• VALEN model with rotation includes a toroidal torque balance
• We investigate the effect of feedback with self consistent torque
• Optimal phasing for feedback determined, essentially no degradation in performance
VALEN rotation includes toroidal torque ($\Gamma$) balance*

\[
\Gamma_{\text{plasma-plasma mode}} + \Gamma_{\text{plasma mode-external}} = 0
\]

\[
\Gamma_{\text{plasma-plasma mode}} = \text{function}\left(\omega_{\text{plasma}} - \Omega_{\text{plasma mode}}\right)
\]

‘Since the magnitude of the torque has many uncertainties, the equations will be formulated so empirical expressions for torque may be used.’

We do not predict $\Gamma_{\text{plasma-plasma mode}}$

The VALEN dimensionless parameter ‘$\alpha$’ is a normalized torque. The VALEN parameters ‘$s$’ & ‘$\alpha$’ together determine growth rate $\gamma$ and rotation $\Omega$ of the plasma mode.

* See Physics of Plasmas, Vol 6, No. 8, Aug. 1999, pg.3180
Torque and energy have same units so $\alpha$ is dimensionless

If we understood the dissipation model and can calculate the torque, then we can make predictions for critical plasma rotation for a given rotation profile shape.

\[
\alpha = \frac{\Gamma_{\text{plasma mode - external}}}{L_B I_B^2 / 2} \propto \mathcal{W}_{\text{plasma}} \nu_{\text{diss}}
\]
VALEN-ROTATION with a torque balance is more realistic

- old VALEN results used a single unstable mode with a fixed toroidal orientation, no change in toroidal orientation of the plasma mode was allowed, toroidal torque was ignored, plasma mode could change growth rate but could not change its initial toroidal orientation.

- VALEN-ROTATION uses two copies of a single unstable mode, start with $\pi/2$ difference in initial position. The application of torque determines growth rate(s), and rotation or orientation(s). (i.e., choose $\alpha$ and possibly apply feedback)

- VALEN-ROTATION with field errors requires a time dependent calculation, we report here only eigenvalue results (without error fields).
We demonstrate VALEN-ROTATION with our most complete VALEN model of ITER, using 7 mid plane port plug control coils.

The model includes double wall vacuum vessel (45 ports), blanket modules, control coils, and interior Bp sensors for mode detection.

Mid and top port plug coils shown in red.
ITER passive growth rate with zero mode rotation (‘$\alpha$’=0). We now add rotation by doing a scan in normalized torque ‘$\alpha$’

We next show growth rate altered by mode rotation near the operating point and near the ideal wall limit.
VALEN prediction of growth rate and mode rotation
for our most complete ITER model, scenario 4 used

\[ \text{Re}(\gamma) = \text{Mode growth rate} - \text{solid lines} \]
\[ \text{Im}(\gamma) = \text{Mode rotation} - \text{dashed lines} \]

Growth rate at \( \alpha=0 \) is ~ growth rate from no-rotation VALEN calculation

All cases shown may be stabilized by adequate rotation

Divide mode rotation in [r/s] by \( 2\pi \) to get hz
Different presentation of same results

VALEN prediction of growth rate vs. mode rotation

lines of constant $\beta_n$ (‘s’) as mode torque ‘$\alpha$’ increases

Same data as previous page

All cases shown are stabilized by adequate rotation
VALEN prediction of growth rate and mode rotation near ideal wall limit

Re($\gamma$) Mode growth rate - solid lines  
Im($\gamma$) Mode rotation - dashed lines

Only cases with $\beta_n < 5.22$, $\beta_n$ below ideal wall limit are stabilized by rotation

$\beta_n = 3.85$
$\beta_n = 4.84$
$\beta_n = 5.23$
$\beta_n = 5.60$

ITER.RWMEMLM.4.07
VALLEN prediction of growth rate and mode rotation near ideal wall limit

\[
\text{Re}(\gamma) \quad \text{Mode growth rate - solid lines}
\]

\[
\text{Im}(\gamma) \quad \text{Mode rotation - dashed lines}
\]

Only cases with \(\beta_n < 5.22\), \(\beta_n\) below ideal wall limit are stabilized by rotation.

If above ideal wall limit (5.22) we can not stabilize with rotation.
RWM growth/rotation modeled with VALEN consistent with measurements on NSTX

- Boozer RWM model in VALEN code
  - Plasma rotation
    - Normalized plasma torque \( a \leftrightarrow W_f \)
  - RWM rotation
    - RWM rotation measured near marginal stability
    - RWM growth rate measured when edge \( W_f \) reduced

\[ \gamma_{\text{RWM}} (1/s), \Omega_{\text{RWM}} (\text{rad/s}) \]

\[ \beta_N \sim 6.4 \]
\[ \beta_N \sim 5.61 \]
\[ \beta_N \sim 5.47 \]
\[ \beta_N \sim 4.7 \]

\[ 116178 \text{ (marginal stability } \beta_N = 5.5) \]

\[ 116178 \text{ (unstable - reduced } \Omega_\phi) \]

\[ \alpha \sim \text{plasma rotation} \]

Solid line: RWM growth rate
Dashed: RWM rotation rate

measured \( \Delta \Omega_\phi \)

sas DPP-05
We use an ‘F-matrix’ approach to adjust feedback logic for toroidal phase of plasma mode i.e., we do a least square fit to n=1 measured by a $B_p$ sensor array.

Measure $B_p$ or $\Phi_p$ in array of sensors (same radius, different toroidal position)

Identify magnitude & phase of sensor signals (n=1) here

Pick phase difference, $\delta$, desired between sensors and control coil V

Choose gain $G_p$ and run feedback,

$$\begin{align*}
\{V_{cc}(t)\}_{cx1} &= G_p \left[ F(\delta) \right]_{cxN} \{\Phi_{\text{sensor}}(t)\}_{Nx1}
\end{align*}$$
We use an ‘F-matrix’ approach to adjust feedback logic for toroidal phase of plasma mode i.e., we do a least square fit to \(n=1\) measured by a Bp sensor array.

\[
S \sin(n \phi_1^{\text{sensor}}) + C \cos(n \phi_1^{\text{sensor}}) = \Phi_1^{\text{sensor}}
\]
\[
S \sin(n \phi_2^{\text{sensor}}) + C \cos(n \phi_2^{\text{sensor}}) = \Phi_2^{\text{sensor}}
\]
\[
\vdots
\]
\[
S \sin(n \phi_N^{\text{sensor}}) + C \cos(n \phi_N^{\text{sensor}}) = \Phi_N^{\text{sensor}}
\]

Pick phase \(\delta\) between sensors and coil \(V\)

\[
\begin{bmatrix} V_1^{\text{coil}} \\ \vdots \\ V_C^{\text{coil}} \end{bmatrix}_{C \times 1} = G_p \begin{bmatrix} \sin(\phi_1^{\text{coil}} + \delta) & \cos(\phi_1^{\text{coil}} + \delta) \\ \vdots & \vdots \\ \sin(\phi_C^{\text{coil}} + \delta) & \cos(\phi_C^{\text{coil}} + \delta) \end{bmatrix}_{C \times 2} \begin{bmatrix} S \\ C \end{bmatrix}_{2 \times 1} = \begin{bmatrix} \Phi_1^{\text{sensor}} \\ \Phi_2^{\text{sensor}} \\ \vdots \\ \Phi_N^{\text{sensor}} \end{bmatrix}_{N \times 1}
\]

\[
\begin{bmatrix} S \\ C \end{bmatrix}_{2 \times 1} = \left(\left[A\right]^t \left[A\right]\right)^{-1} \left[A\right]^t \left\{\Phi^{\text{sensor}}\right\}_{N \times 1}
\]
VALEN results with rotation and feedback ($G_p = 10^8 \text{ [v/w]}$). Growth rate varies with F-matrix phase and $\beta_n$. The plasma has $\alpha = 0$ when feedback is applied.
Feedback applies a torque which results in mode rotation. VALEN with rotation and feedback ($G_p=10^8 [v/w]$), results vary with F-matrix phase and $\beta_n$.

Thin horizontal lines show results with no rotation.
Solid lines show growth rate [1/s]
Dashed lines show mode rotation [r/s]
VALLEN-3D analysis demonstrates optimal relative phase $\Delta \phi_f$ for RWM active control

- First VALLEN-3D analysis with both active and passive stabilization ($\omega_\phi > 0$)
- Unfavorable $\Delta \phi_f$ drives mode growth
- Stable range of $\Delta \phi_f$ increases with increasing $\omega_\phi$
- Optimal $\Delta \phi_f$ for active stabilization at $\omega_\phi = 0$ bracketed by results with $\omega_\phi > 0$. 

![Graph showing growth rate vs. relative phase](image)
Conclusions & next steps

• VALEN with rotational stabilization includes a toroidal torque balance
• Have found good agreement with NSTX data
• We have investigated the effect of feedback using the 7 mid plane port plug coils with self consistent torque in the VALEN ITER model and found no degradation of feedback performance with $\alpha = 0$.
• Can now explore combination of $\alpha > 0$ with feedback for ITER port plug control coils.
• New capabilities in the time domain now available. Could examine ‘ELM’ driven RFA with rotation in torque balance.
growth rate $\Re(\gamma)$ [1/s] | mode rotation $\Im(\gamma)$ [r/s] |
|---|---|

growth rate with FB

no feedback (no FB) growth rate

with FB

mode

NSTX.rot.2007
\gamma\text{ phase angle}