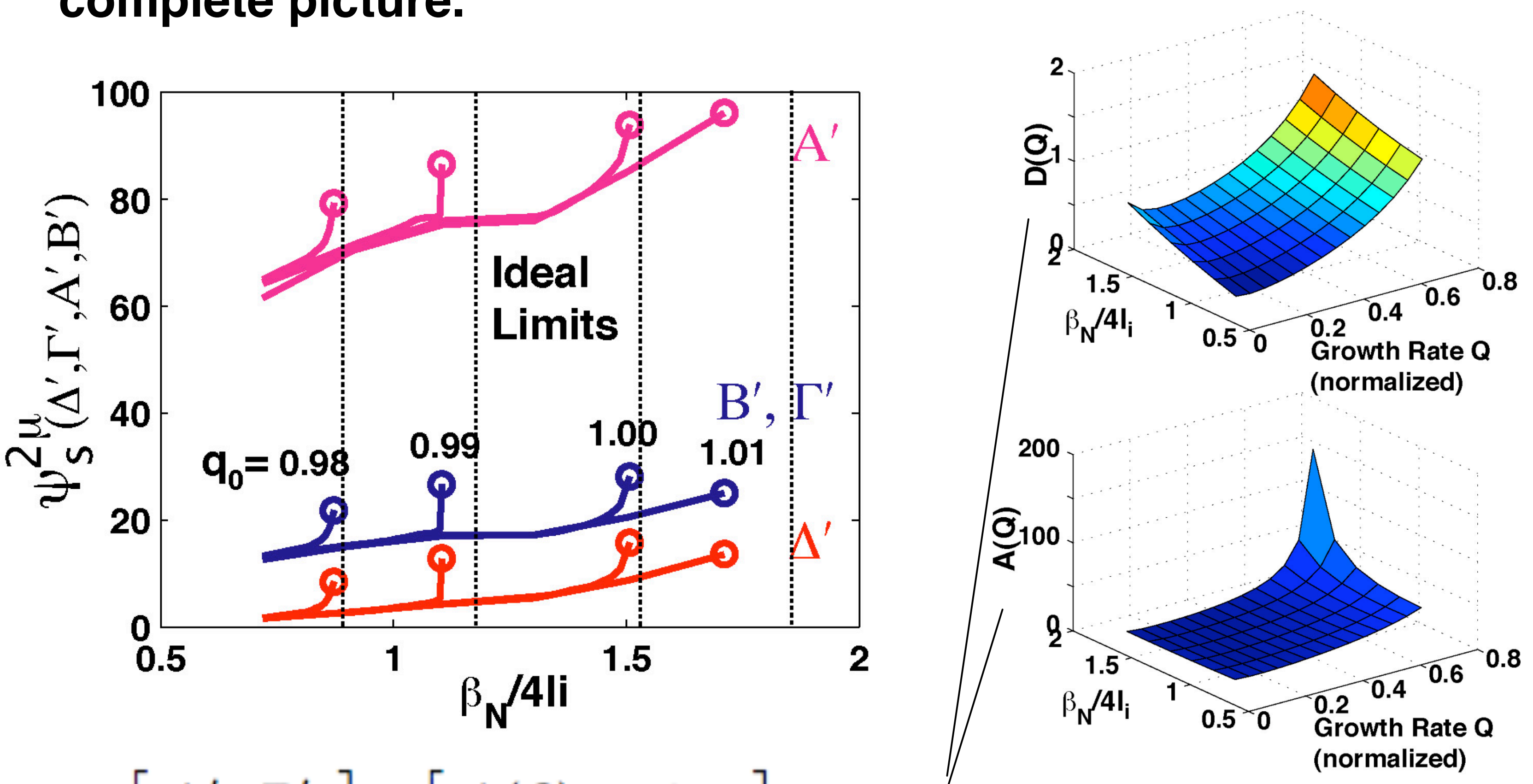
Minimum Δ' for root in Q.





Coupled Tearing/Interchange Analysis Indicates A′,A(Q) Large, Result More Accurate

All four elements, A', B', Γ' and Δ' must be addressed for complete picture.

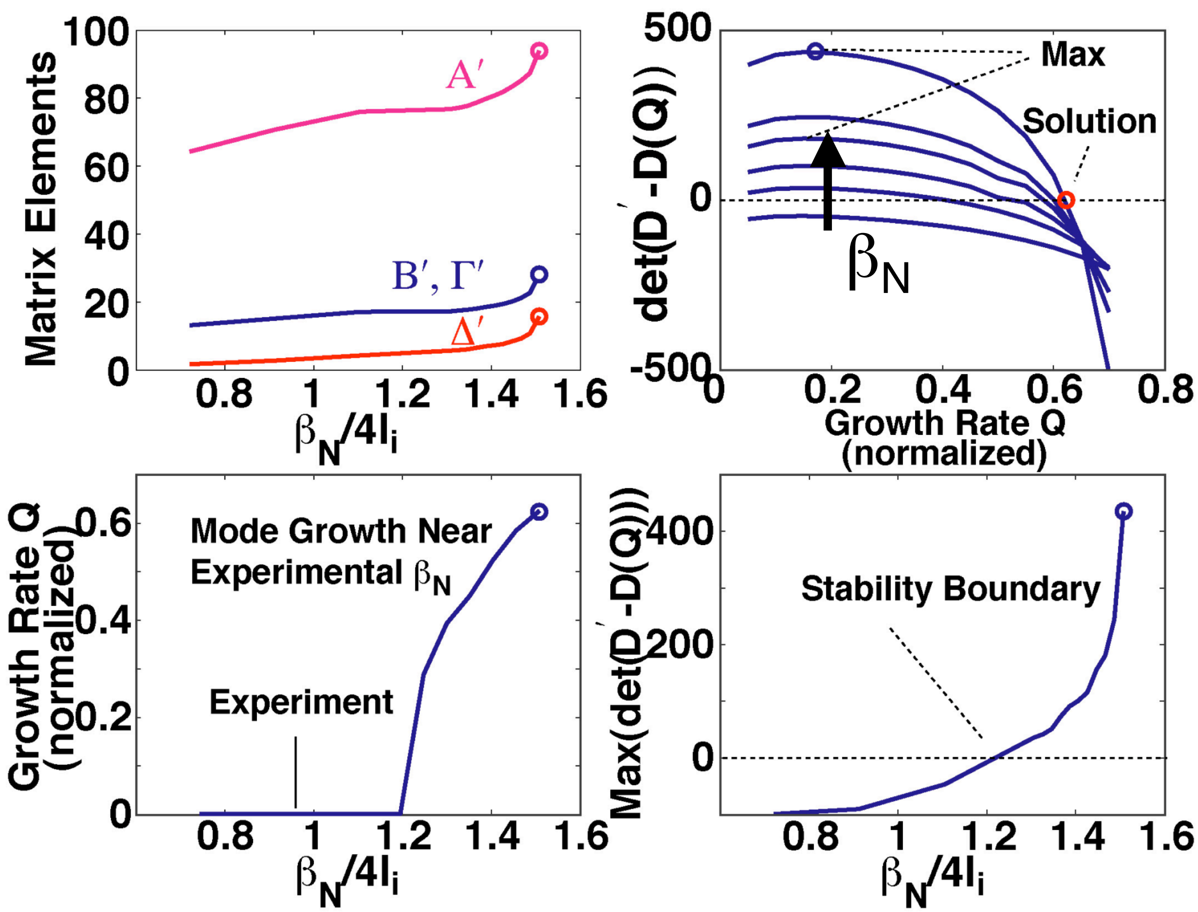


$$\begin{bmatrix} \mathbf{A}' & \mathbf{B}' \\ \mathbf{\Gamma}' & \mathbf{\Delta}' \end{bmatrix} - \begin{bmatrix} \mathbf{A}(Q) & 0 \\ 0 & \mathbf{\Delta}(Q) \end{bmatrix} = 0$$

Both inner layer solutions critical to analysis.

Coupled Tearing/Interchange Analysis Plausible Shows Onset At Lower β_N

q_0=1.01



Maximum of matrix determinant as a function of growth rate Q gives stability boundary.

For lower q_0 lower stability boundary in β_N

Including all four matrix elements is essential for agreement.

Coupled Tearing/Interchange Analysis Explains Onset

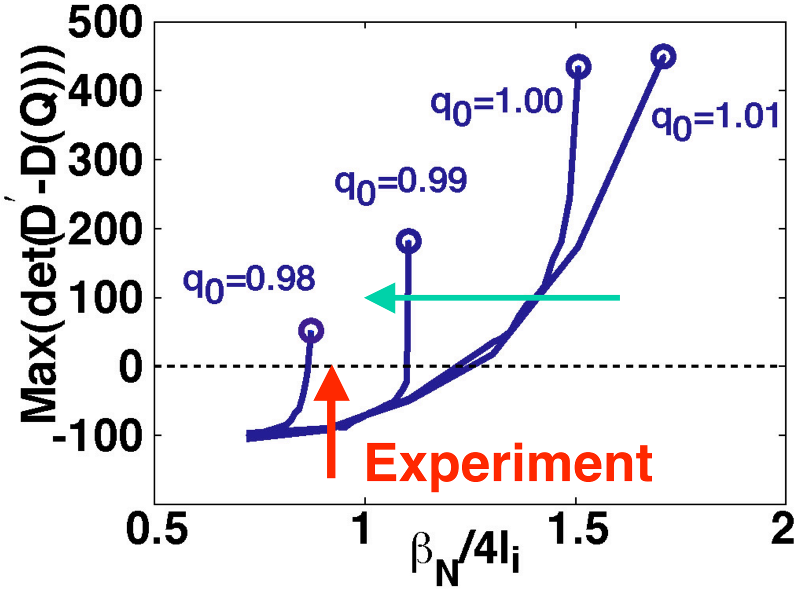
The maximum determinant crosses zero at experimental β_N, causing onset

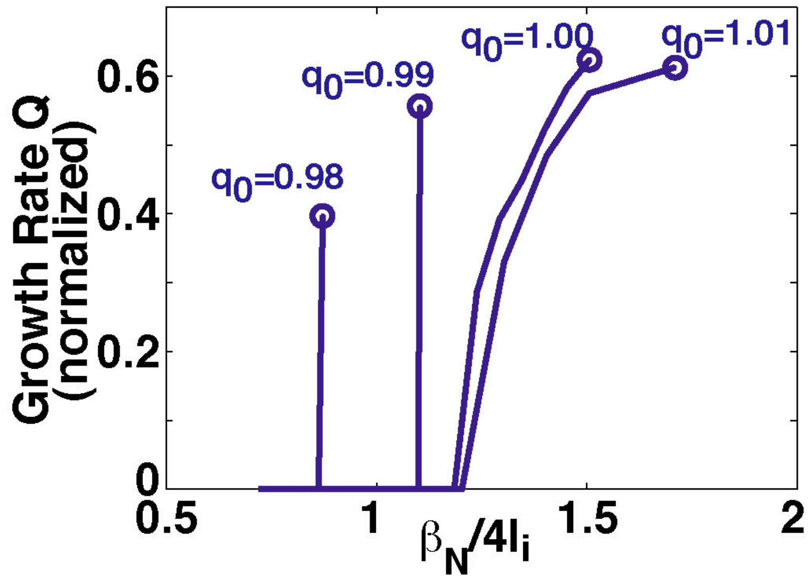
Interchange important at high β_N, and is considered.

Growth rates are within experimental observations.

$\gamma = Q/\tau$
$1/\tau \sim 60 s^{-1}$

Ideal n=1 limit crossed just above $\beta_N/4l_i$ of circle points. Internal kink unstable.



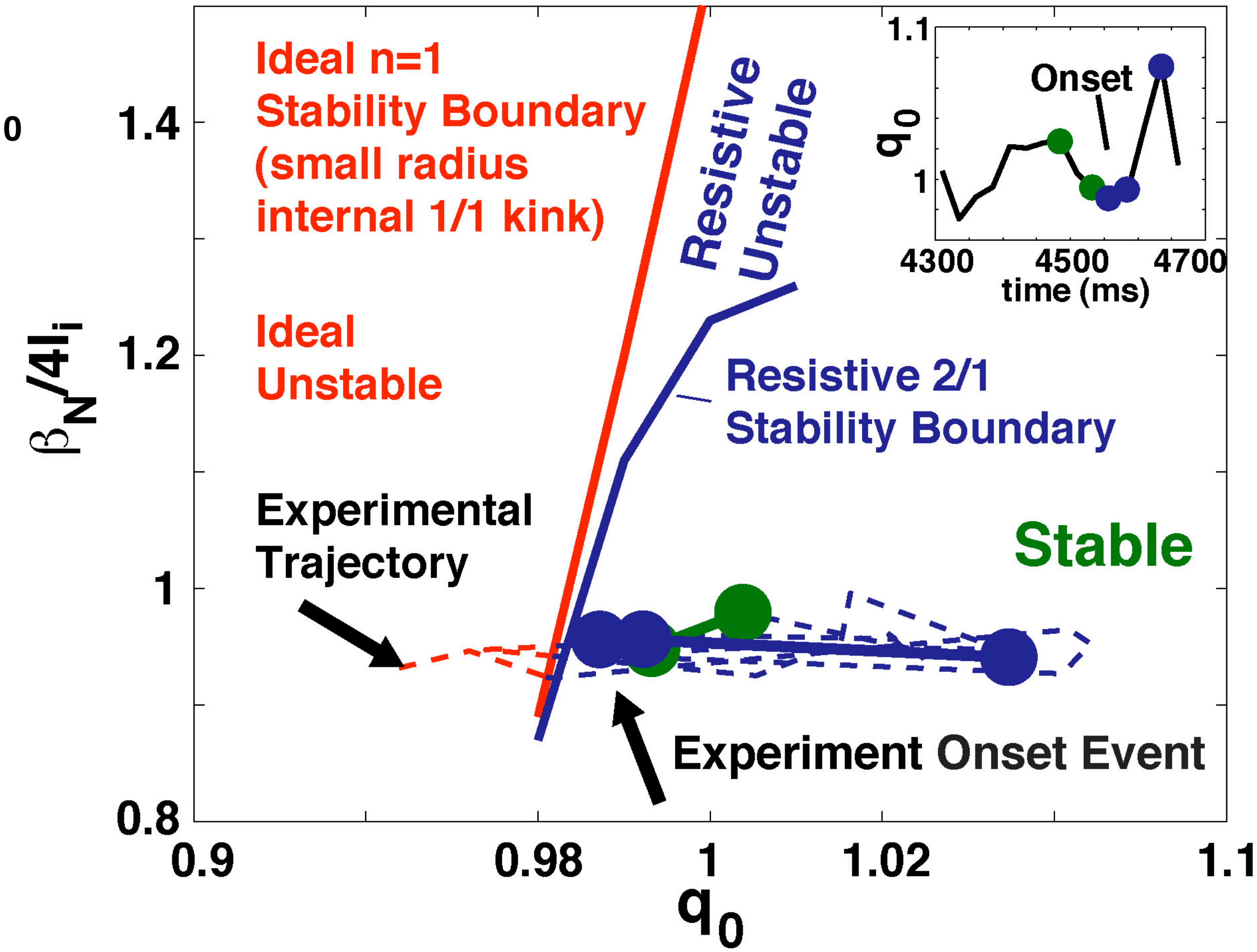


Further Evidence: Experimental Trajectory Crosses the Resistive Limit in q_0 Just Before Onset

Ideal n=1 β_N limit drops strongly as q_0 approaches 1.

2/1 resistive mode linearly unstable when trajectory crosses resistive limit.

Experiment in unstable region at onset.

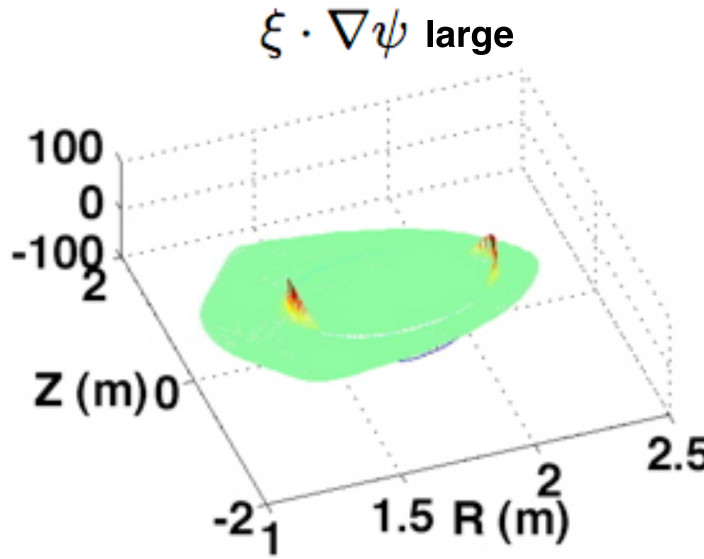


Stability map, generated in advance, could be used as real-time indicator of proximity to stability boundary.

Non-Resonant Small Solution Much Larger Outside Resonant Region Than Resonant Large Solution

Small solution (associated with Ideal instability) is very large on axis near q=1.

Instability, however, is reconnection at q=2.



PEST-III

Resonant

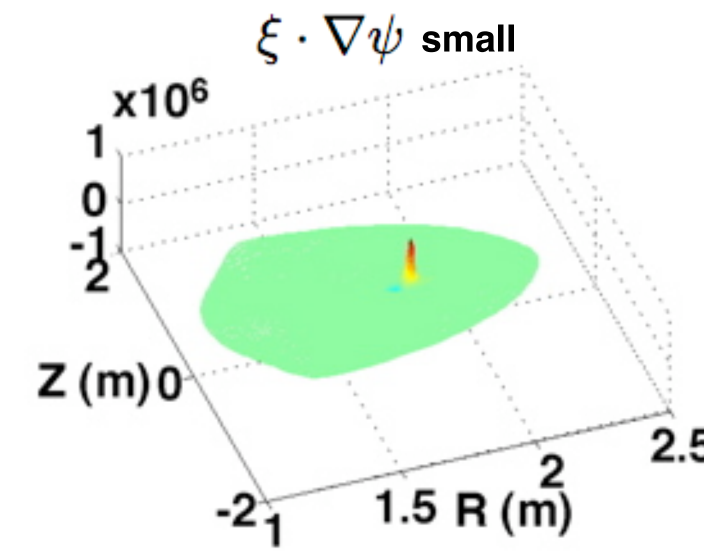
(2/1)

Large Solution

NIMROD calculates

$$\xi = \xi_{small} + \xi_{large}$$

dominated by small solution at axis.



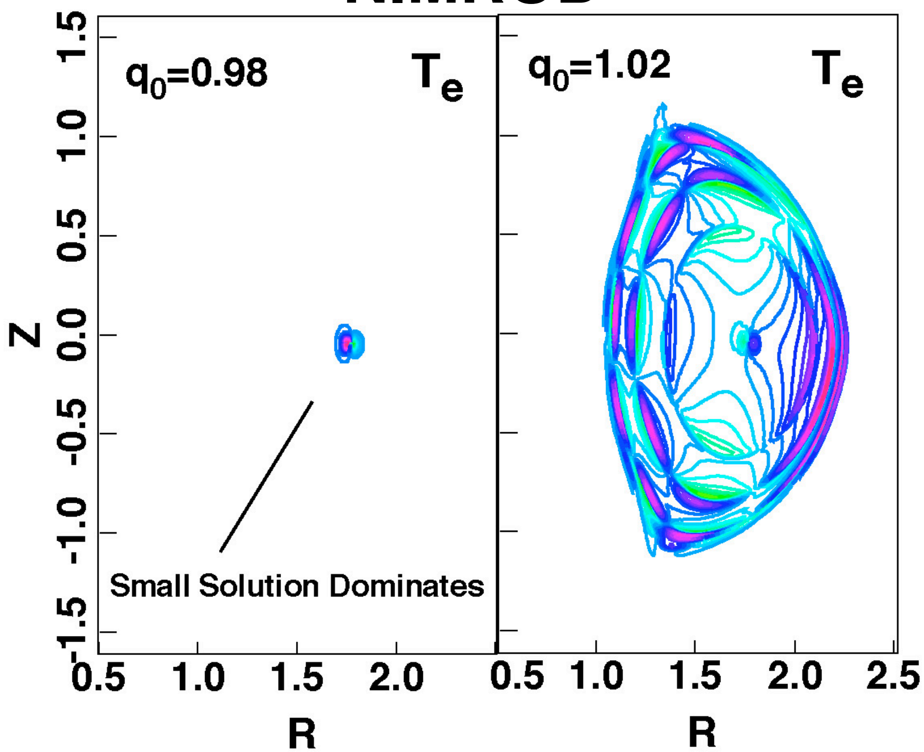
Nonresonant

(1/1)

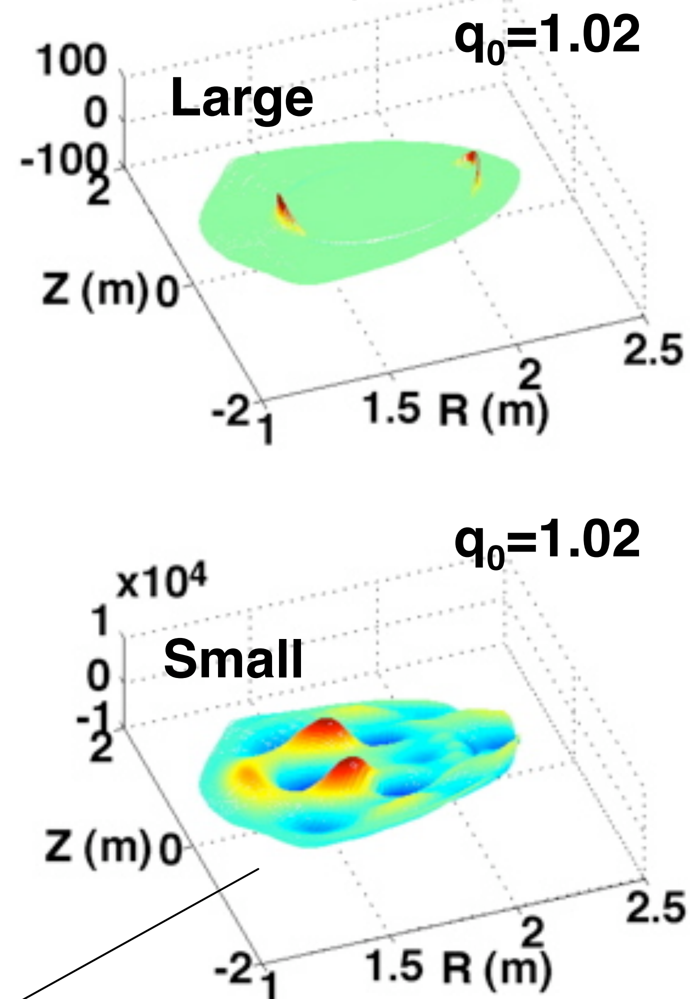
Small solution

Approach to q_0=1 Causes Large Nonresonant Response on Axis.

NIMROD



PEST-III



NIMROD and PEST-III agree that as q0 raises above 1 even slightly, small solution on axis diminishes.

At q0>1 small solution diminished on axis.

Large Nonresonant 1/1 Response on Axis Ubiquitous: Affects Linear Stability and Evolution.

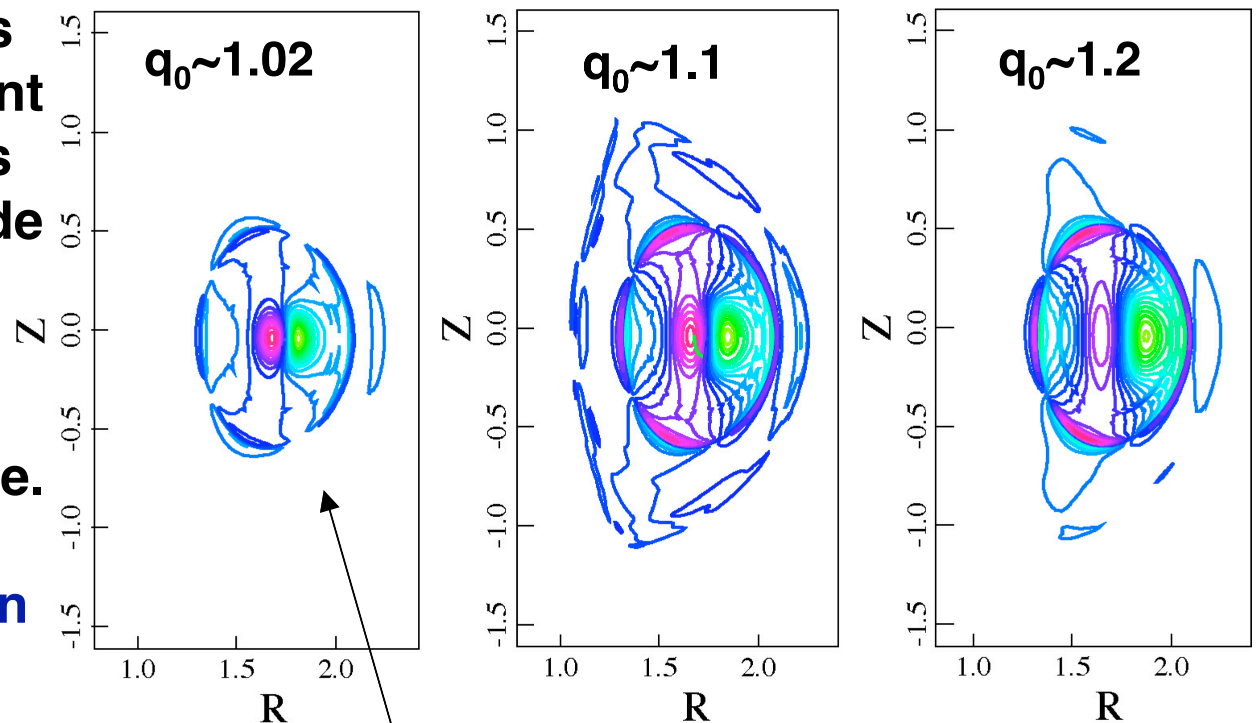
As the q_0 approaches unity, the nonresonant n=1 response on axis increases in amplitude in all cases.

Hybrid discharges "hover" in this regime.

The question is: when will the 2/1 mode be affected by or set off by this resonance?

Similar hybrid case with lower q shear on axis.

n=1



Increased peak amplitudes will nonlinearly drive n=2 coupling.

Observations and Experimental Suggestions

By increasing q_0 even slightly >1, 2/1 can be avoided. Rapid increase in 2/1 β limit with q_0 can be tested experimentally.

Slowing the rate at which the q_0 approaches unity can allow current drive mechanism to prevent further q_0 reduction and resonance.

Example relevance to control:

we can produce a stability map for a specific discharge target.

Stability map can be used for real time control of target discharges => Higher beta values accessible.

Why haven't theorists done this and given the stability maps to the experimentalists?

Because the model is too simple => more physics needed.

How Will Toroidal Flow Affect These Results - Coupling?

After island leaves linear phase, enters Rutherford and nonlinear phases => nonlinear treatment necessary for finite size islands

Previous studies detailed effects of flow on non-ideal MHD instabilities, nonlinear regime current focus

Several papers on effect of shear flow with and without viscosity

Chen and Morrison - Basic approach of FKR with added flow - typical constant ψ regime more unstable with flow

Dewar and Persson / Offman - mixed parity with flow - real and imaginary eigenfunctions - nonlinear theories typically neglect

Menard - progress can be made with equilibrium rotation and resistive MHD (two-fluid, neoclassical physics may not be necessary).

Two highly relevant studies:

S.E. Kruger, PhD. Thesis, U. Wisc. (1999)

A. Sen et al. (2007)

Both reduced MHD initial value computations of coupled modes in toroidal geometry

Summary (combined) of Recent Results of A. Sen et al., May 2007 and S.E. Kruger, PhD Thesis (1999)

- Physics model (GRMHD) based on Kruger, PhD Thesis (1999)
  - Several results in common.
- In the linear regime:
  - flow induces mode rotation
  - differential flow : stabilizing influence
    - modification in Mercier criterion (flow shear)
    - decoupling of rational surfaces
  - negative flow shear: destabilizing influence
- Nonlinear regime
  - Above trend continues for differential and sheared flows
  - Mode acquires real frequency which asymptotes to flow frequency
  - Flow reduces saturated island width
- Effects of sheared flow depends on the sign of flow gradient as + ve flow shear has stabilizing influence

Effects can be somewhat separated into flow shear and differential flow

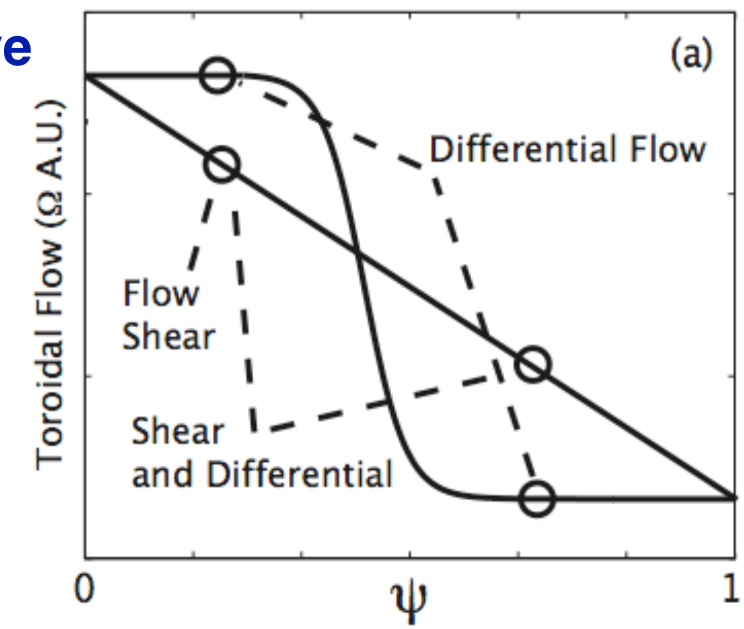
- flow induces mode rotation => complex Q
- differential flow : stabilizing influence
- flow shear: stabilizing or destabilizing influence

Particularly important result in Kruger's thesis: The combination of shear and differential flow can produce complicated results - this is the case for realistic profiles.

Linearly, for two surfaces, we can solve

$$\begin{vmatrix} A'_{11}-A_1(Q) & A'_{21} & & & B'_{11} & B'_{21} \\ A'_{12} & A'_{22}-A_2(Q) & & & B'_{12} & B'_{22} \\ \Gamma'_{11} & \Gamma'_{21} & & \Delta'_{11}-\Delta_1(Q) & & \Delta'_{21} \\ \Gamma'_{12} & \Gamma'_{22} & & \Delta'_{12} & & \Delta'_{22}-\Delta_2(Q) \end{vmatrix} = 0$$

where $Q = Q_R + iQ_i$



First: initial value analysis

The basic physics of coupling between two reconnecting surfaces: flow decouples

Ignoring the interchange part

$$\begin{vmatrix} \Delta'_{11} - \Delta_1(Q) & \Delta'_{21} \\ \Delta'_{12} & \Delta'_{22} - \Delta_2(Q) \end{vmatrix} = 0$$

Moving to the reference frame of the flow $Q = \omega - i\Omega_j$

We can write a quadratic for the complex growth rate

$$2\omega = \Omega_1 + \Omega_2 + i(\gamma_{11} + \gamma_{22}) \pm [\Delta\Omega + i(\gamma_{22} - \gamma_{11})][1 - \frac{4\gamma_{12}\gamma_{21}}{[\Delta\Omega + i(\gamma_{22} - \gamma_{11})]^2}]^{\frac{1}{2}}$$

where $\gamma_{ij} = \frac{\Delta'_{ij}}{\tau_{Li}}$ is the growth rate in the rest frame of the surface I, (IGNORING inner layer effects)

and $\Delta\Omega = \Omega_1 - \Omega_2$ is the differential flow

Taking $\Delta\Omega >> \gamma_{ij}$ and looking at surface 2:

$$\omega \approx \Omega_2 - \frac{\gamma_{21}\gamma_{12}}{\Delta\Omega} + i\gamma_{22}$$

Looking at the growth of mode 2:
(assuming 2 is linear stable)

$$\frac{\psi_2}{\psi_1} \approx \frac{\gamma_{21}}{\Delta\Omega}$$

Intuition for initial value analysis:

At high flow the modes are uncoupled (slightly slower rotation), inner layer dominates.

Resistive MHD Equations Used to Numerically Model Plasma Evolution in NIMROD

http://nimrodteam.org

Single Fluid model ignores drift waves, captures essential physics of MHD stability.

Viscous Stress

$$Mn\frac{d\mathbf{V}}{dt} = -\nabla p + \mathbf{J}\times\mathbf{B} - \nabla\cdot\Pi$$

Resistivity

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{J}$$

$$n\frac{\partial T}{\partial t} + n\mathbf{V}\cdot\nabla T + (\Gamma-1)nT\nabla\cdot\mathbf{V} = -(\Gamma-1)\nabla\cdot\mathbf{q} + (\Gamma-1)Q$$

$$\mathbf{q} = -(\kappa_{\parallel}-\kappa_{\perp})\nabla_{\parallel}T - \kappa_{\perp}\nabla T$$

Thermal Anisotropy

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$

$$\mu_0\mathbf{J} = \nabla\times\mathbf{B}$$

Two fluid treatment readily accessible => understand single fluid first.

Significant mach numbers affect equilibrium via the rotational component of the pressure

Faster flow interesting because linear growth rates faster than experiment.

We apply a flow shear that is constant in ψ

Significant effect on equilibrium

Where we have a mach number

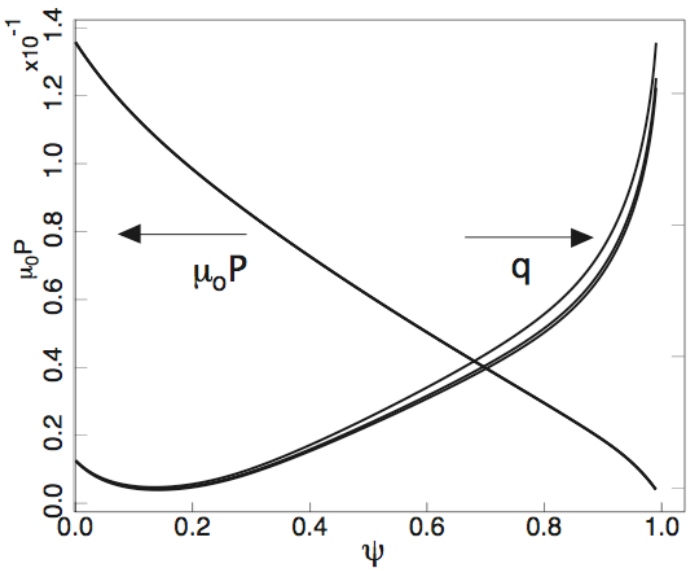
equilibrium mass density ρ_i is constant

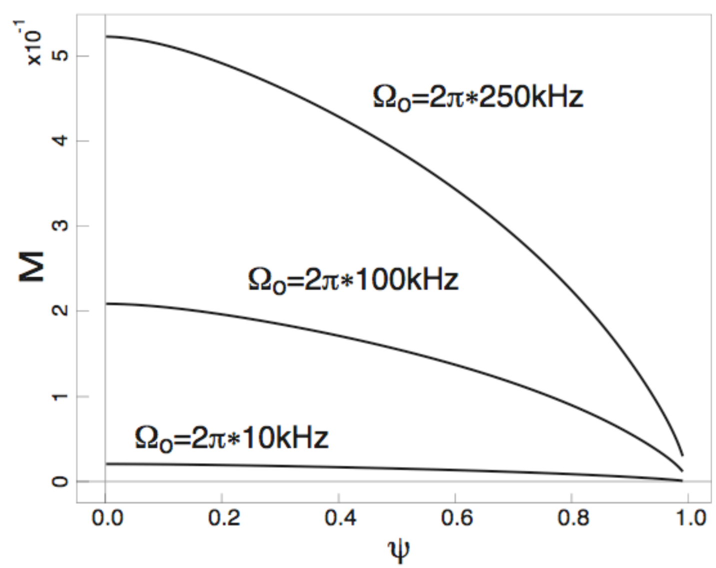
P_0, q and shape based on DIII-D experiment

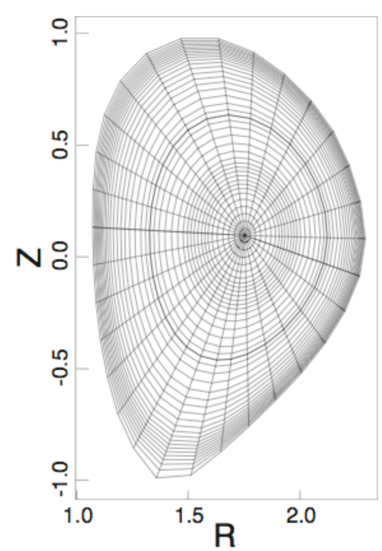
$$\Omega = \frac{d\Omega}{d\psi}(1-\psi)$$

$$P = P_0 e^{((R/R_0)^2-1)M^2}$$

$$M = \Omega R_0 \sqrt{\rho_i/(2P)}$$







At relevant flow and growth rates, modes rotate past each other several times during growth

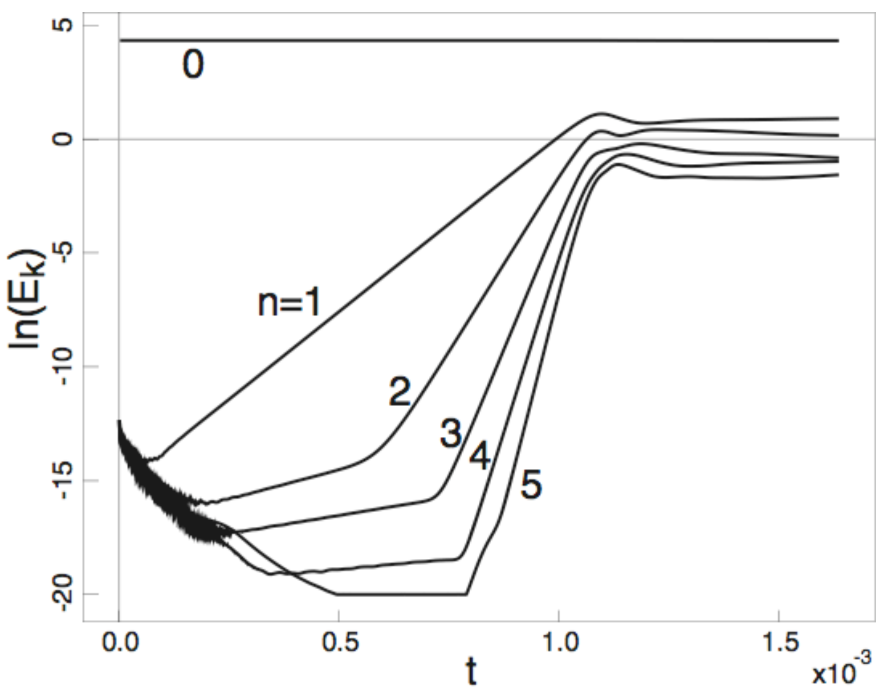
Significant flow differential

1/1 drives 2/2 in $\Delta t \sim 3\times10^{-4}s \sim 10^{3}\tau_A$

$\Delta\Omega \sim 3\times10^{4}$Hz for Ω_o=100kHz

$$\frac{M_1}{M_2} = \frac{\Omega_1}{\Omega_2}\sqrt{\frac{P_2}{P_1}}$$

$$\Delta\Omega \approx \frac{d\Omega}{d\psi}\Delta\psi$$



$$\Delta\Omega \approx 30kHz$$

$$n_\Omega \approx \Delta\Omega\Delta t \approx (3\times10^4)(3\times10^{-4}) \approx 10$$

Naively ~10^1 rotations past each other (2/2 and 3/2) in a growth time

For experimental parameters, growth rates are lower, leading to similar conclusions

Inner layer flow effects can be destabilizing in linear theory

We are in the constant ψ regime where Δ′~-10->10

The linear tearing mode scaling with flow in slab model (without viscosity) suggests a destabilizing influence from the inner layer.

	Constant-ψ tearing modes	Nonconstant-ψ tearing modes
$\left\|\frac{G'(0)}{F'(0)}\right\| \ll 1$	$\gamma \sim \alpha^{2/5}\Delta'^{4/5}S^{-3/5}\hat{\gamma}$ $\delta \sim \alpha S^{-2/5}\Delta'^{1/5}$ Approx. valid if $\delta\|\Delta'\| \ll 1$ Small flow shear G'(0) destabilizes tearing mode $\hat{\gamma} =$ flowcorrection ≥ 1	$\gamma \sim \alpha^{2/3}S^{-1/3}$ $\gamma \sim \alpha S^{-1/3}$ $\delta\|\Delta'\| \gg 1$ Small flow shear stabilizes tearing mode with sufficiently large Δ'
$\left\|\frac{G'(0)}{F'(0)}\right\| \lesssim 1$	$\gamma \sim (\alpha\|\Delta'\|)^{1/2}S^{-1/2}$ $\delta \sim \alpha S^{-1/3} \ll 1$ Approximation valid if $\|\sqrt{Q\delta\|\Delta'\|}\| \ll 1$ If $G'(0)G''(0) - F'(0)F''(0) \neq 0$, Δ' instability criterion is removed	Transition to the ideal K-H instability when Δ' becomes negative through $\Delta' = \infty$ (because of flow in outer region)
$\left\|\frac{G'(0)}{F'(0)}\right\| > 1$	Stabilized	Stabilized

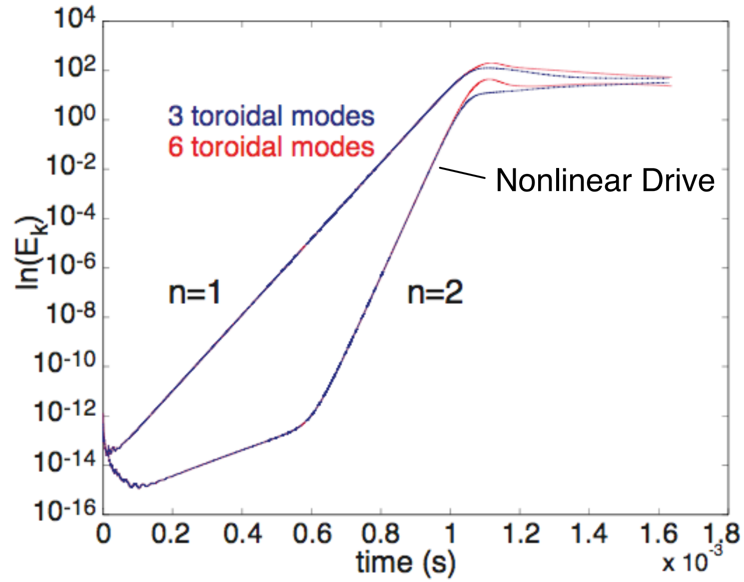
NIMROD results show linearly destabilizing regime

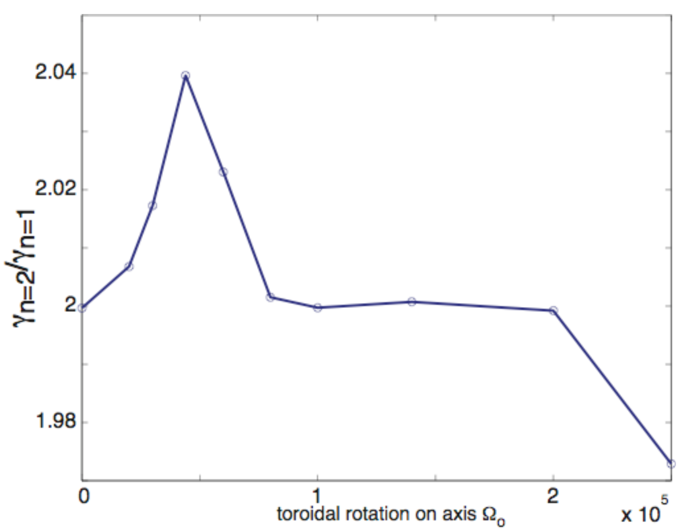
Nonlinear phase resolves > 6 modes. Can study coupled driven phase with 3 modes

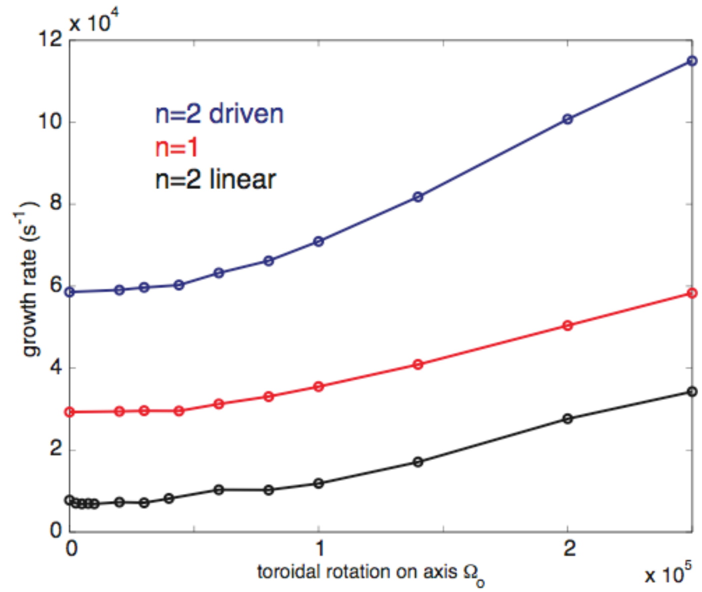
Qualitative in nonlinear phase with 3 modes

Growth rates increase with flow

Slight change with direction







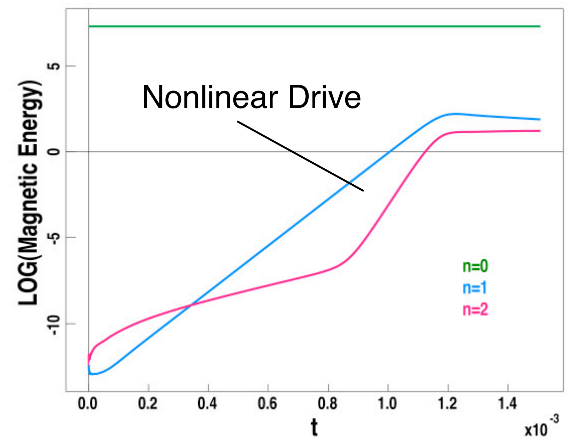
Differential flow damping at low flow, driving from inner layer dominant at high flow

The relative size of the 2/2 to 3/2 B_r fields during nonlinear drive informative

Reduction in relative drive to 3/2 at low flow

At higher flow the drive begins to increase

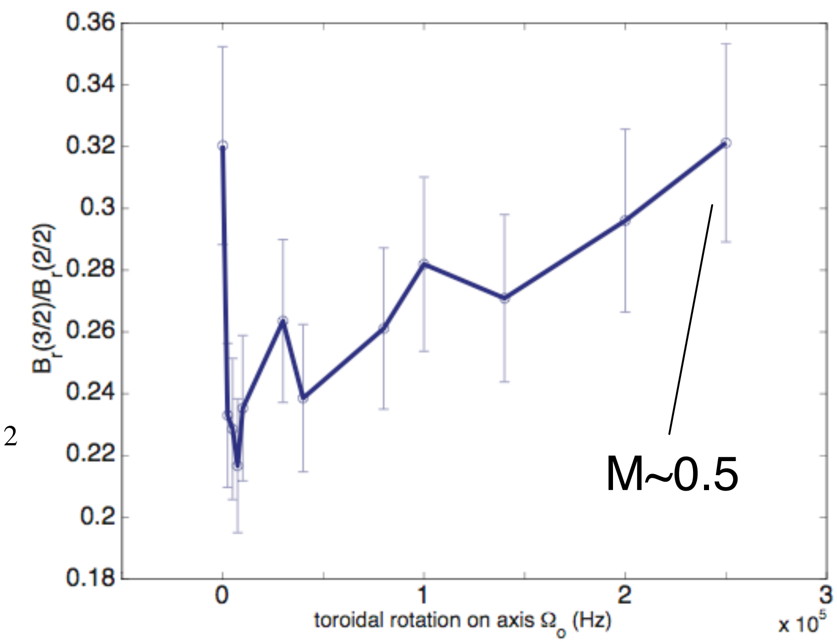
Can measure the island width by the method of Wesson



Wesson island w $$w = \left(\frac{16 r R_o |\tilde{B}_r|}{m s B_{\phi 0}}\right)^{1/2}$$

Shear $$s = \frac{r}{q^2}\frac{dq}{dr}$$

Relative widths $$\frac{w_{3/2}}{w_{2/2}} \approx C\left(\frac{B_r(3/2)}{B_r(2/2)}\right)^{1/2}$$



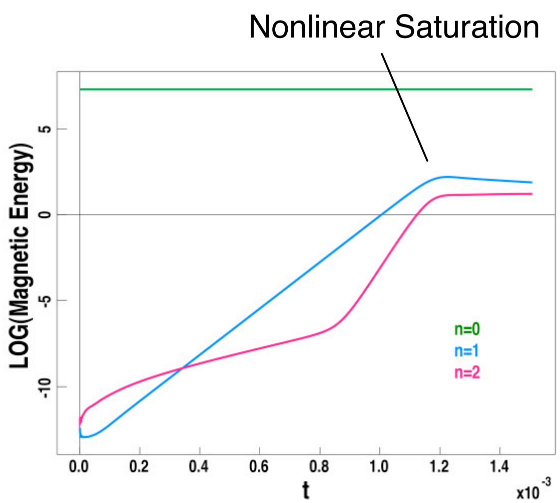
Nonlinear saturation of driven 3/2 increases with flow

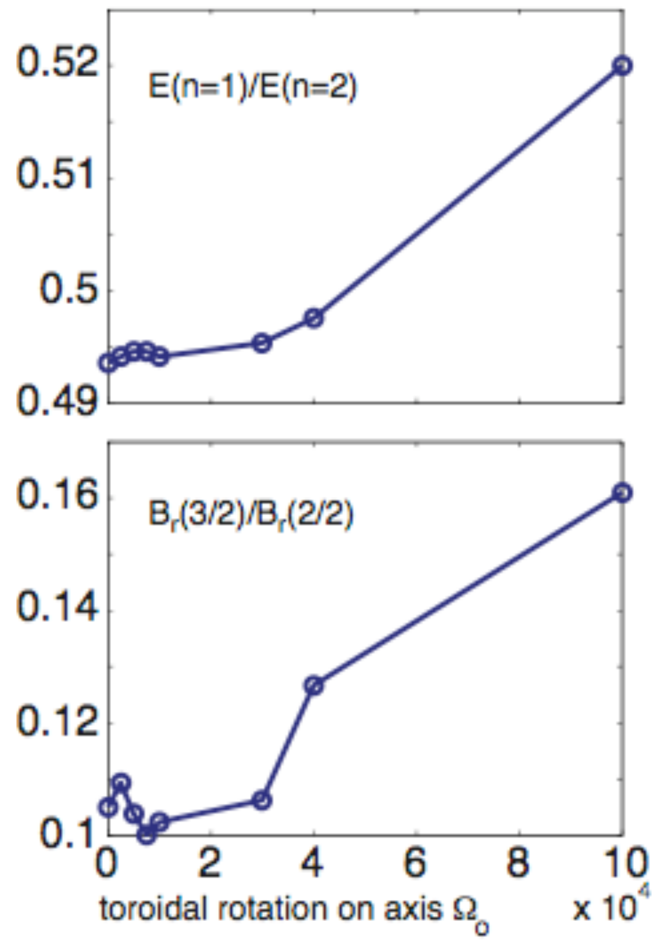
At the point where the 1/1 saturates the 3/2 is driven to slightly larger relative B_r with increasing rotation.

Not surprising with increasing growth rates.

Can this be reconciled with Sen et al?

How will two fluid effects change this?





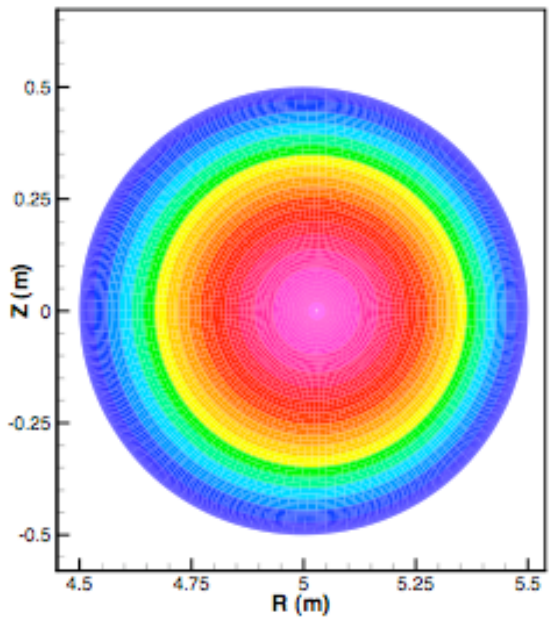
S. Kruger 1999 show a stabilizing influence between 2/1 and 3/1, but analogous nonlinear drive.

Comparative Study with Kruger (1999 Thesis)

Higher field
Larger aspect ratio
Low β
Circular shape

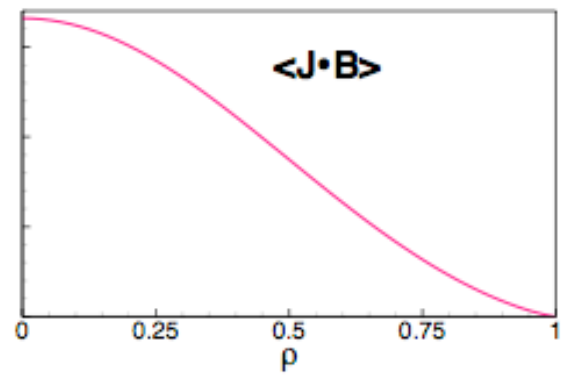
The large radius causes a higher Mach number for similar Ω and ρ than DIII-D study.

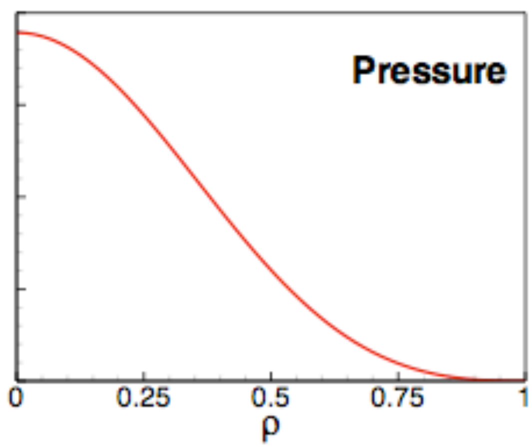
$$M = \Omega R_0 \sqrt{\rho_i/(2P)}$$

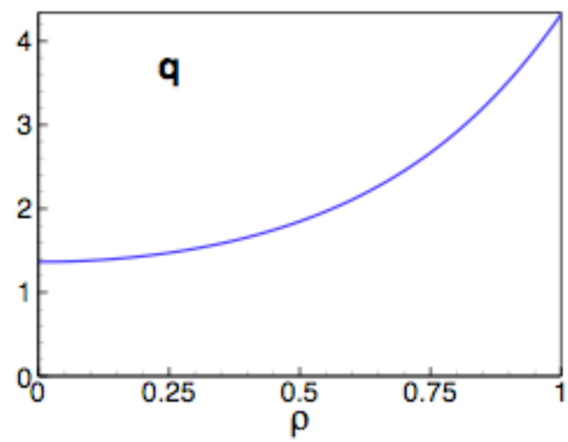


Equilibrium Parameters

B_o = 4.80 T R_o = 5.00 m
I_p = 0.28 MA   a = 0.50 m
β = 0.26%

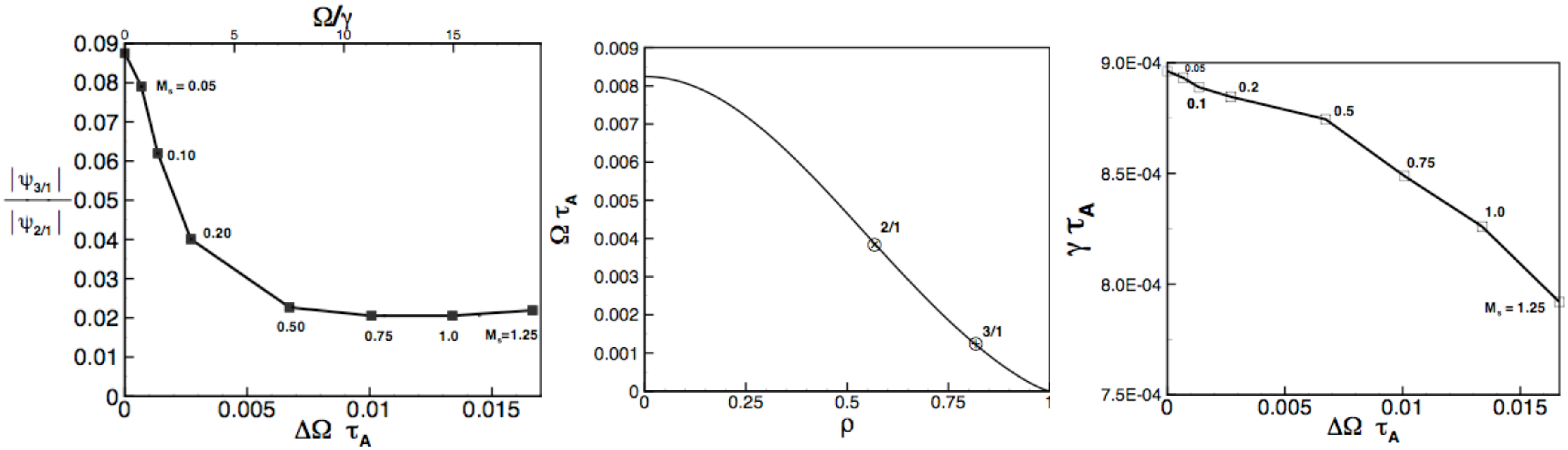






Kruger 1999: Similar results found with GRMHD code for 2/1 driving 3/1

Initial damping of the 2/1 drive to the 3/1 is followed by an increasing driven 3/1 flux at high flow.



Case includes both differential and shear flow, damping is seen. **Growth rates decrease with flow. Contrary to 2/2 driving 3/2 at lower aspect ratio high β.**

Note that at lower aspect ratio <5 and high β, equilibrium flow effects are not separable from stability analyses in toroidal geometry.

2/2 driving 3/2 NIMROD results have maximum $\Delta\Omega\tau_A$~0.06 (higher $\Delta\Omega$ than here for equal mach number. Note the increase above $\Delta\Omega\tau_A$~0.015 and compare to 2/2->3/2 case)

Summary

In linear analysis, ALL matrix elements, coupling tearing and interchange, important for accurate mode onset at high β.

In DIII-D Hybrid discharges the resistive instability at q=2 is sensitive to q_0 approaching 1, as a result of the ideal β_N limit rapidly changing with q_0~1.

Experimental trajectory in stability map indicates this physics mechanism causes 2/1 onset => suggestions for avoidance.

Large nonresonant 2/2 component can drive current and raise q_0, playing important role in evolution to instability.

Including flow: non-resonant and resonant effects.

Summary and Discussion

S. Kruger thesis and Sen et al.=> shear vs. differential flow: realistic profiles not easily comparable to analytic theory - example, increase in driven flux at high flow.

The reconnected flux for a 3/2 mode driven by a 2/2 indicates flow can increase grow rates in single fluid, in finite island coupling physics as well.

Two fluid and neoclassical physics may be critical to analysis

- Competing effects of linear, nonlinear drive vs. shielding

R. Buttery et al. 2007 APS Invited: flow shear stabilizes 3/2 and 2/1 drive. Accurate computational analysis non-trivial - single fluid physics, despite complexity, may not be sufficient to explain.

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