FEEDBACK CONTROL OF INTERNAL MHD MODES

A.K. Sen

Columbia University

I. A NOVEL SUPRESSOR:

MODULATED ECH FEEDBACK FOR INTERNAL MHD MODES

Consider purely radial ECH injection: The transverse energy of electrons $W_{e\perp} \sim p_{e\perp}$ will nearly instaneously (compared to γ, ω_r) increase with resonance response!
$\therefore$ '$\tilde{p}_{e\perp}$' is the suppressor action!

A NAÏVE MODEL OF CONTROL ACTION:

$$\frac{dW_\perp}{dt} + \nu_d W_\perp = P^{ECH} = P_0^{ECH}(1 + \alpha e^{-i\omega_r t})$$, where α is mod. depth

$$\therefore \quad \tilde{p}_{e\perp}^{f.b.} \approx W_\perp = \left(\frac{\alpha P_0^{ECH}}{\nu_d - i\omega_r}\right) e^{-i\omega_r t} \sim i\alpha P_0^{ECH} e^{-i\omega_r t} / \omega_r \qquad (1)$$

A NAÏVE MODEL OF THE INSTABILITY:

$$\rho_0 \frac{d^2\vec{\xi}}{dt^2} = -\nabla\tilde{p} + \tilde{\vec{J}} \times \vec{B}_0 + \vec{J}_0 \times \tilde{\vec{B}}$$, $\vec{\xi}$ = Plasma displacement

$$\approx \rho_0 \gamma_0^2 \vec{\xi} \text{ (models the growth rate) } - \nabla\tilde{p}_{e\perp}^{f.b.} \text{ (ECH)} \qquad (2)$$

Set $\widetilde{p}_{e\perp}^{f.b.} = G\xi, \quad \nabla \sim L_{dep}^{-1}$, and find (3)

D.R. $-\omega^2 = \gamma_0^2 - G/(\rho_0 L_{dep})$

$\therefore$ Marginal stability, if $G = \gamma_0^2 L_{dep} \rho_0$ (4)

Using Eqs. (3), (4) & (1),

$P_0^{ECH} \sim \omega_r \rho_0 \gamma_0^2 L_{dep}\, \xi/\alpha$ (5)

Coupling coefficient C:

$C \approx (Radial\ \ conv.)(m/2\pi a)(n/2\pi R)\,(\pi r_b^2)$ (6)

For a Medium Size Tokamak:

$\gamma_0 \sim 10^4, \quad \omega_r \sim 10^3, \quad \xi = 10^{-2}$

$L_{dep} \sim (2\ to\ 4)\times 10^{-2}$, Beam spot size: radius $r_b \sim (1\ to\ 2)\times 10^{-2}$

From Eqs. (5) & (6):

$P_0^{ECH} \sim 5\ to\ 10\ MW$ Feasible!

Suggestion: Try a simple open loop test to check viability:
Turning ECH ON or OFF, look for magnetic or soft X-ray signature for plasma displacement

LOSS OF CONTROL INFORMATION VIA DRIFTS AND DIFFUSION

NOTE: STRONGLY ECH HEATED ELECTRONS ARE DEEPLY TRAPPED ELECTRONS!

1. TOROIDAL DRIFTS:

$\frac{\omega_{de}}{2\pi} = \frac{k'T_{e\perp}'}{eB_p}\frac{1}{R^2} \sim .3kHz$ for large tokamaks $\leq \gamma$

Not so for small tokamaks ➔ weaker coupling, but still viable!

2. RADIAL THERMAL CONDUCTION:

$$\tau_{cond} \sim \vartheta(10^{-3}\ \text{sec}) \geq \gamma^{-1}$$

3. THERMAL CONDUCTION ON MAGNETIC SURFACE:

$$\tau_{cond} \sim \vartheta((1-2)\times\nu_e^{-1}) > \gamma^{-1}$$

Rewrite Eq. (2) as

$$\rho \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{F}(\vec{\xi}) - g(\vec{r})\vec{f}^{\,f.b.}; \quad \vec{f}^{\,f.b.} = -\nabla \tilde{p}_e^{\,f.b.}$$

where $\vec{F}(\vec{\xi})$ is the MHD force operator; $g(\vec{r})$ is a structure function of the control source coupling ~ ECH foot print on the modes.

For $\vec{f}^{\,f.b.} = 0$, consider orthogonal eigenfunctions $\vec{\xi}_k(\vec{r})$ of the internal modes:

$$\vec{\xi}_k = e^{im\theta} e^{-in\phi} \vec{e}_k(r); \, k = (m, n)$$

Expand:

$$\vec{\xi} = \sum a_k \xi_k, \quad g(\vec{r}) = \sum b_k \xi_k;$$

Substitute the above and take scalar product with $\vec{\xi}^*$ to obtain:

$$\frac{d}{dt}[a] = [A][a] + [B]u(t) + [D]n_s(t) \qquad (7)$$

↑ a_k = modal amplitude

↑ $b_k P_0^{ECH} / \omega_r$

↑ state noise

For convenience, we drop the parenthesis […] sign for matrices below.

II. USE OPTIMAL STATE FEEDBACK WITH KALMAN FILTER

Performance index J to be minimized

$$J = a^T Q a + u^T R u$$

Solution:

$$u(t) = -K_c I(t) = -R^{-1} B^T S I(t).$$

where S is the solution of

$$SA + A^T S - SBR^{-1} B^T S = -Q,$$

Then the resulting modal amplitudes under feedback is the solution of

$$(A - BK_c) a_{RMS}^2 + a_{RMS}^2 (A - BK_c)^T = -DWD^T,$$

$W =$ RMS value of state noise $n_s(t)$.

DESIGN OF THE STATE OBSERVER VIA KALMAN FILTER

The output $\Theta(t)$ (derived from soft x-rays or ECE)
may be written as:

$\Theta(t) = Ha(t) + n_m(t),$ n_m is the measurement noise.

The estimated states $\hat{a}$ satisfy

$$\frac{d}{dt}\hat{a}(t) = A\hat{a}(t) + Bu(t) + K_f[\Theta(t) - H\hat{a}(t)]$$

where the caret indicates estimated state and K_f is Kalman filter gain chosen to minimize the error covariance.

The Kalman filter gain is given by $K_f = \overline{P}H^T V^{-1}$, where $\overline{P}$ is the solution of the equation:

$$0 = (A - K_f H)\overline{P} + \overline{P}(A - K_f H)^T + DWD^T + K_f V K_f^T$$

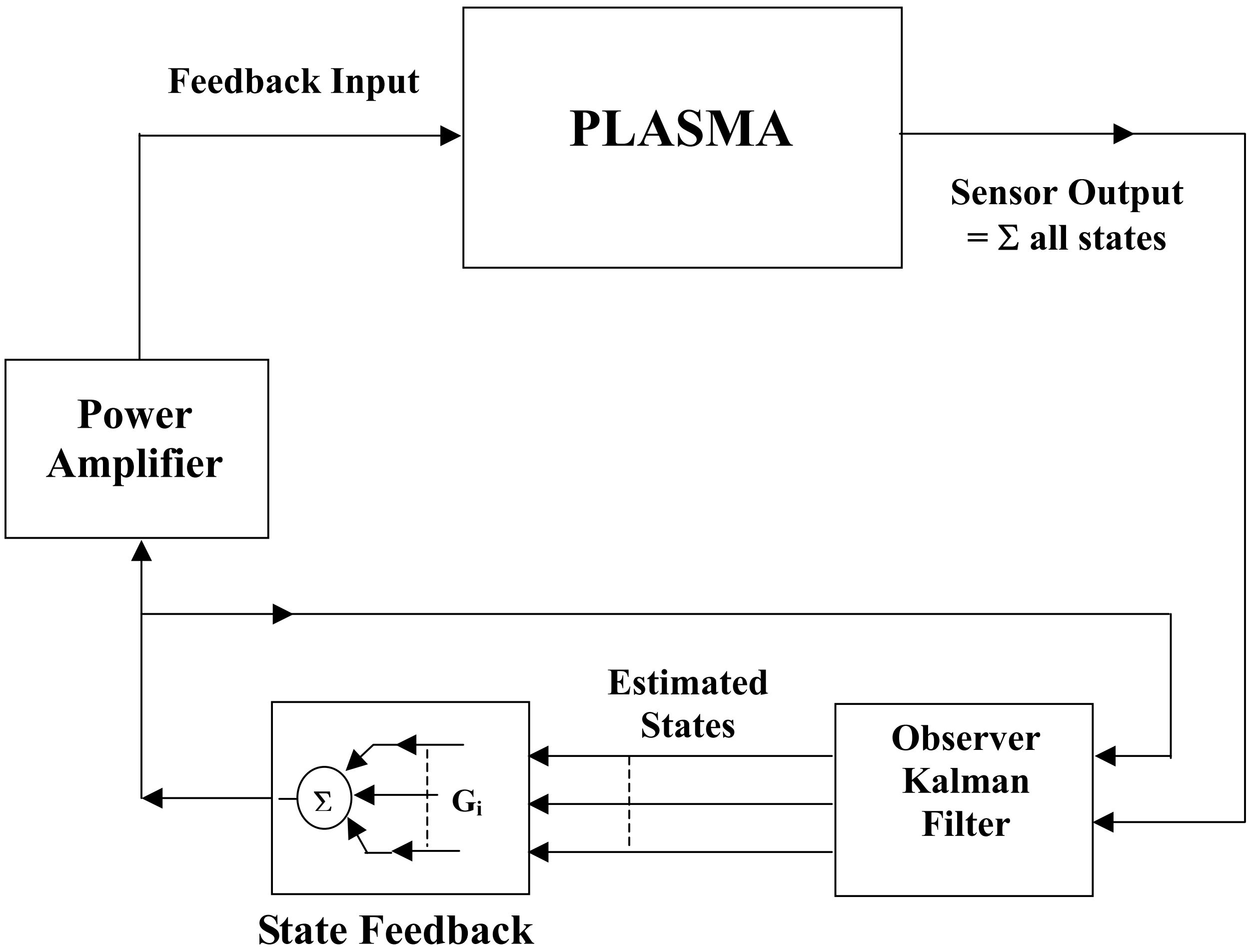
Then, the estimated states $\hat{a}$ are the solution of

$$0 = (A - BK_C)\hat{a}_{RMS}^2 + \hat{a}_{RMS}^2(A - BK_C)^T + K_f V K_f^T,$$

where V is the RMS value of output measurement noise $n_m(t)$.

$$a_{RMS}^2 = \hat{a}_{RMS}^2 + P$$

SCHEMATIC OF STATE FEEDBACK FOR MULTIMODE MHD MODES



III. SENSORS FOR MHD INTERNAL MODES

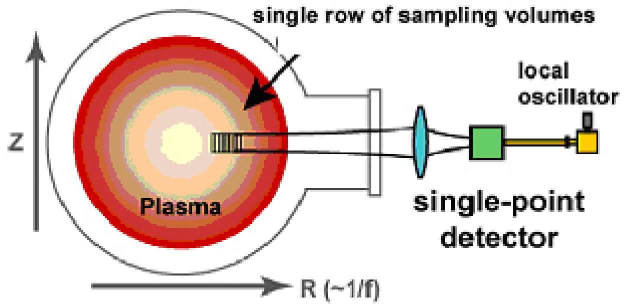
(ECE) EMISSION DETECTORS

- **ECE at electron cyclotron frequency resonance (ECR) layer at frequencies:**
  $l\omega_{ce} = l\omega_{ce}(R) = l\omega_{ce0}\dfrac{R_0}{R}$**,** $l = 1, 2$
  Optically thick for $l = 2$**: Blackbody radiation** $\tilde{I}(\omega) = \tilde{T}_e\omega^2 / 8\pi^3 c^2$**.**
- **In our case the relevant resonance layers are mode rational surfaces:**
  $m = nq;\ q = f(R) = m/n;\ n = 1-3;\ m = 2-6$
- **FFT of total ECE** $\Rightarrow 2\omega_{ce} \Rightarrow$ **Initial setting of LO.**
- **Feature extraction from 2D ECE** $\Rightarrow m, R$
  Stored MSE$\Rightarrow q \Rightarrow 2\omega'_{ce} \Rightarrow$ **frequency tuning of LO**
  Also $q, m \Rightarrow n$
- **Frequency sweep the LO frequency (via varactor diodes) over a small range for optimality, which can be done online.**
- **Chosen LO frequency will be used to set the ECH suppressor frequency** $2\omega_{ce}$**. The incremental change necessary can be accomplished by: varying** B_t **or** R **-- unpractical; modest steering of launching mirrors?? varying ECH frequency – difficult but possible (with multi-frequency ECH) .**

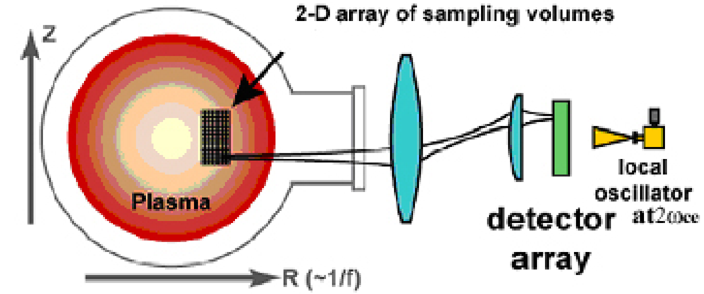
2D ECE imaging system

H.Park & TEXTOR Team

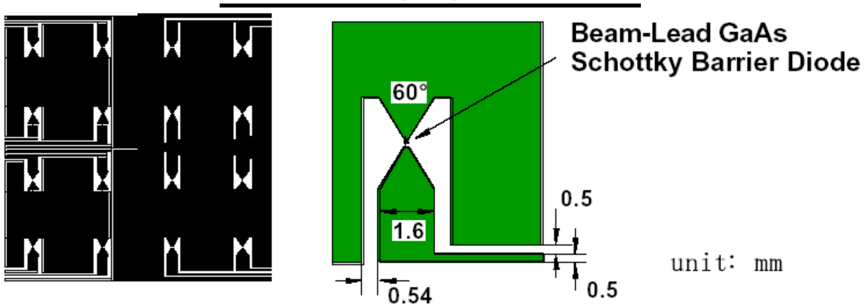
Conventional 1-D ECE system



2-D ECE imaging system

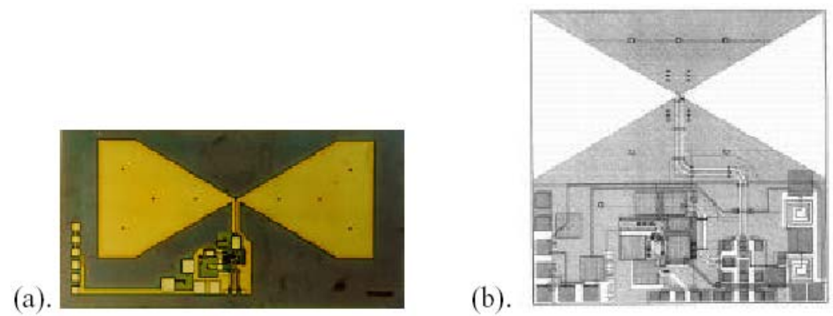


2-D DETECTOR ARRAY



Schematic of the 2-D imaging array

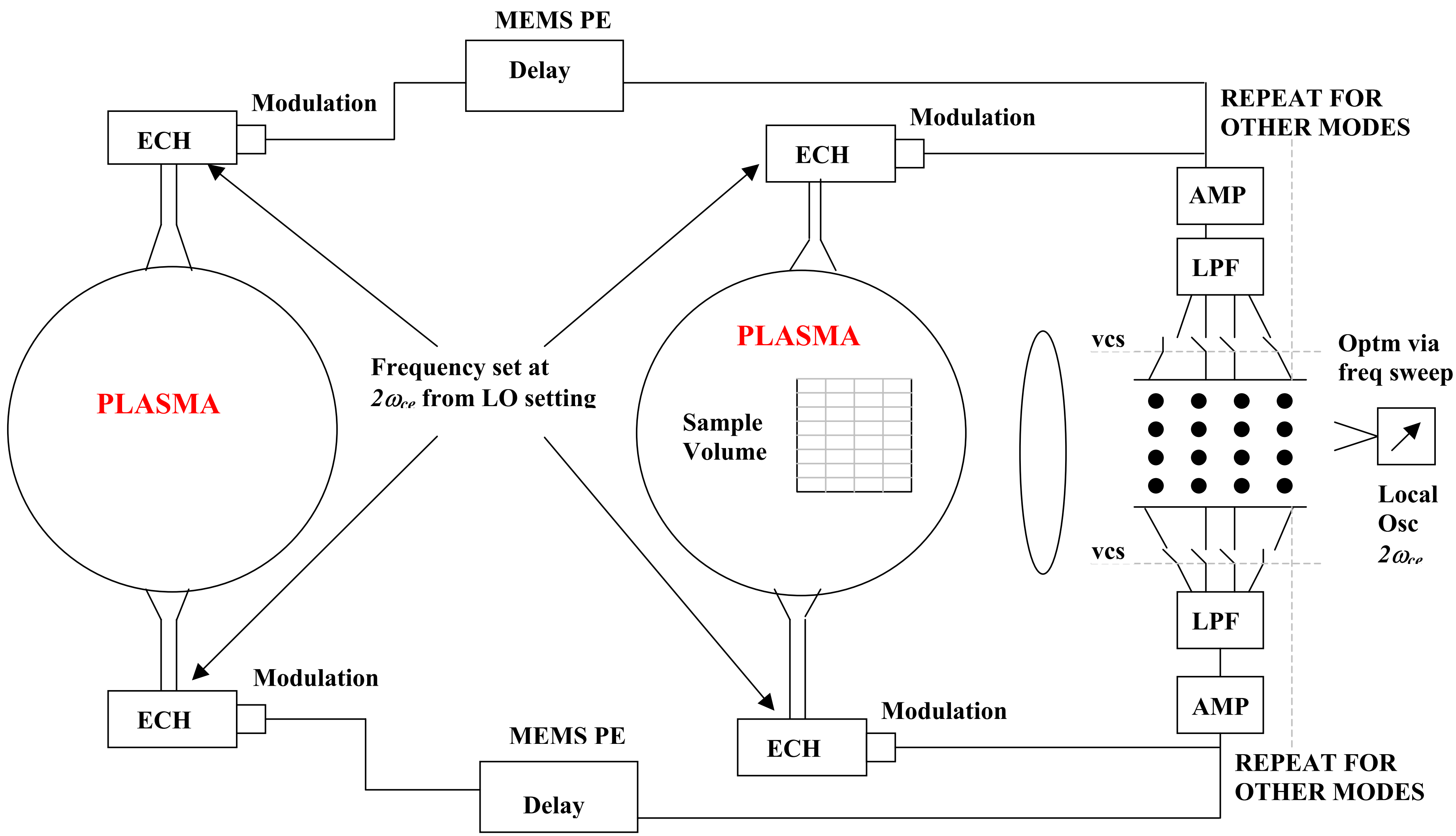
MIMIC DETECTOR Teratec Corp.



Mask patterns of the monolithic detectors integrated with bow tie antennas, Schottky diodes, and HBT amplifiers (a) is the first mask pattern, and (b) is a more recent one.

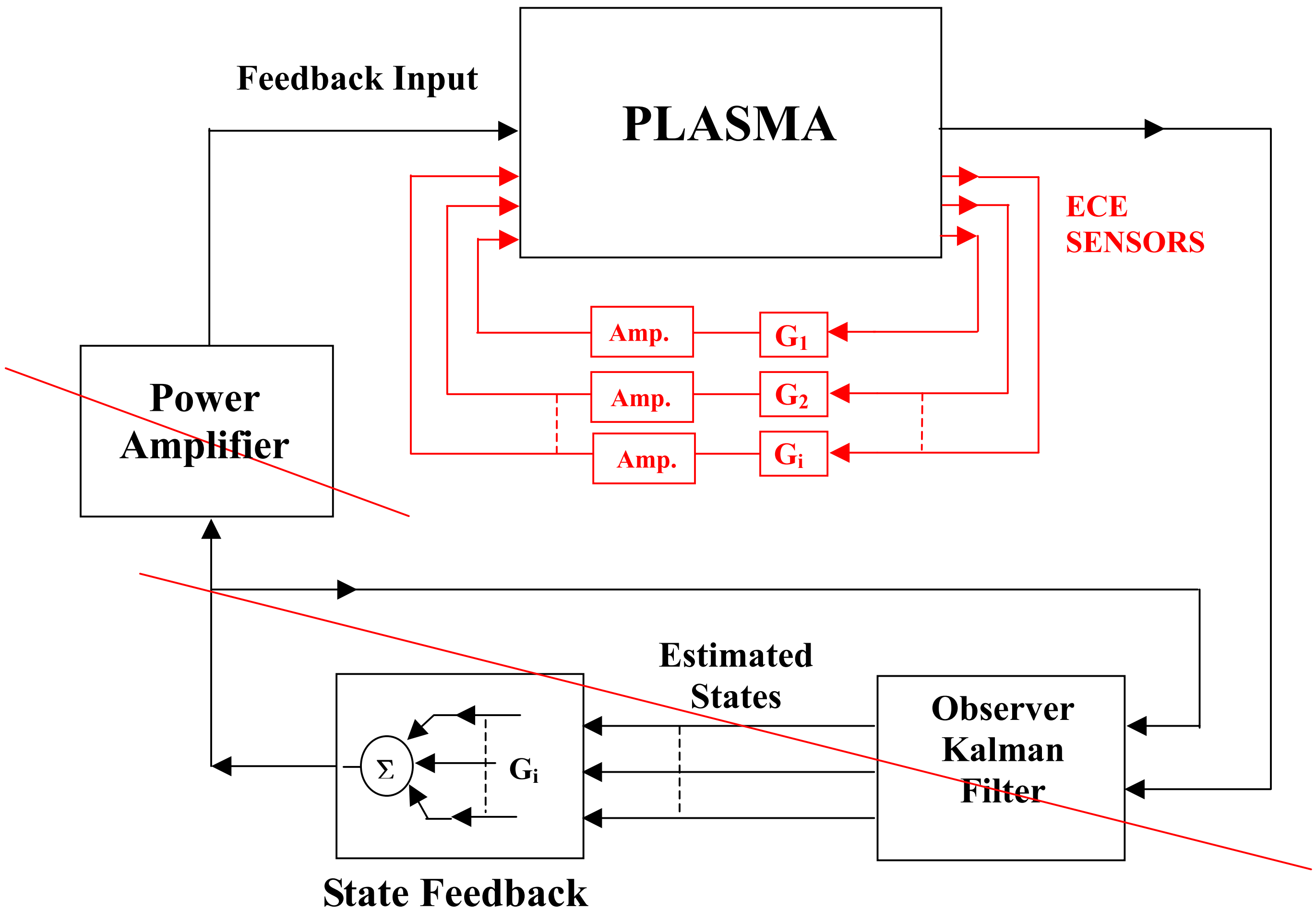
SCHEMATIC OF THE FEEDBACK SYSTEM

Shown for 1 Internal Mode; repeat for others



SIMPLER STATE FEEDBACK FOR MULTIMODE MHD MODES

USING EXPERIMENTALLY MEASURED STATES



A SIMPLE IMPROVEMENT IN THE USE OF KALMAN FILTER

- **Kalman filter strongly depends on a system model incorporated.**
- **This model can be derived simply from the experimental data of ECE.**
- **Assume that the system parameters $[A]$ in the following dynamic model are largely unknown or poorly known:**

$$[\dot{a}] = [A]\cdot[a], \qquad a \;\text{= mode amplitude} \equiv \text{system states}$$

system (↗ $[A]$) **Insert ECE data** (↖ $[a]$)

- **Then solve the above equation numerically iteratively as a system identification problem, as described below.**

  It can be done on line, but may take several ms. Therefore, use the above as a priori model, with minor updates on line.
- **Then Kalman filter with much better system model may produce much better results.**

IV. ADAPTIVE OPTIMAL CONTROL

Z. Sun, A.K. Sen and R. Longman

- **Online system identification (determination of a plasma dynamic model)**

- **Online system identification of a system evolving in time**

- **Optimal stochastic control system based on the above**

SYSTEM IDENTIFICATION

LEAST SQUARE BATCH METHOD

As an example: A system in discrete time form:

$$\psi(k) + a_1\psi(k-1) + a_2\psi(k-2) = b_1u(k-1) + b_2u(k-2)$$

Assume that a sequence of inputs $\{u(1)\ldots u(k)\ldots u(n)\}$ has been applied to the system and the corresponding sequence of outputs $\{\psi(1)\ldots\psi(k)\ldots\psi(n)\}$ has been observed. Use the parameter vector $\theta^T = \begin{pmatrix} a_1 & a_2 & b_0 & b_1 \end{pmatrix}$ and the regression vector $\varphi^T(k-1) = \begin{pmatrix} -\psi(k-1) & -\psi(k-2) & u(k-1) & u(k-2) \end{pmatrix}$. This kind of model is called an autoregressive model.

The model can formally be written as

$$\psi(k) = \varphi^T(k-1)\theta \tag{8}$$

Using the notation

$$\Phi(n-1) = \begin{pmatrix} \vdots \\ \varphi^T(k-1) \\ \vdots \\ \varphi^T(n-1) \end{pmatrix} \qquad \Psi(n) = \begin{pmatrix} \vdots \\ \psi(k) \\ \vdots \\ \psi(n) \end{pmatrix}$$

Eq. (8) can be written as $\Psi(n) = \Phi(n-1)\theta$ (9)

The parameter vector $\hat{\theta}$ should be chosen to minimize the least-square cost function

$$V(\hat{\theta}, n) = \frac{1}{2}\sum_{k=1}^{n}\left(\psi(k) - \varphi^T(k-1)\hat{\theta}\right)^2 \tag{10}$$

If the matrix $\Phi^T\Phi$ is nonsingular, the estimation $\hat{\theta}$ is unique and given by

$$\hat{\theta} = \left(\Phi^T\Phi\right)^{-1}\Phi^T\Psi \tag{11}$$

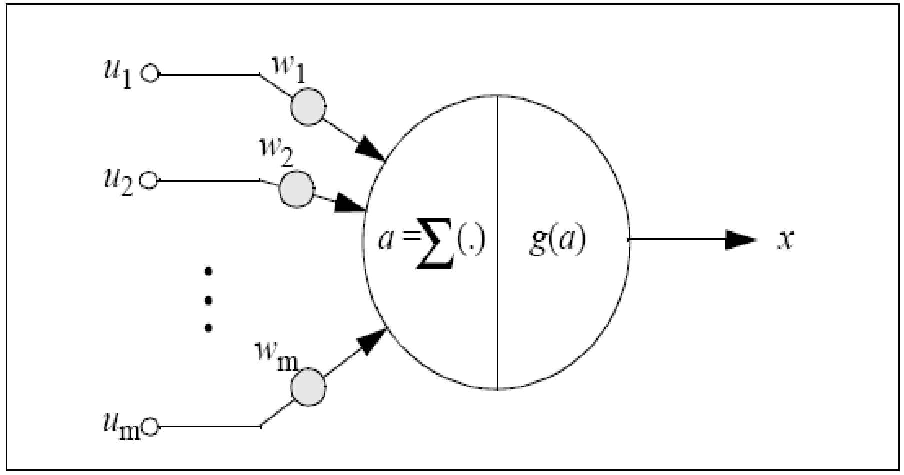
provided $\left(\Phi^T\Phi\right)^{-1}$ exists, otherwise it should be replaced by $\hat{\theta} = \Phi^+\Psi$

V. NEURAL NETWORK CONTROLLER

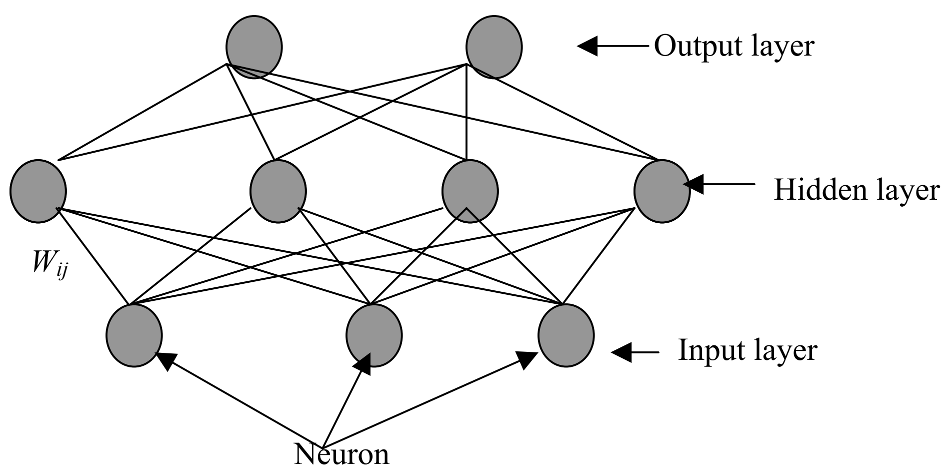
Z. Sun and A.K. Sen

- **Much faster than sequential algorithms**
- **Suitable for multimode MHD internal modes**
- **System identification and control on the same chip**
- **Naturally learning and adaptive**

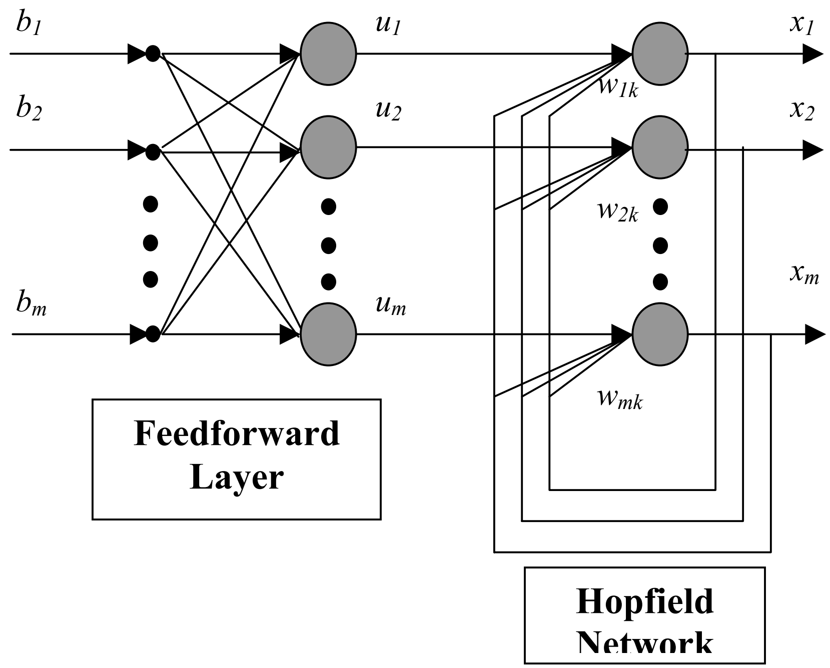
TOPOLOGY OF A NEURON



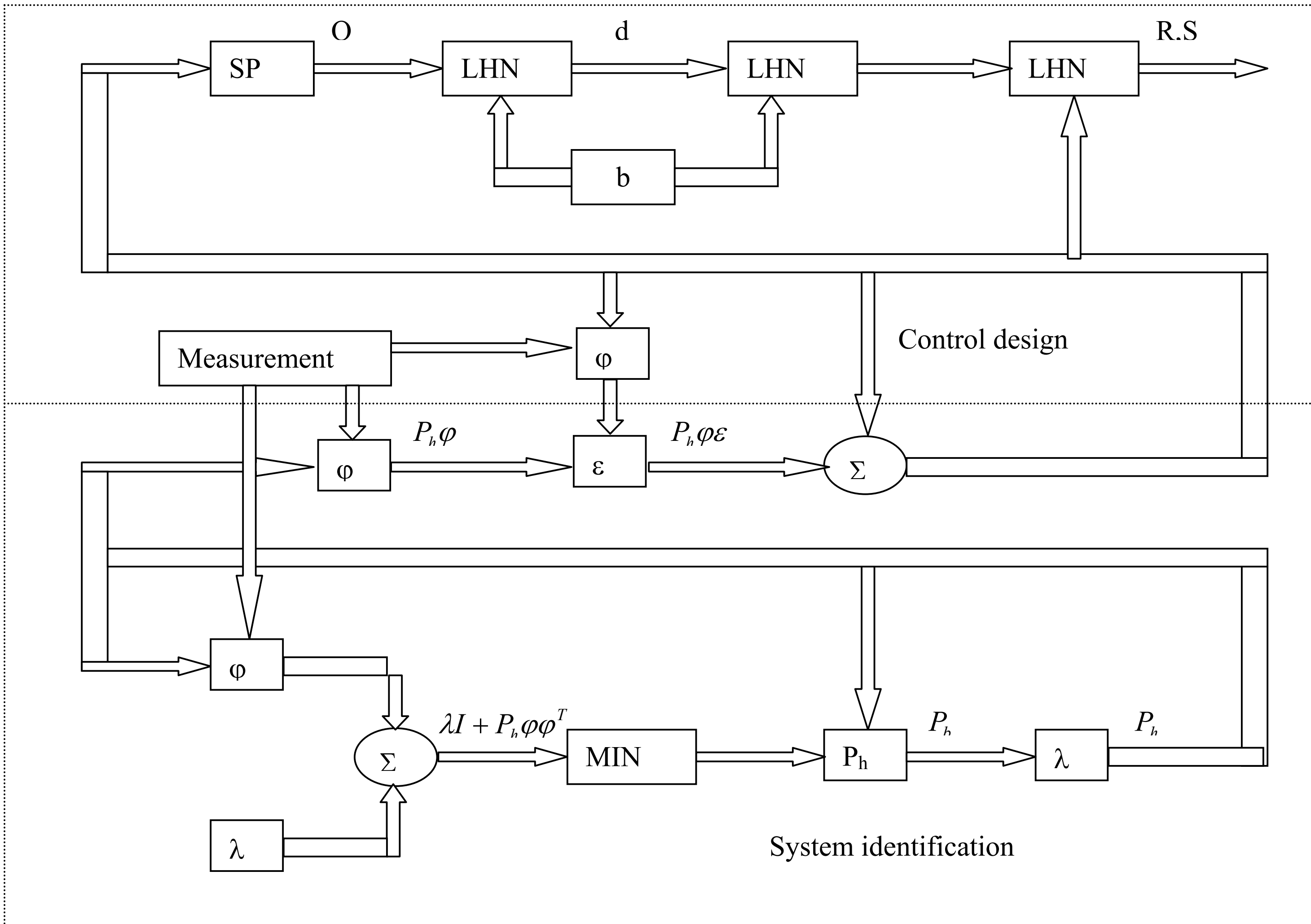
GENERAL ARCHITECTURE OF NN



A DEDICATED NEURAL NETWORK FOR SOLVING LINEAR MATRIX EQUATIONS



A PROBLEM SPECIFIC NEURAL NETWORK ARCHITECTURE



ACKNOWLEDGMENTS:

Fruitful discussions with Allen Boozer, J. Manickam, E. Fredrickson, and H. Park.

The research was supported by DOE Grant DE-FG02-98ER-54464